

**Modification of hyperon masses in nuclear matter**Ki-Hoon Hong,<sup>1,\*</sup> Ulugbek Yakshiev,<sup>1,†</sup> and Hyun-Chul Kim<sup>1,2,3,‡</sup><sup>1</sup>*Department of Physics, Inha University, Incheon 22212, Republic of Korea*<sup>2</sup>*Advanced Science Research Center, Japan Atomic Energy Agency, Shirakata, Tokai, Ibaraki 319-1195, Japan*<sup>3</sup>*School of Physics, Korea Institute for Advanced Study (KIAS), Seoul 02455, Republic of Korea*

(Received 15 June 2018; published 26 March 2019)

We investigate the properties of baryons within the framework of the in-medium modified SU(3) Skyrme model. The modification is performed by a minimal way, the medium functionals in the SU(2) sector being introduced. These functionals are then related to nuclear matter properties near the saturation point. The modifications in the SU(3) sector are performed by changing additionally kaon properties in nuclear matter. The results show that the properties of baryons in the strange sector are sensitive to the in-medium modifications of the kaon properties. We discuss the consistency of the in-medium modifications of hadron properties in this approach, comparing the present results with those from other models.

DOI: [10.1103/PhysRevC.99.035212](https://doi.org/10.1103/PhysRevC.99.035212)**I. INTRODUCTION**

Nucleons are known to undergo changes in nuclear matter due to the strong interaction with the nuclear environment. Since they themselves constitute nuclear matter, the medium modifications of nucleon properties bring about the changes of nuclear matter in a self-consistent manner. Similarly, a hyperon lying in nuclear matter is also altered. It is essential to understand how its attributes become different in a nuclear medium so that neutron stars and hypernuclei can be described in a more realistic way [1–8].

While a plethora of experimental and theoretical works on conventional nonstrange nuclear matter and its constituents in a wide range of nuclear matter densities has been compiled over decades, hyperons in nuclear matter have been relatively less studied [9–15]. Most of the works are based on the hyperon-nucleon ( $YN$ ) interactions. For example, Beane *et al.* [12] computed the  $n\Sigma^-$  scattering phase shifts using lattice QCD to quantify the energy shift of the  $\Sigma^-$  in nuclear matter. In Refs. [13,14], the  $YN$  potential was constructed from effective field theory and the Bruecker-Hartree-Fock (BHF) approximation was employed to investigate hyperons in nuclear matter. Density functional theories were also used to study the hyperons in nuclear matter (see a recent review, Ref. [15]).

In the present work, we propose yet another simple framework of investigating the mass shifts of the hyperons together with the nucleon and the  $\Delta$  isobar. Some years ago, it was studied how they underwent the changes in nuclear matter within the framework of the chiral topological soliton models [16–20], where the mass shifts of the nonstrange baryons were scrutinized and various in-medium modified form factors

were computed. The results were in qualitative agreement with those of other approaches and had interesting physical implications such as the stability and shape of the nucleon in nuclear medium.

Moreover, the in-medium modified SU(2) Skyrme model described very well properties of isospin asymmetric nuclear matter near the saturation point (nuclear density  $\rho_0 = 0.16 \text{ fm}^{-3}$ ). The model yielded successfully the equations of states (EoS) for nuclear matter at ordinary densities [21]. It predicted qualitatively various properties of nuclear matter in comparison with different theoretical approaches and empirical information. In particular, the parameters of the symmetric and asymmetric EoS determined from the present model were in qualitative and quantitative agreement with the empirical data [22], with those from Hartree-Fock approaches based on Skyrme interactions [23,24] and with those from different approaches presented in Refs. [25–28]. Furthermore, the extrapolations of the EoS at higher densities indicate that the model can describe rather well the state of matter that may exist in the interior of neutron stars. The results demonstrate that two solar mass neutron stars can be explained in the framework of the present approach [29].

In this context, it is of great interest and significance to extend the SU(2) version of the model to the SU(3) one in a straightforward and simple manner. So, we will generalize the previous analyses to investigate the hyperons in nuclear matter. We will employ an SU(3) Skyrme model developed in Ref. [30] and modify the relevant parameters of the model in nuclear matter. For simplicity, we first consider only the in-medium modification of meson dynamics in the SU(2) sector. However, the kaon is also known to undergo the changes in nuclear matter [31,32]. Thus, we alter the kaon properties in nuclear medium, assuming a simple linear-density approximation. While the dynamics in the SU(2) sector remained intact in the course of generalization to the SU(3) sector, the model still properly explains the phenomenology in the nonstrange sector as discussed in Refs. [21,29]. The present

\*kihoon@inha.edu

†yakshiev@inha.ac.kr

‡hchkim@inha.ac.kr

approach allows one to draw a simple conclusion as to how the in-medium modified kaon can influence the changes of the SU(3) baryons in nuclear matter.

The paper is organized as follows: In Sec. II, we recapitulate briefly an SU(3) Skyrme model in free space [30], where the SU(2) Skyrme model is extended into the SU(3) by a trivial embedding of the SU(2) chiral soliton into SU(3) [33]. In addition, we show how the strange sector incorporates the quantum fluctuations (see Subsec. II A). Then, we explain how the meson dynamics is altered in nuclear medium, based on the phenomenology in the nonstrange sector, and then discuss the in-medium changes of nucleon and  $\Delta$  isobar properties in Subsec. II B. The modification of kaon properties in nuclear matter is discussed in Subsec. II C. In Sec. III, we present and discuss the results. We first deal with the medium effects in the mesonic sector (Subsec. III A) and then we show how the modification of the mesonic sector brings about the density effects on the hyperons (Subsec. III B). The final section, Sec. IV, is devoted to the summary of the present work and outlook of possible developments of the present model in relation with the strangeness physics in various nuclear environments.

## II. THE MODEL

The SU(3) Skyrme models have been developed over decades. There are many variants of the model [34–40] (see a review [41] for extensive references). The main difference among the models comes mainly from specific methods as to how the strange sector is treated. We will follow an SU(3) Skyrme model developed in Ref. [30], because one can easily and transparently modify the model in nuclear medium.

### A. Baryons in free space

The standard SU(3) Skyrme model is based on the effective chiral Lagrangian written by

$$\begin{aligned} \mathcal{L} = & \mathcal{L}_{\text{WZ}} - \frac{F_\pi^2}{16} \text{Tr} L_\mu L^\mu + \frac{1}{32e^2} \text{Tr} [L_\mu, L_\nu]^2 \\ & + \frac{F_\pi^2}{16} \text{Tr} \mathcal{M} (U + U^\dagger - 2), \end{aligned} \quad (1)$$

where  $L_\mu = U^\dagger \partial_\mu U$  and  $U(\mathbf{x}, t)$  is a chiral field in SU(3). The mass matrix  $\mathcal{M}$  is defined in terms of the pion and kaon masses

$$\mathcal{M} = \begin{pmatrix} m_\pi^2 & 0 & 0 \\ 0 & m_\pi^2 & 0 \\ 0 & 0 & 2m_K^2 - m_\pi^2 \end{pmatrix}, \quad (2)$$

where  $m_\pi$  and  $m_K$  stand for the pion and kaon masses, respectively. The Wess-Zumino term [42]  $\mathcal{L}_{\text{WZ}}$  constrains the soliton to identify as a baryon, which is expressed by the five-dimensional integral over a disk  $D$

$$S_{\text{WZ}} = -\frac{iN_c}{240\pi^2} \int_D d^5\vec{x} \epsilon^{\mu\nu\alpha\beta\gamma} \text{Tr} (L_\mu L_\nu L_\alpha L_\beta L_\gamma). \quad (3)$$

Here the totally antisymmetric tensor  $\epsilon^{\mu\nu\alpha\beta\gamma}$  is defined as  $\epsilon^{01234} = 1$  and  $N_c = 3$  is the number of colors. The input parameters of the model are the pion decay constant

$F_\pi = 108.783$  MeV, the Skyrme parameter  $e = 4.854$ , and the masses of the  $\pi$  and  $K$  mesons, given respectively as  $m_\pi = 134.976$  MeV and  $m_K = 495$  MeV, which are taken close to the experimental data.

