Isospin properties in quark matter and quark stars within isospin-dependent quark mass models

Peng-Cheng Chu,[1,*] Yi Zhou,[2] Xin Qi,[1] Xiao-Hua Li,[3,4,†] Zhen Zhang,[5,‡] and Ying Zhou[6,§]

[1]*School of Science, Qingdao University of Technology, Qingdao 266000, China*
[2]*School of Mathematics and Physics, Qingdao University of Science and Technology, Qingdao 266000, China*
[3]*School of Nuclear Science and Technology, University of South China, 421001 Hengyang, China*
[4]*Cooperative Innovation Center for Nuclear Fuel Cycle Technology & Equipment, University of South China, 421001 Hengyang, China*
[5]*Sino-French Institute of Nuclear Engineering and Technology, Sun Yat-sen University, Zhuhai 519082, China*
[6]*School of Physics and Astronomy and Shanghai Key Laboratory for Particle Physics and Cosmology, Shanghai Jiao Tong University, Shanghai 200240, China*

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We investigate the isospin properties of the strange quark matter (SQM) and quark stars (QSs) in the framework of the isospin-dependent confining quark matter (ICQM) model and confined-isospin and density-dependent mass (CIDDM) model. Within these two isospin-dependent quark mass phenomenological models, we study the quark matter symmetry energy, the stability of strange quark matter, the quark fractions, the isospin asymmetry, and the quark mass asymmetry in SQM, and the mass-radius relation of quark stars. We find that including isospin dependence of the quark mass can significantly influence the isospin properties of the quark matter. Recently, the LIGO-Virgo collaboration reported their detection of gravitational wave (GW) signals GW170817, which are originating from a binary compact star merger. Using the ICQM model and CIDDM model, we describe the compact stars with the new maximum mass limits $2.01^{+0.04}_{-0.04} \leqslant M/M_{\odot} \leqslant 2.16^{+0.17}_{-0.15}$ as quark stars, and the dimensionless tidal deformabilities of QSs are also investigated in this work.

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I. INTRODUCTION

In contemporary nuclear physics, the experiments of high-energy heavy-ion collisions (HICs) in terrestrial laboratories provide the unique approach to explore the properties of strongly interacting matter, which plays a central role in understanding the nuclear structures and reactions. In nature, the compact stars show another way of exploring the strongly interacting matter at high baryon density and low temperature [1,2]. Neutron stars (NSs), which are a class of the densest compact stars, have been shown to provide the natural testing grounds about the equation of state (EOS) of neutron-rich matter [3–6]. Theoretical studies indicate that NSs may be converted to strange quark stars (QSs) [7–9], which are made up of deconfined absolutely stable u, d, and s quark matter with leptons, i.e., strange quark matter (SQM). The possible existence for QSs is still one of the most intriguing aspects of modern astrophysics and cosmology, which has important implications for understanding the strongly interacting matter physics, especially the properties of SQM essentially determining the structure of QSs [10–16]. In order to understand the properties of SQM and QSs, people have built many quantum chromodynamics (QCD)-inspired phenomenological models, such as the MIT bag model [16–18], the pQCD approach [19–21], the Nambu-Jona-Lasinio (NJL) model [22–28], the Dyson-Schwinger approach [29–32], and the confined density-dependent quark mass model [33–36]. At extremely high baryon density, SQM could be in color-flavor-locked (CFL) state [37].

In the star matter of QSs, there exists large u-d quark asymmetry (isospin asymmetry), which indicates that the isospin properties of SQM may play an important role, and large isospin asymmetry can still be found in the previous studies on hybrid stars [38–43]. Therefore, exploring the isospin properties of quark matter is important and useful to understand the properties of QSs as well as the QCD phase diagram. In the recent observations, two heaviest compact stars have been precisely measured. One is the radio pulsar J1614-2230 [44] with a mass of $1.97 \pm 0.04 M_{\odot}$ by using the general relativistic Shapiro delay, and the other is J0348+0432 [45] with a larger mass $2.01 \pm 0.04 M_{\odot}$. In order to describe these two stars as strange QSs, the interaction among quarks should be very strong [18,22,46–54], which may also depend on the isospin properties of the quark star matter.

Recently, the LIGO-Virgo collaboration has reported their first detection of gravitational wave (GW) signals from a low mass compact binary merger GW170817 [55], whose chirp mass is precisely constrained to $1.188^{+0.004}_{-0.002} M_{\odot}$. Moreover, due to the lack of the information on the postmerger remnant [56–59], the possibility of a binary quark star merger cannot be excluded as the origin of GW170817. Since there exists large u-d quark asymmetry in the QSs, it is of interests and importance to investigate what this GW mass observation means to the EOS of QSs and the isospin properties of the quark matter.

