Analysis of nuclear structure in the nuclear chart and improvement to the gross theory of β decay

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The gross theory, a global method for the calculation of nuclear β decay, is improved using single-particle treatment. We analyzed the β-decay channel for the entire region of nuclides and found that the allowed transition is suppressed systematically in the neighborhood of heavier doubly magic nuclei. This is mainly due to the shift of neutron and proton single-particle levels in one major shell, which leads to a change in the spin and parity between parent and daughter nuclei. Under this consideration, the nuclear matrix elements for the allowed transition in the gross theory are suppressed in the above condition. In the vicinity of the doubly magic numbers of neutron-rich nuclides, such as nuclei with $Z = 50$ and 82, the calculated half-lives are longer than those reported in previous work, where only parity mismatch was considered. In superheavy neutron-rich nuclei with $N = 184$, the half-lives are predicted to be longer due to the same mechanism.

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I. INTRODUCTION

β decay is attributed to weak interaction. β decay in neutron-rich nuclei is one of the key processes in r-process nucleosynthesis and nuclear reactors. Regarding r-process nucleosynthesis, a neutron star merger comparable to a supernova explosion has recently been discussed as a candidate site. At such sites, the r-process could reach a heavier and neutron-rich nuclear mass region that includes superheavy nuclei.

To theoretically estimate the β-decay rate in such a wide nuclear mass region, some β-decay models have been presented. The gross theory is a method for calculating the β-decay rate for such purposes. It is based on the sum rule of the β-decay strength function and treats the transitions to all final nuclear levels in a statistical manner. It has been successful in describing β decay for the entire range of nuclear masses [1–8]. Recently, we have developed a model based on the parity exchange of single-particle levels of the ground state [9] and systematically improved the calculated half-lives near heavier neutron-rich doubly magic nuclei, including neutron-rich silver-tin isotopes, for which Lorusso *et al.* [10] reported discrepancies in half-lives obtained from experiments and the previous gross theory. The essence of the improvement is the suppression of the allowed transition under parity mismatch in the ground-to-ground states. This improvement would extract the main part of hindrance of the allowed transition; however, this treatment was insufficient. In this paper, we treat the suppression of the allowed transition under the full condition of forbidden transitions by considering the spin and parity of the ground state of nuclei.

We first surveyed all transitions, including excited states, using a Woods-Saxon-type single-particle potential and estimated the nuclear mass region in which there is no allowed transition in the ground states and low excited states. We periodically found nuclei with no allowed transitions in the chart. We then improved the gross theory by introducing a full spin-parity condition; in previous work, only parity mismatch was adopted. In Sec. II, we describe the β decay and the method of the gross theory. In Sec. III, we present the results of a survey of nuclei with no allowed transitions based on the single-particle level. The results of β-decay half-lives are given in Sec. IV. A summary is presented in Sec. V.

II. β DECAY AND GROSS THEORY

The decay constant of β decay is expressed as the sum of all transition modes as

$$\lambda = \lambda_F + \lambda_{GT} + \lambda_{1st} \dots, \quad (1)$$

where λ_F and λ_{GT} are the allowed transitions and λ_{1st} is the first-forbidden transition. In this paper, we regard the forbidden rank as Ω. The decay constant can be written as

$$\lambda_\Omega = \frac{m_e^5 c^4}{2\pi^4 \hbar^7}\left(\frac{m_e c}{\hbar}\right)^{2\Omega} g^2 \int_{-Q_\beta}^{0} |M_\Omega|^2 f_\Omega(-E)dE, \quad (2)$$

where g is the coupling constant, written as

$$g = \begin{cases} g_V & \text{(V-type)} \\ g_A & \text{(A-type)}, \end{cases} \quad (3)$$

and $g_A/g_V = 1.25$. E is the transition energy measured from the parent state. The integral is performed from $-Q_\beta$ to 0, with the β-decay Q value, Q_β. $f_\Omega(-E)$ is the integrated Fermi function, which represents the distortion of the wave function

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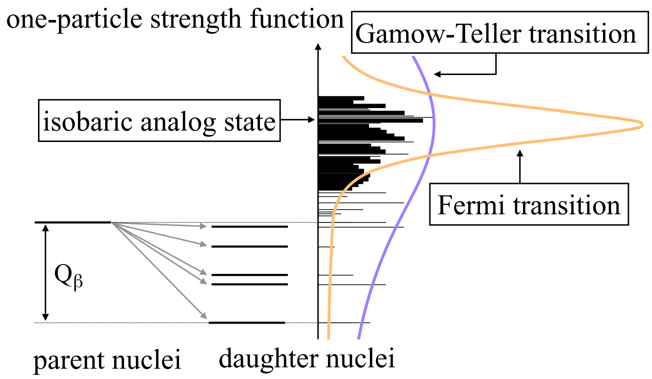

FIG. 1. Schematic view of the one-particle strength function of the allowed transition. The yellow solid line indicates the one-particle strength function of the Fermi transition, $D_F(E, \varepsilon)$, and the purple dashed line indicates that of the Gamow-Teller transition, $D_{GT}(E, \varepsilon)$. The strength function of the Fermi transition has a peak at the isobaric analog state; the peak for the Gamow-Teller transition shifts to higher energy and is a few MeV above the energy of the isobaric analog state.

due to the Coulomb force. To obtain the decay constant, we need the integrated Fermi function and the nuclear matrix elements. The former is rather easy to estimate, whereas the latter generally requires more complicated calculation due to the many-body treatment and complexity of the nuclear force. The sum rule of all the intensities of the β decay must be guaranteed for each nucleus. Considering this property for all nuclei, we can utilize the sum rule of the β decay. The gross theory is one of the ways to estimate the half-lives of β decay from this viewpoint.

