Excitation energy dependence of the moments of inertia of well deformed nuclei

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(Received 30 January 2019; published 29 April 2019)

Background: Experimental data on some nuclei indicate a significant increase of the moment of inertia of excited states in comparison with its value in the ground state. It is interesting to investigate the reasons for this effect based on a microscopic nuclear model.

Purpose: To investigate the excitation energy dependence of the excited state moments of inertia of some even-even well deformed nuclei.

Method: The Hamiltonian of the quasiparticle phonon model with a mean field part, monopole pairing, and multipole-multipole residual interaction is used to calculate the moments of inertia of excited states.

Results: The moments of inertia of the γ-vibrational states of several rare earth nuclei and of the 1^+ states of ^{156,158}Gd and ^{160,162,164}Dy are calculated and compared to experimental data.

Conclusion: It is shown that both the blocking effect and the Coriolis coupling between quasiparticles and the rotating core contribute significantly to the increase of the excited states' moments of inertia in comparison to the ground state value. In the case of the 1^+ states a contribution of the Coriolis interaction of quasiparticles and the rotating core can be considerably larger than the blocking effect. This is one reason for a very large value of the moment of inertia of some 1^+ states which can exceed the rigid body value.

DOI: 10.1103/PhysRevC.99.044319

I. INTRODUCTION

The moments of inertia of the ground and excited states of well deformed nuclei are important characteristics of their structure. There is a huge amount of experimental data on them. The first expression for calculations of the nuclear ground state moment of inertia derived using the language of the microscopic nuclear model is known as the Inglis formula [1]. There are other approaches to deriving an expression for the moment of inertia [2–4]. The values of the ground state moments of inertia resulting from these formulas are usually very close to the rigid body value. However, as is well known the experimental moments of inertia for nuclear ground state bands are a factor 2 to 3 smaller than their rigid body values. In [5] and [6] it was indicated that the residual interactions would lower these values. The most important effect comes from pair correlations. The inclusion of this effect into calculations of the ground state moment of inertia has been realized within the BCS formalism in [7] and [8]. This procedure gives the well-known expression for the moment of inertia arising from the admixture of the two-quasiparticle states to the quasiparticle vacuum in a perturbation treatment of a relatively small Coriolis term. The theory agrees satisfactorily with experiment in most cases.

The same approach which has been used in [7] and [8] for calculations of the ground state moments of inertia of the even-even nuclei can be applied to calculations of the moments of inertia of odd-mass nuclei and of excited states of even-even nuclei. In these cases, additional terms appear in the formula for the moment of inertia. The reason of their appearance is the following. When the ground state of an even-even nucleus, which is considered as a quasiparticle vacuum, is perturbed by the Coriolis interaction, the wave function of the ground state acquires an admixture of two-quasiparticle components. In addition to this mechanism in the case of odd-mass nuclei or of excited states of even-even nuclei, the interaction between quasiparticles and the rotational motion of the core can change the quasiparticle states without changing the total number of quasiparticles. This happens because the single particle angular momentum operator written in the quasiparticle representation contains a term which does not change the number of quasiparticles but describes their scattering. The corresponding formula can be found in [9]. For instance, for the moment of inertia of the

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2469-9985/2019/99(4)/044319(6)

two-quasiparticle state $|\rho_1, \rho_2\rangle$ we have

$$\begin{aligned}\Im^{(\rho_1,\rho_2)} = 2\sum_{s_1,s_2} \frac{|\langle s_1|j_x|s_2\rangle|^2\big[u_{s_1}(\rho_1\rho_2)v_{s_2}(\rho_1\rho_2) - u_{s_2}(\rho_1\rho_2)v_{s_1}(\rho_1\rho_2)\big]^2}{\varepsilon_{s_1}(\rho_1\rho_2) + \varepsilon_{s_2}(\rho_1\rho_2)} \\ + \sum_{s_1} \frac{|\langle \rho_1|j_x|s_1\rangle|^2\big[u_{s_1}(\rho_1\rho_2)u_{\rho_1}(\rho_1\rho_2) + v_{s_1}(\rho_1\rho_2)v_{\rho_1}(\rho_1\rho_2)\big]^2}{\varepsilon_{s_1}(\rho_1\rho_2) - \varepsilon_{\rho_1}(\rho_1\rho_2)} \\ + \sum_{s_2} \frac{|\langle \rho_2|j_x|s_2\rangle|^2\big[u_{s_2}(\rho_1\rho_2)u_{\rho_2}(\rho_1\rho_2) + v_{s_2}(\rho_1\rho_2)v_{\rho_2}(\rho_1\rho_2)\big]^2}{\varepsilon_{s_2}(\rho_1\rho_2) - \varepsilon_{\rho_2}(\rho_1\rho_2)}.\end{aligned} \tag{1}$$

