Covariant nucleon-nucleon contact Lagrangian up to order $\mathcal{O}(q^4)$

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We adopt a covariant version of the naive dimensional analysis and construct the two-nucleon contact Lagrangian constrained by Lorentz, parity, charge conjugation, and Hermitian conjugation symmetries. We show that at $\mathcal{O}(q^0)$, $\mathcal{O}(q^2)$, and $\mathcal{O}(q^4)$, where q denotes a generic small momentum, there are 4, 13, and 23 terms, respectively. We find that by performing $1/m_N$ expansions, the covariant Lagrangian reduces to the conventional nonrelativistic one, which includes 2, 7, and 15 terms at each corresponding order.

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I. INTRODUCTION

Chiral perturbation theory (χPT), as proposed by Weinberg [1,2], has turned out to be very successful in the study of the strong interactions in nuclear physics. This theory of the nuclear forces is consistent with all the relevant symmetries, particularly the chiral symmetry and its breaking pattern. The interactions between the Nambu-Goldstone bosons themselves and with a heavier hadron can be computed in a systematic way order by order as an expansion of q/Λ_χ, where q denotes a generic small momentum and Λ_χ is the chiral symmetry breaking scale. The unknown short-range physics is encoded in the so-called low-energy constants (LECs), which in principle can be obtained from QCD, but in practice are often determined by either fitting to experimental data or lattice QCD simulations. The predictive power of χPT relies on the fact that for specific observables only a selected set of LECs contribute and with the same LECs it relates different observables.

χPT has been successfully applied in the pure mesonic sector [3,4]. However, when extended to the one baryon sector, one encounters a problem since the relatively large baryon mass cannot be counted as a small scale, which does not vanish in the chiral limit and breaks the power counting first introduced in the mesonic sector. In the past two decades, several modified power counting schemes have been developed to overcome this problem. At the very beginning, a nonrelativistic approach, i.e., the heavy baryon (HB) chiral perturbation theory [5,6], inspired by the heavy quark effective field theory [7], was introduced. More recently two relativistic approaches, i.e., the infrared (IR) [8] and the extended-on-mass-shell (EOMS) scheme [9,10], were developed to overcome some of the drawbacks of the nonrelativistic approach. The latter seems to be successful both formally and empirically [11–31].[1]

In the 1990s, Weinberg proposed that one can construct the nucleon-nucleon interaction using heavy baryon chiral perturbation theory [33,34]. It has been remarkably successful and led to an era of high precision since the year of 2003 [35–40]. An ingredient of the chiral nuclear force is the contact nucleon-nucleon Lagrangian, which parametrizes the short-range interactions. In the present work, we would like to study the constraints of Lorentz covariance on the NN contact Lagrangian. At present, there are already a few works in this direction. In Refs. [41,42], the relativistic NN Lagrangian is constructed up to $\mathcal{O}(q^2)$. In Ref. [43], such a work is extended to the SU(3) case. In a series of recent works, using the leading order relativistic Lagrangian, some of us have studied nucleon-nucleon scattering data [44] and hyperon-nucleon (hyperon) data [45] as well as related lattice QCD simulations [46,47]. Quite lately, it was shown in Ref. [48] that one can achieve a satisfactory description of polarized p-d scattering data below the deuteron breakup threshold either with the leading order relativistic $3N$ interaction (in the power counting of Ref. [44]) or with the next-to-leading order nonrelativistic $3N$ interaction.

In this work, we revisit this problem in the NN sector and construct a complete set of covariant NN contact Lagrangian terms up to $\mathcal{O}(q^4)$. Using the equation of motion to eliminate redundant terms, the total number of independent terms is found to be 4 at $\mathcal{O}(q^0)$, 13 at $\mathcal{O}(q^2)$, and 23 at $\mathcal{O}(q^4)$. As an explicit check, we expand our relativistic[2] Lagrangian in terms of $1/m_N$, where m_N is the nucleon mass, and show that in the heavy baryon limit our Lagrangian reduces to that of the HBχPT.

This work is organized as follows. In Sec. II, we explain the general principles to construct a covariant nucleon-nucleon Lagrangian, list the approximate equalities derived from the equation of motion and use them to eliminate redundant

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[1]For a concise review, see, e.g., Ref. [32].

[2]In the present work, "relativistic" and "covariant" are used interchangeably.

terms, and write down the covariant nucleon-nucleon contact Lagrangian up to order $\mathcal{O}(q^4)$. In Sec. III, we compare our results with those of earlier studies and perform the nonrelativistic reduction, which is followed by a short summary in Sec. IV. A concise derivation of the nonrelativistic nucleon-nucleon contact Lagrangian up to order $\mathcal{O}(q^4)$ is given in the Appendix.

II. RELATIVISTIC *NN* LAGRANGIAN UP TO $\mathcal{O}(q^4)$

The relativistic NN contact Lagrangian should fulfill the following requirements. First of all, the Lagrangian must be a Lorentz scalar. Second, it has to be invariant under chiral transformations, parity ($\mathcal{P}$), charge conjugation ($\mathcal{C}$), Hermitian conjugation (H.c.), and time reversal. Third, it needs to satisfy a proper power counting so that one can determine the relative importance of each term and have limited terms at each order in the chiral expansion.