The gross theory of β decay was originally devised by Takahashi and Yamada and has since been revised by other authors [1–8]. The squared nuclear matrix element in the gross theory is written as

$$|M_\Omega(E)|^2 = \int_{\varepsilon_{\min}}^{\varepsilon_{\max}} D_\Omega(E, \varepsilon) W(E, \varepsilon) \frac{dn_1}{d\varepsilon} d\varepsilon, \tag{4}$$

where ε is the one-particle energy of the decaying nucleons. $D_\Omega(E, \varepsilon)$ is the one-particle strength function, which satisfies the sum rule and the energy-weighted sum rules of β-decay intensity as one nucleon. $W(E, \varepsilon)$ is a weighting function that reflects the Pauli exclusion principle. $dn_1/d\varepsilon$ is the one-particle energy distribution of the decaying nucleons.

Figure 1 shows a schematic view of the allowed strength function. For the Fermi transition, the strength function, $D_F(E, \varepsilon)$, forms a sharp peak at the energy of the isobaric analog state (IAS) and has a long-tailed distribution over all E. For the Gamow-Teller transition, the strength function, $D_{GT}(E, \varepsilon)$, has a peak at a few MeV above the energy of IAS and a broad-tailed distribution. The one-particle strength functions are chosen as a simple functional form of Z, N, and A. The form is considered through experimental (p, n) reactions.

Figure 2 shows schematic diagrams of the nuclear matrix element of the β^- decay. The left figure shows the decay transition between levels in the general form. Each transition

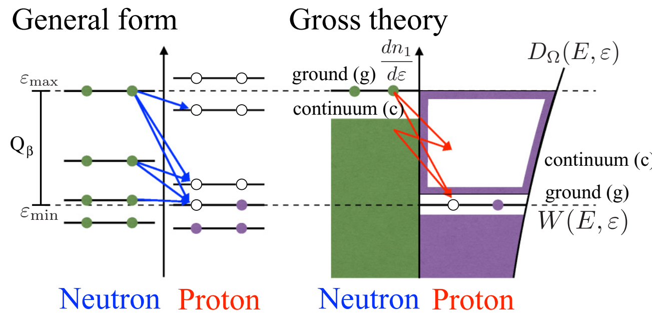

FIG. 2. Schematic diagram of the nuclear matrix element of β^- decay. Left: general form of β^--decay transition. Right: β^--decay transition in the gross theory. Arrows indicate selected transition channels, which give the nuclear matrix elements.

gives the nuclear matrix elements of β^- decay, shown as arrows. When the energy level of a neutron is higher than that of a proton, the neutron decays into a proton. The right figure shows the nuclear levels in the gross theory. Discrete levels other than the Fermi level are replaced by continuum levels (filled region). Consequently, the discrete levels have two cases, namely the Fermi level of neutron states (parent) and that of proton states (daughter), in the gross theory. The continuum levels can be expressed as the level density. In the gross theory, the level density is obtained from the Fermi gas model. The one-particle strength function is also plotted (curved line), which is the same as that shown in Fig. 1.

A nuclear matrix element can be divided into four components (four arrows in Fig. 2):

$$|M_\Omega(E)|^2 = a_1|M_{\Omega g\to g}(E)|^2 + a_2|M_{\Omega g\to c}(E)|^2 + a_3|M_{\Omega c\to g}(E)|^2 + a_4|M_{\Omega c\to c}(E)|^2. \tag{5}$$

Here, g and c stand for the ground-state and the continuum levels, respectively. The coefficients $a_{1\text{–}4}$ are parameters and generally unity. The values may change for transition types and also nuclear states. We neglect the incoherent components [11].

In our previous work, we introduced an allowed-transition hindrance into the nuclear matrix elements by using the parity mismatch of the ground-to-ground (g-g) transition estimated from the Wood-Saxon-type single-particle potential [9]. In the treatment, the matrix elements are suppressed without the continuum-to-continuum (c-c) type of the matrix element when parity mismatch occurs in the transition of g-g states, which is regarded as a forbidden transition. This treatment systematically improved the calculated half-lives near heavier neutron-rich doubly magic nuclei, especially the half-lives of neutron-rich silver-tin isotopes beyond $N = 82$, for which Lorusso *et al.* [10] reported discrepancies in half-lives obtained from experiments and the previous gross theory.

Parity mismatch is one of the principal conditions for hindrance to the allowed transition; however, this is insufficient because the change of spin that occurs in the transition is not considered. In this study, we treat the suppression of each transition type, such as the allowed transition (e.g., first-forbidden transition), with the spin and parity condition instead of parity mismatch. We also consider the low excited levels in addition to the ground-state level.

III. SINGLE-PARTICLE LEVEL

The transition types of β decay depend on the spin parity between parent and daughter nuclei. To systematically estimate them in the entire nuclear mass region, we adopt a modified Woods-Saxon-type single-particle potential [12].