In Eq. (1) ρ_1, ρ_2 are the quantum numbers of the two-quasiparticle states under consideration; $\langle s_1|j_x|s_2\rangle$ is the matrix element of the x component of the single particle angular momentum operator; $\varepsilon_{s_1}(\rho_1\rho_2)$ is the energy of the single quasiparticle state with quantum numbers s_1; and $u_s(\rho_1\rho_2), v_s(\rho_1\rho_2)$ are the coefficients of the $u - v$ Bogoliubov transformation. The collective angular momentum is directed here along the x axis. The single quasiparticle energies and the $u - v$ coefficients in Eq. (1) are marked by the quasiparticle indices $\rho_1\rho_2$ which means that they are calculated taking into account the blocking effect. It is assumed in the derivation of Eq. (1) that the self-consistent mean field does not depend on rotation. This assumption is reasonable for the small values of angular momentum considered below.

To our knowledge much less calculations have been performed for the moment of inertia of excited states of even-even nuclei compared to those for the ground state. Some results of these calculations can be found in [9]. Generally, it is expected that the moments of inertia of excited states exceed the ground state moments of inertia. The analysis of experimental data on the excited 1^+ states of ^{156}Gd shows that their moments of inertia increase with excitation energy [10]. One example is the rotational band based on the mixed-symmetry 1^+ state of ^{156}Gd with an excitation energy of 3070 keV. Also the 2^+ state belonging to this band is known experimentally with an excitation energy of 3089 keV [11]. Although, knowledge of a single energy difference is in general not enough to determine both the moment of inertia and the decoupling parameter, the calculations indicate that here the decoupling parameter is very small and may be neglected. The moment of inertia of the rotational band, determined under this assumption, exceeds not only the ground state moment of inertia in this nucleus significantly but also the rigid body value by more than 50%.

It is the aim of the present paper to investigate the excitation energy dependence of the excited states moment of inertia of even-even well deformed nuclei in the framework of the quasiparticle-phonon model [9].

II. CALCULATIONS OF EXCITED STATE MOMENTS OF INERTIA OF EVEN-EVEN WELL DEFORMED NUCLEI

In this paper we present the results of calculations of the moment of inertia of the γ-vibrational states and of $K^\pi = 1^+$ excited states of several rare earth nuclei and their analysis. The wave functions of the excited states are obtained mainly as a mixture of several two-quasiparticle components. For this reason the moment of inertia of these states is calculated using the equation

$$\Im = \sum_{\rho_1,\rho_2} \psi^2_{\rho_1\rho_2}\Im^{(\rho_1,\rho_2)}, \tag{2}$$

where $\psi^2_{\rho_1\rho_2}$ is the contribution of the corresponding two-quasiparticle component to the norm of the wave function of the state under consideration. The backward amplitudes $\phi^2_{\rho_1\rho_2}$ are small in the case of well deformed nuclei and for this reason they are neglected in Eq. (2).

Equation (1) for $\Im^{(\rho_1,\rho_2)}$ is derived assuming applicability of perturbation theory to the treatment of the Coriolis term in the cranking model Hamiltonian. This assumes the inequality

$$\hbar\Omega_{\rm rot}\left|\frac{\langle s|j_x|s'\rangle}{\varepsilon_{s'} - \varepsilon_s}\right| \ll 1 \tag{3}$$