Classically, the model describes a set of absolutely stable topological solitons with the corresponding topological integer numbers that is identified as a baryon number  $B$ . The lowest-lying baryon states can be obtained by the zero-mode quantization of the soliton with baryon number  $B = 1$

$$U(\mathbf{r}, t) = \mathcal{A}(t) U_0(\mathbf{r}) \mathcal{A}(t)^\dagger, \quad (4)$$

where  $\mathcal{A}(t)$  is rotational matrix in SU(3). The time-independent soliton field  $U_0(\mathbf{r})$  is expressed as the trivial embedding of the SU(2) soliton into SU(3)

$$U_0(\mathbf{r}) = \begin{pmatrix} e^{i\tau \cdot \mathbf{n} F(r)} & 0 \\ 0 & 1 \end{pmatrix}, \quad \mathbf{n} = \frac{\mathbf{r}}{r}. \quad (5)$$

Note that the SU(2) soliton field satisfies the hedgehog ansatz. The profile function  $F(r)$  with the boundary conditions

$$F(0) = \pi, \quad F(\infty) = 0 \quad (6)$$

satisfies the classical field equations corresponding to the baryon number  $B = 1$  solution.

The model [30] is characterized in dealing with the time-dependent rotational matrix  $\mathcal{A}(t)$ . While the SU(2) rotation is restricted to the nonstrange sector represented by  $A(t)$ , the transformation along the strange sector is governed by the new matrix  $S(t)$ . Thus,  $\mathcal{A}(t)$  and  $S(t)$  can be expressed respectively as

$$\mathcal{A}(t) = \begin{pmatrix} A(t) & 0 \\ 0^\dagger & 1 \end{pmatrix} S(t), \quad (7)$$

$$A(t) = k_0(t) \mathbf{1} + i \sum_{a=1}^3 \tau_a k_a(t), \quad (8)$$

$$\begin{aligned} S(t) &= \exp \left\{ i \sum_{p=4}^7 k_p \lambda_p \right\} \\ &\equiv \exp(i\mathcal{D}) = \exp \left\{ \begin{pmatrix} 0 & i\sqrt{2}D \\ i\sqrt{2}D^\dagger & 0 \end{pmatrix} \right\}, \end{aligned} \quad (9)$$

with

$$D(t) = \frac{1}{\sqrt{2}} \begin{pmatrix} k_4(t) - ik_5(t) \\ k_6(t) - ik_7(t) \end{pmatrix}. \quad (10)$$

Here  $\tau_{1,2,3}$  denote the Pauli matrices, whereas  $\lambda_p$  stand for the strange part of the SU(3) Gell-Mann matrices.  $k_a(t)$  ( $a = 0, 1, 2, \dots, 7$ ) represent arbitrary collective coordinates. The matrix  $A(t)$  with the collective coordinates  $k_a$  ( $a = 0, 1, 2, 3$ ) stands for the rotational fluctuation of the SU(2) static soliton in the nonstrange sector. On the other hand, the matrix  $S(t)$  with the collective coordinates  $k_p$  ( $p = 4, 5, 6, 7$ ) describes the zero-mode fluctuation along the strangeness direction. Note that the Wess-Zumino term imposes a constraint on the eighth component which is related to the baryon number.

$S(t)$  in Eq. (9) can be systematically expanded in terms of matrix  $D(t)$  because  $\mathcal{D}(t)$  satisfies the relation

$$\mathcal{D}^3 = d^2 \mathcal{D}, \quad d^2 \equiv 2D^\dagger D.$$

We will perform the expansion and will keep lower orders in power of  $D$  terms (including  $D^4$  terms) in the Lagrangian. Having expanded  $S(t)$ , we obtain the time-dependent Lagrangian in the form

$$\begin{aligned}
 L = & -E_0 + 4\Phi\dot{D}^\dagger\dot{D} - \Gamma M^2 D^\dagger D \\
 & + \frac{iN_c}{2}(D^\dagger\dot{D} - \dot{D}^\dagger D) + \frac{1}{2}\Omega\omega^2 \\
 & + i(\Omega - 2\Phi)[D^\dagger(\boldsymbol{\omega} \cdot \boldsymbol{\tau})\dot{D} - \dot{D}^\dagger(\boldsymbol{\omega} \cdot \boldsymbol{\tau})D] \\
 & + 2\left(\Omega - \frac{4}{3}\Phi\right)(D^\dagger D)(\dot{D}^\dagger\dot{D}) \\
 & - \frac{1}{2}\left(\Omega - \frac{4}{3}\Phi\right)(D^\dagger\dot{D} + \dot{D}^\dagger D)^2 \\
 & + 2\Phi(D^\dagger\dot{D} - \dot{D}^\dagger D)^2 + \frac{2}{3}\Gamma M^2(D^\dagger D)^2 \\
 & - \frac{N_c}{2}[D^\dagger(\boldsymbol{\omega} \cdot \boldsymbol{\tau})D] \\
 & - \frac{iN_c}{3}(D^\dagger D)(D^\dagger\dot{D} - \dot{D}^\dagger D), \tag{11}
 \end{aligned}$$

where  $M^2 = m_K^2 - m_\pi^2$  and  $\boldsymbol{\omega}$  denotes the rotational velocity in SU(2), defined by

$$A^\dagger\dot{A} = \frac{1}{2}\boldsymbol{\omega} \cdot \boldsymbol{\tau}. \tag{12}$$

The energy of the static configuration  $E_0[F]$  is derived as

$$\begin{aligned}
 E_0[F] = & 4\pi \int_0^\infty dr r^2 \left\{ \frac{F_\pi^2}{8} \left( \frac{2\sin^2 F}{r^2} + F_r^2 \right) \right. \\
 & \left. + \frac{1}{2e^2} \frac{\sin^2 F}{r^2} \left( \frac{\sin^2 F}{r^2} + 2F_r^2 \right) + \frac{F_\pi^2 m_\pi^2}{2} \sin^2 \frac{F}{2} \right\}, \tag{13}
 \end{aligned}$$

where  $F_r \equiv \partial_r F$ . Minimizing this functional, we obtain the solutions of the field equations with the boundary conditions defined in Eq. (6).

The functional  $\Omega[F]$  arises from the rotations of the static soliton in the SU(2) sector, whereas the functionals  $\Phi[F]$  and  $\Gamma[F]$  explain the deviation into the strangeness sector. They are expressed as

$$\begin{aligned}
 \Omega[F] = & \frac{2\pi}{3} \int_0^\infty dr r^2 \sin^2 F \\
 & \times \left\{ F_\pi^2 + \frac{4}{e^2} \left( F_r^2 + \frac{\sin^2 F}{r^2} \right) \right\}, \tag{14}
 \end{aligned}$$

$$\begin{aligned}
 \Phi[F] = & \pi \int_0^\infty dr r^2 \sin^2 \frac{F}{2} \\
 & \times \left\{ F_\pi^2 + \frac{1}{e^2} \left( F_r^2 + \frac{2\sin^2 F}{r^2} \right) \right\}, \tag{15}
 \end{aligned}$$

$$\Gamma[F] = 4\pi \int_0^\infty dr r^2 F_\pi^2 \sin^2 \frac{F}{2}. \tag{16}$$

In order to quantize the soliton, one introduces the canonical momenta conjugate to the  $\omega_i$  and  $\dot{D}$ , which correspond respectively to the SU(2) rotation and the deviation to the

strangeness direction

$$(J_{ud})_i = \frac{\partial L}{\partial \omega^i}, \quad \Pi^\gamma = \frac{\partial L}{\partial \dot{D}_\gamma^\dagger}. \tag{17}$$

They satisfy the following commutation relations:

$$[(J_{ud})_i, \alpha^j] = \frac{1}{i}\delta_i^j, \quad \boldsymbol{\alpha} = \boldsymbol{\omega}, \tag{18}$$

$$[\Pi^\gamma, D_\beta^\dagger] = [\Pi_\beta^\dagger, D^\gamma] = \frac{1}{i}\delta_\beta^\gamma. \tag{19}$$