Figure 3 shows the decay scheme of ^{208}Tl$_{127}$ in single-particle levels as an example. In pure single-particle levels, the nucleus ^{208}Tl$_{127}$ has five decay channels. The single-neutron level of the ground state is $2g_{9/2}$ with even parity, and the single-proton level of the ground state is $3s_{1/2}$ with even parity. Thus, the differences of spin and parity are $\Delta J = 4$ and $\Delta\pi = +$, respectively, and the corresponding channel of the *g*-*g* state is the fourth-forbidden transition. Furthermore, there are four first-forbidden transitions. In our previous work, we only considered the parity change, and thus we did not treat this nucleus as having forbidden transitions.

By using the single-particle potential, we surveyed the distribution of β decay modes in the nuclear chart.

Figure 4 shows the β-decay modes determined by the change of spin and parity of the *g*-*g* states for spherical or small deformed nuclei in the entire nuclear mass region. It can be seen that the allowed transition appears in the region up to $Z \approx 40$ and becomes scarce in the region beyond $Z \approx 50$ near stable nuclei. If the proton number of the nuclei is increased to $Z \approx 20$ or more, stable nuclei must have $N > Z$. Consequently, heavier stable nuclei, such as ^{132}Sn and ^{208}Pb, may have different major shells between neutrons and protons. In the vicinity of ^{132}Sn, which has $Z = 50$ and $N = 82$, the β-decay mode below $Z = 50$ changes from the second-forbidden transition (+) to the first-forbidden transition (negative parity change, −) when the number of neutrons exceeds $N = 82$ in the nuclei. In the vicinity of ^{208}Pb, which has $Z = 82$ and $N = 126$, the β-decay mode below $Z = 82$ changes from the first-forbidden transition (−) to the fourth-forbidden transition (+) when the number of neutrons exceeds $N = 126$ in the nuclei. In this way, the heavy and superheavy nuclei tend to lack the allowed transition due to a shift in the major shells between neutrons and protons.

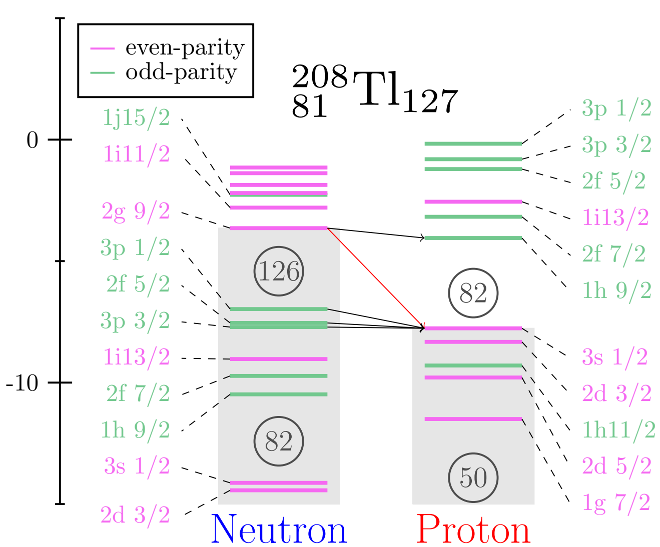

FIG. 3. Decay scheme of ^{208}Tl$_{127}$ in single-particle levels. The thin purple line represents even (positive) parity, and the thin green line represents odd (negative) parity. The black arrows indicate all transition channels for β decay. Single-particle levels were estimated from Ref. [12].

This estimation can be applied for spherical or small deformed nuclei. Deformed nuclei have more levels due to their more complicated configurations. Therefore, we limit the analysis to nuclei with a small deformation parameter of $\alpha_2 = 0.05$, as done in Ref. [9].

When we estimate the total half-life of β^- decay, the contribution from the excited states of a proton should be considered because the decay constant is expressed by integrals over energies ranging from $-Q_\beta$ to 0, as in Eq. (2). For ^{208}Tl$_{127}$, there are no allowed transitions in the five decay channels that include excited states, as shown in Fig. 3. Figure 5 shows nuclei with no allowed transitions of β^- decay even in the excited states, as was the case for ^{208}Tl$_{127}$. In the figure, nuclei in which the allowed transition is suppressed are located in regions of almost doubly magic nuclei: (1) the northeast side from ^{48}Ca, (2) along $N = 50$ between $Z \approx 32$ and 40, (3) the northeast side from ^{132}Sn, (4) the south side from ^{208}Pb, (5) along $N = 184$ between $Z \approx 100$ and 118, (6) along $N = 228$ between $Z \approx 114$ and 120, and (7) along $N = 308$.

Among these regions, there are typical doubly magic nuclei, such as ^{132}Sn in region (4), ^{208}Pb in (5), and ^{298}Fl in (6). Figure 6 shows these single-particle levels. For ^{132}Sn, the neutron single-particle levels above $N = 82$ have odd parity, and most of the proton levels between 82 and 50 have even parity. Consequently, the decay channels of nuclei on the northeast side from ^{132}Sn$_{82}$ are almost nonallowed transitions because there is a parity mismatch. For ^{208}Pb$_{126}$ and ^{114}Fl$_{184}$, each Fermi surface of a neutron and that of a proton for these nuclei are relatively shifted in one major shell. This is the case for the shift of one major shell between 82 and 126 and between 114 and 184. Energy differences between the Fermi surface of a neutron and that of a proton are small due to this shift. Therefore, these nuclei have no allowed transitions.