to be correct, where $\Omega_{\rm rot}$ is the rotational frequency. Considering the values of all terms on the right-hand side of Eq. (1), we have found that in the second and the third sums there are only one or at most two terms separately for neutrons and protons, that dominate the sums. The contribution of the other terms is less than 3% of the total value of the moment of inertia. For some states only a single contribution of the proton or neutron subsystems is essential. At the same time just these terms cannot be treated in perturbation theory even for low I because of the large matrix elements of j_x and because of small energy differences in the denominators. This happens when the single quasiparticle states ρ and s in the matrix element $\langle s|j_x|\rho\rangle$ satisfy the following selection rules for the asymptotic quantum numbers:

$$|n_s - n_\rho| = 1, \quad |\Lambda_s - \Lambda_\rho| = 1. \tag{4}$$

Examples are $\rho = n[642]$ and $s = n[651]$, $\rho = n[514]$ and $s = n[523]$, $\rho = p[541]$ and $s = p[532]$. Nonapplicability of the perturbation theory means that an exact diagonalization of the quasiparticle Hamiltonian with the Coriolis term should be done. At the same time in every concrete case the number of terms with large matrix elements of $|\langle\rho|j_x|s\rangle|$ and small energy denominators $|\varepsilon_\rho - \varepsilon_s|$ is quite restricted. For instance, in the case of ^{156}Gd the neutron single quasiparticle states coupled by strong matrix elements of j_x have the following excitation energies: $\varepsilon(n[606]) = 7.12$ MeV, $\varepsilon(n[615]) = 5.56$ MeV, $\varepsilon(n[624]) = 3.93$ MeV, $\varepsilon(n[633]) = 2.36$ MeV, $\varepsilon(n[642]) = 1.26$ MeV, $\varepsilon(n[651]) = 1.24$ MeV, $\varepsilon(n[660]) = 1.57$ MeV. In this sequence there are only two single quasiparticle levels, namely, $n[642]$ and $n[651]$ lying near the Fermi surface and having very close values of the excitation energies. For protons we have $\varepsilon(p[505]) = 5.12$ MeV, $\varepsilon(p[514]) = 3.18$ MeV, $\varepsilon(p[523]) = 1.52$ MeV, $\varepsilon(p[532]) = 1.39$ MeV, $\varepsilon(p[541]) = 2.46$ MeV, $\varepsilon(p[550]) = 3.18$ MeV. In this case there are also only two single quasiparticle levels with close excitation energies. This implies that a diagonalization of the Hamiltonian matrix can be restricted to the basis including two or three single quasiparticle states, only. For instance in the case of mixing of only two single quasiparticle states the expression for $\Im^{(\rho_1,\rho_2)}$ take the form (see Appendix)

$$\Im^{(\rho_1,\rho_2)} = 2\sum_{s_1,s_2} \frac{|\langle s_1|j_x|s_2\rangle|^2\big[u_{s_1}(\rho_1\rho_2)v_{s_2}(\rho_1\rho_2) - u_{s_2}(\rho_1\rho_2)v_{s_1}(\rho_1\rho_2)\big]^2}{\varepsilon_{s_1}(\rho_1\rho_2) + \varepsilon_{s_2}(\rho_1\rho_2)} + \frac{(\varepsilon_{s_1} - \varepsilon_{\rho_1})}{2\Omega_{\mathrm{rot}}^2}\left(1 - \frac{1}{\sqrt{1 + 4\Omega_{\mathrm{rot}}^2 \frac{|\langle s_1|j_x|\rho_1\rangle|^2(u_{s_1}u_{\rho_1} + v_{s_1}v_{\rho_1})^2}{(\varepsilon_{s_1} - \varepsilon_{\rho_1})^2}}}\right)$$
$$+ \frac{(\varepsilon_{s_2} - \varepsilon_{\rho_2})}{2\Omega_{\mathrm{rot}}^2}\left(1 - \frac{1}{\sqrt{1 + 4\Omega_{\mathrm{rot}}^2 \frac{|\langle s_2|j_x|\rho_2\rangle|^2(u_{s_2}u_{\rho_2} + v_{s_2}v_{\rho_2})^2}{(\varepsilon_{s_2} - \varepsilon_{\rho_2})^2}}}\right). \tag{5}$$