The general expression of a covariant nucleon-nucleon contact Lagrangian[3] reads

$$\frac{1}{(2m)^{N_d}}(\bar{\psi} i \overleftrightarrow{\partial}^{\alpha} i \overleftrightarrow{\partial}^{\beta} \cdots \Gamma_A \psi) \times \partial^{\lambda}\partial^{\mu}\cdots(\bar{\psi} i \overleftrightarrow{\partial}^{\sigma} i \overleftrightarrow{\partial}^{\tau} \cdots \Gamma_B \psi), \quad (1)$$

where ψ and $\bar{\psi}$ denote the relativistic nucleon field, $\overleftrightarrow{\partial}^{\alpha} = \overrightarrow{\partial}^{\alpha} - \overleftarrow{\partial}^{\alpha}$, where $\overrightarrow{\partial}^{\alpha}/\overleftarrow{\partial}^{\alpha}$ refers to the derivatives acting on ψ and $\bar{\psi}$, and $\Gamma_{A,B} \in \{\mathbb{1}, \gamma_5, \gamma^{\mu}, \gamma_5\gamma^{\mu}, \sigma^{\mu\nu}, g^{\mu\nu}, \epsilon^{\mu\nu\rho\sigma}\}$. In the above equation, N_d refers to the number of four-derivatives (both $\overleftrightarrow{\partial}$ and ∂) in the Lagrangian, m refers to the nucleon mass in the chiral limit, and the factor $1/(2m)^{N_d}$ has been introduced so that all the contact terms have the same dimension [41].

For the construction of effective Lagrangians, symmetry constraints are the most important. In our present case, apart from the invariance under Lorentz transformation, the covariant nucleon-nucleon contact Lagrangian has to be invariant under local chiral, parity, charge conjugation, Hermitian conjugation, and time reversal transformations. The Lorentz indices $\alpha, \beta, \ldots$ have to be contracted among themselves to fulfill Lorentz invariance. The local chiral symmetry can be trivially fulfilled because the nucleon field ψ transforms under chiral rotation $\psi \to K\psi$, where $K \in \mathrm{SU(2)_V}$. The Hermitian conjugation symmetry does not impose any constraint since to fulfill it one can always multiply the Lagrangian with a factor i if necessary. According to the CPT (charge, parity, and time reversal) theorem, time reversal symmetry is also automatically fulfilled if parity symmetry and charge conjugation symmetry are fulfilled. Therefore, one only needs to make sure that parity and charge conjugation symmetries are fulfilled.

To construct the NN Lagrangian, one has to specify a proper power counting. In our present case, we need to specify the chiral dimensions of all the building blocks. In the covariant case, the power counting is more involved, compared to the nonrelativistic case (see the Appendix). The chiral dimensions and properties of fermion bilinears, derivative operators, Dirac matrices, and Levi-Civita tensor under parity, charge conjugation, and Hermitian conjugation transformations are listed in Table I. The derivative ∂ acting on the whole bilinear is of order $\mathcal{O}(q^1)$, while the derivative $\overleftrightarrow{\partial}$ acting inside a bilinear is of $\mathcal{O}(q^0)$ due to the presence of the nucleon mass, where q denotes a generic small momentum, such as the nucleon three-momentum. The bilinear $\bar{\psi}\gamma_5\psi$ is of order $\mathcal{O}(q^1)$ because it mixes the large and small components of the Dirac spinor. The Levi-Civita tensor $\epsilon_{\mu\nu\rho\sigma}$ contracting with n derivatives acting inside a bilinear raises the chiral order by $n-1$. If a derivative $\overleftrightarrow{\partial}$ is contracted with one of the Dirac matrices $\gamma_5\gamma^{\mu}$ or $\sigma^{\mu\nu}$ in a different bilinear, the matrix element is of $\mathcal{O}(q^1)$, as can be explicitly checked by means of the equation of motion (EOM). Therefore, at each order in the powering counting, only a finite number of ∂ and $\epsilon_{\mu\nu\rho\sigma}$ appear. However, in principle, any number of pairwise contracted $i\overleftrightarrow{\partial}$ of the form

TABLE I. Chiral dimensions and properties of fermion bilinears, derivative operators, Dirac matrices, and Levi-Civita tensor, under parity ($\mathcal{P}$), charge conjugation ($\mathcal{C}$), and Hermitian conjugation (H.c.) transformations.

	$\mathbb{1}$	γ_5	γ_μ	$\gamma_5\gamma_\mu$	$\sigma_{\mu\nu}$	$\epsilon_{\mu\nu\rho\sigma}$	$\overleftrightarrow{\partial}_\mu$	∂_μ
dim	0	1	0	0	0	−	0	1
$\mathcal{P}$	+	−	+	−	+	−	+	+
$\mathcal{C}$	+	+	−	+	−	+	−	+
H.c.	+	−	+	+	+	+	−	+

$$\widetilde{\mathcal{O}}^{(n)}_{\Gamma_A\Gamma_B} = \frac{1}{(2m)^{2n}}\left(\bar{\psi} i \overleftrightarrow{\partial}^{\mu_1} i \overleftrightarrow{\partial}^{\mu_2} \cdots i \overleftrightarrow{\partial}^{\mu_n} \Gamma_A^{\alpha}\psi\right) \times (\bar{\psi} i \overleftrightarrow{\partial}_{\mu_1} i \overleftrightarrow{\partial}_{\mu_2} \cdots i \overleftrightarrow{\partial}_{\mu_n} \Gamma_{B\alpha}\psi) \quad (2)$$

is allowed, since it is of $\mathcal{O}(q^0)$. Such a term gives $N(p_1) + N(p_2) \to N(p_3) + N(p_4)$. On the other hand, such structures as