For nuclei in the neutron-rich region in the chart, the spacing of the Fermi surfaces between neutrons and protons is larger. Therefore, there are many energy levels between Q values, including levels in which allowed transitions take place. Consequently, nuclei with nonallowed transitions are located near the β-stable region, not far from the β-stability line. If we extend the survey to a much heavier region, we find some regions of nonallowed transitions located at $N \approx 228$ and 308. These neutron numbers were estimated to be closed shells [11].

Returning to β decay, we consider the above properties of single-particle levels to calculate the half-lives in the gross theory. Instead of using parity mismatch, as done in our previous work [9], we adopt the condition of the change of spin and parity in the *g*-*g* transition between a neutron and a proton. As the treatment of higher-rank-forbidden transitions is rather complicated, we simply divide the cases into two types. If the spin and parity of the *g*-*g* transition are matched to the allowed transition, the calculation is the same as that in the original

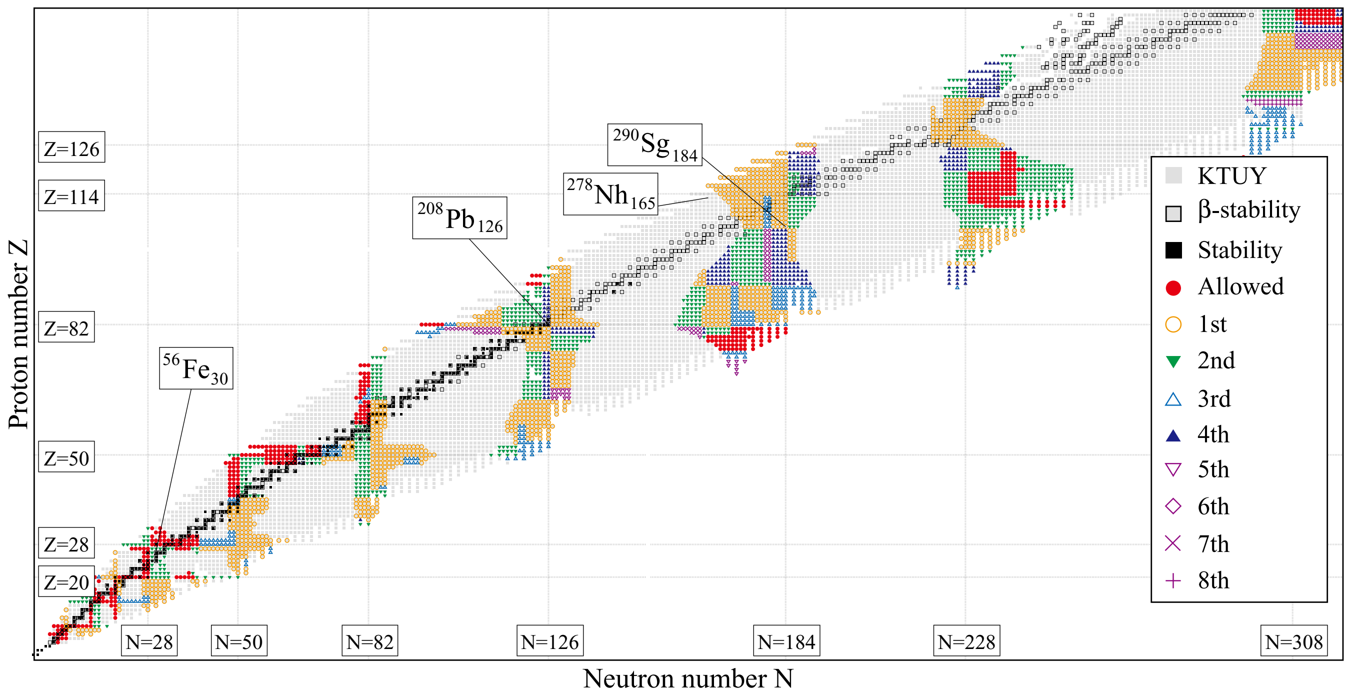

FIG. 4. β-decay mode in the g-g states for spherical or small deformed nuclei. The spin and parity of the ground state were estimated from the modified Woods-Saxon potential [12]. Filled squares (black): β-stable or long-lived nuclides (experiment). Open squares (black): β-stable nuclides according to the Koura-Tachibana-Uno-Yamada (KTUY) mass model [13]. Filled squares (gray): nuclides predicted to exist by the KTUY mass model.

gross theory. If the spin and parity of the g-g transition are matched to each forbidden transition, the allowed transition is suppressed for the g-g, g-c, and c-g components of the nuclear matrix element.