*kyois@126.com
†lixiaohuaphysics@126.com
‡zhangzh275@mail.sysu.edu.cn
§yingchow@sjtu.edu.cn

In this work, we extend the CQM model to include isospin dependence of the quark mass. Compared with the confined isospin- and density-dependent mass (CIDDM) model, we investigate the quark matter symmetry energy in asymmetric quark matter, the stability of SQM, and the mass-radius relation for QSs within the isospin-dependent confining quark matter (ICQM) model.

II. THEORETICAL FORMULISM

In this work, we study the isospin properties for the quark matter by comparing two isospin-dependent quark mass phenomenological models.

A. Confined isospin- and density-dependent mass model

The confined isospin- and density-dependent mass (CIDDM) model [36] is the extension of the confined density-dependent mass (CDDM) model [29,33–35,60–65] for quark matter by including the isospin dependence of the equivalent quark mass. With baryon density n_B and isospin asymmetry δ, the equivalent quark mass can be expressed as

$$m_q = m_{q_0} + m_I + m_{\rm iso} = m_{q_0} + \frac{D}{{n_B}^z} - \tau_q \delta D_I n_B^\alpha e^{-\beta n_B}, \quad (1)$$

where m_{q0} is the bare mass for quarks, $m_I = \frac{D}{{n_B}^z}$ reflects the flavor-independent quark interactions, while $m_{\rm iso} = -\tau_q \delta D_I n_B^\alpha e^{-\beta n_B}$ means the isospin-dependent mass term. For $m_I = \frac{D}{{n_B}^z}$, the constant D is a parameter determined by stability arguments of SQM and the constant z is the equivalent mass scaling parameter. For the isospin-dependent term $m_{\rm iso}$, the constants D_I, α, and β are parameters determining the isospin-density dependence of the effective interactions in quark matter, τ_q is the isospin quantum number of quarks, and we set $\tau_q = 1$ for $q = u$ (u quarks), $\tau_q = -1$ for $q = d$ (d quarks), and $\tau_q = 0$ for $q = s$ (s quarks). The isospin asymmetry is defined from the works [36,66–68] as

$$\delta = 3\frac{n_d - n_u}{n_d + n_u}. \quad (2)$$

In Eq. (1), one can find that the quark confinement condition $\lim_{n_B\to 0} m_q = \infty$ will be guaranteed once the scaling parameter $z > 0$ and $\alpha \geqslant 0$. In addition, if $\beta > 0$, then $\lim_{n_B\to\infty} m_{\rm iso} = 0$ and thus the asymptotic freedom $\lim_{n_B\to\infty} m_q = m_{q0}$ is satisfied. Therefore, the phenomenological parametrization form of the isospin-dependent equivalent quark mass in Eq. (1) is very general and respects the basic features of QCD. To see more details about the CIDDM model, the readers are referred to Ref. [36].

B. Isospin-dependent confining quark matter model

In the original CQM model [69], the Hamiltonian for u-d-s quark matter at finite chemical potential is given by

$$\mathcal{H} = \sum_i (\alpha_i \cdot p_i + \beta_i M_i) + \sum_{i<j} \frac{\lambda(i)\lambda(j)}{4} V_{ij}, \quad (3)$$

where $i(j)$ stands for the $i(j)$th flavor of quarks, α_i and β_i come from Dirac equation, λ_i is SU(3) matrix for quarks, V_{ij} is the vector interaction among quarks and taken as the Richardson potential, and M_i is the quark mass, which is density dependent and parameterized as

$$M_i = m_i + (310\,\text{MeV})\text{sech}\left(\nu\frac{n_B}{n_0}\right), \quad (4)$$

where i stands for the ith flavor of quarks, m_i is the bare quark mass, n_B is the baryon density, $n_0 = 0.17$ fm^{-3} is the nuclear matter saturation density, and ν is the parameter determining the density dependence for quark mass.

Since Eq. (4) also indicates that the value of M_i for u quark is identical to that of d quark, suggesting that there is no isospin dependence in the mass term, one can extend the CQM model to the isospin-dependent confining quark matter (ICQM) model by including the contribution of isovector-scalar channels into M_i in Eq. (4). Due to the form of isospin dependence of the quark mass being still unclear, we adopt the phenomenological parametrization for isospin dependence of the equivalent quark mass in the CIDDM model in Ref. [36], and then the quark mass can be expressed as

$$M_i = m_i + m_i^* \text{sech}\left(\nu_i \frac{n_B}{n_0}\right) - \tau_i \delta D_I n_B^\alpha e^{-\beta n_B}, \quad (5)$$

where D_I, $\alpha > 0$ and $\beta > 0$ are parameters introducing isospin dependence of the quark mass in quark matter, τ_i is the isospin quantum number for quarks, and δ is the isospin asymmetry.

