Antisymmetrized, translationally invariant theory of the nucleon optical potential

R. C. Johnson
Department of Physics, Faculty of Engineering and Physical Sciences, University of Surrey, Guildford, Surrey GU2 7XH, United Kingdom

(Received 24 February 2019; published 18 April 2019)

Earlier work showed how a nucleon optical model wave function could be defined as a projection of a many-nucleon scattering state within a translationally invariant second quantized many-body theory. In this paper, an optical potential operator that generates this optical model wave function is defined through a particular off-shell extension of the elastic transition operator. The theory is expressed explicitly in terms of the many-nucleon Hamiltonian in a mixed representation in which localized target nucleus states feature. No reference to a mean-field concept is involved in the definition. It is shown that the resulting optical model operator satisfies the requirements of rotational invariance and translational invariance and has standard behavior under the time-reversal transformation. The contributions to the optical potential from two different exchange mechanisms are expressed in terms of an effective Hamiltonian involving a nucleon-number-conserving one-body interaction. In the weak-binding limit, the method reduces to a version of Feshbach's projection operator formulation of the optical potential with a truncated nucleon-nucleon potential including exchange terms and recoil corrections. Definitions of the nucleon single-particle Green's function and the corresponding Dyson self-energy modified by corrections for translational invariance are presented, and different definitions of the optical potential operator are compared.

DOI: 10.1103/PhysRevC.99.044608

I. INTRODUCTION

Theories describing nucleon scattering from an A-nucleon target in terms of fundamental two- and three-body inter-nucleon interactions are of considerable current interest. An extensive recent review of the optical potential approach is given in Ref. [1]. Apart from the case of very light targets, all theories cited in Ref. [1] ignore the full implications of translational invariance.

As a step towards correcting this situation, earlier work [2] showed how a nucleon optical model wave function could be defined as a projection of a many-nucleon scattering state within a translationally invariant second quantized many-body theory. In Secs. II A and II B the results of Ref. [2] are reviewed briefly. In Secs. II C and II D, in a natural development of Ref. [2], an optical potential operator that generates the optical model wave function is defined in terms of a particular off-shell extension of the elastic transition operator and expressed explicitly in terms of the many-nucleon Hamiltonian. No reference to a mean-field concept is involved in this definition which is shown to produce an optical potential that satisfies the requirements of rotational invariance and translational invariance and has a standard behavior under the time-reversal transformation.

In Secs. II E–II G the new definition is used to distinguish the contributions from two different exchange mechanisms to the optical potential, knockout exchange and heavy-particle stripping. For an A-nucleon target, these two contributions are expressed in terms of an effective Hamiltonian involving a nucleon-number-conserving interaction acting within A-nucleon and $(A-1)$-nucleon subspaces. Section III shows how the heavy-particle stripping term is related to the hole term in the Lehmann, Symanzik, and Zimmermann (LSZ) representation of the transition operator [3]. In Sec. IV, it is shown that in the weak-binding limit the theory reduces to a modified version of Feshbach's original theory of the optical potential [4].

In Sec. V, a modified single-particle Green's function and Dyson self-energy are derived. The associated definition of an optical potential is compared with that given in Sec. II.

Concluding discussions can be found in Sec. VI, and an acknowledgment follows. Details of some of the derivations are collected in Appendices A–E.

The inclusion of recoil effects in many-body theories of the optical potential was discussed by Redish and Villars [5]. Their work derived corrections to systematic perturbation methods about a mean field. Although much of the analysis in this paper shares with Ref. [5] the use of techniques developed in Ref. [3], no attempt is made here to develop a perturbation theory. The motivation here is rather to write down a theory of the optical potential that explicitly satisfies antisymmetry and translational invariance requirements and brings out the physical content in a way that bridges the gap between many-body theory and standard nuclear reaction theory ideas.

II. THEORY OF THE NUCLEON OPTICAL POTENTIAL

A. The optical model wave function

In Ref. [2], the optical model wave-function $\xi^{\epsilon}_{E,\boldsymbol{k}_0}(\boldsymbol{r})$, corresponding to the elastic scattering of a nucleon of momentum $\boldsymbol{k}_0$ in the overall c.m. system by an A-nucleon target in its ground state was formally defined as a matrix element between many-nucleon states in Fock space through the formula,

$$\xi^{\epsilon}_{E,\boldsymbol{k}_0}(\boldsymbol{r}) = \langle\langle\Psi(0,\boldsymbol{x}=0)|\psi(\boldsymbol{r})|\Psi^{\epsilon}_{E,\boldsymbol{k}_0}\rangle\rangle. \tag{1}$$

The operator $\psi(\boldsymbol{r})$ destroys a nucleon at a point labeled $\boldsymbol{r}$. The notation $\boldsymbol{r}$ will be taken to include spin and isospin coordinates of a nucleon unless it is obvious that only space coordinates are referred to by the context. The notation $|\,\rangle\rangle$ and $\langle\langle\,|$ denotes kets and bras in Fock space.

The ground-state energy will be taken to be the zero of energy so that total c.m. energy E is related to $\boldsymbol{k}_0$ on the energy shell by

$$E = \frac{\hbar^2 k_0^2}{2\mu_{mA}}, \tag{2}$$

where μ_{mA} is the nucleon-target reduction mass,

$$\mu_{mA} = \frac{A}{(A+1)}m. \tag{3}$$

In order to make subsequent formulas have a simpler appearance, the difference between neutron and proton rest masses will be ignored.

The ket on the right of Eq. (1) is the Fock-space scattering state [2],

$$|\Psi^{\epsilon}_{E,\boldsymbol{k}_0}\rangle\rangle = \frac{\imath\epsilon}{E-H+\imath\epsilon}(2\pi)^{3/2}a^{\dagger}_{\boldsymbol{k}_0}|-\boldsymbol{k}_0,\psi_0\rangle\rangle. \tag{4}$$

where H is the many-nucleon Hamiltonian operator in Fock space.

In the limit $\epsilon\to 0^+$, the ket $|\Psi^{\epsilon}_{E,\boldsymbol{k}_0}\rangle\rangle$ describes an antisymmetric $(A+1)$-nucleon scattering state of total momentum zero in the overall c.m. system. The incident channel has an incident nucleon with momentum $\boldsymbol{k}_0$. In this channel, the A-nucleon target has a total momentum $-\boldsymbol{k}_0$ and is in its ground-state ψ_0. All other channel components of $|\Psi^{\epsilon}_{E,\boldsymbol{k}_0}\rangle\rangle$ have purely outgoing waves asymptotically. It will be assumed that all Coulomb interactions are screened at large separations. The factor $(2\pi)^{3/2}$ arises because $a^{\dagger}_{\boldsymbol{k}_0}$ creates a normalized plane-wave state, whereas scattering states are conventionally normalized to an incoming plane wave of unit amplitude.

The bra $\langle\langle\Psi(0,\boldsymbol{x}=0)|$ on the right of Eq. (1) describes a state in which the target is in its ground state and its c.m. is located at the origin of coordinates $\boldsymbol{x}=0$. This is one of the complete set of A-nucleon Fock-space states $|\Psi(n,\boldsymbol{x})\rangle\rangle$, formed from an intrinsic state $\psi_n(\boldsymbol{r}_1,\dots,\boldsymbol{r}_A)$, and and having a c.m. located at position $\boldsymbol{x}$. Explicitly [2],

$$|\Psi(n,\boldsymbol{x})\rangle\rangle = \frac{1}{\sqrt{A!}}\int d\boldsymbol{r}_1 d\boldsymbol{r}_2\cdots d\boldsymbol{r}_A\,\delta(\boldsymbol{R}_A-\boldsymbol{x})\psi_n \times(\boldsymbol{r}_1,\dots,\boldsymbol{r}_A)\psi^{\dagger}(\boldsymbol{r}_A)\cdots\psi^{\dagger}(\boldsymbol{r}_1)|0\rangle\rangle, \tag{5}$$

where

$$\boldsymbol{R}_A = \frac{(\boldsymbol{r}_1+\cdots+\boldsymbol{r}_A)}{A}. \tag{6}$$

State ψ_n is an eigenstate of the intrinsic part of the A-nucleon Hamiltonian H_A,

$$H_A - \frac{(\boldsymbol{P})^2}{2Am}, \tag{7}$$

where $\boldsymbol{P}$ is the momentum operator in Fock space,

$$\boldsymbol{P} = \hbar\int d\boldsymbol{k}\,\boldsymbol{k}a^{\dagger}_{\boldsymbol{k}}a^{\dagger}_{\boldsymbol{k}}. \tag{8}$$

State $\psi_n(\boldsymbol{r}_1,\dots,\boldsymbol{r}_A)$ is an eigenfunction of $\boldsymbol{P}$ with eigenvalue zero. The ket $|-\boldsymbol{k}_0,\psi_0\rangle\rangle$ in Eq. (4) is an eigenfunction of $\boldsymbol{P}$ with eigenvalue $-\boldsymbol{k}_0$.

In Eq. (1), the argument $\boldsymbol{r}$ of $\xi^{\epsilon}_{E,\boldsymbol{k}_0}(\boldsymbol{r})$ can be interpreted as the position of the incident nucleon relative to the c.m. of the target, although of course in the scattering state the target c.m. is not at rest in the overall c.m. system. This is reflected in the complete uncertainty of the momentum of the bra $\langle\langle\Psi(0,\boldsymbol{x}=0)|\psi(\boldsymbol{r})$. The translational invariance of H means that only the zero total momentum component of this bra contributes to the matrix element (1).

B. Source equation for the optical model wave function

The optical model wave function defined by Eq. (1) satisfies [2]

$$\left(\frac{\hbar^2 k_0^2}{2\mu_{mA}}+\imath\epsilon+\frac{\hbar^2}{2\mu_{mA}}\nabla^2_{\boldsymbol{r}}\right)\xi^{\epsilon}_{E,\boldsymbol{k}_0}(\boldsymbol{r}) = F^{\epsilon}_{\boldsymbol{k}_0}(\boldsymbol{r})+\imath\epsilon\,\exp(\imath\boldsymbol{k}_0\cdot\boldsymbol{r}), \tag{9}$$

where

$$F^{\epsilon}_{\boldsymbol{k}_0}(\boldsymbol{r}) = \langle\langle\Psi(0,\boldsymbol{x}=0)|[\psi(\boldsymbol{r}),V]_-|\Psi^{\epsilon}_{E,\boldsymbol{k}_0}\rangle\rangle. \tag{10}$$

The form Eq. (9) takes in the $\epsilon\to 0$ is derived in Sec. V of Ref. [2]. A straightforward modification of the argument given there gives the finite ϵ version, Eq. (9).

The optical potential operator $\hat{V}^{\mathrm{opt}}(E)$ will be defined as an operator in barycentric space (B space), the space of a fictitious particle of mass μ_{mA}, spin-1/2, position operator $\hat{\boldsymbol{r}}$, and momentum operator $\hat{\boldsymbol{p}} = -\imath\hbar\nabla_r$, that enables Eq. (9) to be written in the equivalent form

$$\left(\frac{\hbar^2 k_0^2}{2\mu_{mA}}+\imath\epsilon+\frac{\hbar^2}{2\mu_{mA}}\nabla^2_{\boldsymbol{r}}\right)\xi^{\epsilon}_{E,\boldsymbol{k}_0}(\boldsymbol{r}) = \hat{V}^{\mathrm{opt}}(E)\xi^{\epsilon}_{E,\boldsymbol{k}_0}(\boldsymbol{r})+\imath\epsilon\,\exp(\imath\boldsymbol{k}_0\cdot\boldsymbol{r}). \tag{11}$$

The "hat" notation over a quantity will be used to denote operators in B space as opposed to the bare-headed operators of Fock space.

In general, $V^{\mathrm{opt}}(E)$ will be a nonlocal operator with matrix elements in the $\boldsymbol{r}$ representation in B-space $\hat{V}^{\mathrm{opt}}(E;\boldsymbol{r},\boldsymbol{r}')$ so that the first term on the right in Eq. (9) has the form

$$\hat{V}^{\mathrm{opt}}(E)\xi^{\epsilon}_{E,\boldsymbol{k}_0}(\boldsymbol{r}) = \int d\boldsymbol{r}'\hat{V}^{\mathrm{opt}}(E;\boldsymbol{r},\boldsymbol{r}')\xi^{\epsilon}_{E,\boldsymbol{k}_0}(\boldsymbol{r}'). \tag{12}$$

To achieve the identification of this operator, Eq. (9) must be written as a relation between operators and kets in B space. This can be achieved in many ways. The choice made here results in an operator that conserves the total angular momentum in B space and has features which permits a transparent representation of the physical processes involved.

$F^{\epsilon}_{\boldsymbol{k}_0}(\boldsymbol{r})$, given in Eq. (10), is first rewritten using an expression for the scattering state, Eq. (4), as a plane wave in the incident channel plus outgoing wave components. This step uses the Green's function identity [3], Eq. (3.42), p. 315,

$$G(z)a^{\dagger}_{\boldsymbol{k}_0} = a^{\dagger}_{\boldsymbol{k}_0}G(z-\epsilon_{k_0})+G(z)[V,a^{\dagger}_{\boldsymbol{k}_0}]_-G(z-\epsilon_{k_0}), \tag{13}$$

where $G(z) = \frac{1}{(z-H)}$ for arbitrary complex z and

$$\epsilon_{\boldsymbol{k}_0} = \frac{\hbar k_0^2}{2m}. \tag{14}$$

It has been assumed that H can be expressed as the sum of a nucleon kinetic-energy term T and an internucleon potential-energy term V.

Acting on the ket $|-\boldsymbol{k}_0, \psi_0\rangle\rangle$,

$$\imath\epsilon G\big(E + \imath\epsilon - \epsilon_{k_0}\big)|-\boldsymbol{k}_0, \psi_0\rangle\rangle = \imath\epsilon \frac{1}{\big(E + \imath\epsilon - \epsilon_{k_0} - \frac{1}{A}\epsilon_{k_0}\big)}|-\boldsymbol{k}_0, \psi_0\rangle\rangle = |-\boldsymbol{k}_0, \psi_0\rangle\rangle, \tag{15}$$

when Eq. (2) is satisfied. The definition (4) can therefore be rewritten

$$(2\pi)^{-3/2}\big|\Psi^{\epsilon}_{E,\boldsymbol{k}_0}\big\rangle\!\big\rangle = a^{\dagger}_{\boldsymbol{k}_0}|-\boldsymbol{k}_0, \psi_0\rangle\rangle + G(E + \imath\epsilon)\big[V, a^{\dagger}_{\boldsymbol{k}_0}\big]_{-}|-\boldsymbol{k}_0, \psi_0\rangle\rangle, \tag{16}$$

and the expression (10) for $F^{\epsilon}_{\boldsymbol{k}_0}(\boldsymbol{r})$ becomes

$$F^{\epsilon}_{\boldsymbol{k}_0}(\boldsymbol{r}) = (2\pi)^{3/2}\big[\langle\langle\Psi(0, \boldsymbol{x} = 0)|[\psi(\boldsymbol{r}), V]_{-}a^{\dagger}_{\boldsymbol{k}_0}|-\boldsymbol{k}_0, \psi_0\rangle\rangle + \langle\langle\Psi(0, \boldsymbol{x} = 0)|[\psi(\boldsymbol{r}), V]_{-}G(E + \imath\epsilon)\big[V, a^{\dagger}_{\boldsymbol{k}_0}\big]_{-}|-\boldsymbol{k}_0, \psi_0\rangle\rangle\big]. \tag{17}$$

The first term on the right involves ground-state–ground-state matrix elements of V. In the second term, the ground state is coupled to a complete set of intermediate $(A+1)$-nucleon states.