The Coriolis antipairing effect is discussed in the literature as a mechanism responsible for an increase of the moment of inertia of an excited state in comparison to the ground state [12]. This effect is contained in the first sum in Eq. (5) if the blocking effect generated by the presence of the quasiparticles in the states ρ_1 and ρ_2 is taken into account. If the single particle states ρ_1 and ρ_2 are located near the Fermi surface their blocking can significantly decrease the pairing gap Δ increasing in this way the moment of inertia. The blocking effect is negligible for the ground state moment of inertia although it is in principle also present in this case due to the ground state correlations. But the backward amplitudes, characterizing the ground state correlations, are small in well deformed nuclei and, therefore, the corresponding blocking effect can be neglected here. The second and the third terms in Eq. (5) do not contribute to the ground state moment of inertia. Their presence reflects the possibility for a nucleus to increase angular momentum without increasing the number of quasiparticles: the angular momentum of the nucleus is increased by changing the state occupied by a quasiparticle to the one carrying more angular momentum along the axis of the collective rotation without increasing the number of quasiparticles.

If we expand Eq. (5) in terms of Ω_{rot} and keep only the lowest order term then Eq. (5) is reduced to Eq. (1), however, with only one term in the second and third sums. Since higher order terms in the Coriolis interaction are taken into account in Eq. (5), it explicitly depends on the rotational frequency. In the calculations below the values

$$\hbar\Omega_{\mathrm{rot}} = \frac{E(J+1) - E(J)}{\sqrt{(J+1)(J+2)} - \sqrt{J(J+1)}} \tag{6}$$

are taken from the experimental data on the corresponding rotational bands when possible. J is the angular momentum of a rotational band head and $E(J')$ is the excitation energy of the state with angular momentum J' belonging to this band. A strong impact of the energies of some single quasiparticle states on the excited state moments of inertia by the small energy denominators $(\varepsilon_\rho - \varepsilon_s)$ can be used as an additional test of the correctness of the single particle level scheme used in the calculations.

It is seen from Eqs. (2) and (5) that, in order to calculate the moments of inertia of the excited states, we need to know the single quasiparticle energies, the $u - v$ coefficients of the Bogoliubov transformation, and the amplitudes characterizing the quasiparticle structure of the excited states. To calculate these quantities we have used the Hamiltonian of the quasiparticle phonon model [13] which contains the mean field part for protons and neutrons, the monopole pairing interaction, and the multipole-multipole and the spin-multipole interactions in a separable form [14]. These interactions act in the particle-particle and in the particle-hole channels. Since deformation leads to a more uniform distribution of the single particle states, the density profile of a deformed nucleus is relatively flat inside a nucleus. This resembles the use of the Woods-Saxon single-particle potential. In our calculations we have used the parameters that were determined in numerous previous calculations ([15] and references therein).

We now turn to the results of calculations of the excited states' moments of inertia. We mention that, in fact, we have calculated not the absolute values of the excited states' moments of inertia but the differences between the excited state and ground state moments of inertia. Then the experimental values of the corresponding ground state moment of inertia are added to the calculated values of the differences. The ground state moment of inertia is given in the calculations by the first term in Eq. (5) which has been calculated without inclusion of the blocking effect, because it is practically absent in this case.

The results of our calculations of the moment of inertia of the γ-vibrational states of several well deformed rare earth nuclei are presented in Table I. The expectation is that these moments of inertia should be larger than those for the ground states. It is seen from Table I and Fig. 1 that the calculated values of the γ-band moments of inertia are in qualitative agreement with the experimental data in the following aspects: they exceed systematically the ground state moments of inertia as it happens for the experimental data; their variation from nucleus to nucleus follows the same tendency as the data and

TABLE I. Calculated values of the excitation energies and the moments of inertia of the γ-vibrational state ($\frac{\Im(2_\gamma^+)}{\hbar^2}$) in several rare-earth nuclei. The experimental values of these quantities and of the ground state moment of inertia are shown for comparison.