The quark vector interaction $V_{i,j}$ is taken as the Richardson potential [70], i.e.,

$$V_{ij} = \frac{4\pi}{9}\frac{1}{\ln(1 + [(\mathbf{k}_i - \mathbf{k}_j)^2 + D^{-2}]/\Lambda^2)} \times \frac{1}{(\mathbf{k}_i - \mathbf{k}_j)^2 + D^{-2}}, \quad (6)$$

where $\mathbf{k}_i - \mathbf{k}_j$ is the momentum transfer between the ith and jth particles, Λ is the scale parameter, and D is the screening length (the gluon mass). This potential will be screened in the medium due to the pair creation, and the squared inverse screening length can be expressed as [71]

$$(D^{-1})^2 = \frac{2\alpha_0}{\pi} \sum_{i=u,d,s} k_i^f \sqrt{\left(k_i^f\right)^2 + M_i^2}, \quad (7)$$

where k_i^f is the quark Fermi momentum, and α_0 is the perturbative quark-gluon coupling constant. In the present work, we adopt the original value of the scale parameter $\Lambda = 100$ MeV and $\alpha_0 = 0.2$ as in the original CQM model [69], which are obtained from pQCD for hadron phenomenology. For the bare mass m_i of quarks in Eq. (1), we set $m_u = m_d = 5.5$ MeV and $m_s = 95$ MeV. We also set $m_u^* = m_d^* = 329.5$ MeV and $m_s^* = 432$ MeV in order to match the vacuum values of quark mass of $M_{u0} = M_{d0} = 335$ MeV and $M_{s0} = 527$ MeV obtained in SU(3) NJL model with the parameter set HK [72]. In addition, the values of $\alpha > 0$ and $\beta > 0$ are fixed as $\alpha = 1.5$ and $\beta = 1$ fm^3 to provide a reasonable baryon density dependence for quark matter symmetry energy in Sec. III.

C. Quark matter symmetry energy

Similar to the nuclear matter symmetry energy [73–75], the energy per baryon number for isospin asymmetric quark matter consisting of u, d, and s quarks can be expanded in isospin asymmetry δ as

$$E(n_B,\delta,n_s)=E_0(n_B,n_s)+E_{\rm sym}(n_B,n_s)\delta^2+\mathcal{O}(\delta^4), \quad (8)$$

where n_B is the total baryon number density, and $E_0(n_B,n_s)=E(n_B,\delta=0,n_s)$ is the energy per baryon in isospin asymmetric quark matter with an equal fraction of u and d quarks. Then the quark matter symmetry energy Eq. (8) can be expressed as

$$E_{\rm sym}(n_B,n_s)=\frac{1}{2!}\frac{\partial^2E(n_B,\delta,n_s)}{\partial\delta^2}\bigg|_{\delta=0}. \quad (9)$$

D. Properties of strange quark matter

SQM is assumed to be neutrino free and composed of u, d, s quarks and e^- in β equilibrium with electric charge neutrality. Then the weak β-equilibrium condition can be expressed as

$$\mu_u+\mu_e=\mu_d=\mu_s, \quad (10)$$

where μ_i ($i=u,d,s$, and e^-) is the chemical potential for the quarks and leptons. Furthermore, the electric charge neutrality condition requires

$$\tfrac{2}{3}n_u=\tfrac{1}{3}n_d+\tfrac{1}{3}n_s+n_e. \quad (11)$$

For u-d-s quark matter within ICQM model (the details of the analytic expressions of the thermodynamical quantities for quark matter within ICQM model and CIDDM model can be found in Refs. [36,76]), the kinetic part of the energy density at zero temperature can be written as

$$\begin{aligned}\epsilon_k&=\frac{6}{(2\pi)^3}\sum_{i=u,d,s}\int_0^{k_i^f}d^3k\sqrt{k^2+M_i^2}\\&=\frac{3}{4\pi^2}\sum_{i=u,d,s}\left[k_i^f\left(\left(k_i^f\right)^2+M_i^2/2\right)\sqrt{\left(k_i^f\right)^2+M_i^2}\right.\\&\quad\left.-\frac{M_i^4}{2}\ln\frac{\sqrt{\left(k_i^f\right)^2+M_i^2}+k_i^f}{M_i}\right]. \end{aligned}\quad (12)$$

The potential part ϵ_v of the energy density for u-d-s quark matter at zero temperature can be obtained as

$$\begin{aligned}\epsilon_v=&-\frac{1}{2\pi^3}\sum_{i,j}\int_{-1}^{1}dx\int_0^{k_j^f}k_j^2\int_0^{k_i^f}k_i^2\\&\times V_{ij}f(k_i,k_j,M_i,M_j,x)dk_jdk_i,\end{aligned} \quad (13)$$

where f is

$$\begin{aligned}f(k_i,k_j,M_i,M_j,x)=&\left(e_i\cdot e_j+2\cdot k_i\cdot k_j\cdot x+\frac{k_i^2k_j^2}{e_i\cdot e_j}\right)\\&\times\frac{1}{(e_i-M_i)(e_j-M_j)}\end{aligned} \quad (14)$$

with

$$e_i=\sqrt{k_i^2+M_i^2}+M_i. \quad (15)$$

Then the chemical potential for each flavor of quarks can then be obtained as

$$\mu_i=\frac{d\epsilon}{dn_i}=\frac{\partial\epsilon_k}{\partial M_i}\frac{\partial M_i}{\partial n_i}+\frac{\partial\epsilon_v}{\partial k_i^f}\frac{\partial k_i^f}{\partial n_i}+\frac{\partial\epsilon_v}{\partial M_i}\frac{\partial M_i}{\partial n_i}, \quad (16)$$

where $\epsilon=\epsilon_k+\epsilon_v$ is the total energy density for SQM.