Recall that by definition in state $|-\boldsymbol{k}_0, \psi_0\rangle\rangle$ the A-nucleon c.m. is in a plane-wave state of momentum $-\boldsymbol{k}_0$ and unit amplitude and hence

$$|-\boldsymbol{k}_0, \psi_0\rangle\rangle = \int d\boldsymbol{x}\, \exp(-\imath\boldsymbol{k}_0 \cdot \boldsymbol{x})|\Psi(0, \boldsymbol{x})\rangle\rangle, \tag{18}$$

where $|\Psi(0, \boldsymbol{x})\rangle\rangle$ is one of the states defined in Eq. (5). Equation (17) can now be developed as

$$F^{\epsilon}_{\boldsymbol{k}_0}(\boldsymbol{r}) = (2\pi)^{3/2}\int d\boldsymbol{x}\, \exp(-\imath\boldsymbol{k}_0 \cdot \boldsymbol{x})\big[\langle\langle\Psi(0, \boldsymbol{x} = 0)|[\psi(\boldsymbol{r}), V]_{-}a^{\dagger}_{\boldsymbol{k}_0}|\Psi(0, \boldsymbol{x})\rangle\rangle$$
$$+\langle\langle\Psi(0, \boldsymbol{x} = 0)|[\psi(\boldsymbol{r}), V]_{-}G(E + \imath\epsilon)\big[V, a^{\dagger}_{\boldsymbol{k}_0}\big]_{-}|\Psi(0, \boldsymbol{x})\rangle\rangle\big] = \int d\boldsymbol{r}'\hat{\mathcal{T}}_{0,0}(E + \imath\epsilon; \boldsymbol{r}, \boldsymbol{r}')\exp(\imath\boldsymbol{k}_0.\boldsymbol{r}'), \tag{19}$$

where

$$\hat{\mathcal{T}}_{0,0}(E + \imath\epsilon; \boldsymbol{r}, \boldsymbol{r}') = \int d\boldsymbol{x}[\langle\langle\Psi(0, \boldsymbol{x} = 0)|[\psi(\boldsymbol{r}), V]_{-}\psi^{\dagger}(\boldsymbol{r}' + \boldsymbol{x})|\Psi(0, \boldsymbol{x})\rangle\rangle$$
$$+ \langle\langle\Psi(0, \boldsymbol{x} = 0)|[\psi(\boldsymbol{r}), V]_{-}G(E + \imath\epsilon)[V, \psi^{\dagger}(\boldsymbol{r}' + \boldsymbol{x})]_{-}|\Psi(0, \boldsymbol{x})\rangle\rangle]. \tag{20}$$

The role of the integration over $\boldsymbol{x}$ is to pick out the momentum zero component of the ket $\psi^{\dagger}(\boldsymbol{r}' + \boldsymbol{x})|\Psi(0, \boldsymbol{x})\rangle\rangle$. This is particularly transparent when the properties of the total momentum operator $\boldsymbol{P}$ are used to write

$$\int d\boldsymbol{x}\, \psi^{\dagger}(\boldsymbol{r}' + \boldsymbol{x})|\Psi(0, \boldsymbol{x})\rangle\rangle = \int d\boldsymbol{x}\, \exp(-\imath\boldsymbol{x} \cdot \boldsymbol{P})\psi^{\dagger}(\boldsymbol{r}')|\Psi(0, \boldsymbol{x} = 0)\rangle\rangle = (2\pi)^3\delta(\boldsymbol{P})\psi^{\dagger}(\boldsymbol{r}')|\Psi(0, \boldsymbol{x} = 0)\rangle\rangle. \tag{21}$$

Equation (20) defines a fully off-shell transition matrix that is independent of the direction of the incident momentum $\boldsymbol{k}_0$. This feature will be essential for the angular momentum conserving property of the optical model defined in the next section.

All the terms on the right in Eq. (20) involve matrix elements between spatially localized A-nucleon states. The range of integration of the c.m. coordinate $\boldsymbol{x}$ is also strongly limited in realistic physical situations. The range of nonlocality associated with the distance $|\boldsymbol{r} - \boldsymbol{r}'|$ is limited by the range of nonlocality of V and the spatial dimensions of the target. The range of the variable $\boldsymbol{x}$, i.e., the change in the c.m. position of the target during the collision, can be estimated from

$$\text{(Speed of c.m.)(Time for nucleon to travel } |\boldsymbol{r} - \boldsymbol{r}'|) = \frac{\hbar k_0}{Am}\frac{|\boldsymbol{r} - \boldsymbol{r}'|}{\hbar k_0/m} = \frac{|\boldsymbol{r} - \boldsymbol{r}'|}{A}. \tag{22}$$

This estimate is finite for all A and since $(|\boldsymbol{r} - \boldsymbol{r}'|)_{\max} \propto A^{1/3}$ decreases like $A^{-2/3}$ for large A.

The momentum space matrix element between general normalized plane-wave states of momenta $\boldsymbol{k}, \boldsymbol{k}'$ that correspond to $\hat{\mathcal{T}}_{0,0}(E + \imath\epsilon; \boldsymbol{r}, \boldsymbol{r}')$ is

$$\hat{\mathcal{T}}_{0,0}(E + \imath\epsilon; \boldsymbol{k}, \boldsymbol{k}') = \int d\boldsymbol{r}\int d\boldsymbol{r}'\frac{\exp(-\imath\boldsymbol{k} \cdot \boldsymbol{r})}{(2\pi)^{3/2}}\hat{\mathcal{T}}_{0,0}(E + \imath\epsilon; \boldsymbol{r}, \boldsymbol{r}')\frac{\exp(\imath\boldsymbol{k}' \cdot \boldsymbol{r}')}{(2\pi)^{3/2}}$$
$$= \langle\langle\Psi(0, \boldsymbol{x} = 0)|[a_{\boldsymbol{k}}, V]_{-}a^{\dagger}_{\boldsymbol{k}'}|-\boldsymbol{k}', \psi_0\rangle\rangle + \langle\langle\Psi(0, \boldsymbol{x} = 0)|[a_{\boldsymbol{k}}, V]_{-}G(E + \imath\epsilon)[V, a^{\dagger}_{\boldsymbol{k}'}]_{-}|-\boldsymbol{k}', \psi_0\rangle\rangle. \tag{23}$$

The quantities $\hat{\mathcal{T}}_{0,0}(E+\imath\epsilon;\boldsymbol{r},\boldsymbol{r}')$ and $\hat{\mathcal{T}}_{0,0}(E+\imath\epsilon;\boldsymbol{k},\boldsymbol{k}')$ can be thought of as matrix elements of an operator in B-space $\hat{\mathcal{T}}_{0,0}(E+\imath\epsilon)$. Using this identification, Eq. (9) can know be expressed entirely in terms of operators and kets in B space,

$$\left(\tilde{\epsilon}_{k_0}+\imath\epsilon-\hat{T}\right)\left|\xi^{\epsilon}_{E,\boldsymbol{k}_0}\right\rangle=\hat{\mathcal{T}}_{0,0}(E+\imath\epsilon)(2\pi)^{3/2}|\boldsymbol{k}_0\rangle+\imath\epsilon(2\pi)^{3/2}|\boldsymbol{k}_0\rangle, \tag{24}$$

where

$$\langle\boldsymbol{r}|\hat{T}|\boldsymbol{r}'\rangle=-\frac{\hbar^2}{2\mu_{mA}}\left[\nabla^2_{\boldsymbol{r}}\delta(\boldsymbol{r}-\boldsymbol{r}')\right] \tag{25}$$

is the kinetic operator in B space for a particle of mass μ_{mA}. The notation $|\,\rangle$ is used for kets in B space.

Equation (24) has the solution,

$$\left|\xi^{\epsilon}_{E,\boldsymbol{k}_0}\right\rangle=\hat{g}_0(E+\imath\epsilon)\hat{\mathcal{T}}_{0,0}(2\pi)^{3/2}|\boldsymbol{k}_0\rangle+\imath\epsilon(2\pi)^{3/2}\hat{g}_0(E+\imath\epsilon)|\boldsymbol{k}_0\rangle=\hat{g}_0(E+\imath\epsilon)\hat{\mathcal{T}}_{0,0}(2\pi)^{3/2}|\boldsymbol{k}_0\rangle+\imath(2\pi)^{3/2}|\boldsymbol{k}_0\rangle, \tag{26}$$

where

$$\hat{g}_0(E+\imath\epsilon)=\frac{1}{(E+\imath\epsilon-\hat{T})}. \tag{27}$$

and where, for $E=\hbar k_0^2/2\mu_{mA}$, the result $\imath\epsilon\hat{g}_0(E+\imath\epsilon)|\boldsymbol{k}_0\rangle=|\boldsymbol{k}_0\rangle$ has been used.

Hence,

$$\left|\xi^{\epsilon}_{E,\boldsymbol{k}_0}\right\rangle=\hat{\Omega}(E+\imath\epsilon)(2\pi)^{3/2}|\boldsymbol{k}_0\rangle, \tag{28}$$

where

$$\hat{\Omega}(E+\imath\epsilon)=[1+\hat{g}_0(E+\imath\epsilon)\hat{\mathcal{T}}_{0,0}]. \tag{29}$$

The notation used for $\hat{\mathcal{T}}_{0,0}(E+\imath\epsilon)$ reflects the fact that, in the limit $\epsilon\to 0$, between plane-wave states of unit amplitude and on-energy-shell wave numbers, this operator gives the elastic-scattering transition amplitude and is related to the elastic-scattering amplitude $f_{0,0}(\boldsymbol{k}_0',\boldsymbol{k}_0)$,

$$\begin{aligned}&\langle\boldsymbol{k}_0',\psi_0|T(E)|\boldsymbol{k}_0,\psi_0\rangle\\&=\int d\boldsymbol{r}\int d\boldsymbol{r}'\exp(-\imath\boldsymbol{k}_0'\cdot\boldsymbol{r})\tilde{\mathcal{T}}_{0,0}(E+\imath\epsilon;\boldsymbol{r},\boldsymbol{r}')\exp(\imath\boldsymbol{k}_0\cdot\boldsymbol{r}')\\&=\langle\boldsymbol{k}_0'|V^{\rm opt}\left|\xi^{\epsilon}_{E,\boldsymbol{k}_0}\right\rangle\\&=-2\pi\frac{\hbar^2}{\mu_A}f_{0,0}(\boldsymbol{k}_0',\boldsymbol{k}_0).\end{aligned} \tag{30}$$

C. Definition of the optical potential operator

Standard many-body approaches to the definition of the optical model operator proceed through through the definition of the mass operator associated with the nucleon single-particle Green's function [1]. A different path via the off-shell elastic transition operator is followed here. The optical potential will be defined as the nonlocal operator in B space that is related to $\hat{\mathcal{T}}_{0,0}(E+\imath\epsilon)$ by

$$\begin{aligned}&\hat{V}^{\rm opt}(E+\imath\epsilon)\\&=\hat{\mathcal{T}}_{0,0}(E+\imath\epsilon)-\hat{V}^{\rm opt}(E+\imath\epsilon)\hat{g}_0(E+\imath\epsilon)\hat{\mathcal{T}}_{0,0}(E+\imath\epsilon).\end{aligned} \tag{31}$$

This definition assumes that the operator $\hat{\Omega}$ defined in Eq. (29) has an inverse so that Eq. (31) has the solution,

$$\hat{V}^{\rm opt}(E+\imath\epsilon)=\hat{\mathcal{T}}_{0,0}(E+\imath\epsilon)\hat{\Omega}(E+\imath\epsilon)^{-1}. \tag{32}$$

To verify that the definition (31) produces an equation for $\xi^{\epsilon}_{E,\boldsymbol{k}_0}(\boldsymbol{r})$ of the form (11) note that, if $\hat{V}^{\rm opt}$ satisfies Eq. (31), then

$$\begin{aligned}\hat{V}^{\rm opt}(E+\imath\epsilon)\left|\xi^{\epsilon}_{E,\boldsymbol{k}_0}\right\rangle&=\hat{V}^{\rm opt}(E+\imath\epsilon)\hat{\Omega}(E+\imath\epsilon)(2\pi)^{3/2}|\boldsymbol{k}_0\rangle,\\&=\hat{\mathcal{T}}_{0,0}(E+\imath\epsilon)(2\pi)^{3/2}|\boldsymbol{k}_0\rangle.\end{aligned} \tag{33}$$

Referring to Eq. (24), the equality (33) implies that $\xi^{\epsilon}_{E,\boldsymbol{k}_0}(\boldsymbol{r})$ satisfies

$$\begin{aligned}&(E+\imath\epsilon-\hat{T})\xi^{\epsilon}_{E,\boldsymbol{k}_0}(\boldsymbol{r})\\&=\int d\boldsymbol{r}'\hat{V}^{\rm opt}(E+\imath\epsilon;\boldsymbol{r},\boldsymbol{r}')\xi^{\epsilon}_{E,\boldsymbol{k}_0}(\boldsymbol{r}')+\imath\epsilon\,\exp(\imath\boldsymbol{k}_0\cdot\boldsymbol{r}),\end{aligned} \tag{34}$$

confirming that $\hat{V}^{\rm opt}$ does indeed plays the role of an optical potential operator.

D. Nonuniqueness of $\hat{V}^{\rm opt}$

The optical potential $\hat{V}^{\rm opt}$ as defined by Eq. (31) is not unique because the operator $\hat{\mathcal{T}}_{0,0}(E+\imath\epsilon)$ is not unique. Any operator $\hat{\mathcal{T}}'_{0,0}(E+\imath\epsilon)$ with the property,

$$\hat{\mathcal{T}}'_{0,0}(E+\imath\epsilon)|\boldsymbol{k}_0\rangle=\hat{\mathcal{T}}_{0,0}(E+\imath\epsilon)|\boldsymbol{k}_0\rangle, \tag{35}$$

when the half-on-shell condition $E=\hbar k_0^2/2\mu_{mA}$ is satisfied, will generate the same optical model wave function as $\hat{\mathcal{T}}_{0,0}(E+\imath\epsilon)$ when used in Eq. (11). However, when used in Eq. (31), it will give a different $\hat{V}^{\rm opt}(E)$ if its half-off-shell momentum matrix elements $\hat{\mathcal{T}}'_{0,0}(E+\imath\epsilon)|\boldsymbol{k}\rangle$ with $k\neq k_0$ differ from those of $\hat{\mathcal{T}}_{0,0}(E+\imath\epsilon)$ because these matrix elements will contribute to the explicit solution, Eq. (32). An example of an alternative definition of the optical potential is discussed in Sec. V C.

The particular choice of off-shell extensions for $\hat{\mathcal{T}}_{0,0}(E+\imath\epsilon)$ as defined in Eqs. (20) and (23) have the feature that the resulting operator conserve angular momentum in B space (see Appendix A 1) and have standard transformation properties under time reversal (see Appendix A 2).

E. Interpretation of the optical potential operator defined by Eq. (31)

The formal expressions (20) and (23) for the the transition operator $\hat{\mathcal{T}}_{0,0}$ describe the many different reaction mechanisms that contribute to elastic scattering, such as direct and exchange scatterings and heavy-particle stripping. The relative importance of these mechanisms vary with, for example, the incident energy and the mass of the target. In this section, it is shown how some of these effects can be separated out from the formal expressions.

The definition in Eq. (23) is repeated here for convenience,

$$\begin{aligned}\hat{\mathcal{T}}_{0,0}(E+\imath\epsilon;\boldsymbol{k},\boldsymbol{k}') &= \langle\langle\Psi(0,\boldsymbol{x}=0)|[a_{\boldsymbol{k}},V]_{-}a^{\dagger}_{\boldsymbol{k}'}|-\boldsymbol{k}',\psi_0\rangle\rangle \\ &\quad+\langle\langle\Psi(0,\boldsymbol{x}=0)|[a_{\boldsymbol{k}},V]_{-}G(E+\imath\epsilon)[V,a^{\dagger}_{\boldsymbol{k}'}]_{-}|-\boldsymbol{k}',\psi_0\rangle\rangle.\end{aligned} \tag{36}$$

The first, Born, term on the right of Eq. (36) is the sum of two terms that describe quite different physical processes. Using

$$[a_{\boldsymbol{k}},V]_{-}a^{\dagger}_{\boldsymbol{k}'} = \{[a_{\boldsymbol{k}},V]_{-},a^{\dagger}_{\boldsymbol{k}'}\}_{+} - a^{\dagger}_{\boldsymbol{k}'}[a_{\boldsymbol{k}},V]_{-} \tag{37}$$

gives

$$\hat{\mathcal{T}}^{\rm Born}_{0,0} = \hat{\mathcal{T}}^{\rm Born(0)}_{0,0} + \hat{\mathcal{T}}^{{\rm Born}(HPS)}_{0,0}, \tag{38}$$

where

$$\begin{aligned}\hat{\mathcal{T}}^{\rm Born(0)}_{0,0}&(E+\imath\epsilon;\boldsymbol{k},\boldsymbol{k}') \\ &= \langle\langle\Psi(0,\boldsymbol{x}=0)|\{[a_{\boldsymbol{k}},V]_{-},a^{\dagger}_{\boldsymbol{k}'}\}+|-\boldsymbol{k}',\psi_0\rangle\rangle,\end{aligned} \tag{39}$$

and

$$\begin{aligned}\hat{\mathcal{T}}^{{\rm Born}(HPS)}_{0,0}&(E+\imath\epsilon;\boldsymbol{k},\boldsymbol{k}') \\ &= -\langle\langle\Psi(0,\boldsymbol{x}=0)|a^{\dagger}_{\boldsymbol{k}'}[a_{\boldsymbol{k}},V]_{-}|-\boldsymbol{k}',\psi_0\rangle\rangle.\end{aligned} \tag{40}$$

The significance of this separation becomes clear when the commutator and anticommutator are evaluated for a general additive nonlocal two-body interaction V. In a momentum-space basis,

$$V = \frac{1}{4}\int d\boldsymbol{k}_1 d\boldsymbol{k}_2 d\boldsymbol{k}_3 d\boldsymbol{k}_4 \langle\boldsymbol{k}_1,\boldsymbol{k}_2|V_{\mathcal{A}}|\boldsymbol{k}_3,\boldsymbol{k}_4\rangle a^{\dagger}_{\boldsymbol{k}_1}a^{\dagger}_{\boldsymbol{k}_2}a_{\boldsymbol{k}_4}a_{\boldsymbol{k}_3}, \tag{41}$$

where the subscript $\mathcal{A}$ indicates that the matrix element involves normalized antisymmetrized two-body states.