$$\frac{[(p_1+p_3)\cdot(p_2+p_4)]^n}{(2m)^{2n}}, \quad \frac{[(p_1+p_4)\cdot(p_2+p_3)]^n}{(2m)^{2n}} \quad (3)$$

can be rewritten as

$$\left[1 + \frac{(s-4m^2)-u}{4m^2}\right]^n, \quad \left[1 + \frac{(s-4m^2)-t}{4m^2}\right]^n, \quad (4)$$

with $s - 4m^2 = -(p_1-p_2)^2 = -(p_3-p_4)^2 \sim \mathcal{O}(q^2)$, $u = (p_1-p_4)^2 \sim \mathcal{O}(q^2)$, and $t = (p_1-p_3)^2 \sim \mathcal{O}(q^2)$.[4] Therefore, at $\mathcal{O}(q^0)$, only the terms with $n = 0, 1, 2$ are needed, at $\mathcal{O}(q^2)$ only the terms with $n = 0, 1$ are needed, and at $\mathcal{O}(q^4)$ only the terms with $n = 0$ are needed since no new structures appear for n larger than those specified above.

[3]In the present work, to simplify the derivation, we do not consider the interaction of external fields and the pion with the nucleon.

[4]This can be easily checked by noting that in the center-of-mass frame $p_1^{\mu} = M_N + \mathcal{O}(q)$ and $p_2^{\mu} = M_N - \mathcal{O}(q)$ because of momentum conservation.

TABLE II. Decomposition of the Dirac matrix products $\Gamma \times \gamma_\lambda$ into charge conjugation even (Γ'_λ) and charge conjugation odd (Γ''_λ) parts [43].

Γ	Γ'_λ	Γ''_λ
$\mathbb{1}$	γ_λ	0
γ_μ	$g_{\mu\lambda}\mathbb{1}$	$-i\sigma_{\mu\lambda}$
γ_5	0	$\gamma_5\gamma_\lambda$
$\gamma_5\gamma_\mu$	$\frac{1}{2}\epsilon_{\mu\lambda\rho\tau}\sigma^{\rho\tau}$	$g_{\mu\lambda}\gamma_5$
$\sigma_{\mu\nu}$	$\epsilon_{\mu\nu\lambda\tau}\gamma_5\gamma^\tau$	$-i(g_{\mu\lambda}\gamma_\nu - g_{\nu\lambda}\gamma_\mu)$
$\epsilon_{\mu\nu\rho\tau}\gamma^\tau$	$\epsilon_{\mu\nu\rho\lambda}\mathbb{1}$	$g_{\mu\lambda}\gamma_5\sigma_{\nu\rho} + g_{\rho\lambda}\gamma_5\sigma_{\mu\nu} + g_{\nu\lambda}\gamma_5\sigma_{\rho\mu}$
$\epsilon_{\mu\nu\rho\tau}\gamma_5\gamma^\tau$	$g_{\mu\lambda}\sigma_{\nu\rho} + g_{\rho\lambda}\sigma_{\mu\nu} + g_{\nu\lambda}\sigma_{\rho\mu}$	$\epsilon_{\mu\nu\rho\lambda}\gamma_5$
$\epsilon_{\mu\nu\rho\alpha}\sigma^\alpha_\tau$	$\gamma_5\gamma_\rho(g_{\lambda\nu}g_{\mu\tau} - g_{\lambda\mu}g_{\nu\tau}) + \gamma_5\gamma_\nu(g_{\lambda\mu}g_{\rho\tau} - g_{\lambda\rho}g_{\mu\tau}) + \gamma_5\gamma_\mu(g_{\lambda\rho}g_{\nu\tau} - g_{\lambda\nu}g_{\rho\tau})$	$ig_{\lambda\tau}\epsilon_{\mu\nu\rho\alpha}\gamma^\alpha - i\epsilon_{\mu\nu\rho\lambda}\gamma_\tau$
$\frac{i}{2}\epsilon_{\mu\nu\rho\tau}\sigma^{\rho\tau} = \gamma_5\sigma_{\mu\nu}$	$\frac{1}{i}(g_{\mu\lambda}\gamma_5\gamma_\nu - g_{\nu\lambda}\gamma_5\gamma_\mu)$	$\epsilon_{\mu\nu\lambda\rho}\gamma^\rho$

Following the general principles of constructing effective Lagrangians and guided by Table I, one can write down all the terms of $\mathcal{O}(q^0)$, $\mathcal{O}(q^2)$, and $\mathcal{O}(q^4)$. As we show in the following, not all of them are independent up to the order of our concern, and one can use the EOM to eliminate the nonindependent or redundant terms.