Nucleus	$E(2_\gamma^+)$ in MeV		$\frac{\Im(2_\gamma^+)}{\hbar^2}$ in MeV^{-1}		$\frac{\Im(gs)}{\hbar^2}$ in MeV^{-1}
	exp	cal	exp	cal	exp
^{156}Gd	1.154	1.046	34.8	40.3	33.7
^{158}Gd	1.187	1.238	40.9	46.2	37.7
^{160}Dy	0.966	1.057	36.8	40.1	34.6
^{162}Dy	0.888	0.915	40.5	48.2	37.2
^{166}Er	0.786	0.860	41.8	50.2	37.2
^{178}Hf	1.175	1.196	33.3	35.1	32.2

their deviations from the experimental values are not larger than 22%. Although the presented calculations systematically overestimate the excess of the excited state's moment of inertia over those of the respective ground states, it reproduces satisfactorily the tendency of this excess. The correlation between the calculated and experimental deviations of the moment of inertia of the 2_γ^+ state from the experimental value of the ground state moment of inertia is shown in Fig. 2. It is seen that these values correlate approximately linear with some scatter. The calculated values are as a rule higher than the experimental ones. This is in contrast with the situation for the calculated values of the ground state moments of inertia which are systematically lower than the experimental values [16,17]. We mention that the γ-vibrational states are more collective than the $K^\pi = 1^+$ states considered below in which one or two main components exhaust the norm of the wave function. In the case of the γ-vibrational states the three most important components give typically less than 50% of the norm, only.

Important and comparably large contributions to the calculated values of $\Im(2_\gamma^+)$ come from the blocking effect and the quasiparticle interaction presented by the second and the third terms in Eq. (5) for the case when the consideration of the mixing of the two-quasiparticle states can be restricted to two components. Since the lowest lying two-quasiparticle states give the main contribution to the structure of the γ-vibrational state, the contributions of the second and the third terms in Eq. (5) are positive.

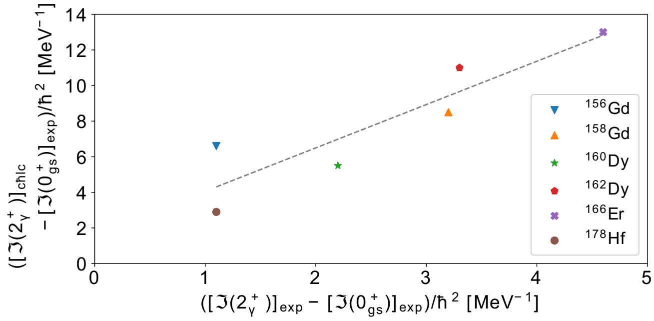

FIG. 2. Correlation between the calculated and the experimental values of the differences of the 2_γ^+ and the ground state moments of inertia.

The second class of excited states whose moments of inertia are calculated in this paper is given by the 1^+ states of ^{156}Gd with excitation energies from 1.9 MeV to 3.1 MeV and by the 1^+ states of ^{158}Gd and ^{160,162,164}Dy which are characterized by strong $M1$ transitions to the ground state. The results of our calculations are presented in Tables II and III.

As it is seen from Table II the calculated moments of inertia of these 1^+ states of ^{156}Gd significantly exceed the moment of inertia of the ground state of the same nucleus which is equal to 33.7 MeV^{-1}. While our calculations have shown that in the case of the γ-vibrational states both, blocking effect and quasiparticle interaction, make comparable contributions, the effect of the quasiparticle interaction can be significantly higher in the case of the 1^+ states. As it is seen from Table II this effect is especially large in the case of the 1^+ state at $E^*_{\rm cal} = 2.909$ MeV. The rotational band based on this state is characterized by the largest strengths for $M1$ transitions to the ground state. Its excitation energy is close to the excitation energy of the band head state for which the moment of inertia measured in [11] has the value 105 MeV^{-1} which exceeds the rigid body value. The reason is the following. An important contribution to the structure of this state comes from the neutron two-quasiparticle component $[651]3/2^+ \otimes [660]1/2^+$. The neutron single quasiparticle states $[651]3/2^+$ and $[642]5/2^+$ that are strongly mixed by the Coriolis

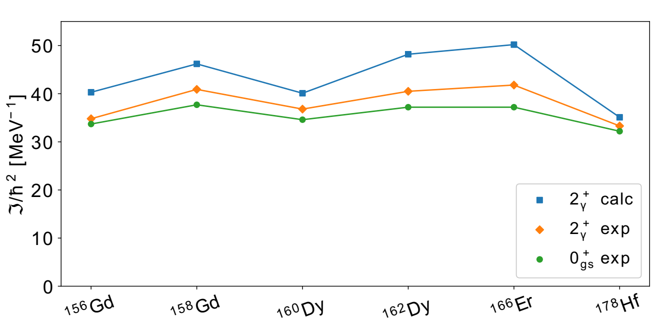

FIG. 1. Calculated and experimental values of the moment of inertia of the 2_γ^+ state of some well deformed rare-earth nuclei. Experimental values of the ground state moment of inertia are shown for comparison.

