


# Thermodynamic behavior of magnetized quark-gluon plasma within the self-consistent quasiparticle model

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The self-consistent quasiparticle model has been successful in studying QCD thermodynamics. In this model, the medium effects are taken into account by considering quarks and gluons as quasiparticles with temperature dependent masses which are proportional to the plasma frequency. The present work involves the extension of this model in the presence of magnetic fields. We have included the effect of the magnetic field by considering relativistic Landau levels. The quasiparticle masses are then found to be dependent on both temperature and magnetic field. The thermomagnetic mass thus defined allows us to obtain the thermodynamics of magnetized quark matter within the self-consistent quasiparticle model. The model then has been applied to the case of two-flavor quark-gluon plasma and the equation of state obtained in the presence of magnetic fields.

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## I. INTRODUCTION

High energy collisions have succeeded in recreating the state of matter called quark gluon plasma (QGP) which is believed to have existed shortly after the big bang. It has been observed that QGP produced in high energy collisions behave very much like a nearly perfect fluid [1]. These collisions mostly occur with a finite impact parameter. During off-central collisions, the charged ions thus can produce very large magnetic fields reaching up to  $eB \approx (1 - 5)m_\pi^2$  [2,3]. These magnetic fields may exist only for a short time but, depending on the transport coefficients, they may reach their maximum value and can be stationary during this time [4–7]. The magnetic field can cause different phenomena, such as magnetic catalysis [8–11], chiral magnetic effect [12–14], etc., in the QGP. The equation of state is important for studying the particle spectra created in heavy-ion collisions. Very strong magnetic fields are estimated to have existed right after the big bang [15]. Effects of external magnetic fields are relevant in the context of strongly magnetized neutron stars too [16]. Therefore, it is of importance to investigate the behavior of QGP under magnetic fields, in particular, the effect on the QCD thermodynamics [17,18]. There have been several investigations as to how these magnetic fields affect the transport coefficients [19,20].

In this work, we intend to understand the effect of magnetic fields on QCD thermodynamics and obtain the equation of state by extending a quasiparticle model for QGP, called the self-consistent quasiparticle model. We incorporate the effect of the magnetic field by modifying the thermal mass using the relativistic Landau levels. Such modified masses in the presence of magnetic fields can be used to calculate thermodynamic quantities, such as energy density, pressure,

entropy density, etc. Besides, by applying this formalism to the two-flavor system, we obtain its thermodynamics in the presence of background magnetic field and examine the qualitative behavior of the equation of state.

## II. THE SELF-CONSISTENT QUASIPARTICLE MODEL

In quasiparticle models, the thermal properties of interacting real particles are modelled by noninteracting quasiparticles. The quasiparticles have an effective mass which is determined by the collective properties of the medium [21,22]. Such models include the Nambu-Jona-Lasinio (NJL) and Polyakov-loop-extended Nambu-Jona-Lasinio Model based quasiparticle models [23] or those that include effective mass with the Polyakov loop [24]. There are also quasiparticle models based on Gribov-Zwanziger quantization [25]. Other effective mass quasiparticle models include self-consistent and single parameter quasiparticle models [26–30]. There are quasiparticle models which incorporate the medium effects by considering quasiparticles with effective fugacities too. Such models have been quite successful in describing the lattice QCD results [31,32].

The self-consistent quasiparticle model here considers QGP as consisting of noninteracting quasiparticles with effective masses which depend on thermodynamic quantities and encode all medium interactions [26,30]. Since the thermal mass depends on thermodynamic quantities which in turn depends on thermal mass, the whole problem is solved self-consistently.

In the self-consistent model, following standard thermodynamics, all thermodynamic quantities are derived from the expressions for energy density and number density. The expression for energy density is

$$\varepsilon = \frac{g_f}{2\pi^2} \int_0^\infty dk k^2 \frac{\omega_k}{z^{-1} e^{\frac{\omega_k}{T}} \mp 1}, \quad (1)$$

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where  $g_f$  is the degeneracy and  $\mp$  refers to bosons and fermions.  $z$  is the fugacity. The expression for number density is

$$n_{g/q} = \frac{g_f}{2\pi^2} \int_0^\infty dk k^2 \frac{1}{z^{-1} e^{\frac{\omega_k}{T}} \mp 1}. \quad (2)$$

The single particle energy  $\omega_k$  is approximated to a simple form,

$$\omega_k = \sqrt{k^2 + m_g^2} \quad (3)$$

and

$$\omega_k = \sqrt{k^2 + m_q^2}, \quad (4)$$

for gluons and quarks, respectively. This approximation is valid at high temperatures only.