For this V, the commutator and anticommutator that appear in the Born terms give

$$[a_{\boldsymbol{k}},V]_{-} = \frac{1}{2}\int d\boldsymbol{k}_2 d\boldsymbol{k}_3 d\boldsymbol{k}_4 \langle\boldsymbol{k},\boldsymbol{k}_2|V_{\mathcal{A}}|\boldsymbol{k}_3,\boldsymbol{k}_4\rangle a^{\dagger}_{\boldsymbol{k}_2}a_{\boldsymbol{k}_4}a_{\boldsymbol{k}_3}, \tag{42}$$

and

$$\{[a_{\boldsymbol{k}},V]_{-},a^{\dagger}_{\boldsymbol{k}'}\}_{+} = \int d\boldsymbol{k}_2 d\boldsymbol{k}_4 \langle\boldsymbol{k},\boldsymbol{k}_2|V_{\mathcal{A}}|\boldsymbol{k}',\boldsymbol{k}_4\rangle a^{\dagger}_{\boldsymbol{k}_2}a_{\boldsymbol{k}_4}, \tag{43}$$

or, in terms of nonantisymmetrized matrix elements of V,

$$\begin{aligned}\{[a_{\boldsymbol{k}},&V]_{-},a^{\dagger}_{\boldsymbol{k}'}\}_{+} \\ &= \int d\boldsymbol{k}_2 d\boldsymbol{k}_4 \langle\boldsymbol{k},\boldsymbol{k}_2|V(|\boldsymbol{k}',\boldsymbol{k}_4\rangle - |\boldsymbol{k}_4,\boldsymbol{k}'\rangle)a^{\dagger}_{\boldsymbol{k}_2}a_{\boldsymbol{k}_4}.\end{aligned} \tag{44}$$

The anticommutator term (43) has the form of a nucleon-number-conserving effective one-body operator including direct and exchange matrix elements of the nucleon-nucleon interaction. Scattering and excitations of the target induced by this operator can proceed through low nucleon momentum components in the target ground-state and momentum transfer components that can be found in the short-range interaction V.

On the other hand $\hat{\mathcal{T}}^{{\rm Born}(HPS)}_{0,0}$, Eq. (40) describes quite different processes. The explicit expression (42) shows that the state $[a_{\boldsymbol{k}},V]|-\boldsymbol{k}',\psi_0\rangle\rangle$ that appears in Eq. (40) only involves interactions between pairs of nucleons that are initially in the target ground state and produce an outgoing nucleon with momentum $\boldsymbol{k}$. The bra $\langle\langle\Psi(0,\boldsymbol{x}=0)|a^{\dagger}_{\boldsymbol{k}'}$ shows that the incoming momentum $\boldsymbol{k}'$ has to be found in the target ground-state wave function. These are heavy-particle stripping terms in standard textbook language [6], p. 93. In contrast, the first interactions in $\hat{\mathcal{T}}^{\rm Born(0)}_{0,0}$ are direct and exchange scatterings of the incident nucleon by a target nucleon. It would be expected that, except at very low incident energies and/or very small A when momentum components arising from target recoil may be comparable to the incident momentum, elastic scattering would be dominated by the processes described in $\hat{\mathcal{T}}^{\rm Born(0)}_{0,0}$ and their iterations.

For given values of $\boldsymbol{k}$ and $\boldsymbol{k}'$ the right-hand side of Eq. (39) is the matrix element of a one-body interaction in Fock-space $\mathcal{V}$,

$$\hat{\mathcal{T}}^{\rm Born(0)}_{0,0}(E+\imath\epsilon;\boldsymbol{k},\boldsymbol{k}') = \langle\langle\Psi(0,\boldsymbol{x}=0)|\mathcal{V}(\boldsymbol{k},\boldsymbol{k}')|-\boldsymbol{k}',\psi_0\rangle\rangle, \tag{45}$$

where

$$\mathcal{V}(\boldsymbol{k},\boldsymbol{k}') = \int d\boldsymbol{k}_2 \int d\boldsymbol{k}_4 \langle\boldsymbol{k},\boldsymbol{k}_2|V_{\mathcal{A}}|\boldsymbol{k}',\boldsymbol{k}_4\rangle a^{\dagger}_{\boldsymbol{k}_2}a_{\boldsymbol{k}_4}. \tag{46}$$

In the next several subsections, it will be shown that the complete transition operator $\hat{\mathcal{T}}_{0,0}$ can be written

$$\hat{\mathcal{T}}_{0,0} = \hat{\mathcal{T}}^{(1)}_{0,0} + \hat{\mathcal{T}}^{{\rm Born}(HPS)}_{0,0}, \tag{47}$$

where $\hat{\mathcal{T}}^{(1)}_{0,0}$ and not only the Born contribution $\hat{\mathcal{T}}^{\rm Born(0)}_{0,0}$ can be expressed entirely in terms of matrix elements of $\mathcal{V}$ in the A-nucleon subspace.

F. Development of $\hat{\mathcal{T}}^{(1)}_{0,0}$

The second term on the right in Eq. (36) for $\hat{\mathcal{T}}_{0,0}$ can be expressed in terms of $\mathcal{V}$, Eq. (46), by examining the explicit formulas for the commutators $[a_{\boldsymbol{k}},V]_{-}$ [see Eq. (42)] and $[V,a^{\dagger}_{\boldsymbol{k}'},]_{-}$,

$$[a_{\boldsymbol{k}},V]_{-} = \frac{1}{2}\int d\boldsymbol{k}_3 \mathcal{V}(\boldsymbol{k},\boldsymbol{k}_3)a_{\boldsymbol{k}_3}. \tag{48}$$

and

$$[V,a^{\dagger}_{\boldsymbol{k}'}]_{-} = \frac{1}{2}\int d\boldsymbol{k}_3 a^{\dagger}_{\boldsymbol{k}_3}\mathcal{V}(\boldsymbol{k}_3,\boldsymbol{k}'). \tag{49}$$

Note that, if the nucleon-nucleon interaction V is Hermitian, then

$$[\mathcal{V}(\boldsymbol{k}',\boldsymbol{k})]^{\dagger} = \mathcal{V}(\boldsymbol{k},\boldsymbol{k}'). \tag{50}$$

Equation (36) for $\hat{\mathcal{T}}_{0,0}$ can now be written

$$\hat{\mathcal{T}}_{0,0} = \hat{\mathcal{T}}^{(1)}_{0,0} + \hat{\mathcal{T}}^{{\rm Born}(HPS)}_{0,0}, \tag{51}$$

where

$$\begin{aligned}\hat{\mathcal{T}}^{(1)}_{0,0}&(E+\imath\epsilon;\boldsymbol{k},\boldsymbol{k}') \\ &= \langle\langle\Psi(0,\boldsymbol{x}=0)|\mathcal{V}(\boldsymbol{k},\boldsymbol{k}')|-\boldsymbol{k}',\psi_0\rangle\rangle\end{aligned}$$

$$+\frac{1}{4}\int dk_3'\int dk_3'\langle\langle\Psi(0,\boldsymbol{x}=0)|\mathcal{V}(\boldsymbol{k},\boldsymbol{k}_3)a_{\boldsymbol{k}_3}\times G(E+\imath\epsilon)a^\dagger_{\boldsymbol{k}_3'}\mathcal{V}(\boldsymbol{k}_3',\boldsymbol{k}')|-\boldsymbol{k}',\psi_0\rangle\rangle. \quad (52)$$

It will be shown below that, acting on the A-nucleon intermediate states that contribute to the Green's function term in Eq. (52), the operator $G(E+\imath\epsilon)a^\dagger_{\boldsymbol{k}_3'}$ can be expressed in terms of matrix elements of $\mathcal{V}$ in the A-nucleon subspace.

For a translationally invariant nucleon-nucleon interaction V matrix, elements of the operator $\mathcal{V}$ satisfy

$$\begin{aligned}&\langle\langle-\boldsymbol{k}'',\psi_n|\mathcal{V}(\boldsymbol{k},\boldsymbol{k}')|-\boldsymbol{k}''',\psi_m\rangle\rangle\\&=(2\pi)^3\delta[\boldsymbol{k}-\boldsymbol{k}''-(\boldsymbol{k}'-\boldsymbol{k}''')]\\&\quad\times\langle\langle\Psi(n,\boldsymbol{x}=0)|\mathcal{V}(\boldsymbol{k},\boldsymbol{k}')|-\boldsymbol{k}''',\psi_m\rangle\rangle. \quad (53)\end{aligned}$$

This result is most simply derived by noting that the δ-function dependence on the momenta $\boldsymbol{k},\boldsymbol{k}'',\boldsymbol{k}',\boldsymbol{k}'''$ follows from the momentum conservation properties of the matrix elements of V and the structure of $\mathcal{V}$ in terms of creation and destruction operators. Choosing $\boldsymbol{k},\boldsymbol{k}',\boldsymbol{k}'''$ as the independent variables and integrating over $\boldsymbol{k}''$ using

$$\int d\boldsymbol{k}''\exp(\imath\boldsymbol{k}''\cdot\boldsymbol{x})|-\boldsymbol{k}'',\psi_n\rangle\rangle=(2\pi)^3|\Psi(n,\boldsymbol{x})\rangle\rangle \quad (54)$$

gives the result (53).

A similar useful formula results when $\boldsymbol{k},\boldsymbol{k}',\boldsymbol{k}''$ are chosen as the independent variables,

$$\begin{aligned}&\langle\langle-\boldsymbol{k}'',\psi_n|\mathcal{V}(\boldsymbol{k},\boldsymbol{k}')|-\boldsymbol{k}''',\psi_m\rangle\rangle\\&=(2\pi)^3\delta[\boldsymbol{k}-\boldsymbol{k}''-(\boldsymbol{k}'-\boldsymbol{k}''')]\\&\quad\times\langle\langle-\boldsymbol{k}'',\psi_n|\mathcal{V}(\boldsymbol{k},\boldsymbol{k}')|\Psi(m,\boldsymbol{x}=0)\rangle\rangle. \quad (55)\end{aligned}$$

Equations (53) and (55) are special cases of a more general result derived in Appendix A 4.

Note that Eqs. (53) and (55) imply that

$$\begin{aligned}&\langle\langle-\boldsymbol{k}'',\psi_n|\mathcal{V}(\boldsymbol{k},\boldsymbol{k}')|\Psi(m,\boldsymbol{x}=0)\rangle\rangle\\&=\langle\langle\Psi(n,\boldsymbol{x}=0)|\mathcal{V}(\boldsymbol{k},\boldsymbol{k}')|-(\boldsymbol{k}''+\boldsymbol{k}'-\boldsymbol{k}),\psi_m\rangle\rangle. \quad (56)\end{aligned}$$

Using Eqs. (53) and (55) together with the result, valid for a translationally invariant H,

$$\begin{aligned}&\langle\langle-\boldsymbol{k}_3''',\psi_n|a_{\boldsymbol{k}_3''}G(E+\imath\epsilon)a^\dagger_{\boldsymbol{k}_3'}|-\boldsymbol{k}_3',\psi_{n'}\rangle\rangle\\&=\delta(\boldsymbol{k}_3'''-\boldsymbol{k}_3'')(2\pi)^3\langle\langle\Psi(n,\boldsymbol{x}=0)|a_{\boldsymbol{k}_3''}G(E+\imath\epsilon)a^\dagger_{\boldsymbol{k}_3'}|-\boldsymbol{k}_3',\psi_{n'}\rangle\rangle \quad (57)\end{aligned}$$

allows Eq. (52) to be written

$$\begin{aligned}&\tilde{\mathcal{T}}^{(1)}_{0,0}(E+\imath\epsilon;\boldsymbol{k},\boldsymbol{k}')\\&=\langle\langle\Psi(0,\boldsymbol{x}=0)|\mathcal{V}(\boldsymbol{k},\boldsymbol{k}')|-\boldsymbol{k}',\psi_0\rangle\rangle\\&\quad+\frac{1}{4}\sum_{n,n'}\int dk_3'\int dk_3''\langle\langle\Psi(0,\boldsymbol{x}=0)|\mathcal{V}(\boldsymbol{k},\boldsymbol{k}_3'')|-\boldsymbol{k}_3'',\psi_n\rangle\rangle\\&\quad\times\langle\langle\Psi(n,\boldsymbol{x}=0)|a_{\boldsymbol{k}_3''}G(E+\imath\epsilon)a^\dagger_{\boldsymbol{k}_3'}|-\boldsymbol{k}_3',\psi_{n'}\rangle\rangle\\&\quad\times\langle\langle\Psi(n',\boldsymbol{x}=0)|\mathcal{V}(\boldsymbol{k}_3',\boldsymbol{k}')|-\boldsymbol{k}',\psi_0\rangle\rangle. \quad (58)\end{aligned}$$

It is convenient here to generalize the concept of B space introduced at the beginning of Sec. II B to include the space of A-nucleon target states ψ_n. These target nucleons interact with the fictitious particle of reduced mass introduced in Sec. II B, but they are not identical to it. A basis in this space is, by definition, an orthonormal set of states $|\boldsymbol{k},n\rangle$, $n=0,\ldots,\infty$, where the fictitious particle has momentum $\boldsymbol{k}$ and the target nucleons have an intrinsic state n and a total momentum $-\boldsymbol{k}$. The interaction between the particle and the target is described by an operator $\hat{\mathcal{U}}$ in this extended space with matrix elements defined in terms of the Fock-space elements of $\bar{\mathcal{V}}$ by

$$\langle\boldsymbol{k},n|\hat{\mathcal{U}}|\boldsymbol{k}',n'\rangle=\tfrac{1}{2}\langle\langle\Psi(n,\boldsymbol{x}=0)|\mathcal{V}(\boldsymbol{k},\boldsymbol{k}')|-\boldsymbol{k}',\psi_{n'}\rangle\rangle, \quad (59)$$

where $\mathcal{V}$ is the interaction defined in Eq. (46). The Hamiltonian of the system "fictitious particle + A target nucleons" is

$$\hat{\mathcal{H}}=\hat{T}+\hat{h}_A+\hat{\mathcal{U}}, \quad (60)$$

where $\hat{T}$ is the kinetic-energy operator associated with a particle of mass $\frac{A}{(A+1)}m$ and $\hat{h}_A$ is diagonal in the $|\boldsymbol{k},n\rangle$ basis with eigenvalues E_n,

$$\langle\boldsymbol{k}',n'|\hat{h}_A|\boldsymbol{k},n\rangle=\delta(\boldsymbol{k}-\boldsymbol{k}')\delta_{n,n'}E_n. \quad (61)$$

The introduction of this extended space is useful because the Fock-space states $G(E+\imath\epsilon)a^\dagger_{\boldsymbol{k}}|-\boldsymbol{k},\psi_n\rangle\rangle$ that appear in Eq. (52) can be expressed in terms of the Green's function associated with the Hamiltonian $\hat{\mathcal{H}}$.