TABLE II. Calculated values of the excitation energies ($E^*_{\rm cal}$) and the moments of inertia of some 1^+ states of ^{156}Gd. The contribution of the blocking effect $(\Delta\frac{\Im(1_n^+)}{\hbar^2})_{\rm block}$ and the quasiparticle interaction $(\Delta\frac{\Im(1_n^+)}{\hbar^2})_{\rm qp\ int}$ are given separately in the last two columns. Energies are given in MeV. Other quantities are given in MeV^{-1}.

$E^*_{\rm cal}$	$(\frac{\Im(1_n^+)}{\hbar^2})_{\rm cal}$	$(\Delta\frac{\Im(1_n^+)}{\hbar^2})_{\rm block}$	$(\Delta\frac{\Im(1_n^+)}{\hbar^2})_{\rm qp\ int}$
1.900	53.6	13.4	6.5
2.182	79.9	9.6	36.6
2.450	60.3	4.4	22.2
2.795	67.2	6.1	27.4
2.909	85.6	6.0	45.9
3.109	48.9	6.2	9.0

TABLE III. Calculated values of the excitation energies ($E^*_{\rm cal}$) and the moments of inertia of some 1^+ states in ^{158}Gd and ^{160,162,164}Dy. The contribution of the blocking effect $(\Delta\frac{\Im(1^+_n)}{\hbar^2})_{\rm block}$ and the quasiparticle interaction $(\Delta\frac{\Im(1^+_n)}{\hbar^2})_{\rm qp\ int}$ are given separately in the last two columns. Energies are given in MeV. Other quantities are given in MeV^{-1}.

Nucleus	$E^*_{\rm cal}$	$(\frac{\Im(1^+_n)}{\hbar^2})_{\rm cal}$	$(\Delta\frac{\Im(1^+_n)}{\hbar^2})_{\rm block}$	$(\Delta\frac{\Im(1^+_n)}{\hbar^2})_{\rm qp\ int}$
^{158}Gd	2.908	75.9	5.5	32.7
^{160}Dy	2.933	127.8	7.0	86.2
^{162}Dy	2.917	54.4	4.3	12.9
^{164}Dy	2.680	63.9	8.4	14.6

interaction have very close energies: 1.244 MeV and 1.269 MeV, correspondingly. We stress that the results obtained can be sensitive in some cases to small variations of the single particle level scheme.

Table III presents our results for the moments of inertia of the selected 1^+ states of ^{158}Gd and ^{160,162,164}Dy that are characterized by the largest calculated $B(M1)$ values for transitions to the ground states [18]. Hence, these states, together with the 1^+ state of ^{156}Gd calculated at 2.909 MeV shown in Table II, may be identified with the largest fragments of the experimental $M1$ excitation strength distribution often called scissors mode.

The largest moment of inertia is obtained for the 1^+ state of ^{160}Dy calculated at 2.933 MeV excitation energy. The main contribution to the structure of this state is provided by the neutron two-quasiparticle component $[642]5/2^+ \otimes [402]3/2^+$. The neutron single quasiparticle state $[642]5/2^+$ is coupled by the large matrix element of the angular momentum operator j_x to the $[651]3/2^+$ single quasiparticle state also lying near the Fermi surface. Also this case features clearly the situation that we have analyzed above.

III. SUMMARY

We have studied excited states' moments of inertia of well deformed axially symmetric nuclei. The moments of inertia of the γ-vibrational states and $K^\pi = 1^+$ excited states of several rare earth nuclei are considered. The blocking effect and the Coriolis interaction between quasiparticles and the rotating core are taken into account.