It is known that quasiparticles acquire ‘‘thermal mass’’ of order  $gT$  at one loop order [33–35]. In the model that we study here, the thermal mass is defined as proportional to the plasma frequencies as

$$m_g^2 = \frac{3}{2} \omega_p^2 \quad \text{and} \quad m_q^2 = 2m_f^2, \quad (5)$$

for massless particles. For massive quarks  $m_q^2$  is written as

$$m_q^2 = (m_0 + m_f)^2 + m_f^2. \quad (6)$$

The plasma frequencies are calculated from the density dependent expressions

$$\omega_p^2 = a_g^2 g^2 \frac{n_g}{T} + a_q^2 g^2 \frac{n_q}{T}, \quad (7)$$

for gluons and,

$$m_f^2 = b_q^2 g^2 \frac{n_q}{T}, \quad (8)$$

for quarks. Here,  $n_q$  is the quark number density and  $n_g$  is the gluon number density.  $g^2 = 4\pi\alpha_s$  is the QCD running coupling constant. The coefficients  $a_g$ ,  $a_q$ ,  $b_q$  are determined by demanding that as  $T \rightarrow \infty$ ,  $\omega_p$  and  $m_f$  both go to the corresponding perturbative results. The motivation for choosing such an expression for plasma frequency is that the plasma frequency for electron-positron plasma is known to be proportional to  $n/T$  in the relativistic limit [36,37]. Since the thermal masses appear in the expression for the density, we need to solve the density equation self-consistently to obtain the thermal mass, which may be used to evaluate the thermodynamic quantities of interest. The result obtained have shown a good fit with lattice data even at temperatures near  $T_c$  [38].

### III. EXTENSION OF THE SELF-CONSISTENT MODEL IN MAGNETIC FIELD

In this section, we extend the self-consistent quasiparticle model to a system with zero chemical potential, in the presence of magnetic fields. The quantization of fermionic theory in magnetic fields has been known. The energy eigenvalues are obtained as Landau levels and have been subject to several

investigations [39,40]. In the Landau gauge  $A_y = Bx$  so that  $\mathbf{B} = B\hat{z}$  and the energy eigenvalues are obtained as

$$E_j = \sqrt{m^2 + k_z^2 + 2j|q_f eB|}. \quad (9)$$

Here,  $q_f e$  is the charge of the fermion and  $j = 0, 1, 2, \dots$  are the Landau energy levels.

In the presence of a magnetic field  $B$ , the integral over the phase space is modified as [41–45]

$$\int \frac{d^3k}{(2\pi)^3} \rightarrow \frac{|q_f eB|}{2\pi} \sum_{j=0}^{\infty} \int \frac{dk_z}{2\pi} (2 - \delta_{0j}), \quad (10)$$

where  $(2 - \delta_{0j})$  is the degeneracy of the  $j$ th Landau level [46].

#### A. Thermomagnetic mass for quarks

Making use of Eqs. (9) and (10), we can write the expression for number density in the presence of magnetic fields, for a system at zero chemical potential as

$$n_q = \frac{g_f q_f eB}{(2\pi)^2} \sum_{j=0}^{\infty} \left[ 2 \int_{-\infty}^{\infty} dk_z \frac{1}{e^{\sqrt{(\frac{k_z}{T})^2 + (\frac{m_{qj}}{T})^2}} + 1} - \int_{-\infty}^{\infty} dk_z \frac{1}{e^{\sqrt{(\frac{k_z}{T})^2 + (\frac{m_{qj}}{T})^2}} + 1} \delta_{0j} \right], \quad (11)$$

where we have assigned for simplicity

$$m_{qj}^2 = m_q^2 + 2j|q_f eB|. \quad (12)$$

Equation (11) can be further simplified to

$$T^2 F_q^2 = \sum_{j=0}^{\infty} \left( 2 \sum_{l=1}^{\infty} (-1)^{(l-1)} \frac{m_{qj}}{T} K_1 \left( l \frac{m_{qj}}{T} \right) - \sum_{l=1}^{\infty} (-1)^{(l-1)} \frac{m_{qj}}{T} K_1 \left( l \frac{m_{qj}}{T} \right) \delta_{0j} \right). \quad (13)$$

Here, we have defined for later convenience

$$F_q^2 = \frac{(2\pi)^2}{2g_f |q_f eB|} \frac{n_q}{T^3}. \quad (14)$$

For massive quarks,  $m_q$  can be obtained from Eq. (6), with  $m_f$  calculated just as in [30]

$$m_f^2 = c_q^2 g^2 \frac{n_q}{T}. \quad (15)$$

Or

$$\begin{aligned} \left( \frac{m_f}{T} \right)^2 &= c_q^2 g^2 \frac{n_q}{T^3} \\ &= \bar{c}_q^2 F_q^2, \end{aligned} \quad (16)$$

where

$$\bar{c}_q^2 = 2c_q^2 g^2 \frac{g_f |q_f eB|}{(2\pi)^2}. \quad (17)$$

Combining Eqs. (6), (12), and (16), we can write

$$\left( \frac{m_{qj}}{T} \right)^2 = \left[ \frac{m_0}{T} + \bar{c}_q F_q \right]^2 + \bar{c}_q^2 F_q^2 + 2j \frac{|q_f eB|}{T^2}. \quad (18)$$

Using Eq. (18) in Eq. (13), and simplifying the Kronecker  $\delta$  we get

$$T^2 F_q^2 = \sum_{l=1}^{\infty} (-1)^{(l-1)} \left[ 2 \sum_{j=0}^{\infty} \sqrt{\left[ \frac{m_0}{T} + \bar{c}_q F q \right]^2 + \bar{c}_q^2 F_q^2} + 2j \frac{|q_f e B|}{T^2} K_1 \left( l \sqrt{\left[ \frac{m_0}{T} + \bar{c}_q F q \right]^2 + \bar{c}_q^2 F_q^2} + 2j \frac{|q_f e B|}{T^2} \right) - \sqrt{\left[ \frac{m_0}{T} + \bar{c}_q F q \right]^2 + \bar{c}_q^2 F_q^2} K_1 \left( l \sqrt{\left[ \frac{m_0}{T} + \bar{c}_q F q \right]^2 + \bar{c}_q^2 F_q^2} \right) \right], \quad (19)$$

where  $K_n(x)$  are the modified Bessel functions of the second kind. Solving this equation for  $F_q$  and using Eq. (18) we get the thermomagnetic mass which depends both on temperature and magnetic field. This thermomagnetic mass can be used to obtain the thermodynamics of the system considering it as a collection of noninteracting quasiparticles with mass depending on temperature and magnetic field.