For the leptons, we use $\mu_l=\sqrt{(k_l^f)^2+m_l^2}$ to obtain the chemical potential, where k_l^f is the fermion momentum for leptons, and the pressure for SQM within ICQM model can be obtained as

$$P=-\epsilon+\sum_{j=u,d,s,l}n_j\mu_j. \quad (17)$$

E. Properties of quark stars

Using the EOS's of SQM, one can obtain the mass-radius relation for QSs by solving the Tolman-Oppenheimer-Volkov (TOV) equation [77]:

$$\frac{dM}{dr}=4\pi r^2\epsilon(r), \quad (18)$$

$$\begin{aligned}\frac{dp}{dr}=&-\frac{G\epsilon(r)M(r)}{r^2}\left[1+\frac{p(r)}{\epsilon(r)}\right]\\&\times\left[1+\frac{4\pi p(r)r^3}{M(r)}\right]\left[1-\frac{2GM(r)}{r}\right]^{-1},\end{aligned} \quad (19)$$

where $M(r)$ is the total mass inside the sphere of radius r, G is Newton's gravitational constant, $\epsilon(r)$ is the corresponding energy density, and $p(r)$ is the corresponding pressure.

III. RESULTS AND DISCUSSIONS

A. Quark matter symmetry energy

In Fig. 1, we demonstrate the baryon density dependence of the quark matter symmetry energy within the ICQM model and the CIDDM model. We also include the results of the symmetry energy for free Fermi gas and the normal quark matter within the conventional Nambu-Jona-Lasinio (NJL) model [23,78] for comparison. In the present work, we choose two sets of parameters for this two models, namely, DI-600 in ICQM model and DI-85 in CIDDM model. For DI-600 in ICQM model, we have $D_I=600\,\mathrm{MeV\,fm}^{3\alpha}$, $\nu_{ud}=0.63$, and $\nu_s=0.7$, while $D_I=85\,\mathrm{MeV\,fm}^{3\alpha}$ and $z=1.8$ [36] for DI-85 in CIDDM model. One can find that the quark matter symmetry energy for all the models increases with the increment of the baryon density. Following Ref. [36], the authors have shown that the quark matter symmetry energy should be at least about twice that of a free Fermi gas or the normal quark matter within the conventional NJL model at the baryon density of 1.5 fm^{-3} (around $10n_0$) in order to describe PSR J0348+0432 as QSs within CIDDM model with DI-85. One can find in Fig. 1 that the quark matter symmetry energy with DI-600 within ICQM model is 82 MeV at 1.5 fm^{-3}, while 71 MeV at 1.5 fm^{-3} with DI-85 within

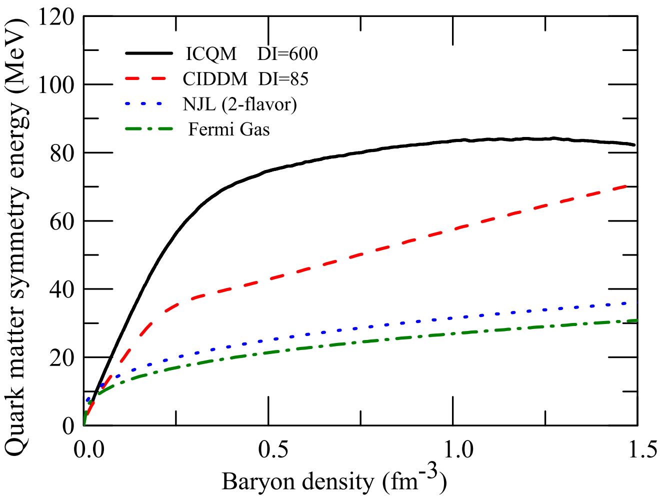

FIG. 1. Quark matter symmetry energy as a function of baryon number density within the ICQM model with DI-600 and the CIDDM model with DI-85. The results of the symmetry energy of a free quark gas and normal quark matter within the conventional NJL model are also included for comparison.