The angular momentum operator  $\mathbf{J}_{ud}$  and the momentum  $\Pi$  are derived as

$$\begin{aligned}
 \mathbf{J}_{ud} = & \Omega\boldsymbol{\omega} + i(\Omega - 2\Phi)(D^\dagger\boldsymbol{\tau}\dot{D} - \dot{D}^\dagger\boldsymbol{\tau}D) \\
 & - \frac{N_c}{2}D^\dagger\boldsymbol{\tau}D, \tag{20} \\
 \Pi = & 4\Phi\dot{D} - \frac{iN_c}{2}D - i(\Omega - 2\Phi)\boldsymbol{\omega} \cdot \boldsymbol{\tau}D \\
 & - \left(\Omega - \frac{4}{3}\right)(D^\dagger\dot{D} + \dot{D}^\dagger D)D \\
 & - 4\Phi(D^\dagger\dot{D} - \dot{D}^\dagger D)D + \frac{1}{3}N_c(D^\dagger D)D \\
 & + 2\left(\Omega - \frac{4}{3}\Phi\right)(D^\dagger D)\dot{D}. \tag{21}
 \end{aligned}$$

Then, we obtain the collective Hamiltonian to order  $N_c^{-1}$  as follows:

$$\begin{aligned}
 H = & E_0 + \frac{1}{4\Phi}\Pi^\dagger\Pi + \left(\Gamma M^2 + \frac{N^2}{16\Phi}\right)D^\dagger D \\
 & - \frac{iN_c}{8\Phi}(D^\dagger\Pi - \Pi^\dagger D) + \frac{1}{2\Omega}\mathbf{J}_{ud}^2 + \frac{N_c}{4\Phi}D^\dagger\mathbf{J}_{ud} \cdot \boldsymbol{\tau}D \\
 & + i\left(\frac{1}{2\Omega} - \frac{1}{4\Phi}\right)(D^\dagger\mathbf{J}_{ud} \cdot \boldsymbol{\tau}\Pi - \Pi^\dagger\mathbf{J}_{ud} \cdot \boldsymbol{\tau}D) \\
 & + \left(\frac{1}{2\Omega} - \frac{1}{3\Phi}\right)\left[(D^\dagger D)(\Pi^\dagger\Pi) - \frac{1}{4}(D^\dagger\Pi + \Pi^\dagger D)^2\right] \\
 & - \frac{1}{8\Phi}(D^\dagger\Pi - \Pi^\dagger D)^2 - i\frac{N_c}{8\Phi}(D^\dagger\Pi - \Pi^\dagger D)D^\dagger D \\
 & + \left(\frac{N_c^2}{12\Phi} - \frac{2}{3}\Gamma M^2\right)(D^\dagger D)^2. \tag{22}
 \end{aligned}$$

The collective Hamiltonian can be diagonalized by introducing the creation and annihilation operators instead of  $D$  and  $\Pi$ ,

$$D = \frac{1}{\sqrt{N_c}}\left(1 + \frac{M^2}{M_0^2}\right)^{-1/4}(a + b^\dagger), \tag{23}$$

$$\Pi = -\frac{i}{2}\sqrt{N_c}\left(1 + \frac{M^2}{M_0^2}\right)^{1/4}(a - b^\dagger), \tag{24}$$

where  $M_0$  is defined as  $M_0 = N_c/(4\sqrt{\Phi\Gamma})$ . The operators  $a^\dagger(a)$  and  $b^\dagger(b)$  denote respectively the creation (annihilation) operators of the strange quark and antiquark, respectively. The strangeness and the angular momentum of the strange quark

are given respectively by

$$s = b^\dagger b - a^\dagger a, \quad \mathbf{J}_s = \frac{1}{2}(a^\dagger \boldsymbol{\tau} a - b \boldsymbol{\tau} b^\dagger). \quad (25)$$

Then the normal-ordered Hamiltonian to order  $N_c^0$  is derived as

$$H = E_0 + \omega_- a^\dagger a + \omega_+ b^\dagger b, \quad (26)$$

where

$$\omega_\pm = \frac{N_c}{8\Phi} \left( \sqrt{1 + \frac{16\Phi\Gamma}{N_c^2} M^2} \pm 1 \right). \quad (27)$$

Since we are interested in baryons containing only the strange quarks, not antiquarks, we will ignore the  $\omega_+$  term in the Hamiltonian. Furthermore, we also neglect the quartic terms in the kaon field because classical dynamics in the mesonic sector is still restricted to the pion-pion interaction. Thus, ignoring the corresponding terms related to the kaon-kaon interaction in Eq. (22), we arrive at the final expressions of the collective Hamiltonian

$$H = E_0 + \omega_- a^\dagger a + \frac{1}{2\Omega} (\mathbf{J}_{ud} + c\mathbf{J}_s)^2, \quad (28)$$

where  $c$  is defined as

$$c = 1 - \frac{4\Omega\omega_-}{8\Phi\omega_- + N_c}. \quad (29)$$

Sandwiching the collective Hamiltonian between the eigenstates  $|n_s\rangle|I, J\rangle$  with the definite quantum numbers such as isospin  $I$ , total angular momentum  $J$ , and given number of strange quarks, we obtain the final mass formula of the SU(3) baryons

$$M = E_0 - s\omega_- + \frac{1}{2\Omega} \left\{ cJ(J+1) + (1-c)I(I+1) + \frac{c(c-1)}{4} s(s-2) \right\}. \quad (30)$$

More details of the model in free space can be found in Refs. [30,43].

### B. Baryons in nuclear matter

We now show how to implement the medium effects into the SU(3) Skyrme model. For simplicity, we will first take into account a modification of meson dynamics in the SU(2) sector, introducing the medium functionals into the effective chiral Lagrangian, based on the low-energy phenomenology in nuclear medium [44]. As we mentioned already, the SU(2) Skyrme model was parametrized in terms of the density functionals and was applied successfully to the description of properties of the nucleon and  $\Delta$  isobar near the normal nuclear matter density  $\rho_0$  [21]. The model was even well extrapolated to higher density regions [29].

In Ref. [21], the in-medium modified SU(2) Skyrme model was discussed in detail, with isospin symmetric and asymmetric infinite nuclear matter being considered. The effective

chiral Lagrangian is modified as follows:

$$\begin{aligned} \mathcal{L} = & -\frac{F_\pi^2}{16} \alpha_2^t(\rho) \text{Tr} L_0 L_0 + \frac{F_\pi^2}{16} \alpha_2^s(\rho) \text{Tr} L_i L_i \\ & - \frac{\alpha_4^t(\rho)}{16e^2} \text{Tr}[L_0, L_i]^2 + \frac{\alpha_4^s(\rho)}{32e^2} \text{Tr}[L_i, L_j]^2 \\ & + \frac{F_\pi^2}{16} \alpha_{\chi_{SB}}(\rho) \text{Tr} \mathcal{M}(U + U^\dagger - 2), \end{aligned} \quad (31)$$

where  $\alpha_2^t(\rho)$ ,  $\alpha_2^s(\rho)$ ,  $\alpha_4^t(\rho)$ ,  $\alpha_4^s(\rho)$ , and  $\alpha_{\chi_{SB}}(\rho)$  denote the functionals of the nuclear matter density, which reflect the changes of meson properties in nuclear medium. In principle, they should be defined in a self-consistent way. However, it will be extremely difficult to determine them self-consistently, in particular when one considers real nuclei with respect to their in-medium modified constituents. Therefore, we simply assume these medium functionals to be external functions of nuclear matter density  $\rho$ . Then we are able to study properties of a single baryon in nuclear matter. This assumption is a rather plausible one, as far as we are interested in homogenous infinite nuclear matter. The medium functionals can indeed be considered as simple external parameters at a given density so that one can carry out the calculations in a easy manner. Furthermore, the density-dependent parameters can be related to the properties of infinite nuclear matter, so that one can partially restore the self-consistency of the model [21].

In the present work, we will generalize the method developed in Ref. [21]. The in-medium modified Lagrangian in SU(3) will be modified as done in Eq. (31), the Wess-Zumino Lagrangian  $\mathcal{L}_{WZ}$  being included. However, we note that the Wess-Zumino term should not be modified in nuclear matter, since the topology of the model must be kept intact such that the baryon number is preserved. So, the Wess-Zumino term is modified in nuclear matter only inexplicitly through the medium modification of the solutions with the same baryon number in nuclear matter.