### B. Thermomagnetic mass for gluons

The density-dependent expression for plasma frequency for gluons is given by Eq. (7). The expression for gluon number density  $n_g$  remains unchanged because gluons are chargeless and are not directly affected by magnetic fields. They are only indirectly affected by their coupling to the electrically charged quarks. Thus the term  $n_q$  changes as explained and so the gluons also acquire a thermomagnetic mass. We have

$$n_g = \frac{g_g}{2\pi^2} \int_0^{\infty} dk k^2 \frac{1}{e^{\omega_k/T} - 1}. \quad (20)$$

Following Eq. (7), we define the plasma frequency as

$$\omega_p^2 = a_g^2 g^2 \frac{n_g}{T} + d_q^2 g^2 \frac{n_q}{T}. \quad (21)$$

Making use of Eq. (5) and simplifying Eq. (20)

$$f_g^2 = (\bar{a}_g^2 f_g^2 + \bar{d}_q^2 F_q^2) \sum_{l=1}^{\infty} \frac{1}{l} K_2 [l (\bar{a}_g^2 f_g^2 + \bar{d}_q^2 F_q^2)^{1/2}], \quad (22)$$

where

$$f_g^2 = \frac{2\pi^2}{g_g} \frac{n_g}{T^3}, \quad (23)$$

$$\bar{a}_g^2 = \frac{3}{4\pi^2} g_g a_g^2 g^2, \quad (24)$$

$$\bar{d}_q^2 = \frac{3}{2} \frac{a_q^2 g^2}{(2\pi)^2} 2g_f |q_f e B|, \quad (25)$$

and  $F_q$  is obtained as a solution to Eq. (19). Now, solving Eq. (22) for  $f_g$  using the above, we obtain the thermomagnetic mass using

$$\left( \frac{m_g}{T} \right)^2 = \bar{a}_g^2 f_g^2 + \bar{d}_q^2 F_q^2. \quad (26)$$

The coefficients  $c_q$ ,  $a_g$ , and  $d_q$  are determined by demanding that as  $T \rightarrow \infty$  the expression for frequency approaches the corresponding perturbative QCD results.

## C. QCD thermodynamics in background magnetic field

### 1. Energy density

We start with the energy density. Using Eqs. (1), (9), and (10) the contribution to the energy density from quarks becomes

$$\varepsilon_q = \frac{12n_q |q_f e B|}{4\pi^2} \sum_{j=0}^{\infty} \left[ 2 \int_{-\infty}^{\infty} dk_z \frac{\omega_{k_z j}}{e^{\omega_{k_z j}/T} + 1} - \int_{-\infty}^{\infty} dk_z \frac{\omega_{k_z j}}{e^{\omega_{k_z j}/T} + 1} \delta_{0j} \right], \quad (27)$$

where

$$\begin{aligned} \omega_{k_z j} &= \sqrt{m_q^2 + k_z^2 + 2j |q_f e B|} \\ &= \sqrt{m_{q_j}^2 + k_z^2}. \end{aligned} \quad (28)$$

Equation (27) becomes, after some algebra,

$$\begin{aligned} \varepsilon_q &= \frac{12n_q |q_f e B|}{2\pi^2} \frac{T^2}{2} \sum_{l=0}^{\infty} \frac{(-1)^{(l-1)}}{l^2} \\ &\times \left\{ 2 \sum_{j=0}^{\infty} [(\beta m_{q_j} l) K_1(\beta m_{q_j} l) + (\beta m_{q_j} l)^2 K_0(\beta m_{q_j} l)] \right. \\ &\left. - [(\beta m_q l) K_1(\beta m_q l) + (\beta m_q l)^2 K_0(\beta m_q l)] \right\}. \end{aligned} \quad (29)$$

The expression for the energy density of gluons remains unchanged as they are chargeless. The energy density is indirectly affected by the interaction with quarks, included through the thermomagnetic mass of gluons:

$$\varepsilon_g = \frac{g_g T^4}{2\pi^2} \sum_{l=1}^{\infty} \frac{1}{l^4} [(\beta m_g l)^3 K_1(\beta m_g l) + 3(\beta m_g l)^2 K_2(\beta m_g l)]. \quad (30)$$