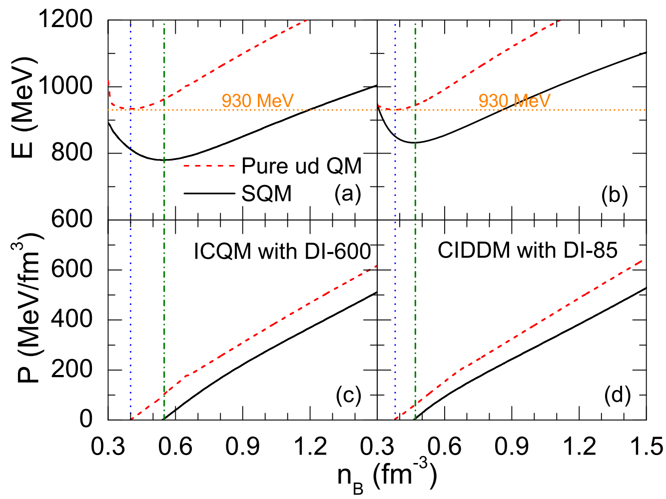

FIG. 2. Energy per baryon and the corresponding pressure as functions of the baryon number density for SQM and two-flavor pure *u*-*d* quark matter in β equilibrium within the ICQM model with DI-600 and the CIDDM model with DI-85.

CIDDM model, both predicting about two times larger quark matter symmetry energy than the conventional NJL model (37 MeV at 1.5 fm^{-3}), and as shown in Fig. 6, the ICQM model with DI-600 can support a two solar mass quark star and be able to describe PSR J0348+0432 as QSs within ICQM model, which is consistent with the results in Ref. [36].

B. Stability of SQM within two models

From Farhi and Jaffe [14], the absolute stability requires that the minimum energy per baryon of SQM should be less than that of the observed stable nuclei, i.e., $M(^{56}\mathrm{Fe})c^2/56 = 930$ MeV. For the minimum energy per baryon of the β-equilibrium *u*-*d* quark matter (pure *u*-*d* quark matter), at the same time, the value should be larger than 930 MeV in order to be consistent with the standard nuclear physics. Figure 2 shows the energy per baryon and the corresponding pressure as functions of the baryon density for SQM and pure *u*-*d* quark matter within ICQM model and the CIDDM model. As one can see from Fig. 2 that the minimum energy per baryon of the pure *u*-*d* quark matter in β-equilibrium condition is larger than 930 MeV for both cases, while the corresponding minimum energy per baryon for SQM is less than 930 MeV, which satisfies the requirement of the absolute stable condition. We can also find that the baryon density at the minimum energy per baryon is exactly the zero pressure point for both models, which is consistent with the requirement of thermodynamical self-consistency. One can see that the zero-pressure densities for SQM and pure *u*-*d* quark matter are 0.55 fm^{-3} and 0.40 fm^{-3} within ICQM model, while within CIDDM model the corresponding zero-pressure densities are and 0.46 fm^{-3} and 0.38 fm^{-3}. Our results indicate that the zero-pressure density of the pure *u*-*d* quark matter is less than the corresponding zero-pressure density of the strange quark matter within the usual isospin-dependent mass model.

C. Density dependence of the quark mass and quark fraction

Shown in Fig. 3 is the density dependence of the quark mass in SQM within the ICQM model with DI-600 and the CIDDM model with DI-85. It can be seen that the quark mass increases drastically with the decreasing baryon density for both two cases, which reflects the confinement feature for quarks. One can also find that there exists an obvious isospin splitting in the *u* and *d* quark masses, and the *d* quark is generally heavier than the *u* quark, which reflects the isospin dependence of quark-quark interactions in isospin asymmetric quark matter within ICQM model and CIDDM model. In addition, it can be seen that the isospin splitting for both cases is large at low densities while becoming weaker and weaker and even disappears at ultrahigh density, which is due to the

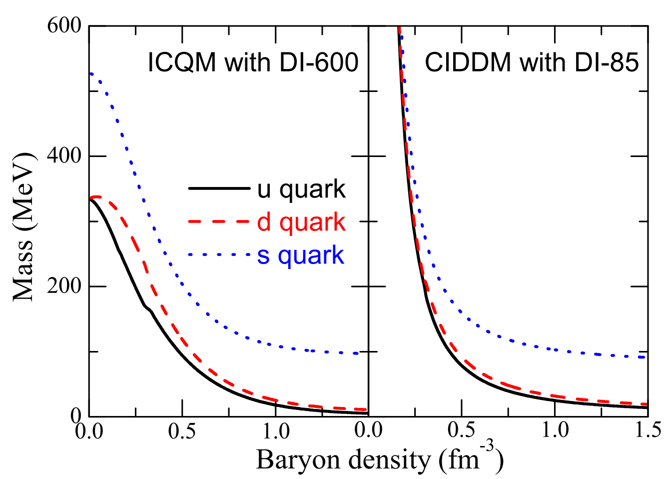

FIG. 3. Quark mass for *u*, *d*, and *s* quarks as functions of baryon density in SQM within the ICQM model with DI-600 and the CIDDM model with DI-85.