We want to mention an important aspect of the present approach. The Skyrme model is based on a truncated version of the most general effective chiral Lagrangian. It indicates that the contributions from higher order terms enter tacitly into the parameter of the Skyrme term. This means that the parameter carries the effects of the higher order contributions effectively. Therefore, the in-medium modified Skyrme model keeps already almost all the necessary ingredients and in principle could be a relevant theoretical framework to study nuclear many-body problems, at least to a qualitative extent. For example, the in-medium Skyrme term plays an essential role in stabilizing the nucleon even in nuclear matter. The Skyrme term brings about the repulsive nature in the inner part of the nucleon [16,19], which assures the stability of the nucleon. It implies that when the density of nuclear matter grows, higher order terms of the effective chiral Lagrangian will definitely come into play and are required so that the collapse of nuclear matter to a singularity [45] be avoided. Therefore, the effect of higher order derivative terms is incorporated by introducing the density-dependent parameter in the Skyrme term.

If the functionals are taken to be functions of nuclear-matter density  $\alpha(\rho)$ , then all the functional parameters are

reduced to the simple external parameters in an infinite and homogeneous nuclear matter approximation. We will follow in this work the method developed in a previous work [21]. First, we introduce a *convenient* relation between the medium functions in the following way:

$$\alpha_2^t = \alpha_4^t \alpha_2^s (\alpha_4^s)^{-1}, \quad (32)$$

which reduces the number of the external density-dependent parameters to four different parameters. Furthermore, these remaining four density-dependent parameters can be related to each other, so that the three independent parameters can be defined as follows:

$$1 + C_1 \lambda = f_1(\lambda) \equiv \sqrt{\alpha_2^s \alpha_4^s}, \quad (33)$$

$$1 + C_2 \lambda = f_2(\lambda) \equiv \sqrt{\frac{\alpha_{\chi SB} \alpha_4^s}{(\alpha_2^s)^2}}, \quad (34)$$

$$1 + C_3 \lambda = f_3(\lambda) \equiv \left(\frac{\alpha_2^s}{\alpha_4^s}\right)^{\frac{3}{2}} \frac{1}{\alpha_2^s}, \quad (35)$$

where  $\lambda = \rho/\rho_0$ . This reduction allows us to keep the medium modification of the parameters simpler and more general.

By defining Eq. (32) and by introducing Eqs. (33)–(35), an algebraic manipulations become much simplified and yield convenient and transparent forms of the final expressions. Then, we can perform the main part of calculations such as the minimization, the quantization, and so on, in terms of the three independent density-dependent functions  $f_{1,2,3}$ .

The numerical values of these parameters are fixed to be  $C_1 = -0.279$ ,  $C_2 = 0.737$ , and  $C_3 = 1.782$ . They reproduce well the EoS for symmetric nuclear matter near  $\rho_0$  and at higher densities that may exist in the interior of a neutron star [21,29]. The parameters of the present model are completely fixed in nuclear matter except for the strangeness direction. So, we will then introduce the modification of the kaon properties after the quantization, which will be discussed in Subsec. II C.

While the form of the baryon mass formula is kept to be the same as in Eq. (30), it becomes now density dependent by the functions  $f_{1,2,3}$ <sup>1</sup>

$$M^* = E_0^* - s\omega_-^* + \frac{1}{2\Omega^*} \left[ c^* J(J+1) + (1 - c^*) I(I+1) + \frac{c^*(c^* - 1)}{4} s(s-2) \right], \quad (36)$$

where  $\omega_-^*$  and  $c^*$  are changed as

$$\omega_-^* = \frac{N_c}{8\Phi^*} \left( \sqrt{1 + \frac{16\Phi^*\Gamma^*}{N_c^2} M^{*2}} - 1 \right), \quad (37)$$

$$c^* = 1 - \frac{4\Omega^*\omega_-^*}{8\Phi^*\omega_-^* + N_c}, \quad (38)$$

<sup>1</sup>Quantities with the asterisk (\*) in the expressions stand for those modified in nuclear medium in terms of the density-dependent functions  $f_{1,2,3}$ .

where  $M^{*2} = m_K^{*2} - m_\pi^2$ . In Eq. (38), the value of the kaon mass is released from the experimental data in free space by considering the medium effects. The classical soliton mass  $E_0^*$ ,  $\omega_-^*$ , and  $c^*$  are expressed respectively as

$$E_0^* = f_1 \frac{4\pi F_\pi}{e} \int_0^\infty dx x^2 \left\{ \frac{1}{8} \left( F_x^2 + \frac{2 \sin^2 F}{x^2} \right) + \frac{\sin^2 F}{x^2} \left( F_x^2 + \frac{\sin^2 F}{2x^2} \right) + \frac{\beta^2}{2} \sin^2 \frac{F}{2} \right\}, \quad (39)$$

$$\Omega^* = f_3^{-1} \frac{2\pi}{3e^3 F_\pi} \int_0^\infty dx x^2 \sin^2 F \left\{ 1 + 4 \left( F_x^2 + \frac{\sin^2 F}{x^2} \right) \right\}, \quad (40)$$

$$\Phi^* = f_3^{-1} \frac{\pi}{e^3 F_\pi} \int_0^\infty dx x^2 \sin^2 \frac{F}{2} \left\{ 1 + \left( F_x^2 + \frac{2 \sin^2 F}{x^2} \right) \right\}, \quad (41)$$

$$\Gamma^* = f_1 f_2^2 \frac{4\pi}{e^3 F_\pi} \int_0^\infty dx x^2 \sin^2 \frac{F}{2}, \quad (42)$$

where we have introduced a parameter  $\beta = f_2 m_\pi / e F_\pi$  and a dimensionless variable  $x = e F_\pi (\alpha_2^s / \alpha_4^s)^{1/2} r$ . Other aspects of the medium modifications can be found in Ref. [21] and references therein.

### C. Kaon properties in nuclear matter

We are now in a position to deal with the change of kaon properties in nuclear matter. As seen in Eq. (38), an additional medium modification was implemented by the kaon mass in nuclear matter. Before we carry out the explicit calculation of the SU(3) baryon masses in nuclear matter, we need to explain how the kaon properties undergo the change in nuclear environment. To be more consistent, one should consider how the kaon propagator is altered in nuclear matter, which arises from the polarization effects, as done for that of the pion in nuclear matter [44]. The polarization operator can be described phenomenologically by introducing a kaon-nucleus optical potential. The properties of this optical potential may be related either to the phenomenology of kaon-nucleus scattering or to the properties of kaonic atoms as done for the nonstrange sector (see, e.g., Ref. [21] and references therein). Because of a lack of the experimental data, it is, however, rather difficult to extract information on how the kaon properties are varied in nuclear matter. Thus, instead of conducting such a complicated analysis, we will rather take into account a simple modification of the kaon properties after the quantization in the present work, keeping dynamics of the mesonic sector intact in nuclear medium. Since it is known that the kaon mass drops off in dense matter [46,47], we will consider only the change of the kaon mass in the present work as a minimal modification of the kaon properties in nuclear matter.

Here we note that the in-medium modified Lagrangian in Eq. (31) can be reformulated in terms of the in-medium modified pion decay constants and the Skyrme



parameters

$$\begin{aligned}
 F_{\pi,t}^* &= F_\pi \sqrt{\alpha_2^t}, & F_{\pi,s}^* &= F_\pi \sqrt{\alpha_2^s}, \\
 e_t^* &= \frac{e}{\sqrt{\alpha_4^t}}, & e_s^* &= \frac{e}{\sqrt{\alpha_4^s}}, \\
 m_\pi^* &= m_\pi \sqrt{\frac{\alpha_{\chi SB}}{\alpha_2^s}}.
 \end{aligned} \tag{43}$$

Then the change of SU(2) dynamics in nuclear matter can be understood as the medium modification of the input parameters.