It is shown in Appendix B that states $G(E+\imath\epsilon)a^\dagger_{\boldsymbol{k}}|-\boldsymbol{k},\psi_n\rangle\rangle$, E fixed $\boldsymbol{k}$, n, arbitrary, satisfy the set of coupled equations Eq. (B5). In terms of matrix elements in extended B space these equations can be written

$$\begin{aligned}&\int d\boldsymbol{k}'\sum_{n'}G(E^+)a^\dagger_{\boldsymbol{k}'}|-\boldsymbol{k}',\psi_{n'}\rangle\rangle\langle\boldsymbol{k}',n'|(E^+-\hat{\mathcal{H}})|\boldsymbol{k},n\rangle\\&=a^\dagger_{\boldsymbol{k}}|-\boldsymbol{k},\psi_n\rangle\rangle, \quad (62)\end{aligned}$$

with the solution,

$$\begin{aligned}&G(E+\imath\epsilon)a^\dagger_{\boldsymbol{k}}|-\boldsymbol{k},\psi_n\rangle\rangle\\&=\sum_{n'}\int d\boldsymbol{k}'a^\dagger_{\boldsymbol{k}'}|-\boldsymbol{k}',\psi_{n'}\rangle\rangle\langle\boldsymbol{k}',n'|\frac{1}{(E+\imath\epsilon-\hat{\mathcal{H}})}|\boldsymbol{k},n\rangle. \quad (63)\end{aligned}$$

The quantity on the left is a ket in Fock space, and the right-hand side is a linear combination of Fock-space kets with coefficients that are matrix elements in the extended B space introduced before Eq. (60).

Using (63) in Eq. (58) for $\tilde{\mathcal{T}}^{(1)}_{0,0}$ gives

$$\begin{aligned}&\tilde{\mathcal{T}}^{(1)}_{0,0}(E+\imath\epsilon;\boldsymbol{k},\boldsymbol{k}')\\&=\langle\langle\Psi(0,\boldsymbol{x}=0)|\mathcal{V}(\boldsymbol{k},\boldsymbol{k}')|-\boldsymbol{k}',\psi_0\rangle\rangle\\&\quad+\frac{1}{4}\sum_{n,n'}\int dk_3'\int dk_3''\langle\langle\Psi(0,\boldsymbol{x}=0)|\mathcal{V}(\boldsymbol{k},\boldsymbol{k}_3'')|-\boldsymbol{k}_3'',\psi_n\rangle\rangle\\&\quad\times\sum_{n''}\int dk_3'''\langle\langle\Psi(n,\boldsymbol{x}=0)|a_{\boldsymbol{k}_3''}a^\dagger_{\boldsymbol{k}_3'''}|-\boldsymbol{k}_3''',\psi_{n''}\rangle\rangle\langle\boldsymbol{k}_3''',n''|\\&\quad\times\frac{1}{(E+\imath\epsilon-\hat{T}-\hat{h}_A-\hat{\mathcal{U}})}|\boldsymbol{k}_3',n'\rangle\\&\quad\times\langle\langle\Psi(n',\boldsymbol{x}=0)|\mathcal{V}(\boldsymbol{k}',\boldsymbol{k}_3')|-\boldsymbol{k}',\psi_0\rangle\rangle, \quad (64)\end{aligned}$$

or, expressed entirely in terms of matrix elements of operators in the extended B space,

$$\tilde{\mathcal{T}}^{(1)}_{0,0}(E+\imath\epsilon;\boldsymbol{k},\boldsymbol{k}') = 2\langle\boldsymbol{k},0|\hat{\mathcal{U}}|\boldsymbol{k}',0\rangle + \langle\boldsymbol{k},0|\hat{\mathcal{U}}\hat{\mathcal{K}}_A\hat{\mathcal{G}}(E+\imath\epsilon)\hat{\mathcal{U}}|\boldsymbol{k}',0\rangle, \quad (65)$$

where

$$\hat{\mathcal{G}}(E+\imath\epsilon) = \frac{1}{(E+\imath\epsilon-\hat{T}-\hat{h}_A-\hat{\mathcal{U}})}, \quad (66)$$

and

$$\langle\boldsymbol{k},n|\hat{\mathcal{K}}_A|\boldsymbol{k}_3',n''\rangle = \langle\langle\Psi(n,\boldsymbol{x}=0)|a_{\boldsymbol{k}}a^\dagger_{\boldsymbol{k}'}|-\boldsymbol{k}',\psi_{n''}\rangle\rangle \quad (67)$$

is the matrix of one-nucleon transition density matrices of the A-nucleon system in momentum space.

G. Summary of Sec. II

From Eqs. (40) and (65) the complete exact formula for the off-shell transition matrix is

$$\begin{aligned}\tilde{\mathcal{T}}_{0,0}(E+\imath\epsilon;\boldsymbol{k},\boldsymbol{k}') &= 2\langle\boldsymbol{k},0|\hat{\mathcal{U}}|\boldsymbol{k}',0\rangle + \langle\boldsymbol{k},0|\hat{\mathcal{U}}\hat{\mathcal{K}}_A\hat{\mathcal{G}}(E+\imath\epsilon)\hat{\mathcal{U}}|\boldsymbol{k}',0\rangle\rangle \\ &\quad - \langle\langle\Psi(0,\boldsymbol{x}=0)|a^\dagger_{\boldsymbol{k}'}[a_{\boldsymbol{k}},V]_-|-\boldsymbol{k}',\psi_0\rangle\rangle. \quad (68)\end{aligned}$$

Explicit expressions relating the matrix elements of $\hat{\mathcal{U}}$ in the $|\boldsymbol{k},n\rangle$ basis and matrix elements of the nucleon-nucleon interaction V between Fock-space states follow from Eqs. (59) and (46):

$$\begin{aligned}\langle\boldsymbol{k},n|\hat{\mathcal{U}}|\boldsymbol{k}',n'\rangle &= \frac{1}{2}\langle\langle\Psi(n,\boldsymbol{x}=0)|\mathcal{V}(\boldsymbol{k},\boldsymbol{k}')|-\boldsymbol{k}',\psi_{n'}\rangle\rangle \\ &= \frac{1}{2}\int d\boldsymbol{k}_2\int d\boldsymbol{k}_4\langle\boldsymbol{k},\boldsymbol{k}_2|V_A|\boldsymbol{k}',\boldsymbol{k}_4\rangle \\ &\quad\times\langle\langle\Psi(n,\boldsymbol{x}=0)|a^\dagger_{\boldsymbol{k}_2}a_{\boldsymbol{k}_4}|-\boldsymbol{k}',\psi_{n'}\rangle\rangle. \quad (69)\end{aligned}$$

The first two terms on the right in Eq. (68) are expressed in terms of matrix elements of a one-body nucleon-number-conserving nucleon-nucleus interaction between eigenstates of the A-nucleon Hamiltonian. In this sense, they have a similar structure to Feshbach's theory of the optical model [4] but with both direct and exchange components of the nucleon-nucleon interaction included in $\hat{\mathcal{U}}$. However, because of the factor 2 in front of the first term and the occurrence of $\hat{\mathcal{K}}_A$, the first two terms do not have the standard form expected for the transition matrix associated with the Hamiltonian $\hat{\mathcal{H}}$ of Eq. (60). If they did, and the heavy-particle stripping term was neglected, the solution of Eq. (31) for the nucleon optical model operator would simply be the operator defined by Feshbach (see Appendix D) corresponding to $\hat{\mathcal{H}}$ but with a nucleon with the appropriate nucleon-target reduced mass and matrix elements of the nucleon-nucleon corrected for recoil as in Eq. (69). It will be shown in Sec. IV that in the weak-binding limit[1] this identification can be made with a modified definition of $\hat{\mathcal{U}}$, but there appears to be no general justification for optical model developments that attempt to take antisymmetry into account in Feshbach's approach by simply adding knockout exchange terms, even when the heavy-particle stripping term is neglected.

[1]The terminology of Ref. [7] is used.

III. THE HEAVY-PARTICLE STRIPPING TERM AND THE LEHMANN, SYMANZIK, AND ZIMMERMANN REPRESENTATION

In standard many-body theories of the optical model that make the link with the nucleon single-particle Green's function, the second term on the right in Eq. (68), referred to here as the heavy-particle stripping term, is transformed using the identity,

$$\begin{aligned}&\langle\langle\Psi(0,\boldsymbol{x}=0)|a^\dagger_{\boldsymbol{k}'}[a_{\boldsymbol{k}},V]_-|-\boldsymbol{k}',\psi_0\rangle\rangle \\ &= \langle\langle\Psi(0,\boldsymbol{x}=0)|[V,a^\dagger_{\boldsymbol{k}'}]\frac{1}{\left(E_0+\frac{\epsilon_k}{A}-\epsilon_{k'}-H\right)} \\ &\quad\times[a_{\boldsymbol{k}},V]_-|-\boldsymbol{k}',\psi_0\rangle\rangle. \quad (70)\end{aligned}$$

This result is derived in Appendix C. The convention of a zero value for the target ground-state intrinsic energy E_0 has been abandoned temporarily to aid comparison with other treatments [1,3]. Note that for a stable ground state the denominator $(\frac{\epsilon_k}{A}-\epsilon_{k'}+E_0-H)$ never vanishes for $A>1$.

Equation (70) displays the heavy-particle stripping contribution in terms of coupling between the A-nucleon ground state and a complete set of $(A-1)$-nucleon intermediate states. Using Eq. (70) and (36)–(40), the complete on-shell $(\epsilon_{k'}=\epsilon_k)$ elastic transition amplitude can be written

$$\begin{aligned}&\hat{\mathcal{T}}_{0,0}\left(E_0+\frac{(A+1)}{A}\epsilon_k;\boldsymbol{k},\boldsymbol{k}'\right) \\ &= \langle\langle\Psi(0,\boldsymbol{x}=0)|\{[a_{\boldsymbol{k}},V]_-,a^\dagger_{\boldsymbol{k}'}\}|-\boldsymbol{k}',\psi_0\rangle\rangle \\ &\quad+\langle\langle\Psi(0,\boldsymbol{x}=0)|[a_{\boldsymbol{k}},V]_-G\left(E_0+\frac{(A+1)}{A}\epsilon_k+\imath\epsilon\right) \\ &\quad\times[V,a^\dagger_{\boldsymbol{k}'}]_-|-\boldsymbol{k}',\psi_0\rangle\rangle\langle\langle\Psi(0,\boldsymbol{x}=0)| \\ &\quad\times[V,a^\dagger_{\boldsymbol{k}'}]\frac{1}{\left(E_0-\frac{(A-1)}{A}\epsilon_k-H\right)}[a_{\boldsymbol{k}},V]_-|-\boldsymbol{k}',\psi_0\rangle\rangle. \quad (71)\end{aligned}$$

The structure of the denominator in the third term on the right can be understood as follows. A state of energy $E_0+\epsilon_{k'}/A-\epsilon_k$ is obtained when a nucleon of momentum $\boldsymbol{k}$ is knocked out of an incident channel A-nucleon state that has momentum $-\boldsymbol{k}$ and energy $E_0+\epsilon_{k'}/A$. The appropriate on-shell intermediate $(A-1)$-nucleon energy that should appear in the denominator is therefore $E_0+\epsilon_{k'}/A-\epsilon_k$. For on-shell elastic scattering $(\epsilon_{k'}=\epsilon_k)$, this reduces to $E_0-\frac{(A-1)}{A}\epsilon_k$.

Equation (71) is the LSZ formula for the elastic transition operator, when the third contribution is referred to as the "hole term." The LSZ formalism usually appears in a time-dependent version of scattering theory. The development here closely follow the methods of Ref. [3] modified to incorporate the requirements of translational invariance along the lines of Ref. [2]. The LSZ formula lends itself well to a systematic development in terms of all contributions from the nucleon-nucleon interaction V, including those in the ground-state wave function. A new definition of the one-particle Green's function used in this approach, including recoil corrections not included in standard treatments, is described in Sec. V A below.

Using the techniques described in Sec. II F, the heavy-particle stripping term, as expressed in the LSZ form, Eq. (70), can also be written in terms of in terms of matrix elements of the one-body nucleon-nucleus interaction $\mathcal{V}$ but between eigenstates of the $(A-1)$-nucleon Hamiltonian.

IV. THE WEAK-BINDING LIMIT

The weak-binding limit is discussed at length in Ref. [7], pp. 775–780. The essential idea is that for sufficiently high incident momentum $\boldsymbol{k}'$ and a sufficiently weakly bound target it is improbable that a nucleon in the target nucleus will be found with momentum greater than $\boldsymbol{k}'$. In the present context, these ideas can be exploited by using an alternative exact form for the commutator $[a_{\boldsymbol{k}}, V]_-$, Eq. (42),

$$[V, a^\dagger_{\boldsymbol{k}'}]_- \ = \int d\boldsymbol{k}_2 d\boldsymbol{k}_3 d\boldsymbol{k}_4 \theta(k_3 - k_2)\langle \boldsymbol{k}_3, \boldsymbol{k}_2|V_{\mathcal{A}}|\boldsymbol{k}', \boldsymbol{k}_4\rangle a^\dagger_{\boldsymbol{k}_3} a^\dagger_{\boldsymbol{k}_2} a_{\boldsymbol{k}_4}. \tag{72}$$

The θ-function $\theta(x)$ is defined by

$$\begin{aligned}\theta(x) &= 1, \quad x > 0\\ &= 0, \quad x < 0.\end{aligned} \tag{73}$$

The θ function means that the integration is restricted to the region $|\boldsymbol{k}_3| > |\boldsymbol{k}_2$. There is no restriction on the limits on any of the other eigenvalues (momentum direction, spin, and isospin) involved in the definition of the single nucleon states $|\boldsymbol{k}\rangle$.

Under weak-binding assumptions, in the ket $[V, a^\dagger_{\boldsymbol{k}'}]_-|-\boldsymbol{k}', \psi_0\rangle\rangle$ that appears in the second term on the right in Eq. (36), the expression (72) can be replaced by the approximate form

$$[V, a^\dagger_{\boldsymbol{k}'}]_- \ \approx \int d\boldsymbol{k}_3 a^\dagger_{\boldsymbol{k}_3} \int d\boldsymbol{k}_2 d\boldsymbol{k}_4 \theta(k_3 - k_2)\theta(k' - k_4)\langle \boldsymbol{k}_3, \boldsymbol{k}_2|V_{\mathcal{A}}|\boldsymbol{k}', \boldsymbol{k}_4\rangle a^\dagger_{\boldsymbol{k}_2} a_{\boldsymbol{k}_4}. \tag{74}$$

Similarly, Eq. (42) can be replaced by

$$[a_{\boldsymbol{k}}, V]_- \approx \int d\boldsymbol{k}_2 d\boldsymbol{k}_3 d\boldsymbol{k}_4 \theta(k - k_2)\theta(k_3 - k_4)\langle \boldsymbol{k}, \boldsymbol{k}_2|V_{\mathcal{A}}|\boldsymbol{k}_3, \boldsymbol{k}_4\rangle a^\dagger_{\boldsymbol{k}_2} a_{\boldsymbol{k}_4} a_{\boldsymbol{k}_3}. \tag{75}$$

To this approximation both $[a_{\boldsymbol{k}}, V]_-$ and $[V, a^\dagger_{\boldsymbol{k}'}]_-$ can be written in terms of a new one-body operator $\bar{\mathcal{V}}$ defined by

$$\bar{\mathcal{V}}(\boldsymbol{k}, \boldsymbol{k}') = \int d\boldsymbol{k}_2 \int d\boldsymbol{k}_4 \theta(k - k_2)\theta(k' - k_4)\langle \boldsymbol{k}, \boldsymbol{k}_2|V_{\mathcal{A}}|\boldsymbol{k}', \boldsymbol{k}_4\rangle a^\dagger_{\boldsymbol{k}_2} a_{\boldsymbol{k}_4}, \tag{76}$$

as

$$[V, a^\dagger_{\boldsymbol{k}'}]_- \approx \int d\boldsymbol{k}_3 a^\dagger_{\boldsymbol{k}_3} \bar{\mathcal{V}}(\boldsymbol{k}_3, \boldsymbol{k}'), \quad [a_{\boldsymbol{k}}, V]_- \approx \int d\boldsymbol{k}_3 \bar{\mathcal{V}}(\boldsymbol{k}, \boldsymbol{k}_3) a_{\boldsymbol{k}_3}. \tag{77}$$

Similarly, from Eqs. (39) and (43), the Born term $\hat{\mathcal{T}}^{\mathrm{Born}(0)}_{0,0}$ can also be written in terms of $\bar{\mathcal{V}}$,

$$\begin{aligned}&\hat{\mathcal{T}}^{\mathrm{Born}(0)}_{0,0}(E + \imath\epsilon; \boldsymbol{k}, \boldsymbol{k}')\\ &= \langle\langle \Psi(0, \boldsymbol{x} = 0)|\{[a_{\boldsymbol{k}}, V]_-, a^\dagger_{\boldsymbol{k}'}\}_+|-\boldsymbol{k}', \psi_0\rangle\rangle\\ &= \int d\boldsymbol{k}_2 d\boldsymbol{k}_4[\theta(k - k_2)\theta(k' - k_4) + \theta(k_2 - k)\theta(k_4 - k')\\ &\quad + \theta(k_2 - k)\theta(k' - k_4) + \theta(k - k_2)\theta(k_4 - k')]\langle \boldsymbol{k}, \boldsymbol{k}_2|V_{\mathcal{A}}|\boldsymbol{k}', \boldsymbol{k}_4\rangle\langle\langle \Psi(0, \boldsymbol{x} = 0)|a^\dagger_{\boldsymbol{k}_2} a_{\boldsymbol{k}_4}|-\boldsymbol{k}', \psi_0\rangle\rangle\\ &\approx \langle\langle \Psi(0, \boldsymbol{x} = 0)|\bar{\mathcal{V}}(\boldsymbol{k}, \boldsymbol{k}')|-\boldsymbol{k}', \psi_0\rangle\rangle,\end{aligned} \tag{78}$$

where in the neglected terms in the last line at least one nucleon of an interacting pair that has a momentum to be found in the ground state and greater than $\boldsymbol{k}$ or $\boldsymbol{k}'$. The same approximation eliminates the heavy-particle stripping term.