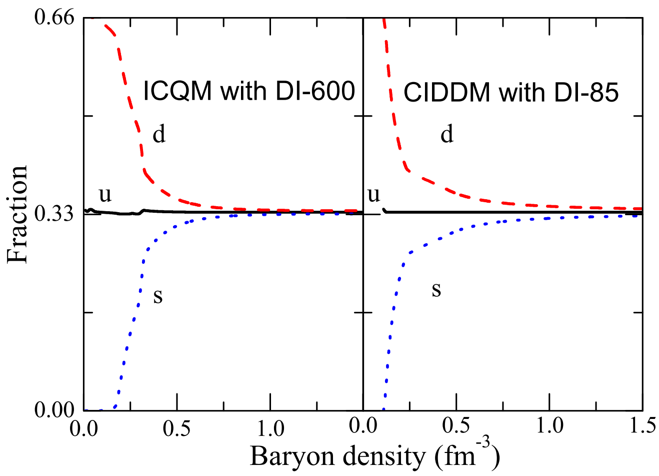

FIG. 4. Quark fraction as a function of the baryon density in SQM within the ICQM model with DI-600 and the CIDDM model with DI-85.

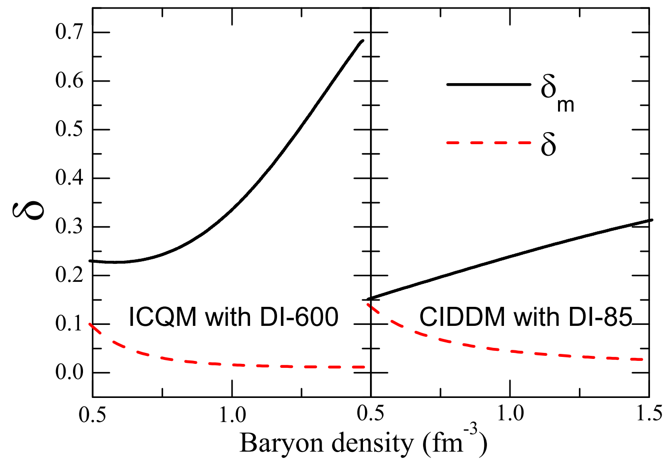

FIG. 5. The isospin asymmetry δ and the quark mass asymmetry as functions of baryon density within the ICQM model with DI-600 and the CIDDM model with DI-85.

fact that the isospin asymmetry in SQM becomes weaker and finally disappears at high density.

In Fig. 4, we show the quark fraction as a function of the baryon density in SQM within the ICQM model with DI-600 and the CIDDM model with DI-85. One can find that for both cases, when the quark matter symmetry energy is not so large, the u, d, and s quark fraction are quite different at low-density region, leading to a large isospin asymmetry in SQM. With the increment of the baryon density, the quark matter symmetry energy gets larger, and then the u, d, and s quark fractions become essentially equal and approach the value of about 0.33 when $n_B > 0.8\ \mathrm{fm}^{-3}$ for ICQM model with DI-600 and $n_B > 1.2\ \mathrm{fm}^{-3}$ for CIDDM model with DI-85. One can also find that the difference among u, d, and s quark fractions for ICQM model with DI-600 is reduced more significantly than that for CIDDM model with DI-85, which is due to the larger quark matter symmetry energy within ICQM model as shown in Fig. 1, and this symmetry energy effect can also been observed in neutron star matter in Ref. [79], which indicates that a larger symmetry energy will reduce the difference between neutron and proton fractions in the β-equilibrium neutron star matter.

In Fig. 5, we show the isospin asymmetry δ and the quark mass asymmetry δ_m as functions of baryon density within the ICQM model with DI-600 and the CIDDM model with DI-85. We define the quark mass asymmetry as

$$\delta_m = \frac{m_d - m_u}{(m_d + m_u)/2}. \tag{20}$$

One can find in Fig. 5 that the isospin asymmetry δ decreases with the increment of the baryon density in both cases, and the isospin asymmetry δ for ICQM model is less than the isospin asymmetry for CIDDM model, which is due to the larger isospin effects in ICQM model caused by the quark matter symmetry energy. One can also obtain that the quark mass asymmetry δ_m increases with the increment of the baryon density from the both panels, which implies that the mass of u quarks has a stronger density dependence than the mass of d quarks for the isospin-dependent mass models. In addition, the density dependence of the quark mass asymmetry δ_m within the ICQM model with DI-600 in this figure is stronger than the density dependence of the quark mass asymmetry within the CIDDM model with DI-85.