The equation of motion for the nucleon refers to the well-known Dirac equation at LO

$$\partial\!\!\!/\psi = \gamma^\mu\partial_\mu\psi = -im\psi + \mathcal{O}(q), \tag{5}$$

and its hermitian conjugate. Up to higher order corrections one can replace $\partial\!\!\!/\psi$ by $-im\psi$ and $\bar\psi\overleftarrow{\partial\!\!\!/}$ by $im\bar\psi$. To fully utilize this EOM, one needs to transform terms that do not contain $\partial\!\!\!/$ into forms that contain it. Such a technique has been extensively discussed in the construction of the πN Lagrangian [49] and baryon-baryon Lagrangian [43]. The details can be found in Refs. [43,49]. The master formula is

$$-2im(\bar\psi\Gamma\psi) \approx 2(\bar\psi\Gamma\times\gamma_\lambda\partial^\lambda\psi) = (\bar\psi\Gamma'_\lambda\overleftrightarrow{\partial}^\lambda\psi) + \partial^\lambda(\bar\psi\Gamma''_\lambda\psi), \tag{6}$$

where Γ, Γ', and Γ'' are sets of Dirac matrices listed in Table II and $\approx$ indicates equal up to higher orders. Using the EOM together with the decomposition of Dirac matrices, one can obtain the following approximate equalities:

$$\partial^\mu(\bar\psi\gamma_\mu\psi) \approx 0, \tag{7.1}$$

$$\partial^\mu(\bar\psi\overleftrightarrow{\partial}_\mu\psi) \approx 0, \tag{7.2}$$

$$\partial^\mu(\bar\psi\gamma_5\gamma_\mu\psi) \approx -2m(\bar\psi i\gamma_5\psi), \tag{7.3}$$

$$\partial^\mu(\bar\psi\sigma_{\mu\nu}\psi) \approx (\bar\psi i\overleftrightarrow{\partial}_\nu\psi) - 2m(\bar\psi\gamma_\nu\psi), \tag{7.4}$$

$$(\bar\psi\gamma^\mu i\overleftrightarrow{\partial}_\mu\psi) \approx 2m(\bar\psi\psi), \tag{7.5}$$

$$(\bar\psi\overleftrightarrow{\partial}^2\psi) \approx -4m^2(\bar\psi\psi) - \partial^2(\bar\psi\psi), \tag{7.6}$$

$$(\bar\psi\gamma_5\gamma^\mu i\overleftrightarrow{\partial}_\mu\psi) \approx 0, \tag{7.7}$$

$$(\bar\psi i\overleftrightarrow{\partial}_\mu\sigma^{\mu\nu}\psi) \approx -\partial^\nu(\bar\psi\psi), \tag{7.8}$$

$$-2im(\bar\psi\gamma_5\gamma^\mu\psi) \approx \left(\bar\psi\tfrac{1}{2}\epsilon^{\mu\lambda\rho\tau}\sigma_{\rho\tau}\overleftrightarrow{\partial}_\lambda\psi\right) + \partial^\mu(\bar\psi\gamma_5\psi), \tag{7.9}$$

$$-2im(\bar\psi\sigma^{\mu\nu}\psi) \approx (\bar\psi\epsilon^{\mu\lambda\rho\tau}\gamma_5\gamma_\tau\overleftrightarrow{\partial}_\lambda\psi) + \partial_\lambda(\bar\psi - i(g^{\mu\lambda}\gamma^\nu - g^{\nu\lambda}\gamma^\mu)\psi), \tag{7.10}$$

$$-2im(\bar\psi\epsilon_{\mu\nu\rho\tau}\gamma^\tau\psi) \approx (\bar\psi\epsilon_{\mu\nu\rho\lambda}\overleftrightarrow{\partial}^\lambda\psi) + \partial^\lambda(\bar\psi(g_{\mu\lambda}\gamma_5\sigma_{\nu\rho} + g_{\rho\lambda}\gamma_5\sigma_{\mu\nu} + g_{\nu\lambda}\gamma_5\sigma_{\rho\mu})\psi), \tag{7.11}$$

$$-2im(\bar\psi\epsilon_{\mu\nu\rho\tau}\gamma_5\gamma^\tau\psi) \approx [\bar\psi(\sigma_{\nu\rho}\overleftrightarrow{\partial}_\mu + \sigma_{\mu\nu}\overleftrightarrow{\partial}_\rho + \sigma_{\rho\mu}\overleftrightarrow{\partial}_\nu)\psi] + \partial^\lambda(\bar\psi\epsilon_{\mu\nu\rho\lambda}\gamma_5\psi), \tag{7.12}$$

$$\begin{aligned}-2im(\bar\psi\epsilon_{\mu\nu\rho\alpha}\sigma^\alpha{}_\tau\psi) \approx{}& \{\bar\psi[\gamma_5\gamma_\rho(\overleftrightarrow{\partial}_\nu g_{\mu\tau} - \overleftrightarrow{\partial}_\mu g_{\nu\tau}) + \gamma_5\gamma_\nu(\overleftrightarrow{\partial}_\mu g_{\rho\tau} - \overleftrightarrow{\partial}_\rho g_{\mu\tau}) + \gamma_5\gamma_\mu(\overleftrightarrow{\partial}_\rho g_{\nu\tau} - \overleftrightarrow{\partial}_\nu g_{\rho\tau})]\psi\}\\ &+ \partial^\lambda(\bar\psi(ig_{\lambda\tau}\epsilon_{\mu\nu\rho\alpha}\gamma^\alpha - i\epsilon_{\mu\nu\rho\lambda}\gamma_\tau)\psi),\end{aligned} \tag{7.13}$$

$$m(\bar\psi\epsilon_{\mu\nu\rho\tau}\sigma^{\rho\tau}\psi) \approx [\bar\psi(\gamma_5\gamma_\mu i\overleftrightarrow{\partial}_\nu - \gamma_5\gamma_\nu i\overleftrightarrow{\partial}_\mu)\psi] + \partial^\lambda(\bar\psi\epsilon_{\mu\nu\lambda\rho}\gamma^\rho\psi), \tag{7.14}$$

$$(\bar\psi\epsilon_{\mu\nu\alpha\beta}i\overleftrightarrow{\partial}^\mu i\overleftrightarrow{\partial}^\nu\cdots\psi) = 0. \tag{7.15}$$

TABLE III. A complete set of relativistic NN contact Lagrangian terms up to $\mathcal{O}(q^4)$.