In the SU(3) case, we need to modify the kaon decay constant and the kaon mass in addition to the pion observables. Within the present framework, the kaon decay constant  $F_K$  is assumed to be equal to the pion decay constant  $F_\pi$  in free space. This is a reasonable consideration, though the value of  $F_K$  is larger than that of  $F_\pi$ . At least, these two constants become equal in the SU(3) symmetric case. If we assume that the modified kaon decay constant will have exactly the same form of the pion decay constant in nuclear medium  $F_K^* = F_{\pi,s}^*$ , Eq. (43) implies that the kaon mass may be also modified in nuclear matter as  $m_K^* = m_K \sqrt{\alpha_{\chi SB}/\alpha_2^s} = m_K f_2 \alpha_2^s / f_1$ . Hence, we consider the following parametrization:

$$m_K \rightarrow \frac{f_1}{f_2 (\alpha_2^s)^{3/2}} (1 - C\lambda) m_K, \tag{44}$$

which has a simple meaning and can be interpreted as follows:  $C = 0$  corresponds to the situation in which the kaon properties do not change in nuclear matter at all, i.e.,  $F_K^* m_K^* = F_K m_K$ . On the other hand, if  $C \neq 0$ , then those of the kaon are linearly varied in nuclear matter, which is in line with what was observed in Refs. [46,47]. Thus, the mass term in the effective chiral Lagrangian is changed as

$$F_\pi^2 \mathcal{M} \rightarrow F_\pi^{*2} m_\pi^{*2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{2F_K^{*2} m_K^{*2}}{F_\pi^{*2} m_\pi^{*2}} - 1 \end{pmatrix}, \tag{45}$$

where

$$F_K^* m_K^* = F_\pi m_K (1 - C\lambda). \tag{46}$$

In the present work, we consider  $C$  as an arbitrary external parameter. In a more consistent approach, its value can be adjusted according to the data on kaon-nucleus scattering or can be related to those on the kaonic atom. The medium modification in Eq. (46) can be explained in terms of the alteration of the kaon decay constant and/or of the kaon mass.

### III. RESULTS AND DISCUSSIONS

#### A. Density dependence of the low-energy constants

In order to discuss the density dependence of input parameters in the mesonic sector according to the definitions in Eq. (43), one should fix the forms of the density-dependent functions. We see from Eqs. (33)–(35) that at least one of the density-dependent functions must be adjusted to fit the explicit forms of the four functions  $\alpha_2^s$ ,  $\alpha_4^s$ ,  $\alpha_{\chi SB}$ , and  $\alpha_2^t$ . There are many possible ways of modifying the functions, since

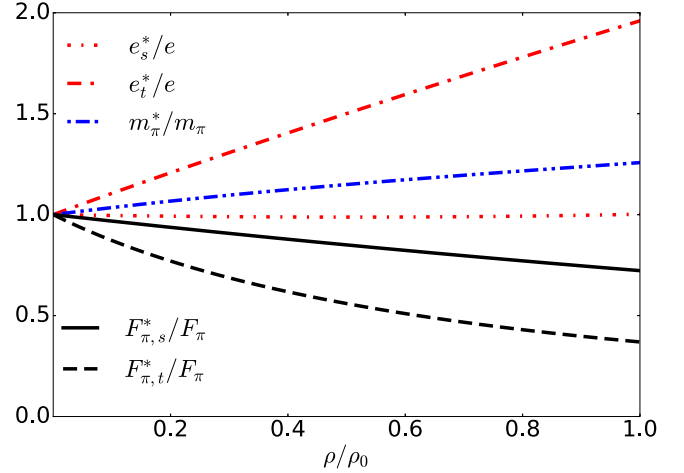


FIG. 1. The density dependence of the input parameters that are defined in Eq. (43). The results shown are those normalized relatively to their values in free space.

we have only three independent relations between the four density-dependent functions. Once the forms of the functions are fixed, then we can discuss the density dependence of the input parameters of the model. Following Ref. [48], we try the following form:

$$\alpha_2^s = \exp(-0.65\lambda). \tag{47}$$

Then, the other three density-dependent functions can be also fixed. Moreover, one can fix the form of  $\alpha_4^t$  from Eq. (32). The parametrization of Eq. (47) is consistent with the data on low-energy pion-nucleus scattering and pionic atoms at low densities [44].

Now we discuss the density dependence of low-energy constants that come into play as the input parameters in the present model. The results are drawn in Fig. 1. One can see that the parametrization given in Eq. (47) makes the pion mass increased as the density increases. Both the temporal and spatial parts of the in-medium pion decay constant,  $F_{\pi,t}^*$  and  $F_{\pi,s}^*$ , fall off as the density increases (see the solid and dashed curves). These results are in qualitative agreement with those from chiral perturbation theory [49,50] and QCD sum rules [51]. We refer to Ref. [21] for a detailed discussion about the consistency of the results in the present approach and the comparison with other works. Nevertheless, we want to note that in contrast to the mentioned works, the temporal part of the pion decay constant falls off faster than the spatial one in the present work. This comes from the fact that  $\alpha_2^s$  is chosen as in Eq. (47), which is consistent with the data on pionic atoms only at low densities [44]. If one changes the density dependence of  $\alpha_2^s$ , then the dependence of the pion decay constants also will be altered.

Since the Skyrme parameter  $e$  is related to the  $g_{\rho\pi\pi}$  coupling constant, its change in nuclear matter is deeply related to those of the  $\rho$ -meson width and its mass [17,19,20]. Interestingly,  $e_s^*$  is almost constant up to normal nuclear matter density  $\rho_0$ . This result is a plausible one, because the in-medium change of  $e_s^*$  characterizes how the inner core of the Skyrmion undergoes the change. On the other hand, the spatial

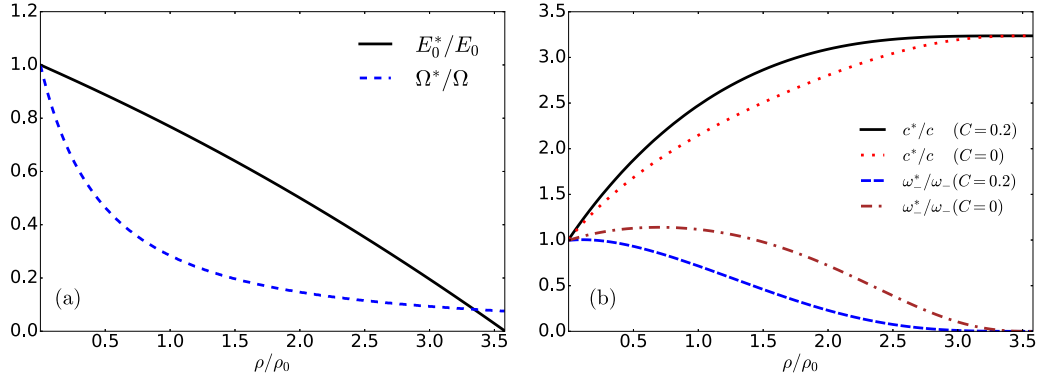


FIG. 2. The ratio of functionals to their free space values as functions of  $\rho/\rho_0$ : (a)  $E_0^*/E_0$  (solid curve) and  $\Omega^*/\Omega$  (dashed curve) are independent of the kaon properties; (b)  $c^*/c$  and  $\omega_-^*/\omega_-$  are dependent on the kaon properties. In the figure (b), the solid and dashed curves correspond to the results of  $c^*/c$  and  $\omega_-^*/\omega_-$  with  $C = 0.2$ , whereas the dotted and dot-dashed ones illustrate those of  $c^*/c$  and  $\omega_-^*/\omega_-$  with  $C = 0$ , respectively.

part of the pion decay constant  $F_{\pi,s}^*$  governs the outer shell of Skyrmion. Figure 1 shows that the parameter  $e_s^*$  remains almost constant by the surrounding nuclear environment up to the normal nuclear matter density (see the dotted curve in Fig. 1). Further,  $e_s^*$  starts to increase faster at higher densities. At  $\rho_0$ , we get the result  $(F_{\pi,s}^*/F_\pi)^2 \approx 0.52$ , which crudely explains that the mass contribution from the outer shell of the soliton in a static approximation is decreased by about 50%.<sup>2</sup> For comparison, we mention that the corresponding contribution from the inner core remains almost the same, i.e.,  $(e_s^*/e)^{-2} \approx 1$ .<sup>3</sup>

The parameter  $e_t^*$ , which is related to the quantum fluctuations of the core of the spinning Skyrmion, rises faster, as  $\rho/\rho_0$  increases. However, its change in nuclear matter is smaller in comparison with the density dependence of  $F_{\pi,t}^*$ . The latter one is related to the quantum fluctuations of the outer shell of the Skyrmion. The temporal part of the pion decay constant is changed to be  $(F_{\pi,t}^*/F_\pi)^2 \approx 0.14$  at  $\rho_0$  while the corresponding temporal part is altered to be as  $(e_t^*/e)^{-2} \approx 0.26$ . Thus, in general, we conclude that the outer shell of the Skyrmion is modified larger than its inner core. Figure 1 reveals clearly that the temporal parts of the constants change more strongly in nuclear medium than the spatial ones. These results demonstrate that the quantum fluctuations  $\approx 1/\Omega^*$  become more pronounced in nuclear medium than in free space.