D. Quark stars

Shown in Fig. 6 is the mass-radius relation for quark stars within the ICQM model with DI-600 and the CIDDM model with DI-85 by solving the Tolman-Oppenheimer-Volkov equation [77]. In addition, the shadowed box with cyan color in Fig. 6 is the recently measured mass and radius of the pulsars in the rapid burster MXB 1730-335, which shows the region constrained to be $M = 1.1 \pm 0.3 M_\odot$ and $R = 9.6 \pm 1.5$ km(1σ) by using the analysis of Swift/XRT time-resolved spectra of the burst [80]. We can find that the

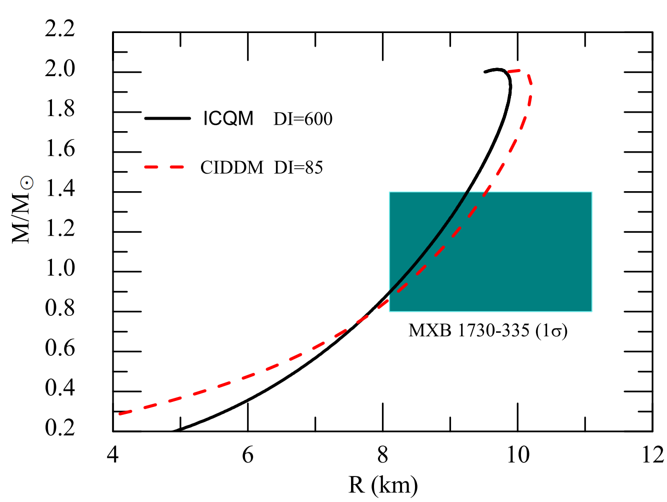

FIG. 6. Mass-radius relation for static quark stars within the ICQM model with DI-600 and the CIDDM model with DI-85. For comparison, $R = 9.6 \pm 1.5$ km(1σ) for MXB 1730-335 (cyan box) [80] is also included.

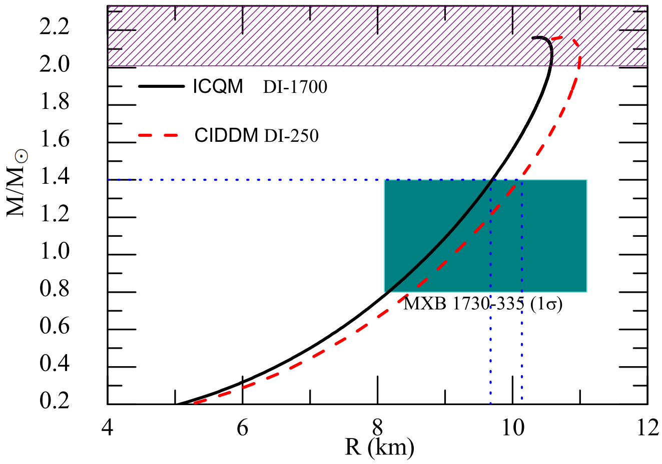

FIG. 7. Mass-radius relation for static quark stars within the ICQM model with $D_I = 1700$, $\nu_{ud} = 0.71$, and the CIDDM model with $DI = 250$.

results of the maximum mass of the quark stars within the ICQM model with DI-600 and the CIDDM model with DI-85 are both consistent with the observations and can describe PSR J0348+0432 as quark stars (2.01 $M_\odot$), when the quark matter symmetry energy within the ICQM model with DI-600 and CIDDM model with DI-85 (in Fig. 1) are both twice that of a free Fermi quark gas or normal quark matter within NJL model, which is consistent with the results in Ref. [36]. We would like to point out that there are several quark mass models, such as the extended quark bag model [52] and SU(3) quark-meson model [81], which can also describe massive compact stars of 2 $M_\odot$ as QSs.