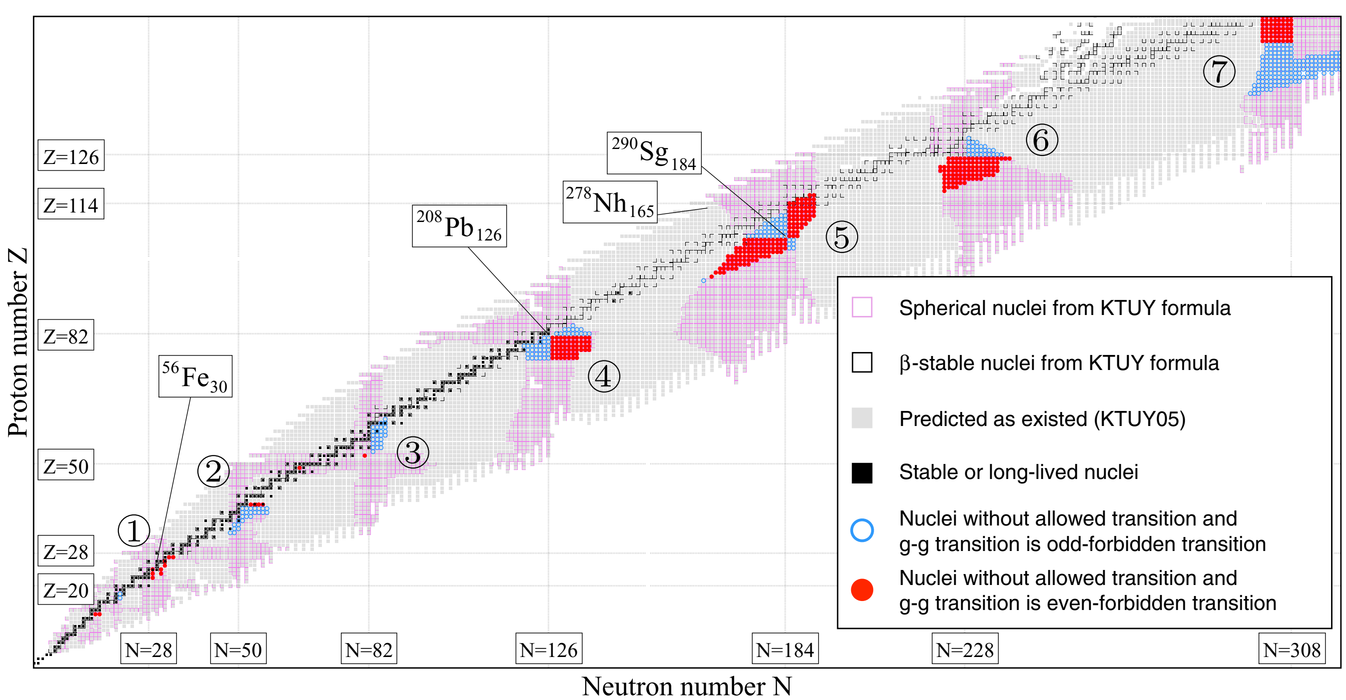

FIG. 5. Nuclei in which there is no allowed transition for β^- decay in spherical single-particle levels. The nuclei are categorized as (open circles) those for which the g-g transition is odd-forbidden and (filled circles) those for which the g-g transition is even-forbidden. Purple open squares are the spherical nuclei of the KTUY mass model. Other symbols are the same as those in Fig. 4.

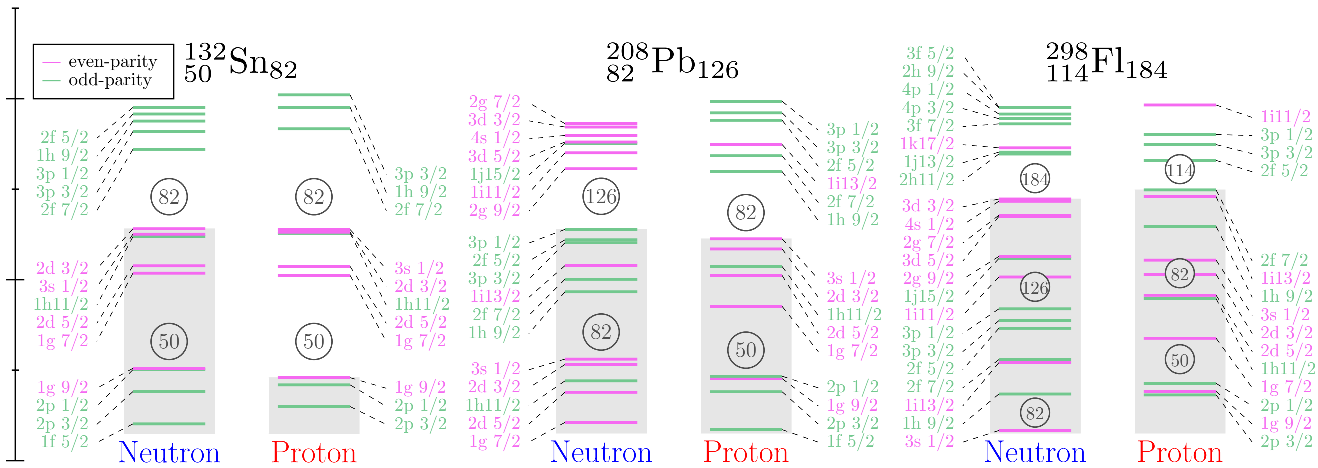

FIG. 6. Single-particle levels in doubly magic nuclei: $^{132}Sn_{82}$ (left), $^{208}Pb_{126}$ (middle), and $^{298}Fl_{184}$ (right). These levels were obtained using the modified Wood-Saxon potential [12].

If the spin and parity of the g-g transition are matched to the first-forbidden transition, the intensity of the allowed transition must be suppressed. The suppression can be related to the coefficients a_{1-4} in Eq. (5). In this study, we simply treat the suppression by adopting $a_{1,2,3} = 0$ and $a_4 = 1$ in Eq. (5) for the allowed transition; otherwise, $a_{1-4} = 1$. The c-c transition may include various states that match the allowed transition even though the g-g transition is the forbidden matching. The estimation of the ratio for each nucleus is more complicated. Therefore, we simply use $a_4 = 1$ only for the c-c component. For the case of the second-forbidden transition, the coefficients of both the allowed and first-forbidden transitions are treated as above. For the case of the high-rank-forbidden transition, all the lower rank transitions, including the allowed transition, are suppressed in the same manner.