The contribution to energy density from quarks and gluons is obtained as

$$\varepsilon = \varepsilon_g + \sum_q \varepsilon_q. \quad (31)$$

### 2. Pressure

It has been known that the presence of magnetic fields breaks the  $O(3)$  rotational symmetry resulting in a pressure

anisotropy [46–49]. There have also been arguments suggesting that the total pressure is indeed isotropic and the issue has been subject to some debate [50–52]. The scheme dependence of pressure anisotropy has been discussed in [53,54]. Here, the authors have distinguished between two schemes. The  $B$  scheme, which corresponds to a setup in which the magnetic field  $B$  is kept constant during the compression, and the  $\Phi$  scheme corresponding to a setup in which the magnetic flux is kept constant during the compression. They showed that pressure anisotropy appears only in the  $\Phi$  scheme, i.e.,  $P_{\perp}^{(B)} = P_{\parallel}^{(B)}$  and  $P_{\perp}^{(\Phi)} \neq P_{\parallel}^{(\Phi)}$ , where  $P_{\perp}$  and  $P_{\parallel}$  denote the transverse and longitudinal pressure, respectively. They also showed that the longitudinal pressure is scheme independent. Thus,  $P_{\parallel}^{(\Phi)} = P_{\parallel}^{(B)}$ . In the  $\Phi$  scheme the longitudinal and transverse

pressures were found to be related by

$$P_{\perp}^{\Phi} = P_{\parallel} - eMB. \quad (32)$$

The magnetization can be calculated in the canonical ensemble using the equation

$$M = \frac{T}{V} \frac{1}{e} \frac{\partial \ln Z}{\partial B}, \quad (33)$$

where  $Z$  is the partition function.

The contribution to  $P_{\parallel}$  from quarks and gluons can be calculated from the expression for thermodynamic pressure in the self-consistent quasiparticle model [27]. We will denote this as  $P$ :

$$\frac{P_{g/q}}{T} = \mp \frac{g_f}{8\pi^3} \int_0^{\infty} d^3k \ln(1 \mp e^{-\beta\epsilon_k}) + \int d\beta \beta \frac{g_f}{2\pi^2} m \frac{dm}{d\beta} \int_0^{\infty} d^3k \frac{k^2}{\epsilon_k(e^{\beta\epsilon_k} \mp 1)}, \quad (34)$$

where  $\mp$  for bosons and fermions, respectively. The contribution to the thermodynamic pressure from quarks can be obtained by changing the integral and energy eigenvalues according to Eqs. (10) and (9). Making these changes and simplifying the integral we obtain

$$\begin{aligned} \frac{P_q}{T} = & \frac{g_f |q_f eB|}{2\pi^2} \sum_{l=1}^{\infty} (-1)^{l-1} \left\{ 2 \sum_{j=0}^{\infty} \left[ \frac{1}{\beta l^2} (\beta m_{q_j} l) K_1(\beta m_{q_j} l) + \int d\beta \beta m_{q_j} \frac{\partial m_{q_j}}{\partial \beta} K_0(\beta m_{q_j} l) \right] \right. \\ & \left. - \left[ \frac{1}{\beta l^2} (\beta m_q l) K_1(\beta m_q l) + \int d\beta \beta m_q \frac{\partial m_q}{\partial \beta} K_0(\beta m_q l) \right] \right\}. \end{aligned} \quad (35)$$

The contribution to the pressure from gluons can be obtained by replacing the thermal mass by thermomagnetic mass. From Eq. (34), we can write the contribution from gluons as

$$\begin{aligned} \frac{P_g}{T} = & \frac{g_f}{2\pi^2} \sum_{l=1}^{\infty} \frac{1}{l^4} \left[ T^3 (\beta m_g l)^2 K_2(\beta m_g l) \right. \\ & \left. + \int d\beta \beta \frac{\partial m_g}{\partial \beta} (\beta m_g l)^3 K_1(\beta m_g l) \right]. \end{aligned} \quad (36)$$

It can easily be shown that the expressions for thermodynamic pressure in Eqs. (36) and (35) are related to the corresponding expressions for energy densities as

$$\varepsilon = T \frac{\partial P}{\partial T} - P, \quad (37)$$

ensuring thermodynamic consistency. Thus, the thermodynamic pressure can also be obtained from the energy density. Solving Eq. (37),

$$\frac{P}{T} = \frac{P_0}{T_0} + \int_{T_0}^T dT \frac{\varepsilon}{T^2}. \quad (38)$$

Here,  $P_0$  and  $T_0$  are pressure and temperature at some reference points [28]. The entropy density can be calculated from

$$s = \frac{\varepsilon + P}{T}. \quad (39)$$

### 3. The pure-field contribution

In addition to the contribution from magnetized matter, there are pure-field contributions to the energy density and pressure that must be taken into account [47,52,55,56]. These contributions are again different in the parallel and perpendicular directions:

$$\varepsilon^{\text{total}} = \varepsilon + \frac{B^2}{2}. \quad (40)$$

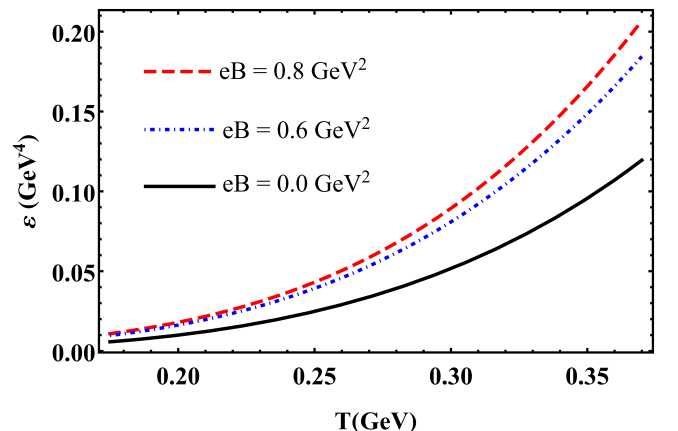


FIG. 1. Energy density for different magnetic fields as a function of temperature.