$\widetilde{O}_1$	$(\bar\psi\psi)(\bar\psi\psi)$	$\widetilde{O}_{21}$	$\frac{1}{16m^4}(\bar\psi i\overleftrightarrow{\partial}^{\mu}\psi)\partial^2\partial^\nu(\bar\psi\sigma_{\mu\nu}\psi)$
$\widetilde{O}_2$	$(\bar\psi\gamma^\mu\psi)(\bar\psi\gamma_\mu\psi)$	$\widetilde{O}_{22}$	$\frac{1}{16m^4}(\bar\psi\sigma^{\mu\alpha}\psi)\partial^2\partial_\alpha\partial^\nu(\bar\psi\sigma_{\mu\nu}\psi)$
$\widetilde{O}_3$	$(\bar\psi\gamma_5\gamma^\mu\psi)(\bar\psi\gamma_5\gamma_\mu\psi)$	$\widetilde{O}_{23}$	$\frac{1}{16m^4}(\bar\psi\sigma^{\mu\nu}i\overleftrightarrow{\partial}^{\alpha}\psi)\partial^\beta\partial_\nu(\bar\psi\sigma_{\alpha\beta}i\overleftrightarrow{\partial}_{\mu}\psi)$
$\widetilde{O}_4$	$(\bar\psi\sigma^{\mu\nu}\psi)(\bar\psi\sigma_{\mu\nu}\psi)$	$\widetilde{O}_{24}$	$\frac{1}{16m^4}(\bar\psi\psi)\partial^4(\bar\psi\psi)$
$\widetilde{O}_5$	$(\bar\psi\gamma_5\psi)(\bar\psi\gamma_5\psi)$	$\widetilde{O}_{25}$	$\frac{1}{16m^4}(\bar\psi\gamma^\mu\psi)\partial^4(\bar\psi\gamma_\mu\psi)$
$\widetilde{O}_6$	$\frac{1}{4m^2}(\bar\psi\gamma_5\gamma^\mu i\overleftrightarrow{\partial}^{\alpha}\psi)(\bar\psi\gamma_5\gamma_\alpha i\overleftrightarrow{\partial}_{\mu}\psi)$	$\widetilde{O}_{26}$	$\frac{1}{16m^4}(\bar\psi\gamma_5\gamma^\mu\psi)\partial^4(\bar\psi\gamma_5\gamma_\mu\psi)$
$\widetilde{O}_7$	$\frac{1}{4m^2}(\bar\psi\sigma^{\mu\nu}i\overleftrightarrow{\partial}^{\alpha}\psi)(\bar\psi\sigma_{\mu\alpha}i\overleftrightarrow{\partial}_{\nu}\psi)$	$\widetilde{O}_{27}$	$\frac{1}{16m^4}(\bar\psi\sigma^{\mu\nu}\psi)\partial^4(\bar\psi\sigma_{\mu\nu}\psi)$
$\widetilde{O}_8$	$\frac{1}{4m^2}(\bar\psi i\overleftrightarrow{\partial}^{\mu}\psi)\partial^\nu(\bar\psi\sigma_{\mu\nu}\psi)$	$\widetilde{O}_{28}$	$\frac{1}{4m^2}(\bar\psi\gamma_5 i\overleftrightarrow{\partial}^{\alpha}\psi)(\bar\psi\gamma_5 i\overleftrightarrow{\partial}_{\alpha}\psi)-\widetilde{O}_5$
$\widetilde{O}_9$	$\frac{1}{4m^2}(\bar\psi\sigma^{\mu\alpha}\psi)\partial_\alpha\partial^\nu(\bar\psi\sigma_{\mu\nu}\psi)$	$\widetilde{O}_{29}$	$\frac{1}{16m^4}(\bar\psi\gamma_5\gamma^\mu i\overleftrightarrow{\partial}^{\alpha}i\overleftrightarrow{\partial}^{\beta}\psi)(\bar\psi\gamma_5\gamma_\alpha i\overleftrightarrow{\partial}_{\mu}i\overleftrightarrow{\partial}_{\beta}\psi)-\widetilde{O}_6$
$\widetilde{O}_{10}$	$\frac{1}{4m^2}(\bar\psi\psi)\partial^2(\bar\psi\psi)$	$\widetilde{O}_{30}$	$\frac{1}{16m^4}(\bar\psi\sigma^{\mu\nu}i\overleftrightarrow{\partial}^{\alpha}i\overleftrightarrow{\partial}^{\beta}\psi)(\bar\psi\sigma_{\mu\alpha}i\overleftrightarrow{\partial}_{\nu}i\overleftrightarrow{\partial}_{\beta}\psi)-\widetilde{O}_7$
$\widetilde{O}_{11}$	$\frac{1}{4m^2}(\bar\psi\gamma^\mu\psi)\partial^2(\bar\psi\gamma_\mu\psi)$	$\widetilde{O}_{31}$	$\frac{1}{16m^4}(\bar\psi i\overleftrightarrow{\partial}^{\mu}i\overleftrightarrow{\partial}^{\beta}\psi)\partial^\alpha(\bar\psi\sigma_{\mu\alpha}i\overleftrightarrow{\partial}_{\beta}\psi)-\widetilde{O}_8$
$\widetilde{O}_{12}$	$\frac{1}{4m^2}(\bar\psi\gamma_5\gamma^\mu\psi)\partial^2(\bar\psi\gamma_5\gamma_\mu\psi)$	$\widetilde{O}_{32}$	$\frac{1}{16m^4}(\bar\psi\sigma^{\mu\alpha}i\overleftrightarrow{\partial}^{\beta}\psi)\partial_\alpha\partial^\nu(\bar\psi\sigma_{\mu\nu}i\overleftrightarrow{\partial}_{\beta}\psi)-\widetilde{O}_9$