Proceeding as in Sec. II F, Eq. (68) is replaced by the the weak-binding elastic transition matrix,

$$\tilde{\mathcal{T}}^{WB}_{0,0}(E + \imath\epsilon; \boldsymbol{k}, \boldsymbol{k}') = \langle \boldsymbol{k}, 0|\hat{\mathcal{U}}_{WB}|\boldsymbol{k}', 0\rangle + \langle \boldsymbol{k}, 0|\hat{\mathcal{U}}_{WB}\hat{\mathcal{G}}_{WB}(E + \imath\epsilon)\hat{\mathcal{U}}_{WB}|\boldsymbol{k}', 0\rangle\rangle, \tag{79}$$

where

$$\hat{\mathcal{G}}_{WB}(E + \imath\epsilon) = \frac{1}{(E + \imath\epsilon - \hat{\mathcal{H}}_{WB})}, \tag{80}$$

and

$$\hat{\mathcal{H}}_{WB} = \hat{T} + \hat{h}_A + \hat{\mathcal{U}}_{WB}. \tag{81}$$

It is consistent with the weak-binding assumptions to use the approximation,

$$\begin{aligned}\langle \boldsymbol{k}, n|\hat{\mathcal{K}}_A|\boldsymbol{k}'_3, n'\rangle &= \langle\langle \Psi(n, \boldsymbol{x}=0)|a_{\boldsymbol{k}} a^\dagger_{\boldsymbol{k}'}|-\boldsymbol{k}', \psi_{n'}\rangle\rangle \\ &= \langle\langle \Psi(n, \boldsymbol{x}=0)|\delta(\boldsymbol{k}-\boldsymbol{k}')|-\boldsymbol{k}', \psi_{n'}\rangle\rangle - \langle\langle \Psi(n, \boldsymbol{x}=0)|a^\dagger_{\boldsymbol{k}'} a_{\boldsymbol{k}}|-\boldsymbol{k}', \psi_{n''}\rangle\rangle \\ &\approx \delta(\boldsymbol{k}-\boldsymbol{k}')\delta_{n,n'}. \end{aligned} \tag{82}$$

The interaction $\hat{\mathcal{U}}_{WB}$ that appears in $\tilde{\mathcal{T}}^{WB}_{0,0}$ is an operator in extended B space and is defined in terms of the nucleon-nucleon interaction V by, c.f. Eq. (69),

$$\begin{aligned}\langle \boldsymbol{k}, n|\hat{\mathcal{U}}_{WB}|\boldsymbol{k}', n'\rangle &= \langle\langle \Psi(n, \boldsymbol{x}=0)|\bar{\mathcal{V}}(\boldsymbol{k}, \boldsymbol{k}')|-\boldsymbol{k}', \psi_{n'}\rangle\rangle \\ &= \int d\boldsymbol{k}_2 \int d\boldsymbol{k}_4 \theta(k-k_2)\theta(k'-k_4)\langle \boldsymbol{k}, \boldsymbol{k}_2|V_{\mathcal{A}}|\boldsymbol{k}', \boldsymbol{k}_4\rangle\langle\langle \Psi(n, \boldsymbol{x}=0)|a^\dagger_{\boldsymbol{k}_2} a_{\boldsymbol{k}_4}|-\boldsymbol{k}', \psi_{n'}\rangle\rangle. \end{aligned} \tag{83}$$

Equation (79) will be recognized as the standard formula for the off-shell elastic transition matrix associated with the Hamiltonian (81), including all target excitations induced by the interaction $\hat{\mathcal{U}}_{WB}$. This is just the physical system studied by Feshbach [4], except that here all nucleon-nucleon interactions include both direct and exchange terms and with appropriate modifications of matrix elements to take into account recoil and the weak-binding assumption. It can immediately be deduced using the techniques outlined in Appendix D that an alternative expression for the operator in B-space $\hat{T}^{WB}_{0,0}$ is

$$\hat{T}^{WB}_{0,0} = U^{WB\,\mathrm{opt}}_{0,0} + U^{WB\,\mathrm{opt}}_{0,0} \frac{1}{E + \imath\epsilon - \hat{T} - U^{WB\,\mathrm{opt}}_{0,0}} U^{WB\,\mathrm{opt}}_{0,0}, \tag{84}$$

where, in the notation of Eq. (83),

$$\langle \boldsymbol{k}|U^{WB\,\mathrm{opt}}_{0,0}|\boldsymbol{k}'\rangle = \langle \boldsymbol{k}, n=0|\hat{U}_{WB\,\mathrm{opt}}|\boldsymbol{k}', n=0\rangle, \tag{85}$$

and $\hat{U}_{WB\,\mathrm{opt}}$ is defined as in Eq. (D8) of Appendix D with $\hat{V}$ replaced by $\hat{\mathcal{U}}_{WB}$. This implies that according to the definition (31) of Sec. II C, in the weak-binding limit the optical potential operator is

$$V^{\mathrm{opt}} = U^{WB\,\mathrm{opt}}_{0,0}. \tag{86}$$

In particular, if the incident energy is below the threshold for exciting the target, the operator $\hat{U}_{WB\,\mathrm{opt}}$ is Hermitian and so is the predicted optical potential V^{opt}. If all target excitations are neglected, the corresponding optical potential is the ground-state expectation value of the truncated nucleon-nucleon potential defined in $\hat{\mathcal{U}}_{WB}$, Eq. (83).

It also follows from this analysis that in the weak-binding limit Watson's multiple-scattering theory and the associated optical model can be modified to include antisymmetry and translational invariance by simply replacing the nucleon-nucleon interaction by the antisymmetrized and truncated form that appears in Eq. (83). This result is consistent with the work of Ref. [8] on the weak-binding limit and described in Ref. [7], pp. 775–780 but without recoil corrections.

V. NUCLEON SINGLE-PARTICLE GREEN'S FUNCTION AND THE DYSON SELF-ENERGY

A. The single-particle Green's function

The off-shell elastic transition operator defined by Eqs. (36), (37), and (70) has the form

$$\begin{aligned}\hat{\mathcal{T}}_{0,0}(E+\imath\epsilon; \boldsymbol{k}, \boldsymbol{k}') &= \langle\langle \Psi(0, \boldsymbol{x}=0)|\{[a_{\boldsymbol{k}}, V]_-, a^\dagger_{\boldsymbol{k}'}\}|-\boldsymbol{k}', \psi_0\rangle\rangle + \langle\langle \Psi(0, \boldsymbol{x}=0)|[a_{\boldsymbol{k}}, V]_- \frac{1}{(E+\imath\epsilon - H)}[V, a^\dagger_{\boldsymbol{k}'}]_-|-\boldsymbol{k}', \psi_0\rangle\rangle \\ &\quad + \langle\langle \Psi(0, \boldsymbol{x}=0)|[V, a^\dagger_{\boldsymbol{k}'}] \frac{1}{\left(\epsilon_{k'} - \frac{\epsilon_k}{A} + H\right)}[a_{\boldsymbol{k}}, V]_-|-\boldsymbol{k}', \psi_0\rangle\rangle. \end{aligned} \tag{87}$$

The requirements of translation invariance lead to the different energy parameters in the denominators of the two terms on the right. Correspondingly, the usual definition of the nucleon single-particle Green's function that ignores translational invariance must be modified. The definition used here for general complex ω is

$$G_{0,0}(\boldsymbol{k}, \boldsymbol{k}'; \omega) = \langle\langle \Psi(0, \boldsymbol{x}=0)|a_{\boldsymbol{k}} \frac{1}{\omega - H} a^\dagger_{\boldsymbol{k}'}|-\boldsymbol{k}', \psi_0\rangle\rangle + \langle\langle \Psi(0, \boldsymbol{x}=0)|a^\dagger_{\boldsymbol{k}'} \frac{1}{\omega - \frac{(\epsilon_{k'}+\epsilon_k)}{A} + H} a_{\boldsymbol{k}}|-\boldsymbol{k}', \psi_0\rangle\rangle. \tag{88}$$

This differs from the usual definition, e.g., Ref. [3], Eq. (3.65), p. 321, by the $1/A$ terms in the energy denominator in the second term, and the appearance of a ground state with its c.m. localized at the origin of coordinates in the bra and a ground state of total momentum $-\boldsymbol{k}'$ in the ket. With this choice shown in Eq. (88) the two energy denominators reduce to the appropriate values shown in the transition matrix formula Eq. (71) in the on-shell limit $\omega = \frac{(A+1)}{A}\epsilon_k$, $\epsilon_{k'} = \epsilon_k$. It will be shown below that these differences are essential if the Green's function is to have the standard relation to the on-shell transition matrix [3], Eq. (3.66), p. 321.

B. Relation between the Green's function and the transition operator

The derivation of the relation among this Green's function, the free Green's function for a particle of reduced mass $\frac{A}{(A+1)}m$, and the transition operator defined in a momentum basis by Eq. (23) uses similar techniques to Ref. [3]. Frequent use is made of the relation (13) but with an abbreviated notation. The integers 1 and 2 will replace $\boldsymbol{k}'$ and $\boldsymbol{k}$, respectively, and J_1 and $J_1^\dagger$ will denote

$$\begin{aligned} J_1 &= [a_1, V], \\ J_1^\dagger &= [V, a_1^\dagger]. \end{aligned} \tag{89}$$

In this notation Eq. (13) becomes

$$\begin{aligned} Ha_1^\dagger &= a_1^\dagger H + \epsilon_1 a_1^\dagger + J_1^\dagger, \\ Ha_1 &= a_1 H - \epsilon_1 a_1 - J_1. \end{aligned} \tag{90}$$

The second term in $G_{0,0}(2,1;\omega)$, Eq. (88) gives

$$\begin{aligned}
&\left(\omega - \frac{(A+1)}{A}\epsilon_2\right) G^{(2)}_{0,0}(2,1;\omega) \\
&= \langle\langle \Psi(0,\boldsymbol{x}=0)|a_1^\dagger \frac{1}{\omega - \frac{(\epsilon_1+\epsilon_2)}{A} + H}\left(\omega - \frac{(A+1)}{A}\epsilon_2\right) a_2|-\boldsymbol{k}_1,\psi_0\rangle\rangle \\
&= \langle\langle \Psi(0,\boldsymbol{x}=0)|a_1^\dagger \frac{1}{\omega - \frac{(\epsilon_2+\epsilon_2)}{A} + H}\left(\omega - \frac{(\epsilon_1+\epsilon_2)}{A} + H - \frac{(A+1)}{A}\epsilon_2 + \frac{(\epsilon_1+\epsilon_2)}{A} - H\right) a_2|-\boldsymbol{k}_1,\psi_0\rangle\rangle \\
&= \langle\langle \Psi(0,\boldsymbol{x}=0)|a_1^\dagger a_2|-\boldsymbol{k}_1,\psi_0\rangle\rangle + \langle\langle \Psi(0,\boldsymbol{x}=0)|a_1^\dagger \frac{1}{\omega - \frac{(\epsilon_1+\epsilon_2)}{A} + H}\left(-\frac{(A+1)}{A}\epsilon_2 a_2 - a_2 H \right. \\
&\quad \left. + \epsilon_2 a_2 + J_2 + \frac{(\epsilon_1+\epsilon_2)}{A} a_2\right)|-\boldsymbol{k}_1,\psi_0\rangle\rangle \\
&= \langle\langle \Psi(0,\boldsymbol{x}=0)|a_1^\dagger a_2|-\boldsymbol{k}_1,\psi_0\rangle\rangle + \langle\langle \Psi(0,\boldsymbol{x}=0)|a_1^\dagger \frac{1}{\omega - \frac{(\epsilon_1+\epsilon_2)}{A} + H} J_2|-\boldsymbol{k}_1,\psi_0\rangle\rangle,
\end{aligned} \tag{91}$$

where use has been made of

$$H|-\boldsymbol{k}_1,\psi_0\rangle\rangle = \frac{\hbar^2(-\boldsymbol{k}_1)^2}{2Am}|-\boldsymbol{k}_1,\psi_0\rangle\rangle = \frac{1}{A}\epsilon_1|-\boldsymbol{k}_1,\psi_0\rangle\rangle. \tag{92}$$

Similarly,

$$\begin{aligned}
&\left(\omega - \frac{(A+1)}{A}\epsilon_1\right)\left(\omega - \frac{(A+1}{A}\epsilon_2\right) G^{(2)}_{0,0}(2,1;\omega) \\
&= \left(\omega - \frac{(A+1)}{A}\epsilon_1\right)\langle\langle \Psi(0,\boldsymbol{x}=0)|a_1^\dagger a_2|-\boldsymbol{k}_1,\psi_0\rangle\rangle \\
&\quad + \langle\langle \Psi(0,\boldsymbol{x}=0)|a_1^\dagger\left[\left(\omega - \frac{(\epsilon_1+\epsilon_2)}{A} + H\right) - \frac{(A+1)}{A}\epsilon_1 + \frac{(\epsilon_1+\epsilon_2)}{A} - H\right]\frac{1}{\omega - \frac{(\epsilon_1+\epsilon_2)}{A} + H} J_2|-\boldsymbol{k}_1,\psi_0\rangle\rangle \\
&= \left(\omega - \frac{(A+1)}{A}\epsilon_1\right)\langle\langle \Psi(0,\boldsymbol{x}=0)|a_1^\dagger a_2|-\boldsymbol{k}_1,\psi_0\rangle\rangle + \langle\langle \Psi(0,\boldsymbol{x}=0)|a_1^\dagger J_2|-\boldsymbol{k}_1,\psi_0\rangle\rangle \\
&\quad + \langle\langle \Psi(0,\boldsymbol{x}=0)|\left(-\frac{(A+1)}{A}\epsilon_1 a_1^\dagger + \frac{(\epsilon_1+\epsilon_2)}{A} a_1^\dagger - Ha_1^\dagger + \epsilon_1 a_1^\dagger + J_1^\dagger\right)\frac{1}{\omega - \frac{(\epsilon_1+\epsilon_2)}{A} + H} J_2|-\boldsymbol{k}_1,\psi_0\rangle\rangle \\
&= \left(\omega - \frac{(A+1)}{A}\epsilon_1\right)\langle\langle \Psi(0,\boldsymbol{x}=0)|a_1^\dagger a_2|-\boldsymbol{k}_1,\psi_0\rangle\rangle + \langle\langle \Psi(0,\boldsymbol{x}=0)|a_1^\dagger J_2|-\boldsymbol{k}_1,\psi_0\rangle\rangle \\
&\quad + \langle\langle \Psi(0,\boldsymbol{x}=0)|J_1^\dagger \frac{1}{\omega - \frac{(\epsilon_1+\epsilon_2)}{A} + H} J_2|-\boldsymbol{k}_1,\psi_0\rangle\rangle,
\end{aligned} \tag{93}$$

where use has been made of

$$H|\Psi(0,\boldsymbol{x}=0)\rangle\rangle = \frac{\hbar^2(\boldsymbol{P})^2}{2Am}|\Psi(0,\boldsymbol{x}=0)\rangle\rangle, \tag{94}$$

together with the fact that inside the matrix element in Eq. (93) the Fock-space momentum operator $\boldsymbol{P}$ has the eigenvalue $-\boldsymbol{k}_2$. Hence,

$$\left(\omega-\frac{(A+1)}{A}\epsilon_1\right)\left(\omega-\frac{(A+1}{A}\epsilon_2\right)G^{(2)}_{0,0}(2,1;\omega)$$
$$=\left(\omega-\frac{(A+1)}{A}\epsilon_1\right)\langle\langle\Psi(0,\boldsymbol{x}=0)|a_1^\dagger a_2|-\boldsymbol{k}_1,\psi_0\rangle\rangle$$
$$+\langle\langle\Psi(0,\boldsymbol{x}=0)|a_1^\dagger J_2|-\boldsymbol{k}_1,\psi_0\rangle\rangle+\langle\langle\Psi(0,\boldsymbol{x}=0)|J_1^\dagger\frac{1}{\omega-\frac{(\epsilon_1+\epsilon_2)}{A}+H}J_2|-\boldsymbol{k}_1,\psi_0\rangle\rangle. \tag{95}$$