In the calculation of the β decay of the gross theory in this work, we only consider the change of the ground-state spin and parity. The contribution from excited states is currently not considered in the model. As shown in Fig. 5, nuclei without the allowed transition in excited states are located near the β-stable nuclei in the neutron-rich region marked as (1)–(7) in Fig. 5. These regions almost coincide with the forbidden regions estimated from the ground-state spin and parity, as shown in Fig. 4. Therefore, the improvement reported in this paper is considered to be reliable for these nuclei. Some results and discussions are given below.

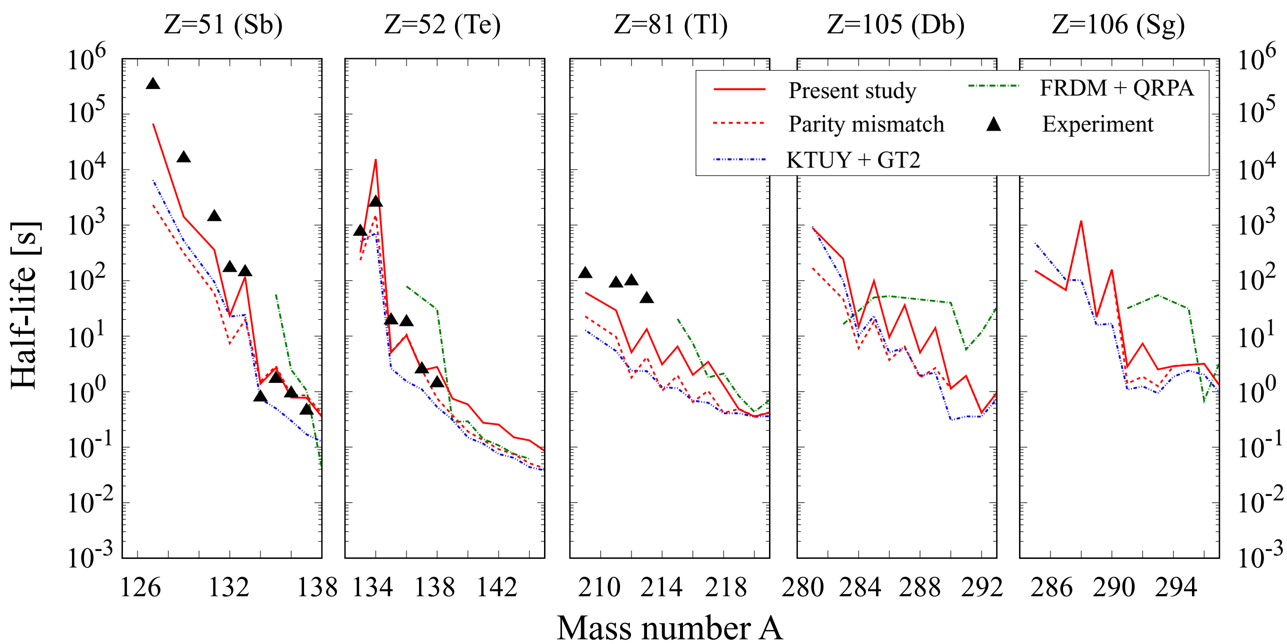

FIG. 7. β^- decay half-lives of $Z = 51$, 52, 81, 105, and 106 isotopes in the neutron-rich mass region. Solid red line: present study. Dashed blue line: KTUY + second version of the gross theory (GT2) [8]. Dashed red line: KTUY + GT2 + parity mismatch [9]. Dashed green line: finite-range droplet model (FRDM) + quasiparticle random-phase approximation (QRPA) [14]. Filled triangles: experimental results reported in 2015 [10].

TABLE I. Properties of nuclear structure for β-decay study of selected nuclei. "Single-particle orbitals" (third and fourth columns) show single-proton and -neutron orbitals at the Fermi surface estimated from the modified Woods-Saxon potential [12]. "Number of transition modes" shows the number of transitions available for each transition from excited levels between the Fermi surfaces of neutrons and protons. "Forbidden" includes all of the forbidden transitions (i.e., the number of forbidden transitions). The ratios of calculated half-lives to experimental ones are also listed.

(Z, N)	^{A}El	Single-particle orbital		Number of transitions		$T_{\rm th}/T_{\rm exp}$	
		Proton	Neutron	Allowed	Forbidden	This work	Previous
(51,80)	^{131}Sb	$1g_{7/2}(+)$	$2d_{3/2}(+)$	1	4	0.26	0.04
(51,81)	^{132}Sb	$1g_{7/2}(+)$	$2d_{3/2}(+)$	1	6	0.14	0.05
(51,82)	^{133}Sb	$1g_{7/2}(+)$	$2d_{3/2}(+)$	1	6	0.84	0.14
(51,83)	^{134}Sb	$1g_{7/2}(+)$	$2f_{7/2}(-)$	1	10	0.60	0.60
(51,84)	^{135}Sb	$1g_{7/2}(+)$	$2f_{7/2}(-)$	1	10	0.34	0.34
(51,85)	^{136}Sb	$1g_{7/2}(+)$	$2f_{7/2}(-)$	2	11	0.28	0.28
(81,130)	^{211}Tl	$3s_{1/2}(+)$	$2g_{9/2}(+)$	0	6	0.33	0.06
(81,131)	^{212}Tl	$3s_{1/2}(+)$	$2g_{9/2}(+)$	0	6	0.05	0.02
(81,132)	^{213}Tl	$3s_{1/2}(+)$	$2g_{9/2}(+)$	0	7	0.29	0.06