$\widetilde{O}_{13}$	$\frac{1}{4m^2}(\bar\psi\sigma^{\mu\nu}\psi)\partial^2(\bar\psi\sigma_{\mu\nu}\psi)$	$\widetilde{O}_{33}$	$\frac{1}{16m^4}(\bar\psi i\overleftrightarrow{\partial}^{\alpha}\psi)\partial^2(\bar\psi i\overleftrightarrow{\partial}_{\alpha}\psi)-\widetilde{O}_{10}$
$\widetilde{O}_{14}$	$\frac{1}{4m^2}(\bar\psi i\overleftrightarrow{\partial}^{\alpha}\psi)(\bar\psi i\overleftrightarrow{\partial}_{\alpha}\psi)-\widetilde{O}_1$	$\widetilde{O}_{34}$	$\frac{1}{16m^4}(\bar\psi\gamma^\mu i\overleftrightarrow{\partial}^{\alpha}\psi)\partial^2(\bar\psi\gamma_\mu i\overleftrightarrow{\partial}_{\alpha}\psi)-\widetilde{O}_{11}$
$\widetilde{O}_{15}$	$\frac{1}{4m^2}(\bar\psi\gamma^\mu i\overleftrightarrow{\partial}^{\alpha}\psi)(\bar\psi\gamma_\mu i\overleftrightarrow{\partial}_{\alpha}\psi)-\widetilde{O}_2$	$\widetilde{O}_{35}$	$\frac{1}{16m^4}(\bar\psi\gamma_5\gamma^\mu i\overleftrightarrow{\partial}^{\alpha}\psi)\partial^2(\bar\psi\gamma_5\gamma_\mu i\overleftrightarrow{\partial}_{\alpha}\psi)-\widetilde{O}_{12}$
$\widetilde{O}_{16}$	$\frac{1}{4m^2}(\bar\psi\gamma_5\gamma^\mu i\overleftrightarrow{\partial}^{\alpha}\psi)(\bar\psi\gamma_5\gamma_\mu i\overleftrightarrow{\partial}_{\alpha}\psi)-\widetilde{O}_3$	$\widetilde{O}_{36}$	$\frac{1}{16m^4}(\bar\psi\sigma^{\mu\nu}i\overleftrightarrow{\partial}^{\alpha}\psi)\partial^2(\bar\psi\sigma_{\mu\nu}i\overleftrightarrow{\partial}_{\alpha}\psi)-\widetilde{O}_{13}$
$\widetilde{O}_{17}$	$\frac{1}{4m^2}(\bar\psi\sigma^{\mu\nu}i\overleftrightarrow{\partial}^{\alpha}\psi)(\bar\psi\sigma_{\mu\nu}i\overleftrightarrow{\partial}_{\alpha}\psi)-\widetilde{O}_4$	$\widetilde{O}_{37}$	$\frac{1}{16m^4}(\bar\psi i\overleftrightarrow{\partial}^{\alpha}i\overleftrightarrow{\partial}^{\beta}\psi)(\bar\psi i\overleftrightarrow{\partial}_{\alpha}i\overleftrightarrow{\partial}_{\beta}\psi)-2\widetilde{O}_{14}-\widetilde{O}_1$
$\widetilde{O}_{18}$	$\frac{1}{4m^2}(\bar\psi\gamma_5\psi)\partial^2(\bar\psi\gamma_5\psi)$	$\widetilde{O}_{38}$	$\frac{1}{16m^4}(\bar\psi\gamma^\mu i\overleftrightarrow{\partial}^{\alpha}i\overleftrightarrow{\partial}^{\beta}\psi)(\bar\psi\gamma_\mu i\overleftrightarrow{\partial}_{\alpha}i\overleftrightarrow{\partial}_{\beta}\psi)-2\widetilde{O}_{15}-\widetilde{O}_2$
$\widetilde{O}_{19}$	$\frac{1}{16m^4}(\bar\psi\gamma_5\gamma^\mu i\overleftrightarrow{\partial}^{\nu}\psi)\partial^2(\bar\psi\gamma_5\gamma_\nu i\overleftrightarrow{\partial}_{\mu}\psi)$	$\widetilde{O}_{39}$	$\frac{1}{16m^4}(\bar\psi\gamma_5\gamma^\mu i\overleftrightarrow{\partial}^{\alpha}i\overleftrightarrow{\partial}^{\beta}\psi)(\bar\psi\gamma_5\gamma_\mu i\overleftrightarrow{\partial}_{\alpha}i\overleftrightarrow{\partial}_{\beta}\psi)-2\widetilde{O}_{16}-\widetilde{O}_3$
$\widetilde{O}_{20}$	$\frac{1}{16m^4}(\bar\psi\sigma^{\mu\nu}i\overleftrightarrow{\partial}^{\alpha}\psi)\partial^2(\bar\psi\sigma_{\mu\alpha}i\overleftrightarrow{\partial}_{\nu}\psi)$	$\widetilde{O}_{40}$	$\frac{1}{16m^4}(\bar\psi\sigma^{\mu\nu}i\overleftrightarrow{\partial}^{\alpha}i\overleftrightarrow{\partial}^{\beta}\psi)(\bar\psi\sigma_{\mu\nu}i\overleftrightarrow{\partial}_{\alpha}i\overleftrightarrow{\partial}_{\beta}\psi)-2\widetilde{O}_{17}-\widetilde{O}_4$