Using a very similar analysis, the first term in $G_{0,0}(2,1;\omega)$ gives

$$\left(\omega-\frac{(A+1)}{A}\epsilon_1\right)\left(\omega-\frac{(A+1)}{A}\epsilon_2\right)G^{(1)}_{0,0}(2,1;\omega_1)$$
$$=\left(\omega-\frac{(A+1)}{A}\epsilon_1\right)\langle\langle\Psi(0,\boldsymbol{x}=0)|a_2 a_1^\dagger|-\boldsymbol{k}_1,\psi_0\rangle\rangle$$
$$+\langle\langle\Psi(0,\boldsymbol{x}=0)|J_2 a_1^\dagger|-\boldsymbol{k}_1,\psi_0\rangle\rangle+\Psi(0,\boldsymbol{x}=0)|J_2\frac{1}{\omega-H}J_1^\dagger|-\boldsymbol{k}_1,\psi_0\rangle\rangle, \tag{96}$$

Putting together the results (96) and (95) gives

$$\left(\omega-\frac{(A+1)}{A}\epsilon_1\right)\left(\omega-\frac{(A+1}{A}\epsilon_2\right)G_{0,0}(2,1;\omega)=\left(\omega-\frac{(A+1)}{A}\epsilon_1\right)\delta(\boldsymbol{k}_2-\boldsymbol{k}_1)+\hat{\mathcal{T}}'_{0,0}(\omega;2,1), \tag{97}$$

where $\hat{\mathcal{T}}'_{0,0}$ is defined by

$$\hat{\mathcal{T}}'_{0,0}(\omega;2,1)=\langle\langle\Psi(0,\boldsymbol{x}=0)|\{J_2,a_1^\dagger\}|-\boldsymbol{k}',\psi_0\rangle\rangle+\Psi(0,\boldsymbol{x}=0)\left|J_2\frac{1}{\omega-H}J_1^\dagger|-\boldsymbol{k}_1,\psi_0\rangle\right\rangle$$
$$+\langle\langle\Psi(0,\boldsymbol{x}=0)\left|J_1^\dagger\frac{1}{\omega-\frac{(\epsilon_1+\epsilon_2)}{A}+H}J_2|-\boldsymbol{k}_1,\psi_0\rangle\right\rangle. \tag{98}$$

This differs from the off-shell transition matrix defined by Eq. (87), which in the present notation reads

$$\hat{\mathcal{T}}_{0,0}(\omega;\boldsymbol{k},\boldsymbol{k}')=\langle\langle\Psi(0,\boldsymbol{x}=0)|\{J_2,a_1^\dagger\}|-\boldsymbol{k}_1,\psi_0\rangle\rangle+\langle\langle\Psi(0,\boldsymbol{x}=0)|J_2\frac{1}{(\omega-H)}J_1^\dagger|-\boldsymbol{k}_1,\psi_0\rangle\rangle$$
$$+\langle\langle\Psi(0,\boldsymbol{x}=0)|J_1^\dagger\frac{1}{(\epsilon_1-\frac{\epsilon_2}{A}+H)}J_2|-\boldsymbol{k}_1,\psi_0\rangle\rangle. \tag{99}$$

Note the different denominators in the third terms on the right in Eqs. (98) and (99). However, fully on shell, when $\epsilon_1=\epsilon_2$ and $\omega=\frac{(A+1)}{A}\epsilon_1+\imath\epsilon$,

$$\hat{\mathcal{T}}'_{0,0}\left(\frac{(A+1)}{A}\epsilon_1+\imath\epsilon;\boldsymbol{k},\boldsymbol{k}'\right)=\hat{\mathcal{T}}_{0,0}\left(\frac{(A+1)}{A}\epsilon_1+\imath\epsilon;\boldsymbol{k},\boldsymbol{k}'\right), \tag{100}$$

in the limit $\epsilon\to 0^+$. In addition, in the same limit, Eq. (97) gives

$$\left(\omega-\frac{(A+1)}{A}\epsilon_1\right)^2 G_{0,0}(2,1;\omega)\to\hat{\mathcal{T}}_{0,0}\left(\frac{(A+1)}{A}\epsilon_1+\imath\epsilon;2,1\right), \tag{101}$$

in agreement with the standard relation between $G_{0,0}$ and the on-shell elastic transition matrix given in Ref. [3], Eq. (3.66), p. 321 and Ref. [1], Eq. (66), p. 8.

Equation (97) can be written as a relation between operators in the subspace of the B space with the target in its ground state as

$$\hat{G}_{0,0}(\omega)=\hat{g}_0(\omega)+\hat{g}_0(\omega)\hat{\mathcal{T}}'_{0,0}(\omega)\hat{g}_0(\omega), \tag{102}$$

where g_0 is the Green's function for a free particle of mass $\frac{A}{(A+1)}m$ defined in Eq. (27).

C. The Dyson self-energy and an alternative definition of the optical potential

In Ref. [9], Sec. II Eqs. (5) and (6), the optical potential operator is defined (in the plane-wave basis used here) as the Dyson self-energy Σ' through the relation,

$$\Sigma' = \hat{g}_0^{-1} - \hat{G}_{0,0}^{-1}. \tag{103}$$

In Sec. II C, the optical model operator is defined in Eq. (32) in terms of the transition operator $\hat{\mathcal{T}}_{0,0}$. The connection between the optical potential defined through Eq. (103) and the transition operator $\hat{\mathcal{T}}'_{0,0}$ follows from

$$\hat{g}_0^{-1} - \hat{G}_{0,0}^{-1} = \hat{\mathcal{T}}'_{0,0}\hat{g}_0\hat{G}_{0,0}^{-1}$$
$$\hat{g}_0\hat{G}_{0,0}^{-1} = (1 + \hat{g}_0\hat{\mathcal{T}}'_{0,0})^{-1}. \tag{104}$$

These can be deduced from Eq. (102) and together with the definition (103) give

$$\Sigma' = \hat{\mathcal{T}}'_{0,0}(1 + \hat{g}_0\hat{\mathcal{T}}'_{0,0})^{-1}. \tag{105}$$

This result means Σ' is related to $\hat{\mathcal{T}}'_{0,0}$ by the same formula as the optical model operator defined in Eq. (32) of Sec. II C is related to $\hat{\mathcal{T}}_{0,0}$.

The Dyson self-energy Σ' is defined here using a one-particle Green's function Eq. (88) and a free Green's function g_0, both modified to take into account the requirements of translational invariance. The resulting optical potential differs from the quantity defined in Eq. (32) of Sec. II C because $\hat{\mathcal{T}}'_{0,0} \neq \hat{\mathcal{T}}_{0,0}$ off shell. Note however that when $\omega = \frac{(A+1)}{A}\epsilon_1,\ \epsilon_2$ arbitrary, i.e., half on shell, the two transition matrices are equal and therefore the distorted waves generated by the two optical potentials will be identical according to Eq. (26). Of course, both these optical potentials will differ from the calculations of Ref. [9] because the Green's function that appears in Eq. (103) differs in two ways to those used in Ref. [9]:

(i) The $1/A$ factors that appear in the denominator in the second term in Eq. (88).

(ii) The matrix elements defining $\hat{G}_{0,0}$ in Eq. (88) and used to define an optical potential through Eq. (103) involve a mixed basis with a localized ground state in the bra and a state of definite total momentum in the ket.

Expressions for the time-dependent one-particle Green's function equivalent to the definition (88) can be found in Appendix E.

VI. DISCUSSION AND CONCLUSIONS

It has been shown how a nucleon optical model operator for an A-nucleon target can be consistently defined within a translation invariant, a completely antisymmetrized many-body theory without reference to a mean-field concept. The distorted wave generated by the optical model potential defined in this way satisfies a quasi-one-body scattering equation for a particle with a mass equal to the nucleon-target reduced mass. The distorted wave incorporates other recoil effects exactly within a vector space referred as B space where the configuration space operator $\hat{\boldsymbol{r}}$ can be interpreted as the separation of the incident nucleon and the target center of mass. The basis of the method is the definition of a specific off-shell extension of the many-body transition matrix. This is used to define the optical model operator as the solution of an integral equation in barycentric space.

The particular off-shell extension chosen is shown to satisfy rotational invariance and to have properties under time reversal that agree with standard conventions. It is also shown that, when heavy-particle stripping is ignored, the transition matrix can be expressed entirely in terms of matrix elements in the A-nucleon subspace of a one-body interaction constructed from the nucleon-nucleon interaction with exchange. Similarly, the heavy-particle stripping term can be expressed in terms of matrix elements of the nucleon-nucleon interaction in the $(A-1)$-nucleon subspace.

Because the method is based on the transition matrix, it is straightforward to relate it to standard methods based on the one-nucleon G matrix and the Dyson self-energy. The modifications of the definition of the G matrix necessitated by translational invariance result in an optical potential that differs from the one defined in Sec. II, although the corresponding distorted waves are identical if a translationally invariant transition matrix is used in both cases.

In the method described here, any theory that generates a calculation of the off-shell extension of the many-body elastic transition matrix defined in Eqs. (20) or (23) leads to a corresponding optical model operator through Eq. (31). Knowledge of the off-shell elastic transition matrix alone is sufficient to calculate the optical model distorted wave through Eqs. (24) or (26), and there is then no need to make the final step to calculate the optical potential. However, an important application of the nucleon optical model concept is to few-body theories of composite particle reactions, e.g., the $A(d,p)B$ reaction, as a tool for nuclear structure studies. For recent reviews for theoretical and experimental work, see Refs. [10,11] and references therein. For these developments, knowledge of the nonlocal nucleon optical model operator itself is essential.

ACKNOWLEDGMENT

Support from the UK Science and Technology Facilities Council through Grant No. STFC ST/000051/1 is acknowledged.

APPENDIX A: SYMMETRY PROPERTIES OF THE OPTICAL POTENTIAL OPERATOR

In understanding the symmetry properties of $V^{\rm opt}$, it is important to distinguish between symmetry transformations in B space and the corresponding transformations in Fock space. To this end, it is convenient to define the B-space operator $\hat{\mathcal{T}}_{0,0}(E+\imath\epsilon)$ with matrix elements $\hat{\mathcal{T}}_{0,0}(E+\imath\epsilon;\boldsymbol{k},\boldsymbol{k}')$ through the relation,

$$\hat{\mathcal{T}}_{0,0}(E+\imath\epsilon) = \int d\boldsymbol{k}\int d\boldsymbol{k}'|\boldsymbol{k}\rangle\langle\boldsymbol{k}'| \times \hat{\mathcal{T}}_{0,0}(E+\imath\epsilon;\boldsymbol{k},\boldsymbol{k}'). \tag{A1}$$

As already noted, operators in B space are indicated with a hat, unless other notation make this unnecessary.

1. Rotation invariance

Consider the transform under rotation of Eq. (A1) by the unitary operator for an arbitrary rotation $\hat{\mathcal{R}}$,

$$\hat{\mathcal{R}}\hat{\mathcal{T}}_{0,0}(E+\imath\epsilon)\hat{\mathcal{R}}^{-1} = \int d\boldsymbol{k}\int d\boldsymbol{k}'\hat{\mathcal{R}}|\boldsymbol{k}\rangle\langle\boldsymbol{k}'|\hat{\mathcal{R}}^{-1}\hat{\mathcal{T}}_{0,0}(E+\imath\epsilon;\boldsymbol{k},\boldsymbol{k}')$$
$$= \int d\boldsymbol{k}\int d\boldsymbol{k}'|\mathcal{R}\boldsymbol{k}\rangle\langle\mathcal{R}\boldsymbol{k}'|\hat{\mathcal{T}}_{0,0}(E+\imath\epsilon;\boldsymbol{k},\boldsymbol{k}'). \tag{A2}$$

Changing the variables of integration to $\boldsymbol{k}''=\mathcal{R}\boldsymbol{k}$ and $\boldsymbol{k}'''=\mathcal{R}\boldsymbol{k}'$ gives

$$\hat{\mathcal{R}}\hat{\mathcal{T}}_{0,0}(E+\imath\epsilon)\hat{\mathcal{R}}^{-1} = \int d\boldsymbol{k}''\int d\boldsymbol{k}'''|\boldsymbol{k}''\rangle\langle\boldsymbol{k}'''|\times\hat{\mathcal{T}}_{0,0}(E+\imath\epsilon;\mathcal{R}^{-1}\boldsymbol{k}'',\mathcal{R}^{-1}\boldsymbol{k}'''). \tag{A3}$$

The contribution from the second term on the right of Eq. (23) to $\hat{\mathcal{T}}_{0,0}(E+\imath\epsilon;\mathcal{R}^{-1}\boldsymbol{k}'',\mathcal{R}^{-1}\boldsymbol{k}''')$ is

$$\hat{\mathcal{T}}_{0,0}(E+\imath\epsilon;\mathcal{R}^{-1}\boldsymbol{k}'',\mathcal{R}^{-1}\boldsymbol{k}''')_{(2)} = \langle\langle\Psi(0,\boldsymbol{x}=0)|[a_{\mathcal{R}^{-1}\boldsymbol{k}''},V]\frac{1}{(E+\imath\epsilon-H)}[V,a^{\dagger}_{\mathcal{R}^{-1}\boldsymbol{k}'''}]|-\mathcal{R}^{-1}\boldsymbol{k}''',\psi_0\rangle\rangle$$
$$= \langle\langle\Psi(0,\boldsymbol{x}=0)|\mathcal{R}^{-1}[a_{\boldsymbol{k}''},V]\frac{1}{(E+\imath\epsilon-H)}[V,a^{\dagger}_{\boldsymbol{k}'''}]\mathcal{R}|-\mathcal{R}^{-1}\boldsymbol{k}''',\psi_0\rangle\rangle$$
$$= \langle\langle\Psi(0,\boldsymbol{x}=0)|[a_{\boldsymbol{k}''},V]\frac{1}{(E+\imath\epsilon-H)}[V,a^{\dagger}_{\boldsymbol{k}'''}]|-\boldsymbol{k}'''\psi_0\rangle\rangle, \tag{A4}$$

where now $\mathcal{R}$ means the rotation operator in Folk space corresponding to $\hat{\mathcal{R}}$ and it is assumed H and V are invariant under rotations. For simplicity, the ground-state ψ_0 has been taken to have zero spin so that

$$\mathcal{R}|\Psi(0,\boldsymbol{x}=0)\rangle\rangle = |\Psi(0,\boldsymbol{x})=0\rangle\rangle. \tag{A5}$$