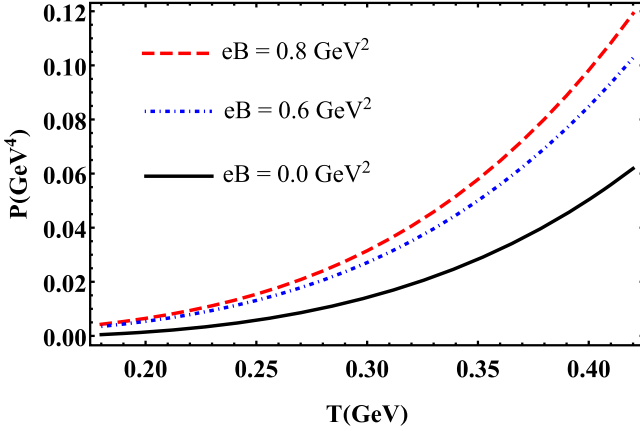


FIG. 2. Pressure for different magnetic fields as a function of temperature.

$$P_{\perp}^{\text{total}} = P_{\perp} + \frac{B^2}{2}. \quad (41)$$

$$P_{\parallel}^{\text{total}} = P - \frac{B^2}{2}. \quad (42)$$

#### D. Thermodynamics for two-flavor system in the presence of magnetic fields

Using the above formalism, we obtain the thermodynamics for the two-flavor system at zero chemical potential in the presence of different magnetic fields. To this end, we need a running coupling constant that depends both on temperature and magnetic field. The running coupling constant needs modification in the presence of magnetic fields. It has been well known that the coupling constant is affected by magnetic fields [57,58]. Different ansatz for the dependence of coupling constant on magnetic fields, in the presence of magnetic fields, have been proposed [59–62]. In [62] the coupling constant depending on both temperature and magnetic fields was introduced in the SU(2) NJL models as

$$G(B, T) = c(B) \left[ 1 - \frac{1}{1 + e^{\beta(B)[T_c(B) - T]}} \right] + s(B), \quad (43)$$

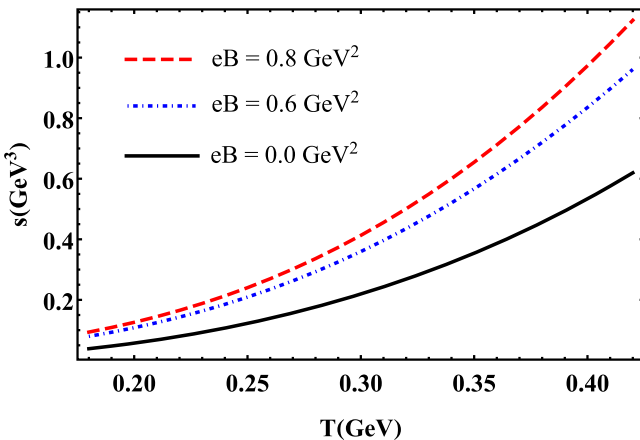


FIG. 3. Entropy density for different magnetic fields as a function of temperature.

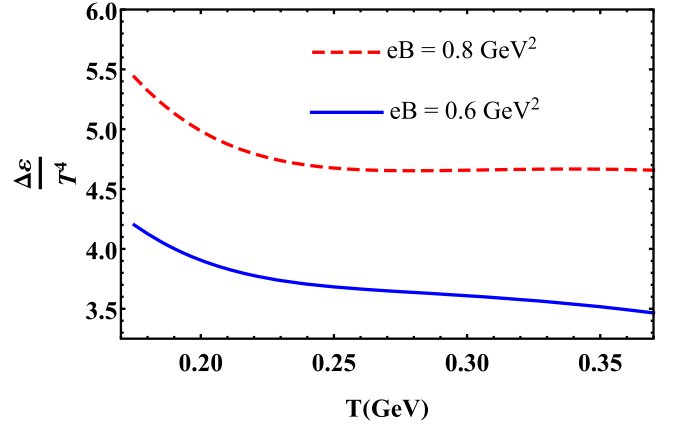


FIG. 4.  $\Delta\epsilon/T^4$  for two different magnetic fields, plotted as a function of temperature.

where the four parameters  $c$ ,  $\beta$ ,  $T_c$ , and  $s$  were obtained by fitting the lattice data. It was shown that the thermodynamic quantities showed correct qualitative behavior, in the presence of magnetic fields, with this parametrization.

We make use of this ansatz for the coupling constant with the parameters as obtained in [62], to calculate the thermomagnetic mass according to Eqs. (7) and (8) and hence obtain the thermodynamics.

Calculation of the contribution of quarks and gluons to  $P_{\parallel}$  requires the value of thermodynamic pressure at some fixed temperature  $T_0$ . If lattice data is available  $P_0$  can be chosen as the value of pressure at transition temperature  $T_c$ . Here, we have chosen the value of thermodynamic pressure at  $T_c$  from [62]. For all calculations, we have taken the physical masses of quarks as in [38].