Concerning the change of the kaon properties, we will regard  $C$  as a free parameter and will examine how it affects the masses of SU(3) baryons in nuclear matter.

### B. Density dependence of the masses of the lowest lying SU(3) baryons

Since the classical energy of the Skyrmion and its moment of inertia constitute essential parts of the baryon masses, we

first discuss the density dependence of these two quantities. Figure 2(a) depict the relative classical energy  $E_0^*/E_0$  and the relative moment of inertia  $\Omega^*/\Omega$  as functions of  $\rho/\rho_0$ . While  $E_0^*/E_0$  decreases slowly as the density increases,  $\Omega^*/\Omega$  falls off drastically until  $\rho$  reaches a half value of normal nuclear matter density. With the further increasing density,  $E_0^*/E_0$  decreases in the same manner and  $\Omega^*/\Omega$  starts to diminish slowly. At normal nuclear matter density,  $E_0^*$  is decreased by about 20%. On the other hand,  $\Omega^*$  drops off by about 80%, which shows that the rotational  $1/N_c$  corrections increases as the density increases. As a result, the nuclear matter becomes stabilized around the saturation density  $\rho_0$ . At higher densities, these functions describe the stiffness of the equations of state for nuclear matter. Note that these two quantities,  $E_0$  and  $\Omega$ , are not at all influenced by the change of the kaon properties. The consequence of this behavior will be discussed soon.

Concerning the parameters in the strangeness sector, i.e.,  $c^*$  and  $\omega_-^*$ , we will present the results for two different cases:

- (i) We do not change the kaon properties in nuclear matter, i.e.,  $F_\pi^* m_K^* = F_\pi m_K$  or  $C = 0$ .
- (ii) We make  $F_\pi^* m_K^*$  decreased linearly as the density of nuclear matter increases. This corresponds to the value  $C = 0.2$ .

By doing this, we can see how the change of the kaon properties affect the mass shift of the SU(3) baryons. In Fig. 2(b), the results of  $c^*/c$  and  $\omega_-^*/\omega_-$  are depicted as functions of  $\rho/\rho_0$  with the above-mentioned two different cases considered. When we turn on the value of  $C$ ,  $c^*/c$  increases faster than that with  $C = 0$ . The behavior of  $\omega_-^*/\omega_-$  is also changed when  $C = 0.2$  is taken. If one switches off  $C$ ,  $\omega_-^*/\omega_-$  starts to increase first and then falls off slowly, as the density increases. However, when one uses  $C = 0.2$ ,  $\omega_-^*/\omega_-$  drops off monotonically, which is distinguished from the case with  $C = 0$ . As will be shown below, this change with the finite value of  $C$ , i.e., the change of the kaon properties, will have a clear effect on the mass splitting of the baryon octet in nuclear matter. The physical meaning of  $\omega_-^*$  is the quantum fluctuation along the strangeness direction. So, it plays an essential role in determining the hyperon masses as shown

<sup>2</sup>There will be also an inexplicit change due to the in-medium modified profile function, which is found to be small.

<sup>3</sup>Note that the contribution from the Skyrme term is proportional to the inverse square of the Skyrme parameter.

TABLE I. Values of the density-dependent Skyrmion functionals at normal nuclear matter density  $\rho_0$  in comparison with those in free space. Those of the functionals with nonzero strangeness are presented with the two different values of  $C$  taken into account.

Skyrmion functionals	Free space values	Values at $\rho = \rho_0$	
		$C = 0$	$C = 0.2$
$E_0^*$ [MeV]	865.60	665.04	665.04
$\Omega^*$ [MeV $^{-1}$ ]	$5.116 \times 10^{-3}$	$1.453 \times 10^{-3}$	$1.453 \times 10^{-3}$
$\Phi^*$ [MeV $^{-1}$ ]	$1.852 \times 10^{-3}$	$5.000 \times 10^{-4}$	$5.000 \times 10^{-4}$
$\Gamma^*$ [MeV $^{-1}$ ]	$3.995 \times 10^{-3}$	$5.442 \times 10^{-3}$	$5.442 \times 10^{-3}$
$\omega_-^*$ [MeV]	202.44	226.03	144.53
$c^*$	0.309	0.664	0.765

in Eq. (36). On the other hand,  $c^*$  is related to the isospin splitting within the same multiplet when the strangeness is equal to zero. Of course, it provides a certain contribution to the hyperon masses [see Eq. (36)].

In Table I, we list the values of the density-dependent Skyrmion functionals at normal nuclear matter density  $\rho_0$ , comparing them with those in free space. As mentioned already,  $E_0^*$  and  $\Omega^*$  are reduced approximately by 20% and 80%, respectively, at  $\rho_0$  in comparison with the corresponding values in free space. Note that the functionals  $\Phi^*$  and  $\Gamma^*$  have no explicit influence on the baryon masses but they influence the values of the other two functionals  $\omega_-^*$  and  $c^*$ . Moreover, they do not depend on  $C$ . When  $C$  is turned off, the value of  $\omega_-^*$  increases by about 12% at  $\rho_0$ , compared with that in free space. However, if one considers the in-medium changes of the kaon properties by taking  $C = 0.2$ ,  $\omega_-^*$  is reduced by about 29%. On the other hand,  $c^*$  increases for both cases. As expected, the changes of the kaon properties in nuclear matter indeed influence the quantities in the strangeness direction and will consequently affect the characteristics of the hyperons in nuclear matter.

In Table II, we list the results of the masses of the baryon octet and decuplet both in free space and in nuclear matter at  $\rho_0$ . The values of the nucleon mass in free space and in

TABLE II. Results of the masses of the baryon octet and decuplet both in free space and in nuclear matter at  $\rho_0$  in units of MeV. Note that the masses of the nucleon and the  $\Delta$  isobar are used as input, which are marked by asterisk (\*) as the superscripts of the corresponding numbers.

Baryon	Experimental mass	Free space mass	Mass at $\rho = \rho_0$	
			$C = 0$	$C = 0.2$
$N$	939	939*	923*	923*
$\Lambda$	1115	1075	1004	960
$\Sigma$	1189	1210	1236	1122
$\Xi$	1315	1302	1221	1088
$\Delta$	1232	1232*	1956	1956
$\Sigma^*$	1385	1301	1921	1912
$\Xi^*$	1530	1392	1906	1878
$\Omega$	1672	1508	1911	1854

nuclear matter at  $\rho_0$ , and the mass of the  $\Delta$  isobar in free space are used as input parameters of the model. The masses of the nucleon and  $\Delta$  in free space are employed to fix the values of pion decay constant and Skyrme parameter in free space. The in-medium mass of nucleon at  $\rho_0$  fixes the one of the density-dependent functions  $f_{1,2,3}$ .

We find that in general the masses of the baryon octet tend to decrease in nuclear matter except for that of  $\Sigma$ , which increases with  $C = 0$  but drops off with  $C = 0.2$  considered. The mass of the  $\Lambda$  is changed as  $m_\Lambda^*/m_\Lambda \approx 0.93$  for  $C = 0$ , whereas  $m_\Lambda^*/m_\Lambda \approx 0.89$  for  $C = 0.2$ . It is interesting to compare the present results with that from SU(3) chiral effective field theory [14] in which  $m_\Lambda^*/m_\Lambda \approx 0.73$  was obtained. The mass of the  $\Xi$  hyperon is changed in a similar manner:  $m_\Xi^*/m_\Xi \approx 0.94$  for  $C = 0$  and  $m_\Xi^*/m_\Xi \approx 0.84$  for  $C = 0.2$ , respectively, both of which are more reduced in nuclear matter in comparison with that from the quark-meson coupling model with the bag radius of the free nucleon  $R_0 = 0.8$  fm, i.e.,  $m_\Xi^*/m_\Xi \approx 0.98$  [9]. Thus, the present results of the  $\Lambda$  and  $\Xi$  mass dropping are in qualitatively agreement with those from the other approaches.