The other term in Eq. (23) transforms in the same way, and Eq. (A2) can be written

$$\hat{\mathcal{R}}\hat{\mathcal{T}}_{0,0}(E+\imath\epsilon)\hat{\mathcal{R}}^{-1} = \hat{\mathcal{T}}_{0,0}(E+\imath\epsilon). \tag{A6}$$

It follows from Eqs. (A6) and (32) that V^{opt} has the analogous property,

$$\hat{\mathcal{R}}\hat{V}^{\text{opt}}\hat{\mathcal{R}}^{-1} = \hat{V}^{\text{opt}}. \tag{A7}$$

The rotational invariance of the elastic-scattering T-matrix $\langle\boldsymbol{k}_0',0|T(E)|\boldsymbol{k}_0,0\rangle$ appearing in Eq. (30) also follows from Eq. (A6):

$$\langle(\mathcal{R}\boldsymbol{k}_0'),\psi_0|T(E)|(\mathcal{R}\boldsymbol{k}_0),\psi_0\rangle = \langle\boldsymbol{k}_0',\psi_0|T(E)|\boldsymbol{k}_0,\psi_0\rangle. \tag{A8}$$

2. Time-reversal properties of $\hat{\mathcal{T}}_{0,0}$

In applications, it is convenient to work with operators that behave in a specific way under transformation by the antiunitary time-reversal operator $\mathcal{K}$. The conventional transformation property for transition operators in B space is

$$\hat{\mathcal{K}}\hat{\mathcal{T}}_{0,0}(E+\imath\epsilon)\hat{\mathcal{K}}^{-1} = [\hat{\mathcal{T}}_{0,0}(E+\imath\epsilon)]^{\dagger}. \tag{A9}$$

The proof of the result (A9) starts from the representation given in Eq. (A1). Consider the second term on the right of Eq. (23) as an example,

$$[\hat{\mathcal{T}}_{0,0}(E+\imath\epsilon)]^{\dagger}_{(2)} = \int d\boldsymbol{k}\int d\boldsymbol{k}'|\boldsymbol{k}'\rangle\langle\boldsymbol{k}|\left(\langle\langle\Psi(0,\boldsymbol{x}=0)|[a_{\boldsymbol{k}},V]\frac{1}{(E+\imath\epsilon-H)}[V,a^{\dagger}_{\boldsymbol{k}'}]|-\boldsymbol{k}',\psi_0\rangle\rangle\right)^{*}$$
$$= \int d\boldsymbol{k}\int d\boldsymbol{k}'|\boldsymbol{k}'\rangle\langle\boldsymbol{k}|\langle\langle-\boldsymbol{k}',\psi_0|\left([a_{\boldsymbol{k}},V]\frac{1}{(E+\imath\epsilon-H)}[V,a^{\dagger}_{\boldsymbol{k}'}]\right)^{\dagger}|\Psi(0,\boldsymbol{x}=0)\rangle\rangle$$
$$= \int d\boldsymbol{k}\int d\boldsymbol{k}'|\boldsymbol{k}\rangle\langle\boldsymbol{k}'|\langle\langle-\boldsymbol{k},\psi_0|\left([a_{\boldsymbol{k}},V]\frac{1}{(E-\imath\epsilon-H)}[V,a^{\dagger}_{\boldsymbol{k}'}]\right)|\Psi(0,\boldsymbol{x}=0)\rangle\rangle, \tag{A10}$$

where in the last line the integration or summation variables $\boldsymbol{k},\boldsymbol{k}'$ have been interchanged.

On the other hand, the effect of an antilinear time-reversal transform operator on this term is

$$\hat{\mathcal{K}}\hat{\mathcal{T}}^{\epsilon}_{0,0}(E+\imath\epsilon)_{(2)}\hat{\mathcal{K}}^{-1} = \int d\boldsymbol{k}\int d\boldsymbol{k}'|-\boldsymbol{k}\rangle\langle-\boldsymbol{k}'|\left(\langle\langle\Psi(0,\boldsymbol{x}=0)|[a_{\boldsymbol{k}},V]\frac{1}{(E+\imath\epsilon-H)}[V,a^{\dagger}_{\boldsymbol{k}'}]|-\boldsymbol{k}',\psi_0\rangle\rangle\right)^{*}, \tag{A11}$$

where the defining property of the time-reversal operator $\hat{\mathcal{K}}|\boldsymbol{k}\rangle = |-\boldsymbol{k}\rangle$ has been used.

Changing the variables of integration or summation from $-\boldsymbol{k}$ to $\boldsymbol{k}$, $-\boldsymbol{k}'$ to $\boldsymbol{k}'$, and using $\mathcal{K}^{-1}a_{\boldsymbol{k}}\mathcal{K} = a_{-\boldsymbol{k}}$ gives

$$\hat{\mathcal{K}}\hat{\mathcal{T}}^{\epsilon}_{0,0}(E+\imath\epsilon)_{(2)}\hat{\mathcal{K}}^{-1} = \int d\boldsymbol{k}\int d\boldsymbol{k}'|\boldsymbol{k}\rangle\langle\boldsymbol{k}'|\left(\langle\langle\Psi(0,\boldsymbol{x}=0)|[a_{-\boldsymbol{k}},V]\frac{1}{(E+\imath\epsilon-H)}[V,a^{\dagger}_{-\boldsymbol{k}'}]|\boldsymbol{k}',\psi_0\rangle\rangle\right)^{*}$$
$$= \int d\boldsymbol{k}\int d\boldsymbol{k}'|\boldsymbol{k}\rangle\langle\boldsymbol{k}'|\left[\langle\langle\Psi(0,\boldsymbol{x}=0)|\left(\hat{\mathcal{K}}^{-1}[a_{\boldsymbol{k}},V]\frac{1}{(E-\imath\epsilon-H)}[V,a^{\dagger}_{\boldsymbol{k}'}]\hat{\mathcal{K}}\right)|\boldsymbol{k}',\psi_0\rangle\rangle\right]^{*}. \quad (A12)$$

where $\mathcal{K}^{-1}\psi(\boldsymbol{r})\mathcal{K} = \psi(\mathcal{K}^{-1}\boldsymbol{r})$ has been used.

A general property of the matrix elements of an arbitrary linear operator O and its time reverse transform $\mathcal{K}^{-1}O\mathcal{K}$ are

$$\langle\langle a|O|b\rangle\rangle^{*} = \langle\langle a'|(\mathcal{K}^{-1}O\mathcal{K})|b'\rangle\rangle, \quad (A13)$$

where

$$|a'\rangle\rangle = \mathcal{K}^{-1}|a\rangle\rangle, \quad |b'\rangle\rangle = \mathcal{K}^{-1}|b\rangle\rangle. \quad (A14)$$

Applying this to the Fock-space matrix element in Eq. (A12) gives

$$\langle\langle\Psi(0,\boldsymbol{x}=0)|\left(\hat{\mathcal{K}}^{-1}[a_{\boldsymbol{k}},V]\frac{1}{(E-\imath\epsilon-H)}[V,a^{\dagger}_{\boldsymbol{k}'}]\hat{\mathcal{K}}\right)|\boldsymbol{k}',\psi_0\rangle\rangle)^{*}$$
$$= \langle\langle\Psi(0,\boldsymbol{x}=0)|[a_{\boldsymbol{k}},V]\frac{1}{(E-\imath\epsilon-H)}[V,a^{\dagger}_{\boldsymbol{k}'}]|-\boldsymbol{k}',\psi_0\rangle\rangle. \quad (A15)$$

The target ground state has been taken to have zero spin for simplicity. Arbitrary phases can then be chosen so that the ground-state wave-function ψ_0 is unchanged under the action of $\mathcal{K}$.

Applying the result (A15), Eq. (A12) becomes

$$\hat{\mathcal{K}}\hat{\mathcal{T}}^{\epsilon}_{0,0}(E+\imath\epsilon)_{(2)}\hat{\mathcal{K}}^{-1} = \int d\boldsymbol{k}\int d\boldsymbol{k}'|\boldsymbol{k}\rangle\langle\boldsymbol{k}'|\langle\langle\Psi(0,\boldsymbol{x}=0)|[a_{\boldsymbol{k}},V]\frac{1}{(E-\imath\epsilon-H)}[V,a^{\dagger}_{\boldsymbol{k}'}]|-\boldsymbol{k}',\psi_0\rangle\rangle. \quad (A16)$$

Using the techniques set out in Appendix A 4 it follows that:

$$\langle\langle\Psi(0,\boldsymbol{x}=0)|[a_{\boldsymbol{k}},V]\frac{1}{(E-\imath\epsilon-H)}[V,a^{\dagger}_{\boldsymbol{k}'}]|-\boldsymbol{k}',\psi_0\rangle\rangle = \langle\langle-\boldsymbol{k},\psi_0|[a_{\boldsymbol{k}},V]\frac{1}{(E-\imath\epsilon-H)}[V,a^{\dagger}_{\boldsymbol{k}'}]|\Psi(0,\boldsymbol{x}=0)\rangle\rangle, \quad (A17)$$

and hence by comparison with Eq. (A10),

$$\hat{\mathcal{K}}\hat{\mathcal{T}}^{\epsilon}_{0,0}(E+\imath\epsilon)_{(2)}\hat{\mathcal{K}}^{-1} = [\hat{\mathcal{T}}_{0,0}(E+\imath\epsilon)]^{\dagger}_{(2)}. \quad (A18)$$

The proof of the analogous result for the first term of Eq. (23) uses similar techniques. The final result is

$$\hat{\mathcal{K}}\hat{\mathcal{T}}^{\epsilon}_{0,0}(E+\imath\epsilon)\hat{\mathcal{K}}^{-1} = [\hat{\mathcal{T}}_{0,0}(E+\imath\epsilon)]^{\dagger}. \quad (A19)$$

It follows from the definition of $\hat{V}^{\text{opt}}(E+\imath\epsilon)$ in terms of $\hat{\mathcal{T}}^{\epsilon}_{0,0}(E+\imath\epsilon)$ given in Eq. (31) that

$$\hat{\mathcal{K}}\hat{V}^{\text{opt}}(E+\imath\epsilon)\hat{\mathcal{K}}^{-1} = [\hat{V}^{\text{opt}}(E+\imath\epsilon)]^{\dagger}. \quad (A20)$$

The implications of these results for the properties of distorted waves associated with $\hat{V}^{\text{opt}}(E+\imath\epsilon)$ are described in the next subsection.

It should be noted that, in the interest of conciseness important phases associated with the effect of the time-reversal operator on spin, eigenstates have been ignored in the derivation presented in this Appendix.

The implications of these results for the properties of distorted waves associated with $\hat{V}^{\text{opt}}(E+\imath\epsilon)$ are described in the next subsection.

3. The scattering states $|\xi^{(\pm)}_{E,\boldsymbol{k}_0}\rangle$ and time-reversal requirements

In applications of the optical model to reaction models, two different scattering states, $|\xi^{(\pm)}_{E,\boldsymbol{k}_0}\rangle$, associated with $\hat{V}^{\text{opt}}$ are used. These are defined as the limit $\epsilon \to 0^{+}$ of $|\xi^{\pm\epsilon}_{E,\boldsymbol{k}_0}\rangle$ where

$$[E+\imath\epsilon-\hat{T}-\hat{V}^{\text{opt}}(E+\imath\epsilon)]\big|\xi^{+\epsilon}_{E,\boldsymbol{k}_0}\big\rangle = \imath\epsilon(2\pi)^{3/2}|\boldsymbol{k}_0\rangle,$$
$$\{E-\imath\epsilon-\hat{T}-[\hat{V}^{\text{opt}}(E+\imath\epsilon)]^{\dagger}\}\big|\xi^{-\epsilon}_{E,\boldsymbol{k}_0}\big\rangle = -\imath\epsilon(2\pi)^{3/2}|\boldsymbol{k}_0\rangle. \quad (A21)$$

The behavior of the operators $\hat{\mathcal{T}}_{0,0}$ and $\hat{V}^{\text{opt,}}_{\text{LSZ}}$ under the time-reversal transformation operator $\hat{\mathcal{K}}$ is discussed in Appendix A 2. Because (A20) is satisfied, the two solutions $|\xi^{\pm\epsilon}_{E,\boldsymbol{k}_0}$ are related by

$$\hat{\mathcal{K}}\big|\xi^{(-)}_{E,\boldsymbol{k}_0}\big\rangle = \big|\xi^{(+)}_{E,-\boldsymbol{k}_0}\big\rangle. \quad (A22)$$

Note also that

$$[\hat{V}^{\text{opt}}(E+\imath\epsilon)]^\dagger = \hat{V}^{\text{opt}}(E-\imath\epsilon), \quad [\hat{\mathcal{T}}_{0,0}(E+\imath\epsilon)]^\dagger = \hat{\mathcal{T}}_{0,0}(E-\imath\epsilon). \quad \hat{\mathcal{K}}\big|\xi^{-\epsilon}_{E,\boldsymbol{k}_0}\big\rangle = \big|\xi^{+\epsilon}_{E,-\boldsymbol{k}_0}\big\rangle. \tag{A23}$$

Another pair of solutions $|\tilde{\xi}^{(\pm)}_{E,\boldsymbol{k}_0}\rangle$ are also needed when orthogonal sets of distorted waves corresponding to non-Hermitian optical potentials are required. These are defined as the limit $\epsilon \to 0^+$ of $|\tilde{\xi}^{\pm\epsilon}_{E,\boldsymbol{k}_0}\rangle$ where

$$\{E+\imath\epsilon-\hat{T}-[\hat{V}^{\text{opt}}(E+\imath\epsilon)]^\dagger\}\big|\tilde{\xi}^{+\epsilon}_{E,\boldsymbol{k}_0}\big\rangle = \imath\epsilon(2\pi)^{3/2}|\boldsymbol{k}_0\rangle,$$
$$(E-\imath\epsilon-\hat{T}-\hat{V}^{\text{opt}}(E+\imath\epsilon))\big|\tilde{\xi}^{-\epsilon}_{E,\boldsymbol{k}_0}\big\rangle = -\imath\epsilon(2\pi)^{3/2}|\boldsymbol{k}_0\rangle. \tag{A24}$$

The states $|\tilde{\xi}^{(\pm)}_{E,\boldsymbol{k}_0}\rangle$ satisfy

$$\big\langle\tilde{\xi}^{(+)}_{E,\boldsymbol{k}_0}\big|\xi^{(+)}_{E,\boldsymbol{k}_0}\big\rangle = (2\pi)^3\delta(\boldsymbol{k}'_0-\boldsymbol{k}_0), \quad \big\langle\tilde{\xi}^{(-)}_{E,\boldsymbol{k}_0}\big|\xi^{(-)}_{E,\boldsymbol{k}_0}\big\rangle = (2\pi)^3\delta(\boldsymbol{k}'_0-\boldsymbol{k}_0). \tag{A25}$$

4. Matrix elements of a class of momentum-conserving operators

This Appendix is concerned with operators of the form

$$O_1 a^\dagger_{\boldsymbol{k}_1} O_2 a_{\boldsymbol{k}_2} O_3, \tag{A26}$$

where the O_i are arbitrary momentum-conserving operators in Fock space. The operators (A26) have a simple form in the basis $|-\boldsymbol{k}, \psi_n\rangle\rangle$ in which the A-nucleon intrinsic state n has a total momentum $-\boldsymbol{k}$. These states are normalized so that

$$\langle\langle -\boldsymbol{k}', \psi_{n'}|-\boldsymbol{k}, \psi_n\rangle\rangle = (2\pi)^3\delta_{n',n}\delta(\boldsymbol{k}'-\boldsymbol{k}), \tag{A27}$$

and they are related to states $|\Psi(n,\boldsymbol{x})\rangle\rangle$ in which the c.m. is located at $\boldsymbol{x}$ and defined in Ref. [2] by

$$|-\boldsymbol{k}, \psi_n\rangle\rangle = \int d\boldsymbol{x}\, \exp(-\imath\boldsymbol{k}\cdot\boldsymbol{x})|\Psi(n,\boldsymbol{x})\rangle\rangle. \tag{A28}$$

In this basis,

$$\langle\langle -\boldsymbol{k}, \psi_n|O_1 a^\dagger_{\boldsymbol{k}_1} O_2 a_{\boldsymbol{k}_2} O_3|-\boldsymbol{k}', \psi_{n'}\rangle\rangle = \int d\boldsymbol{x}\, \exp(\imath\boldsymbol{k}\cdot\boldsymbol{x})\langle\langle\Psi(n,\boldsymbol{x})|O_1 a^\dagger_{\boldsymbol{k}} O_2 a_{\boldsymbol{k}'} O_3|-\boldsymbol{k}', \psi_{n'}\rangle\rangle$$
$$= \int d\boldsymbol{x}\, \exp(\imath\boldsymbol{k}\cdot\boldsymbol{x})\langle\langle\Psi(n,\boldsymbol{x}=0)|\exp(\imath\boldsymbol{P}\cdot\boldsymbol{x})O_1 a^\dagger_{\boldsymbol{k}_1} O_2 a_{\boldsymbol{k}_2} O_3|-\boldsymbol{k}', \psi_{n'}\rangle\rangle, \tag{A29}$$

where $\boldsymbol{P}$ is the momentum operator in Fock space and

$$|\Psi(n,\boldsymbol{x})\rangle\rangle = \exp(-\imath\boldsymbol{P}\cdot\boldsymbol{x})|\Psi(n,\boldsymbol{x}=0)\rangle\rangle \tag{A30}$$

has been used.