IV. HALF-LIFE OF β DECAY

The calculated half-lives of selected isotopes are shown in Fig. 7. For neutron-rich antimony isotopes ($Z = 51$), the half-lives calculated using the gross theory in our previous work are underestimated. Considering the single-neutron orbitals at the Fermi surface, some of the β-decay transitions must be suppressed. For the $A = 130$–133 isotopes of antimony, the single-neutron orbital is estimated to be $2d_{3/2}(+)$, as shown in Table I (see also the single-particle levels for ^{132}Sn in Fig. 6, as the neighboring doubly magic nucleus). Because the single-proton orbit is $1g7/2(+)$, the transition type in the g-g state for these nuclei is the second-forbidden transition. For $A = 134$–142, the single-neutron orbital is estimated to be $2f_{3/2}(-)$; therefore, the transition type in the g-g state for these nuclei is the first-forbidden transition. For the latter case ($A \geqslant 134$), we improved the calculated half-lives using a treatment based on parity mismatch [9]. For the former case ($A < 134$), the calculated half-lives from our previous study (parity mismatch) are still underestimated, whereas those in the present study, for which the change of spin and parity is considered, are close to the experimental values. For neutron-rich tellurium isotopes ($Z = 52$), in the range of $N > 137$, g-g transitions are the first-forbidden transition. However, our previous study did not treat these nuclei as the first-forbidden transition in the region. We fixed this mistake in the present study. For neutron-rich thallium isotopes ($Z = 81$), the single-neutron orbital is estimated to be $2g_{9/2}(+)$ in the range of $A =$ 208–217, as shown in Table I and Fig. 6 for ^{208}Pb. This is an example of a fourth-forbidden transition. We also considered the contribution of excited states to half-lives. Table I shows the number of transition modes, including those from or to excited levels. In these nuclei, there are few or no allowed transitions. Now we focus on neutron-rich isotone nuclei in the neighborhood of $N = 126$. Figure 8 shows the decay schemes of β decay in the single-particle level and the allowed transition channel in $N = 127$.

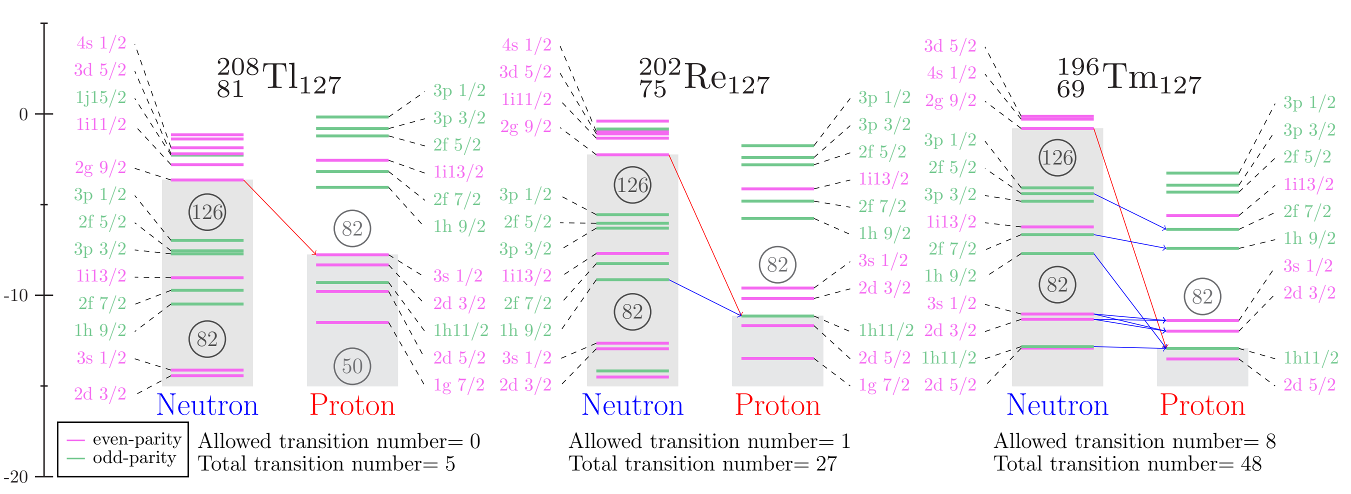

FIG. 8. Decay schemes of selected isotone nuclei with $N = 127$, ^{208}Tl$_{127}$, ^{202}Re$_{127}$, and ^{196}Tm$_{127}$ in the pure single-particle levels. A nucleus becomes neutron rich when it passes to right figure. Solid arrow (red): g-g transition. Dashed arrow (blue): allowed transition. Forbidden transitions are not illustrated in the figure because there are many such transitions.