In contrast with the masses of the baryon octet, those of the decuplet are increased drastically, as the density of nuclear matter increases. This can be understood from Eq. (36). The second term of Eq. (36) makes the baryon decuplet split from the octet. As shown already in Fig. 2(a), the moment of inertia  $\Omega^*$  drops off rapidly as the density of nuclear matter increases, which makes the second term of Eq. (36) increase very fast. This brings about the drastic increment of the spin-3/2 hyperon masses. When  $C = 0.2$  is used, the masses of the hyperon decuplet still increase but are found to be smaller than the case with  $C = 0$ .

Theoretically, it is of more interest to study the density effects on the mass splittings of the hyperons, since soliton models predict them quantitatively in comparison with the experimental data. We first express the formulas for the mass splittings of the hyperon octet, given as

$$m_\Sigma^* - m_\Lambda^* = \frac{1 - c^*}{\Omega^*}, \quad (48)$$

$$m_\Xi^* - m_\Sigma^* = \omega_-^* + \frac{5(c^* - 1)(c^* + 1)}{8\Omega^*}, \quad (49)$$

$$m_\Lambda^* - m_N^* = \omega_-^* + \frac{3(c^* - 1)(c^* + 1)}{8\Omega^*}, \quad (50)$$

and the hyperon decuplet, written by

$$m_{\Sigma^*}^* - m_\Delta^* = \omega_-^* + \frac{(c^* - 1)(3c^* + 7)}{8\Omega^*}, \quad (51)$$

$$m_{\Xi^*}^* - m_{\Sigma^*}^* = \omega_-^* + \frac{5(c^* - 1)(c^* + 1)}{8\Omega^*}, \quad (52)$$

$$m_\Omega^* - m_{\Xi^*}^* = \omega_-^* + \frac{(c^* - 1)(7c^* + 3)}{8\Omega^*} \quad (53)$$

in nuclear matter.

In Fig. 3(a), the mass splittings of the hyperon octet are drawn without changing the kaon properties in nuclear matter, whereas in Fig. 3(b), those are depicted with  $C = 0.2$  used.



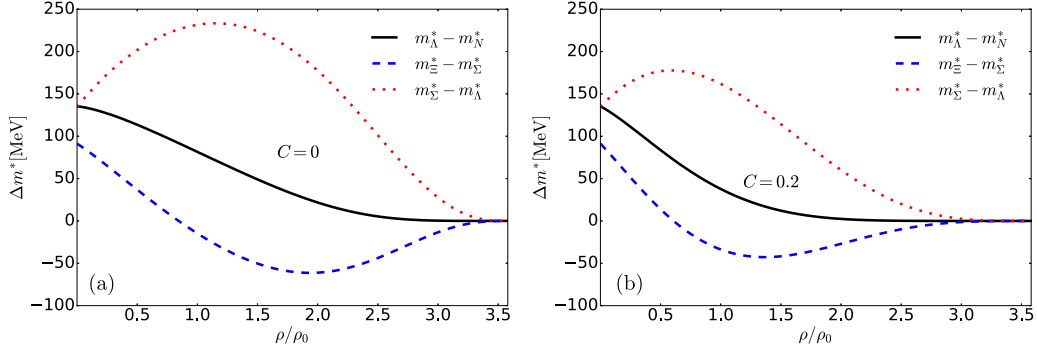


FIG. 3. The density dependence of the mass splittings of the baryon octet,  $m_{\Lambda}^* - m_N^*$ ,  $m_{\Sigma}^* - m_{\Sigma}^*$ , and  $m_{\Sigma}^* - m_{\Lambda}^*$ , are illustrated by the solid, dotted, and dashed curves, respectively. The figure (a) corresponds to  $C = 0$  and (b) corresponds to  $C = 0.2$ , respectively.

The general density dependence of the mass splittings is not much changed by introducing the finite  $C$ ; one can clearly see that the magnitude of the mass splittings is reduced when  $C = 0.2$  is employed. As seen in Fig. 3, the result of  $m_{\Sigma}^* - m_{\Lambda}^*$  illustrated in the dotted curve exhibits a different density dependence. Since  $\Sigma$  and  $\Lambda$  have the same strangeness,  $\omega_-^*$  is not involved in this splitting. As shown in Eq. (48),  $m_{\Sigma}^* - m_{\Lambda}^*$  is proportional to  $1 - c^*$  and  $1/\Omega^*$ . As the density increases, both the numerator and denominator start to decrease but  $\Omega^*$  falls off much faster. Thus, the mass splitting  $m_{\Sigma}^* - m_{\Lambda}^*$  grows larger until the density reaches  $\rho \approx 1.2\rho_0$  ( $\rho \approx 0.7\rho_0$ ) in the case of  $C = 0$  ( $C = 0.2$ ), and then it drops off until  $\rho \approx 3.5\rho_0$  is reached.

The mass splitting  $m_{\Lambda}^* - m_N^*$  falls off monotonically as the density increases, whereas  $m_{\Sigma}^* - m_{\Sigma}^*$  lessens until  $\rho \approx 2\rho_0$  with  $C = 0$  ( $\rho \approx 1.5\rho_0$  with  $C = 0.2$ ). Note that all the masses of baryon octet become degenerate when the density reaches  $\rho \approx 3.5\rho_0$ . It implies that the SU(3) flavor symmetry is restored around  $3.5\rho_0$  within the present framework. Interestingly, if we include the in-medium changes of the kaon properties, it brings about the degeneracy of the masses at lower densities.

Figure 4 represents the numerical results of the mass splittings of the baryon decuplet. The general tendency of the results is in line with that of the  $m_{\Sigma} - m_{\Sigma}$  shown in Fig. 3. This can be understood by examining the formulas given in

Eqs. (48) and (53). Interestingly, there is an identity

$$m_{\Xi} - m_{\Sigma} = m_{\Xi^*} - m_{\Sigma^*} \quad (54)$$

which is kept in nuclear matter too. All other mass splittings of the baryon decuplet exhibit similar behaviors as the density increases. Compared to the case of the baryon octet, the degeneracy takes place at lower densities.

#### IV. SUMMARY AND OUTLOOK

In this work, we investigated the density dependence of the baryon octet and decuplet masses in nuclear matter within the framework of the in-medium modified SU(3) Skyrme model. For simplicity, we first concentrated on the medium modifications arising from the in-medium changes of the pion properties, which encodes the modification of the pion propagation in nuclear matter. The parameters were determined by describing the properties of nuclear matter near the saturation point  $\rho_0$ . In particular, the in-medium modified meson parameters provide the equation of states in the wide range of nuclear matter densities. In addition to this, we introduced the changes of the *produced* kaon properties in nuclear matter, which are in line with Refs. [46,47], and examined their effects on the masses of the baryon octet and decuplet.

We discussed also that the changes of the mesonic properties are generally in qualitative agreement with those from in-medium chiral perturbation theory [49,50] and the QCD

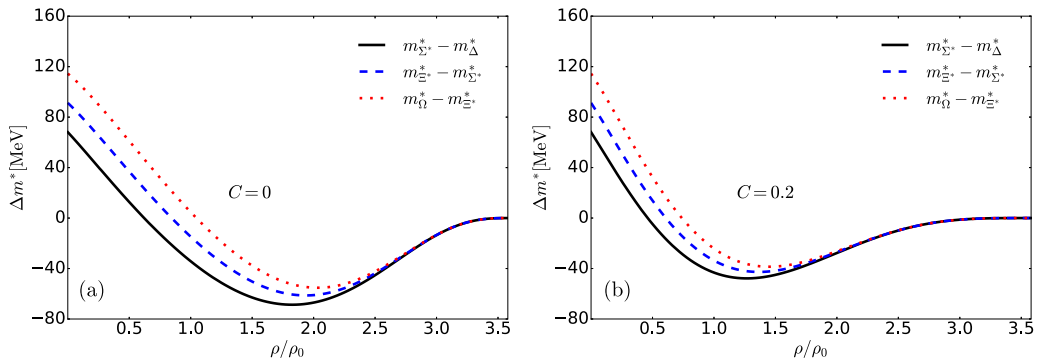


FIG. 4. The density dependence of the mass splittings of the baryon decuplet,  $m_{\Sigma^*} - m_{\Lambda^*}$ ,  $m_{\Xi^*} - m_{\Sigma^*}$ , and  $m_{\Omega^*} - m_{\Xi^*}$ , are illustrated by the solid, dotted, and dashed curves, respectively. Figure (a) corresponds to  $C = 0$  and (b) corresponds to  $C = 0.2$ , respectively.

sum rules [51] except for the relative density dependencies of  $F_{\pi,s}^*$  and  $F_{\pi,t}^*$ . The present results of the SU(3) baryon masses in nuclear matter are also in qualitative agreement with those from in-medium chiral effective field theory [14] and quark-meson coupling model [9].