Recently, the LIGO-Virgo collaboration reported their first detection of gravitational wave (GW) signals from a binary compact star merger GW170817 [55], whose chirp mass is precisely constrained to $1.188^{+0.004}_{-0.002} M_\odot$. Based on the GW observations and the quasiuniversal relations, a new constraint on the maximum mass of the compact stars has been obtained in Refs. [82,83] as $2.01^{+0.04}_{-0.04} \leqslant M/M_\odot \leqslant 2.16^{+0.17}_{-0.15}$, where the lower limit comes from the observation PSR J0348+0432. There exist several other predictions about the upper bound on the M_{TOV} such as $M_{\max} < 2.17 M_\odot$ in Ref. [84], which is also in the constraint in Fig. 7. From the observation of GW170817, the LIGO-Virgo collaboration also set a clean upper limit on the tidal deformability of the compact stars of 1.4 solar mass compact star that is $\Lambda_{1.4} < 800$ for the low-spin priors [55]. In Ref. [85], the new constraints for the tidal deformability parameter $\tilde{\Lambda}$ have been updated as (0,630) for large component spins and 300^{+420}_{-230} by using the highest posterior density interval. Many works have used these new constraints to set new limitation on the EOSs of strongly interacting matter [86–89]. In the works [90–92], the properties of the EOSs for hybrid stars (HSs) and quark stars have been calculated. The results show that GW170817 has the possibility of originating from a binary quark star merger or a binary hybrid star merger, and the GW observation of tidal deformability can put strong constraints for high-density QS and HS EOSs. In Fig. 7, we show the mass-radius relation for static quark stars within the ICQM model with $D_I = 1700\,\mathrm{MeV\,fm}^{3\alpha}$, $\nu_{ud} = 0.71\,(DI - 1700)$ and the CIDDM model with $D_I = 250\,\mathrm{MeV\,fm}^{3\alpha}$, $z = 1.8\,(DI - 250)$. The equations of state for the star matter with the two parameter sets of the two models also satisfy the absolute stability of SQM, and the shaded band in Fig. 7 is the new constraint on the maximum mass of the compact stars $M/M_\odot = 2.16^{+0.17}_{-0.15}$ by using the GW observations and the quasiuniversal relations [82,83]. We can find that both the results of the ICQM model with $DI - 1700$ and the CIDDM model with $DI - 250$ can describe the 2.16 $M_\odot$ compact stars as quark stars, and the corresponding quark matter symmetry energy is 249 MeV with $DI - 1700$ for the ICQM model at $n_B = 1.5\,\mathrm{fm}^{-3}$ while 366 MeV with $DI - 250$ at $n_B = 1.5\,\mathrm{fm}^{-3}$ for the CIDDM model, which indicates very strong isospin interactions among asymmetric quark matter. The tidal deformability $\Lambda_{1.4}$ is also calculated by using this two parameter sets in ICQM model and CIDDM model, and the results show that $\Lambda_{1.4} = 264.032$ and $R_{1.4} = 9.67$ km for ICQM model with $DI - 1700$, while $\Lambda_{1.4} = 353.421$ and $R_{1.4} = 10.08$ km for CIDDM model with $DI - 250$. These results satisfy the results of the tidal deformability in Refs. [55,83,85], and one can find that the $\Lambda_{1.4}$ for QSs within the ICQM model and CIDDM model are both very small. This phenomenon is due to the fact that QSs usually have small radii than NSs at a certain star mass, which can reduce the value of $\Lambda_{1.4}$ in QSs cases (in Ref. [89], the authors have shown that the stars with a quark-matter core usually have smaller radii and, hence higher compactness, which will cause smaller values of the tidal deformability, and this is the reason for this phenomenon). In addition, we have checked the sound speed in the quark matter on the basis of the calculated pressure and energy density with $DI - 1700$ and $DI - 250$, and we find that the sound speed in both cases is less than the speed of light in vacuum, thus satisfying the causality condition. Our results indicate that quark matter symmetry energy should be very large in isospin-dependent strongly interacting matter in order to describe 2.16 $M_\odot$ compact stars as QSs within ICQM model and CIDDM model, and the parameter sets in this work are both consistent with the new constraints of GW170817.

IV. CONCLUSION AND DISCUSSION

In this work, we have investigated the isospin properties of the strange quark matter (SQM) and quark stars (QSs) in the framework of the ICQM model and CIDDM model. Within this two isospin-dependent quark mass phenomenological models, we have studied the stability of strange quark matter, the quark matter symmetry energy, the quark fractions, the isospin asymmetry and the quark mass asymmetry in SQM, and the mass-radius relation of quark stars. The parameter sets used in this work all satisfy the absolutely stable condition for SQM and the thermodynamical self-consistency in phenomenological models. We have found that the quark matter symmetry energy can significantly influence the isospin properties in quark matter as well as the thermodynamical properties of SQM and quark stars.

We have demonstrated that within the two models, the quark mass decreases with the increment of the baryon density, and between the u and d quark mass there exists the quark mass isospin splitting, which disappears at high density due to the increment of the quark matter symmetry energy. We have also found that the isospin asymmetry (quark mass asymmetry) decreases (increases) with the increment of the baryon density within the two models, and the quark mass asymmetry within the ICQM model has a stronger density dependence than the quark mass asymmetry within the CIDDM model.

Furthermore, we have investigated the mass-radius relations of quark stars within ICQM model and CIDDM model. It is found that both the models can support two solar mass quark stars, and satisfy the very recent constraint of the maximum mass of the compact stars from GW observations, when their quark matter symmetry energy are larger than two times that of normal quark matter within the conventional Nambu-Jona-Lasinio model. The tidal deformability $\Lambda_{1.4}$ of $1.4M_{\odot}$ QSs have also been studied by using the ICQM model with DI-1700 and the CIDDM model with DI-250, which both predict the maximum mass of QSs as $2.16M_{\odot}$. The obtained $\Lambda_{1.4}$ are 264.032 and 353.421, respectively, and are thus consistent with the new constraints of the tidal deformability from GW170817.

Therefore, our results have shown that including isospin dependence of the quark mass can significantly influence the isospin properties of the quark matter and quark stars, and the quark matter symmetry energy should be large in strongly isospin-dependent quark matter in order to satisfy the new maximum mass constraints $2.01^{+0.04}_{-0.04} \leqslant M/M_{\odot} \leqslant 2.16^{+0.17}_{-0.15}$ within the ICQM model and the CIDDM model.

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