We have shown the temperature dependence of different thermodynamic quantities at different values of magnetic fields in Figs. 1–3. Figure 1 shows that the energy density increases, at a given temperature, as the magnetic field increases. This is expected as, in the presence of magnetic field the total energy density goes as  $\epsilon_{\text{total}} = \epsilon + q\mathbf{M} \cdot \mathbf{B}$ , where  $\mathbf{M}$  is the magnetization [63]. We notice that the plots show correct qualitative behavior. The quark/gluon contribution

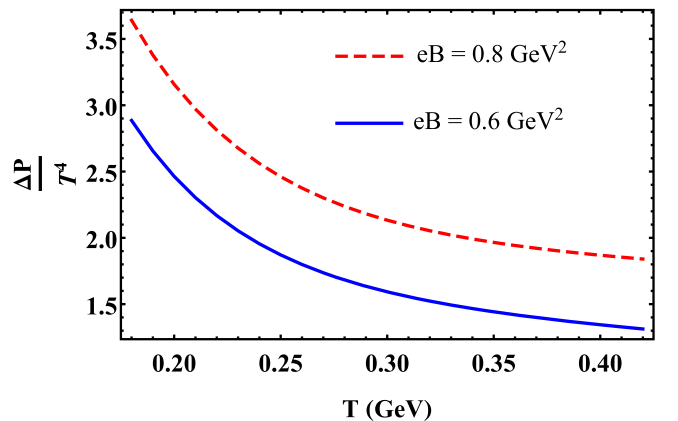


FIG. 5.  $\Delta P/T^4$  for two different magnetic fields, plotted as a function of temperature.



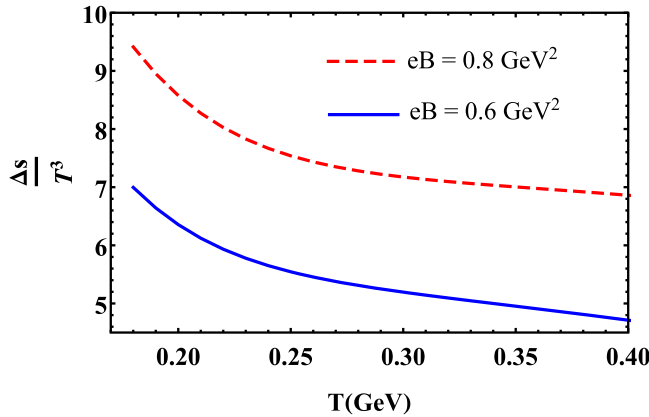


FIG. 6.  $\Delta s/T^3$  for two different magnetic fields, plotted as a function of temperature.

to the thermodynamic pressure, energy density, and entropy density increase with the increase in  $eB$ . This behavior is consistent with that obtained using lattice QCD simulations [53] and [64]. The same behavior has been obtained using an effective fugacity quasiparticle model in [63], within the  $SU(2)$  NJL model in [62] and using the bag model in [41]. There are several other studies which study magnetized quark matter. The QCD equation of state in the presence of magnetic field has been studied numerically in [65,66]. The effect of the magnetic field on QCD thermodynamics has been studied using the hard-thermal-loop perturbation theory both at strong [67] and in weak [68] magnetic fields.

$\Delta\varepsilon$  is the difference between energy density in the presence of magnetic fields with that in the absence of any magnetic field. This depicts the increment of energy density in the presence of the magnetic field. The temperature dependence of  $\Delta\varepsilon/T^4$  has been plotted in Fig. 4. In addition, we have plotted  $\Delta P/T^4$  and  $\Delta s/T^3$  as functions of temperature in Fig. 5 and Fig. 6 respectively. As expected, the higher the magnetic field, the higher their values are too.

The transverse pressure can be obtained using Eqs. (32) and (33). The calculation involves taking the derivatives with respect to the magnetic field. We are not able to calculate it here because the functional dependence of the coupling constant on the magnetic field in Eq. (43) is unknown. We emphasize that this is not a shortcoming of our model and that with the knowledge of the exact functional dependence of the coupling constant on the magnetic field we will be able to calculate the transverse pressure too.

#### IV. CONCLUSIONS

We have extended the self-consistent quasiparticle model for hot QCD in the presence of magnetic fields to understand the behavior of magnetized quark matter. The effect of magnetic fields has been included by redefining the thermal mass of quasiparticles. The definition of thermal mass in the self-consistent model has been extended to define a thermomagnetic mass through Landau level quantization for fermions. The thermodynamic quantities are evaluated by starting with the modified momentum distributions and the energy dispersion relations. The modification of these quantities has been brought about by incorporating relativistic Landau levels.

Using this modified quasiparticle model, we have studied the two-flavor system in the temperature range 170–400 MeV, in the presence of magnetic fields. To this end, we made use of a parametrization of the coupling constant that depends both on temperature and magnetic field, obtained in the context of the  $SU(2)$  NJL model. We found that the energy density, pressure, and entropy density increase in the presence of a magnetic field as expected. Our results are qualitatively consistent with the results obtained using other approaches including lattice QCD simulations.

The correct behavior of the equation of state shows that the self-consistent quasiparticle model can be extended to study the thermodynamics of quark-gluon plasma in the presence of magnetic fields. For a quantitative study that can be compared with the lattice data, we need a coupling constant depending both on temperature and magnetic field. With a proper parametrization of the coupling constant for  $2 + 1$  flavor, we can easily extend this work to obtain the equation of state for  $2 + 1$  flavor QGP in the presence of magnetic field and also calculate the transverse components of pressure. We intend to do this in our future work. Another area that we plan to investigate further is how the modified equation of state affects the transport coefficients of QGP.