In order to study the effects of the modified kaon properties, we have to go beyond the present simple scheme. We need to associate with kaon dynamics in nuclear matter in close relation with experimental data on kaon-nucleus scattering and kaonic atoms. It is also of great importance to investigate the equation of states for strange matter with regards to the interior structure of neutron stars. The relevant studies are under way.

## ACKNOWLEDGMENTS

The authors are grateful to P. Gubler, A. Hosaka, T. Maruyama, and M. Oka for useful discussions. The authors want to express their gratitude to the members of the Advanced Science Research Center at Japan Atomic Energy Agency for the hospitality during their visit, where part of the present work was done. The work is supported by Basic Science Research Program through the National Research Foundation (NRF) of Korea funded by the Korean government (Ministry of Education, Science and Technology, MEST), Grants No. 2016R1D1A1B03935053 (U.Y.) and No. NRF-2018R1A2B2001752 (H.-C.K.).

- 
- [1] S. Balberg, I. Lichtenstadt, and G. B. Cook, *Astrophys. J. Suppl.* **121**, 515 (1999).
- [2] H. Đapo, B.-J. Schaefer, and J. Wambach, *Phys. Rev. C* **81**, 035803 (2010).
- [3] P. Demorest, T. Pennucci, S. Ransom, M. Roberts, and J. Hessels, *Nature (London)* **467**, 1081 (2010).
- [4] I. Bednarek, P. Haensel, J. L. Zdunik, M. Bejger, and R. Manka, *Astron. Astrophys.* **543**, A157 (2012).
- [5] T. Katayama and K. Saito, *Phys. Lett. B* **747**, 43 (2015).
- [6] J. M. Lattimer and M. Prakash, *Phys. Rep.* **621**, 127 (2016).
- [7] A. Gal, E. V. Hungerford, and D. J. Millener, *Rev. Mod. Phys.* **88**, 035004 (2016).
- [8] M. Oertel, M. Hempel, T. Klähn, and S. Typel, *Rev. Mod. Phys.* **89**, 015007 (2017).
- [9] K. Saito and A. W. Thomas, *Phys. Rev. C* **51**, 2757 (1995).
- [10] M. J. Savage and M. B. Wise, *Phys. Rev. D* **53**, 349 (1996).
- [11] J. Schaffner and I. N. Mishustin, *Phys. Rev. C* **53**, 1416 (1996).
- [12] S. R. Beane, E. Chang, S. D. Cohen, W. Detmold, H.-W. Lin, T. C. Luu, K. Orginos, A. Parreño, M. J. Savage, and A. Walker-Loud (NPLQCD Collaboration), *Phys. Rev. Lett.* **109**, 172001 (2012).
- [13] J. Haidenbauer and U. G. Meißner, *Nucl. Phys. A* **936**, 29 (2015).
- [14] S. Petschauer, J. Haidenbauer, N. Kaiser, U. G. Meißner, and W. Weise, *Eur. Phys. J. A* **52**, 15 (2016).
- [15] H. Lenske, M. Dhar, T. Gaitanos, and X. Cao, *Prog. Part. Nucl. Phys.* **98**, 119 (2018).
- [16] H.-C. Kim, P. Schweitzer, and U. T. Yakhshiev, *Phys. Lett. B* **718**, 625 (2012).
- [17] J. H. Jung, U. T. Yakhshiev, and H.-C. Kim, *Phys. Lett. B* **723**, 442 (2013).
- [18] U. T. Yakhshiev and H.-C. Kim, *Phys. Lett. B* **726**, 375 (2013).
- [19] J. H. Jung, U. T. Yakhshiev, H.-C. Kim, and P. Schweitzer, *Phys. Rev. D* **89**, 114021 (2014).
- [20] J. H. Jung, U. T. Yakhshiev, and H.-C. Kim, *Phys. Rev. D* **93**, 054016 (2016).
- [21] U. T. Yakhshiev, *Phys. Rev. C* **88**, 034318 (2013).
- [22] T. Li *et al.*, *Phys. Rev. Lett.* **99**, 162503 (2007).
- [23] E. Chabanat, J. Meyer, P. Bonche, R. Schaeffer, and P. Haensel, *Nucl. Phys. A* **627**, 710 (1997).
- [24] L. Trippa, G. Colò, and E. Vigezzi, *Phys. Rev. C* **77**, 061304(R) (2008).
- [25] L. Chen, *Sci. China G* **52**, 1494 (2009).
- [26] M. M. Sharma, W. T. A. Borghols, S. Brandenburg, S. Crona, A. van der Woude, and M. N. Harakeh, *Phys. Rev. C* **38**, 2562 (1988).
- [27] S. Shlomo and D. H. Youngblood, *Phys. Rev. C* **47**, 529 (1993).
- [28] B. A. Li and L. W. Chen, *Phys. Rev. C* **72**, 064611 (2005).
- [29] U. T. Yakhshiev, *Phys. Lett. B* **749**, 507 (2015).
- [30] K. M. Westerberg and I. R. Klebanov, *Phys. Rev. D* **50**, 5834 (1994).
- [31] T. Waas, N. Kaiser, and W. Weise, *Phys. Lett. B* **365**, 12 (1996).
- [32] T. Waas, M. Rho, and W. Weise, *Nucl. Phys. A* **617**, 449 (1997).
- [33] E. Witten, *Nucl. Phys. B* **223**, 433 (1983).
- [34] E. Guadagnini, *Nucl. Phys. B* **236**, 35 (1984).
- [35] P. O. Mazur, M. A. Nowak, and M. Praszalowicz, *Phys. Lett.* **147B**, 137 (1984).
- [36] M. Chemtob, *Nucl. Phys. B* **256**, 600 (1985).
- [37] C. G. Callan, Jr. and I. R. Klebanov, *Nucl. Phys. B* **262**, 365 (1985).
- [38] C. G. Callan, Jr., K. Hornbostel, and I. R. Klebanov, *Phys. Lett. B* **202**, 269 (1988).
- [39] H. Yabu and K. Ando, *Nucl. Phys. B* **301**, 601 (1988).
- [40] H. Weigel, *Int. J. Mod. Phys. A* **11**, 2419 (1996).
- [41] H. Weigel, in *Chiral Soliton Models for Baryons*, Lecture Notes in Physics Vol. 743 (Springer, Berlin Heidelberg, 2008), p. 1.
- [42] J. Wess and B. Zumino, *Phys. Lett.* **37B**, 95 (1971).
- [43] D. B. Kaplan and I. R. Klebanov, *Nucl. Phys. B* **335**, 45 (1990).
- [44] T. Ericson and W. Weise, *Pions and Nuclei* (Clarendon, Oxford, UK, 1988).
- [45] U. T. Yakhshiev and H. C. Kim, *Phys. Rev. C* **83**, 038203 (2011).
- [46] D. B. Kaplan and A. E. Nelson, *Nucl. Phys. A* **479**, 273c (1988).
- [47] E. E. Kolomeitsev, B. Kampfer, and D. N. Voskresensky, *Nucl. Phys. A* **588**, 889 (1995).
- [48] U. T. Yakhshiev, *PTEP* **2014**, 123D03 (2014).
- [49] U. G. Meissner, J. A. Oller, and A. Wirzba, *Ann. Phys.* **297**, 27 (2002).
- [50] M. Kirchbach and A. Wirzba, *Nucl. Phys. A* **616**, 648 (1997).
- [51] H. C. Kim and M. Oka, *Nucl. Phys. A* **720**, 368 (2003).