The set of relations in Eq. (7) lead to the following four simplification rules:

(1) Terms with $\epsilon_{\mu\nu\rho\tau}$ can always be transformed into those without it, so no terms with $\epsilon_{\mu\nu\rho\tau}$ are needed.
(2) The derivative ∂_μ acting on the whole fermion bilinear cannot be contracted with any elements of the Clifford algebra except for $\sigma^{\mu\nu}$.
(3) The derivative $i\overleftrightarrow{\partial}^{\mu}$ cannot be contracted with any elements of the Clifford algebra inside the same fermion bilinear.
(4) Terms with γ^μ can be transformed into terms with $i\overleftrightarrow{\partial}^{\mu}$ except for the cases where it is contracted with $\sigma^{\mu\nu}$.

Using the four rules listed above, we obtain minimal and complete sets of relativistic NN contact Lagrangian terms $\bar{O}_i$ ($i = 1,\dots,40$), which are summarized in Table III.

III. DISCUSSIONS

A. Comparison with previous works

The Lagrangian terms listed in Table III are different from those of Refs. [41–43]. Moreover, the results of Refs. [41–43] can be reduced to ours using the approximate equalities listed in the last section. Compared with the results of Refs. [41,42], in our case all the terms with $\epsilon_{\mu\nu\alpha\beta}$ are eliminated using the EOM. In Ref. [43], the expressions contain at most two pairwise contracted $i\overleftrightarrow{\partial}$ of the form $\frac{1}{(2m)^{2n}}(\bar\psi i\overleftrightarrow{\partial}^{\mu_1}i\overleftrightarrow{\partial}^{\mu_2}\cdots i\overleftrightarrow{\partial}^{\mu_n}\Gamma^\alpha_A\psi)(\bar\psi i\overleftrightarrow{\partial}_{\mu_1}i\overleftrightarrow{\partial}_{\mu_2}\cdots i\overleftrightarrow{\partial}_{\mu_n}\Gamma_{B\alpha}\psi)$ up to $\mathcal{O}(q^2)$, while we only include at most one, which is consistent with Ref. [41]. We note that the terms with $\partial^\mu(\bar\psi\sigma_{\mu\nu}\psi)$ are included in our Lagrangian but are not in those of Refs. [41–43]. This is just a different choice of independent terms, because $\partial^\mu(\bar\psi\sigma_{\mu\nu}\psi) = (\bar\psi\sigma_{\mu\nu}\overrightarrow{\partial}^{\mu}\psi) + (\bar\psi\overleftarrow{\partial}^{\mu}\sigma_{\mu\nu}\psi) \approx (\bar\psi i\overleftrightarrow{\partial}_{\nu}\psi) - 2m(\bar\psi\gamma_\nu\psi)$. In Refs. [41,42], terms containing $(\bar\psi i\overleftrightarrow{\partial}^{\mu}\psi)(\bar\psi\gamma_\mu\psi)$ are included, while in our case, we replace them with $(\bar\psi i\overleftrightarrow{\partial}^{\mu}\psi)\partial^\nu(\bar\psi\sigma_{\mu\nu}\psi)$ using the EOM. Note further that in Ref. [43] the terms with $\partial^\mu(\bar\psi\sigma_{\mu\nu}\psi)$ are not included because such terms are argued to be of higher order. However, we prefer to keep these terms because they are of unique Lorentz structure which satisfies our power counting rules. The Γ_i's in our work are also different from those in Ref. [43]. In our work we take $\Gamma_i \in \{\mathbb{1}, \gamma_5, \gamma^\mu, \gamma_5\gamma^\mu, \sigma^{\mu\nu}, g^{\mu\nu}, \epsilon^{\mu\nu\rho\sigma}\}$ to keep the complete Dirac algebra, while in Ref. [43] $\Gamma'_i \in \{\mathbb{1}, \gamma_5\gamma^\mu, \sigma^{\mu\nu}, g^{\mu\nu}, \epsilon^{\mu\nu\rho\sigma}\}$ by means of the EOM. Nevertheless, these are simply different choices and can be transformed into each other using the EOM. Note that $-2im(\bar\psi\gamma_5\psi) \approx \partial^\mu(\bar\psi\gamma_5\gamma_\mu\psi)$ and $2m(\bar\psi\gamma^\mu\psi) \approx (\bar\psi i\overleftrightarrow{\partial}^{\mu}\psi)$. The two structures on the left-hand side of the above equations are contained in our work while the structures on the right-hand side are included in Ref. [43].