The results obtained for the moments of inertia of the γ-vibrational states are in a qualitative agreement with the experimental data. They exceed systematically the ground state moments of inertia and their calculated variation from nucleus to nucleus reproduces the observed tendency. The calculated values deviate from the experimental ones no more than for 22%. Comparable contributions to the increase of the moment of inertia of the γ band over the one of the ground band come from the blocking effect and from the Coriolis interaction of quasiparticles.

The calculated moments of inertia of the 1^+ states significantly exceed the moments of inertia of the ground states. In contrast to the results obtained for the γ-vibrational states, the effect of the Coriolis interaction between quasiparticles and the rotating core can be significantly larger than the blocking effect. This results in some cases in very large values for the moment of inertia of the 1^+ states that can even exceed the rigid body value. This situation is realized if near the Fermi surface there are two single quasiparticle states coupled by a strong matrix element of the Coriolis interaction and, at the same time, one of these states contributes significantly to the structure of the excited state under consideration.

ACKNOWLEDGMENTS

The authors are grateful to Dr. M. Spieker for discussions. This work was supported in part by the Heisenberg–Landau Program, and by the RFBR (Russia) (Grant No. 16-02-00068), and by DFG under Grant No. SFB 1245 and by the BMBF under Grant No. 05P18RDEN9.

APPENDIX

In this Appendix we describe shortly a derivation of Eq. (5). The Schrödinger equation in the rotating system has the form

$$(h - \hbar\Omega j_x)\Phi^\omega = e^\omega \Phi^\omega, \tag{A1}$$

where h is a Hamiltonian of the noninteracting quasiparticles

$$h = \sum_\rho \varepsilon_\rho \alpha^+_\rho \alpha_\rho, \tag{A2}$$

Ω is a rotational frequency, and j_x is the x component of the single particle angular momentum operator. In the quasiparticle representation j_x has the following structure:

$$j_x = \sum_{\rho,\rho'} a_{\rho,\rho'}\alpha^+_\rho \alpha_{\rho'} + \sum_{\rho,\rho'} b_{\rho,\rho'}(\alpha^+_\rho \alpha^+_{\rho'} + \alpha_{\rho'}\alpha_\rho). \tag{A3}$$

Here, the matrix elements $a_{\rho,\rho'}$ and $b_{\rho,\rho'}$ are determined by the single particle matrix elements of j_x and the coefficients of the $u - v$ Bogoliubov transformation.

The second term in Eq. (A3) changes the number of quasiparticles by two units. Because of the large value of the energy differences between the states connected by this term the corresponding part of the Coriolis interaction can be treated by perturbation theory. A contribution of this part of the Coriolis term in the Hamiltonian to the moment of inertia is given by the first sum in Eq. (1). Below we consider only a contribution of the first term in Eq. (A3).

The eigenvalues e^ω from Eq. (A1) are referred to as the single quasiparticle energies in the rotating frame [19]. They are obtained by a diagonalization of the matrix

$$\varepsilon_\rho \delta_{\rho\rho'} - \hbar\Omega a_{\rho,\rho'}. \tag{A4}$$

The energies of the single quasiparticle state measured in the laboratory frame are calculated as [19]

$$e = \langle \Phi^\omega | h | \Phi^\omega \rangle = e^\omega + \hbar\Omega \langle \Phi^\omega | j_x | \Phi^\omega \rangle. \tag{A5}$$

Consider the case when only two single quasiparticle levels $|\rho\rangle$ and $|s\rangle$ are strongly mixed and the other mixing matrix elements give a small correction which can be neglected in the simplest approximation. Then the matrix Eq. (A4) can be diagonalized analytically and using Eq. (A5) we obtain

$$e_\rho = \varepsilon_\rho + \frac{1}{2}(\varepsilon_s - \varepsilon_\rho)\left(1 - \frac{1}{\sqrt{1 + \frac{4(\hbar\Omega a_{s\rho})^2}{(\varepsilon_s - \varepsilon_\rho)^2}}}\right). \tag{A6}$$

Only the second term in Eq. (A6) contains a dependence on Ω and therefore only this term contributes to the expression for the moment of inertia.

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