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- [1] B. Back *et al.*, *Nucl. Phys. A* **757**, 28 (2005); J. Adams *et al.*, *ibid.* **757**, 102 (2005); K. Adcox *et al.*, *ibid.* **757**, 184 (2005); I. Arsene *et al.*, *ibid.* **757**, 1 (2005).  
[2] V. V. Skokov, A. Y. Illarionov, and V. D. Toneev, *Int. J. Mod. Phys. A* **24**, 5925 (2009).  
[3] Z. Yang, Y. Chun-Bin, C. Xu, and F. Sheng-Qin, *Chin. Phys. C* **39**, 104105 (2015).  
[4] K. Tuchin, *Phys. Rev. C* **83**, 017901 (2011).  
[5] K. Marasinghe and K. Tuchin, *Phys. Rev. C* **84**, 044908 (2011).  
[6] K. Tuchin, *Phys. Rev. C* **82**, 034904 (2010).  
[7] K. Fukushima and J. M. Pawłowski, *Phys. Rev. D* **86**, 076013 (2012).  
[8] V. P. Gusynin, V. A. Miransky, and I. A. Shovkovy, *Phys. Rev. Lett.* **73**, 3499 (1994).  
[9] *Strongly Interacting Matter in Magnetic Fields* (Springer, Berlin/Heidelberg, 2013).  
[10] V. Gusynin, V. Miransky, and I. Shovkovy, *Nucl. Phys. B* **462**, 249 (1996).  
[11] K. Fukushima and Y. Hidaka, *Phys. Rev. Lett.* **110**, 031601 (2013).