If the O_i's are translationally invariant, the state appearing to the right of $\boldsymbol{P}$ in Eq. (A29) will have momentum $\boldsymbol{k}_1-\boldsymbol{k}_2-\boldsymbol{k}'$. Integrating over $\boldsymbol{x}$, Eq. (A29) reduces to

$$\langle\langle -\boldsymbol{k}, \psi_n|O_1 a^\dagger_{\boldsymbol{k}_1} O_2 a_{\boldsymbol{k}_2} O_3|-\boldsymbol{k}', \psi_{n'}\rangle\rangle = (2\pi)^3\delta(\boldsymbol{k}+\boldsymbol{k}_1-\boldsymbol{k}'-\boldsymbol{k}_2)\langle\langle\Psi(n,\boldsymbol{x}=0)|O_1 a^\dagger_{\boldsymbol{k}_1} O_2 a_{\boldsymbol{k}_2} O_3|-\boldsymbol{k}', \psi_{n'}\rangle\rangle. \tag{A31}$$

This result gives the matrix element of the special operators (A26) between nonlocalized states in terms of a momentum-conserving δ function and a matrix element involving a localized state in the bra.

APPENDIX B: DERIVATION OF THE GREEN'S FUNCTION PROPERTY, EQ. (62)

This Appendix concerns a property of the set of states $G(E^+)a^\dagger_{\boldsymbol{k}}|-\boldsymbol{k}, \psi_n\rangle\rangle$ with E fixed, $\boldsymbol{k}$, n, arbitrary, and $E^+ = E+\imath\epsilon$. Using the identity Eq. (13),

$$G(E^+)a^\dagger_{\boldsymbol{k}}|-\boldsymbol{k}, \psi_n\rangle\rangle = a^\dagger_{\boldsymbol{k}}G(E^+-\epsilon_k)|-\boldsymbol{k}, \psi_n\rangle\rangle + G(E^+)[V, a^\dagger_{\boldsymbol{k}}]_- G(E^+-\epsilon_k)|-\boldsymbol{k}, \psi_n\rangle\rangle$$
$$= \frac{1}{\left(E^+ - \frac{(A+1)}{A}\epsilon_k - E_n\right)}(a^\dagger_{\boldsymbol{k}}|-\boldsymbol{k}, \psi_n\rangle\rangle + G(E^+)[V, a^\dagger_{\boldsymbol{k}}]_-|-\boldsymbol{k}, \psi_n\rangle\rangle)$$
$$= \frac{1}{\left(E^+ - \frac{(A+1)}{A}\epsilon_k - E_n\right)}\left(a^\dagger_{\boldsymbol{k}}|-\boldsymbol{k}, \psi_n\rangle\rangle + \frac{1}{2}\int d\boldsymbol{k}_3 G(E^+)a^\dagger_{\boldsymbol{k}_3}\mathcal{V}(\boldsymbol{k}_3,\boldsymbol{k})|-\boldsymbol{k}, \psi_n\rangle\rangle\right), \tag{B1}$$

where Eq. (49) has been used to express the commutator $[V, a^\dagger_{\boldsymbol{k}}]_-$ in terms of the nucleon-number-conserving interaction $\mathcal{V}$ defined in Eq. (46). Introducing the complete set of A-nucleon states $|-\boldsymbol{k}', \psi_{n'}\rangle\rangle$ and using the result (A31) of Appendix A 4 gives

$$G(E^+)a^\dagger_{\boldsymbol{k}}|-\boldsymbol{k},\psi_n\rangle\rangle = \frac{1}{\left(E^+ - \frac{(A+1)}{A}\epsilon_k - E_n\right)}(a^\dagger_{\boldsymbol{k}}|-\boldsymbol{k},\psi_n\rangle\rangle + \int d\boldsymbol{k}'\sum_{n'} G(E^+)a^\dagger_{\boldsymbol{k}'}|-\boldsymbol{k}',\psi_{n'}\rangle\rangle\frac{1}{2}\langle\langle\Psi(n',\boldsymbol{x}=0)|\mathcal{V}(\boldsymbol{k}',\boldsymbol{k})|-\boldsymbol{k},\psi_n\rangle\rangle). \tag{B2}$$

Defining the matrix,

$$\mathcal{V}_{\boldsymbol{k}',n';\boldsymbol{k},n} = \tfrac{1}{2}\langle\langle\Psi(n',\boldsymbol{x}=0)|\mathcal{V}(\boldsymbol{k}',\boldsymbol{k})|-\boldsymbol{k},\psi_n\rangle\rangle, \tag{B3}$$

Eq. (B2) can be written

$$\left(E^+ - \frac{(A+1)}{A}\epsilon_k - E_n\right)G(E^+)a^\dagger_{\boldsymbol{k}}|-\boldsymbol{k},\psi_n\rangle\rangle = a^\dagger_{\boldsymbol{k}}|-\boldsymbol{k},\psi_n\rangle\rangle + \sum_{n'}\int d\boldsymbol{k}'\, G(E^+)a^\dagger_{\boldsymbol{k}'}|-\boldsymbol{k}',\psi_{n'}\rangle\rangle\mathcal{V}_{\boldsymbol{k}',n';\boldsymbol{k},n}. \tag{B4}$$

This equation can be regarded as an in-homogenous set of couple equations for the ket vectors $G(E^+)a^\dagger_{\boldsymbol{k}}|-\boldsymbol{k},\psi_n\rangle\rangle$ and can be rewritten as

$$\int d\boldsymbol{k}'\sum_{n'} G(E^+)a^\dagger_{\boldsymbol{k}'}|-\boldsymbol{k}',\psi_{n'}\rangle\rangle\left[\left(E^+ - \frac{(A+1)}{A}\epsilon_k - E_n\right)\delta(\boldsymbol{k}'-\boldsymbol{k})\delta_{n,n'} - \mathcal{V}_{\boldsymbol{k}',n';\boldsymbol{k},n}\right] = a^\dagger_{\boldsymbol{k}}|-\boldsymbol{k},\psi_n\rangle\rangle. \tag{B5}$$

A convenient way of expressing the solution of these equations in terms of the inverse of the matrix,

$$\left(E^+ - \frac{(A+1)}{A}\epsilon_k - E_n\right)\delta(\boldsymbol{k}'-\boldsymbol{k})\delta_{n,n'} - \mathcal{V}_{\boldsymbol{k}',n';\boldsymbol{k},n} \tag{B6}$$

is discussed in Sec. II F following Eq. (58).

APPENDIX C: THE IDENTITY EQ. (70)

$$\begin{aligned}\langle\langle\Psi(0,\boldsymbol{x}=0)|a^\dagger_{\boldsymbol{k}'}H &= \langle\langle\Psi(0,\boldsymbol{x}=0)|(Ha^\dagger_{\boldsymbol{k}'} - [H,a^\dagger_{\boldsymbol{k}'}])\\ &= \langle\langle\Psi(0,\boldsymbol{x}=0)|(Ha^\dagger_{\boldsymbol{k}'} - \epsilon_{k'}a^\dagger_{\boldsymbol{k}'} - [V,a^\dagger_{\boldsymbol{k}'}])\\ &= \langle\langle\Psi(0,\boldsymbol{x}=0)|\left[\left(\frac{\boldsymbol{P}^2}{2Am}+E_0\right)a^\dagger_{\boldsymbol{k}'} - \epsilon_{k'}a^\dagger_{\boldsymbol{k}'} - [V,a^\dagger_{\boldsymbol{k}'}]\right],\end{aligned} \tag{C1}$$

where the fact that $|\Psi(0,\boldsymbol{x}=0)\rangle\rangle$ is an eigenfunction with eigenvalue E_0 of the A-nucleon intrinsic Hamiltonian $H - \frac{\boldsymbol{P}^2}{2Am}$ has been used in the second line. Within the matrix element in Eq. (70) $\boldsymbol{P}$ has the eigenvalue $-\boldsymbol{k}$ and Eq. (C1) gives

$$\langle\langle\Psi(0,\boldsymbol{x}=0)|a^\dagger_{\boldsymbol{k}'} = \langle\langle\Psi(0,\boldsymbol{x}=0)|[V,a^\dagger_{\boldsymbol{k}'}]\frac{1}{\left(\frac{\epsilon_k}{A} - \epsilon_{k'} + E_0 - H\right)}. \tag{C2}$$

For $A = 1$, the bra $\langle\langle\Psi(0,\boldsymbol{x}=0)|a^\dagger_{\boldsymbol{k}'}\hat{H}$ vanishes, and the identity (C1) is not useful.

APPENDIX D: FESHBACH THEORY OF THE OPTICAL POTENTIAL APPLIED TO *B*-SPACE TRANSITION OPERATOR $\hat{\mathcal{T}}$

$\hat{\mathcal{T}}$ is the operator in B space defined for a general $\hat{V}$ by

$$\begin{aligned}\hat{\mathcal{T}}(E+\imath\epsilon) &= \hat{V} + \hat{V}\frac{1}{(E+\imath\epsilon-\hat{T}-\hat{h}_A-\hat{V})}\hat{V}\\ &= \hat{V} + \hat{V}\frac{1}{(E+\imath\epsilon-\hat{T}-\hat{h}_A)}\hat{\mathcal{T}}(E+\imath\epsilon).\end{aligned} \tag{D1}$$

The objective of the following algebra is to give uncoupled equations for the matrix elements $P_0\hat{\mathcal{T}}P_0$ and $Q_0\hat{\mathcal{T}}P_0$ where P_0 projects onto states $|\boldsymbol{k},0\rangle$ in B space in which the target is in its ground-state $n = 0$ and Q_0 projects on the orthogonal subspace of states with $n \neq 0$. For this purpose, it is convenient to introduce the operator,

$$\hat{\Omega}(E+\imath\epsilon) = 1 + \frac{1}{(E+\imath\epsilon-\hat{T}-\hat{h}_A-\hat{V})}\hat{V}, \tag{D2}$$

so that Eq. (D1) can be written

$$\hat{\mathcal{T}}(E+\imath\epsilon)=\hat{V}\hat{\Omega}(E+\imath\epsilon). \tag{D3}$$

Using the properties of P_0 and Q_0 gives (with $E^+=E+\imath\epsilon$)

$$(E^+-E_0-\hat{T}-P_0\hat{V}P_0)P_0\hat{\Omega}P_0=(E^+-E_0-\hat{T})P_0+P_0\hat{V}Q_0\hat{\Omega}P_0, \tag{D4}$$

and

$$(E^+-\hat{T}-\hat{h}_A-Q_0\hat{V}Q_0)Q_0\hat{\Omega}P_0=Q_0\hat{V}P_0\hat{\Omega}P_0. \tag{D5}$$

Using Eq. (D5) to give a formula for $Q_0\hat{\Omega}P_0$ and inserting this into Eqs. (D4) and (D3) gives

$$(E^+-E_0-\hat{T}-P_0\hat{U}_{\rm opt}P_0)P_0\hat{\Omega}P_0=(E^+-E_0-\hat{T})P_0, \tag{D6}$$

and

$$P_0\hat{\mathcal{T}}P_0=P_0\hat{U}_{\rm opt}P_0+P_0\hat{U}_{\rm opt}P_0\frac{1}{(E^+-E_0-\hat{T}-P_0\hat{U}_{\rm opt}P_0)}P_0\hat{U}_{\rm opt}P_0, \tag{D7}$$

where

$$\hat{U}_{\rm opt}=\hat{V}+\hat{U}Q_0\frac{1}{(E+\imath\epsilon-\hat{T}-\hat{h}_A-Q_0\hat{V}Q_0)}Q_0\hat{V}. \tag{D8}$$

Note also that according to Eq. (D6),

$$P_0\hat{\Omega}P_0=P_0+\frac{1}{(E^+-E_0-\hat{T}-P_0\hat{U}_{\rm opt}P_0)}P_0\hat{U}_{\rm opt}P_0. \tag{D9}$$

APPENDIX E: TIME-DEPENDENT SINGLE-PARTICLE GREEN'S FUNCTION

When expressed in the time domain, the Green's function relation that is consistent with Eq. (88) is

$$G_{0,0}(\boldsymbol{k},\boldsymbol{k}';\omega)=\frac{1}{\imath}\int_{-\infty}^{+\infty}dt\;\exp(-\imath\omega t)\exp(-\epsilon|t|)\exp\left(\imath\frac{\epsilon_{k'}}{A}t\right)\langle\langle\Psi(0,\boldsymbol{x}=0)|\mathcal{T}\{a^\dagger_{\boldsymbol{k}'}(t),\;a_{\boldsymbol{k}}(t=0)\}|-\boldsymbol{k}',\psi_0\rangle\rangle, \tag{E1}$$

where the time-ordering operator $\mathcal{T}$ for fermions is defined by

$$\begin{aligned}\mathcal{T}\{A(t_2),B(t_1)\}&=A(t_2)B(t_1)\qquad\text{for } t_2>t_1,\\&=-B(t_1)A(t_2)\qquad\text{for } t_1>t_2.\end{aligned} \tag{E2}$$

The Heisenberg operators appearing in Eq. (E1) are defined by

$$a^\dagger_{\boldsymbol{k}}(t)=\exp(\imath Ht)a^\dagger_{\boldsymbol{k}}\exp(-\imath Ht). \tag{E3}$$

[1] W. H. Dickhoff and R. J. Charity, Recent developments for the optical model of nuclei, Progr. Part. Nucl. Phys. **105**, 252 (2019).
[2] R. C. Johnson, Translation invariance and antisymmetry in the theory of the nucleon optical potential, Phys. Rev. C. **95**, 064610 (2017).
[3] F. Villars, Collision theory, in *Fundamentals in Nuclear Theory*, edited by A. de-Shalit and C. Villi, Lectures Presented at an International Course, Trieste, 1966, organized by the International Centre for Theoretical Physics, Trieste (IAEA, Vienna, 1967), Vol. ST1/PUB/145, pp. 260–333.
[4] H. Feshbach, Unified theory of nuclear reactions, Ann. Phys. (NY) **5**, 357 (1958)
[5] E. F. Redish and F. Villars, A perturbation theory of nuclear scattering including recoil, Ann. Phys. (NY) **56**, 355 (1970).
[6] N. Austern, *Direct Nuclear Reaction Theories* (Wiley, New York, 1970).
[7] M. L. Goldberger and K. M. Watson, *Collision Theory* (Wiley, New York, 1964), republished by (Dover, New York, 2004).
[8] Gyo Takeda and K. M. Watson, Scattering of fast neutrons and protons by atomic nuclei, Phys. Rev. **97**, 1336 (1955).
[9] J. Rotureau, P. Danielewicz, G. Hagen, F. M. Nunes, and T. Papenbrock, Optical potential from first principles, Phys. Rev. C **95**, 024315 (2017).
[10] R. C. Johnson, Theory of the $A(d,p)B$ reaction as a tool for nuclear structure studies, J. Phys. G: Nucl. Part. Phys. **41**, 094005 (2014).
[11] K. Wimmer, Nucleon transfer reactions with radioactive beams, J. Phys. G: Nucl. Part. Phys. **45**, 033002 (2018).