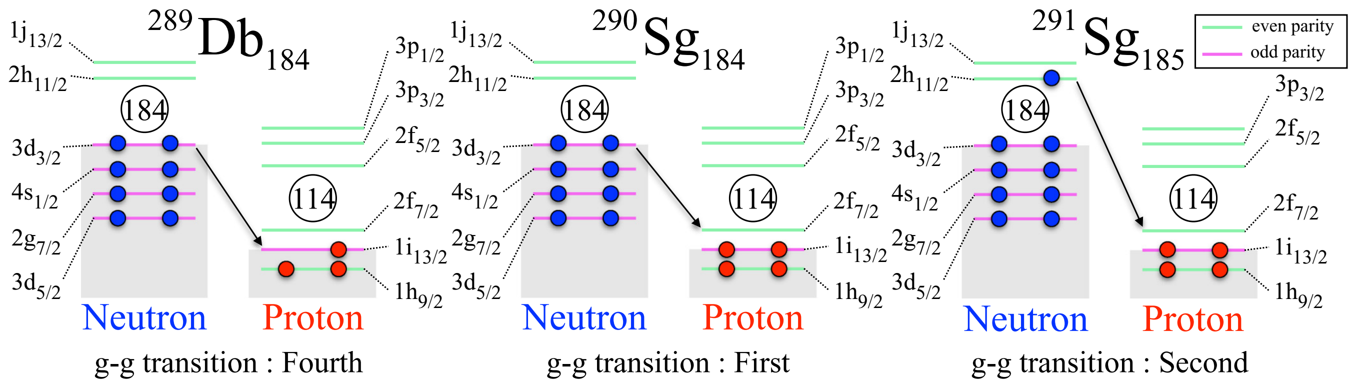

FIG. 9. Decay scheme of superheavy nuclei around ^{290}Sg$_{184}$ [11,12]. The thin purple lines represent even (positive) parity and the thin green lines represent odd (negative) parity.

For ^{208}Tl$_{127}$, there are no allowed transitions, as mentioned. For ^{202}Re$_{127}$, a more neutron-rich nucleus, which has 27 decay channels, there is only one allowed transition. For ^{196}Tm$_{127}$, which has 48 decay channels, 8 channels are allowed transitions. The doubly magic nucleus ^{208}Pb (and neighboring nucleus ^{208}Tl$_{127}$) has a shift of single-particle levels between neutrons and protons in one major shell, resulting in mismatch, as explained in the previous section. For ^{202}Re$_{127}$, this mismatch still remains even though there is only one allowed transition. A much more neutron-rich nucleus, ^{196}Tm$_{127}$, however, does not have a shift in one major shell due to decreasing proton energy with Coulomb repulsion and increasing neutron energy with isospin symmetry. Because of the disappearance of the shift in one major shell and increasing Q value, there are many channels that have an allowed transition.

We applied this β-decay calculation to heavy and superheavy nuclei. In these nuclei, the single-particle levels with high angular momentum, l, mix, and these levels shift to lower energy from higher energy levels, coming even close to the Fermi energy or its neighborhood because of the ls-splitting force, which is proportional to l. Doubly magic superheavy nuclei, such as ^{298}Fl$_{184}$, as shown in Fig. 6, may exhibit a shift of one major shell of single-particle levels, as discussed in Sec III. In addition, the superheavy nuclei have levels with high angular momentum near the Fermi surface or lower excited states. In this case, the combination between neutron and proton levels for the β-decay transition may have a large difference in spin, even though they have the same parity: It is not the allowed transition. Figure 9 shows the β-decay scheme of a superheavy nucleus, namely ^{290}Sg$_{184}$. The neutron Fermi level is $3d_{3/2}(+)$ (184th occupied) and the proton Fermi level is $1i_{13/2}(+)$ (106th occupied). In the β-decay transition, the g-g transition is between $3d_{3/2}(+)$ of a neutron and $2f_{7/2}(-)$ of a proton because the β^- decay increases the number of protons by one. Between these energy levels, there are no allowed transitions.

We consider the β decay of ^{289}Db$_{184}$, which excludes one proton from ^{290}Sg$_{184}$. A $3d_{3/2}(+)$ neutron in this nucleus can decay to the $1i_{13/2}(+)$ of a proton in the g-g transition. This is also a forbidden transition. The situation is similar to that for ^{291}Sg$_{185}$, which excludes one neutron from ^{290}Sg$_{184}$. The Fermi level of a neutron is $2h_{11/2}(+)$ and that of a proton is $2f_{7/2}(-)$.

Figure 7 also shows the results of calculated half-lives for dubnium ($Z = 105$) and seaborgium ($Z = 106$) isotopes. Because of the suppression of forbidden transitions in this work, half-lives are estimated to be longer than those obtained in previous calculations, in which spin parity was not considered (only parity mismatch was considered).

V. SUMMARY

We surveyed the change of spin and parity between neutron and proton single-particle levels for β decay in the entire nuclear chart and improved the gross theory of β decay based on this spin-parity property. We found that regions where the forbidden transition is dominant are distributed periodically in the nuclear chart, especially in the vicinity of ^{132}Sn, ^{208}Pb, and ^{298}Fl. By introducing the spin-parity treatment to the gross theory, the half-lives of nuclei in which the allowed transition is hindered in the g-g state increased, becoming close to experimental values. We showed the case of the neighborhood of ^{132}Sn and ^{208}Pb, including some predictions for neutron-rich nuclei such as Db and Sg isotopes for which experimental data are unavailable.

ACKNOWLEDGMENTS

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