- [12] K. Fukushima, D. E. Kharzeev, and H. J. Warringa, *Phys. Rev. D* **78**, 074033 (2008).
- [13] D. E. Kharzeev, L. D. McLerran, and H. J. Warringa, *Nucl. Phys. A* **803**, 227 (2008).
- [14] D. E. Kharzeev, *Prog. Part. Nucl. Phys.* **75**, 133 (2014).
- [15] D. Grasso and H. R. Rubinstein, *Phys. Rep.* **348**, 163 (2001).
- [16] R. C. Duncan and C. Thompson, *Astrophys. J. Lett.* **392**, L9 (1992); A. Broderick, M. Prakash, and J. M. Lattimer, *Astrophys. J.* **537**, 351 (2000); C. Y. Cardall, M. Prakash, and J. M. Lattimer, *ibid.* **554**, 322 (2001); A. Rabhi, C. Providência, and J. D. Providência, *J. Phys. G: Nucl. Part. Phys.* **35**, 125201 (2008); A. Rabhi, P. K. Panda, and C. Providência, *Phys. Rev. C* **84**, 035803 (2011).
- [17] M. Strickland, V. Dexheimer, and D. P. Menezes, *Phys. Rev. D* **86**, 125032 (2012).
- [18] S. S. Avancini, V. Dexheimer, R. L. S. Farias, and V. S. Timóteo, *Phys. Rev. C* **97**, 035207 (2018).
- [19] M. Kurian and V. Chandra, *Phys. Rev. D* **97**, 116008 (2018).
- [20] M. Kurian, S. Mitra, and V. Chandra, *Eur. Phys. J. C* **79**, 134 (2019).
- [21] V. Goloviznin and H. Satz, *Z. Phys. C* **57**, 671 (1993).
- [22] A. Peshier, B. Kämpfer, O. Pavlenko, and G. Soff, *Phys. Lett. B* **337**, 235 (1994).
- [23] A. Dumitru and R. D. Pisarski, *Phys. Lett. B* **525**, 95 (2002); K. Fukushima, *ibid.* **591**, 277 (2004); S. K. Ghosh, T. K. Mukherjee, M. G. Mustafa, and R. Ray, *Phys. Rev. D* **73**, 114007 (2006); H. Abuki and K. Fukushima, *Phys. Lett. B* **676**, 57 (2009); H.-M. Tsai and B. Müller, *J. Phys. G: Nucl. Part. Phys.* **36**, 075101 (2009).
- [24] M. D'Elia, A. D. Giacomo, and E. Meggiolaro, *Phys. Lett. B* **408**, 315 (1997); P. Castorina and M. Mannarelli, *ibid.* **644**, 336 (2007); *Phys. Rev. C* **75**, 054901 (2007).
- [25] N. Su and K. Tywoniuk, *Phys. Rev. Lett.* **114**, 161601 (2015).
- [26] V. M. Bannur, *Phys. Rev. C* **75**, 044905 (2007).
- [27] V. M. Bannur, *Phys. Lett. B* **647**, 271 (2007).
- [28] V. M. Bannur, *J. High Energy Phys.* **09** (2007) 046.
- [29] V. M. Bannur, *Eur. Phys. J. C* **50**, 629 (2007).
- [30] V. M. Bannur, *Phys. Rev. C* **78**, 045206 (2008).
- [31] V. Chandra and V. Ravishankar, *Eur. Phys. J. C* **64**, 63 (2009).
- [32] V. Chandra and V. Ravishankar, *Phys. Rev. D* **84**, 074013 (2011).
- [33] R. D. Pisarski, *Phys. Rev. Lett.* **63**, 1129 (1989).
- [34] R. D. Pisarski, *Phys. Rev. D* **47**, 5589 (1993).
- [35] A. Peshier, B. Kämpfer, O. P. Pavlenko, and G. Soff, *Phys. Rev. D* **54**, 2399 (1996).
- [36] M. V. Medvedev, *Phys. Rev. E* **59**, R4766 (1999).
- [37] V. M. Bannur, *Phys. Rev. E* **73**, 067401 (2006).
- [38] V. M. Bannur, *Int. J. Mod. Phys. E* **21**, 1250090 (2012).
- [39] K. Bhattacharya, [arXiv:0705.4275v2](https://arxiv.org/abs/0705.4275v2) [hep-th].
- [40] F. Bruckmann, G. Endrődi, M. Giordano, S. D. Katz, T. G. Kovács, F. Pittler, and J. Wellenhofer, *EPJ Web Conf.* **175**, 07014 (2018).
- [41] E. S. Fraga and L. F. Palhares, *Phys. Rev. D* **86**, 016008 (2012).
- [42] A. J. Mizher, M. N. Chernodub, and E. S. Fraga, *Phys. Rev. D* **82**, 105016 (2010).
- [43] S. Chakrabarty, *Phys. Rev. D* **54**, 1306 (1996).
- [44] F. Bruckmann, G. Endrődi, M. Giordano, S. D. Katz, T. G. Kovács, F. Pittler, and J. Wellenhofer, *Phys. Rev. D* **96**, 074506 (2017).
- [45] A. N. Tawfik, *J. Phys.: Conf. Series* **668**, 012082 (2016).
- [46] K. Kohri, S. Yamada, and S. Nagataki, *Astropart. Phys.* **21**, 433 (2004).
- [47] E. J. Ferrer, V. de la Incera, J. P. Keith, I. Portillo, and P. L. Springsteen, *Phys. Rev. C* **82**, 065802 (2010).
- [48] A. P. Martínez, H. P. Rojas, and H. J. M. Cuesta, *Eur. Phys. J. C* **29**, 111 (2003).
- [49] M. Chaichian, S. S. Masood, C. Montonen, A. P. Martínez, and H. P. Rojas, *Phys. Rev. Lett.* **84**, 5261 (2000).
- [50] A. Y. Potekhin and D. G. Yakovlev, *Phys. Rev. C* **85**, 039801 (2012).
- [51] E. J. Ferrer, V. de la Incera, J. P. Keith, I. Portillo, and P. L. Springsteen, *Phys. Rev. C* **85**, 039802 (2012).
- [52] V. Dexheimer, D. P. Menezes, and M. Strickland, *J. Phys. G: Nucl. Part. Phys.* **41**, 015203 (2013).
- [53] G. S. Bali, F. Bruckmann, G. Endrődi, S. D. Katz, and A. Schäfer, *J. High Energy Phys.* **08** (2014) 177.
- [54] G. S. Bali, F. Bruckmann, G. Endrődi, F. Gruber, and A. Schäfer, *J. High Energy Phys.* **04** (2013) 130.
- [55] R. D. Blandford and L. Hernquist, *J. Phys. C: Solid State Physics* **15**, 6233 (1982).
- [56] D. P. Menezes, M. B. Pinto, S. S. Avancini, A. P. Martínez, and C. Providência, *Phys. Rev. C* **79**, 035807 (2009).
- [57] E. J. Ferrer, V. de la Incera, and X. J. Wen, *Phys. Rev. D* **91**, 054006 (2015).
- [58] A. Ayala, C. A. Dominguez, S. Hernandez-Ortiz, L. A. Hernandez, M. Loewe, D. M. Paret, and R. Zamora, *Phys. Rev. D* **98**, 031501(R) (2018).
- [59] V. A. Miransky and I. A. Shovkovy, *Phys. Rev. D* **66**, 045006 (2002).
- [60] M. Ferreira, P. Costa, O. Lourenço, T. Frederico, and C. Providência, *Phys. Rev. D* **89**, 116011 (2014).
- [61] R. L. S. Farias, K. P. Gomes, G. Krein, and M. B. Pinto, *Phys. Rev. C* **90**, 025203 (2014).
- [62] R. L. S. Farias, V. S. Timóteo, S. S. Avancini, M. B. Pinto, and G. Krein, *Eur. Phys. J. A* **53**, 101 (2017).
- [63] M. Kurian and V. Chandra, *Phys. Rev. D* **96**, 114026 (2017).
- [64] L. Levkova and C. DeTar, *Phys. Rev. Lett.* **112**, 012002 (2014).
- [65] C. Bonati, M. D'Elia, M. Mariti, F. Negro, and F. Sanfilippo, *Phys. Rev. Lett.* **111**, 182001 (2013).
- [66] C. Bonati, M. D'Elia, M. Mariti, F. Negro, and F. Sanfilippo, *Phys. Rev. D* **89**, 054506 (2014).
- [67] S. Rath and B. K. Patra, *J. High Energy Phys.* **12** (2017) 098.
- [68] A. Bandyopadhyay, B. Karmakar, N. Haque, and M. G. Mustafa, [arXiv:1702.02875v2](https://arxiv.org/abs/1702.02875v2).