As in the one-baryon sector, one encounters also in the two-baryon sector the problem that nominally higher order terms contain lower order terms that break the power counting [50]. For instance, according to our criteria listed above, we should include $\widetilde{O}_{14}' = \frac{1}{4m^2}(\bar\psi i \overleftrightarrow{\partial}^{\mu}\psi)(\bar\psi i \overleftrightarrow{\partial}_{\mu}\psi)$ at order $\mathcal{O}(q^2)$. However, this term actually starts to contribute at $\mathcal{O}(q^0)$ so that it breaks our power counting. Therefore, we redefine $\widetilde{O}_{14} = \frac{1}{4m^2}(\bar\psi i \overleftrightarrow{\partial}^{\mu}\psi)(\bar\psi i \overleftrightarrow{\partial}_{\mu}\psi) - (\bar\psi\psi)(\bar\psi\psi)$ to recover the power counting. The same also applies to $\widetilde{O}_{14\text{–}17}$ and $\widetilde{O}_{28\text{–}40}$. Notice that this procedure is very similar to the EOMS scheme [9,10],[5] except, in the one-baryon sector, power counting breaking terms only appear in the loop calculation with propagating baryons. In the nucleon-nucleon sector, they already appear at tree level because the nucleon momentum is involved to increase the chiral order.

B. Nonrelativistic reductions

The relativistic results, when reduced to the nonrelativistic ones in the heavy baryon limit, should recover the well-known $2 + 7 + 15$ linear independent nonrelativistic terms up to $\mathcal{O}(q^4)$ [36]. We checked that this is indeed the case.

To perform the nonrelativistic reduction of the covariant Lagrangian constructed in our present work, one has to replace the relativistic nucleon field operator ψ with the nonrelativistic nucleon field operator N and then expand the relativistic Lagrangian in terms of $1/m$. The relativistic nucleon field operator $\psi(x)$ is[6]

$$\psi(x) = \sum_{s=\pm 1/2} \int \frac{d^3p}{(2\pi)^3}\frac{m}{E_p}\widetilde{b}_s(\boldsymbol{p})\,u^{(s)}(\boldsymbol{p})\,e^{-ip\cdot x}, \tag{8}$$

with the following normalization:

$$[\widetilde{b}_s(\boldsymbol{p}), \widetilde{b}^{\dagger}_{s'}(\boldsymbol{p}')]_+ = \frac{E_p}{m}(2\pi)^3\delta^3(\boldsymbol{p}-\boldsymbol{p}')\delta_{ss'},\ \ \bar u^{(s)}(\boldsymbol{p})u^{(s')}(\boldsymbol{p}) = \delta_{ss'}, \tag{9}$$

where $\widetilde{b}_s(\boldsymbol{p})$ and $\widetilde{b}^{\dagger}_{s'}$ are annihilation and creation operators for a nucleon in spin state s and s'. A sum over the repeated index $s(s') = \pm\frac{1}{2}$ is implied. The Dirac spinors of u and $\bar u$ have the following form:

$$\bar u(\boldsymbol{p}', s) = \sqrt{\frac{E_{p'}+m}{2m}}\left(\mathbb{1},\ -\frac{\boldsymbol{\sigma}\cdot\boldsymbol{p}'}{E_{p'}+m}\right)\chi_s^{\dagger},$$
$$u(\boldsymbol{p}, s') = \sqrt{\frac{E_p+m}{2m}}\begin{pmatrix}\mathbb{1}\\ \frac{\boldsymbol{\sigma}\cdot\boldsymbol{p}}{E_p+m}\end{pmatrix}\chi_{s'}, \tag{10}$$

with

$$E_{p'} = \sqrt{m^2+\boldsymbol{p}'^2}, \quad E_p = \sqrt{m^2+\boldsymbol{p}^2}. \tag{11}$$

The nonrelativistic nucleon filed $N(x)$ is

$$N(x) = \sum_{s=\pm 1/2}\int \frac{d^3p}{(2\pi)^3}\,b_s(\boldsymbol{p})\,\chi_s\,e^{-ip\cdot x}. \tag{12}$$

Here, the isospin indices have been suppressed by using the Fierz rearrangement [41]. Note that $b_s(\boldsymbol{p}) = \sqrt{m/E_p}\,\widetilde{b}_s(\boldsymbol{p})$. To order $\mathcal{O}(q^4)$ at which we are working, the relativistic nucleon field operator ψ can be expanded in terms of the nonrelativistic field $N(x)$, defined in Eq. (12), as

$$\psi(x) = \left[\begin{pmatrix}1\\0\end{pmatrix} - \frac{i}{2m}\begin{pmatrix}0\\ \boldsymbol{\sigma}\cdot\boldsymbol{\nabla}\end{pmatrix} + \frac{1}{8m^2}\begin{pmatrix}\boldsymbol{\nabla}^2\\0\end{pmatrix} - \frac{3i}{16m^3}\begin{pmatrix}0\\ \boldsymbol{\sigma}\cdot\boldsymbol{\nabla}\boldsymbol{\nabla}^2\end{pmatrix} + \frac{11}{128m^4}\begin{pmatrix}\boldsymbol{\nabla}^4\\0\end{pmatrix}\right]N(x) + \mathcal{O}(q^5). \tag{13}$$

In the standard representation, the Dirac matrices are given by

$$\gamma^0 = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix},\quad \gamma^5 = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix},$$
$$\boldsymbol{\gamma} = \begin{pmatrix}0 & \boldsymbol{\sigma}\\ -\boldsymbol{\sigma} & 0\end{pmatrix},\quad \sigma^{\mu\nu} = \frac{i}{2}[\gamma^{\mu},\gamma^{\nu}], \tag{14}$$

with $\boldsymbol{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ the Pauli spin matrices.

Using these relations and the properties of Pauli matrices, one can perform the nonrelativistic expansion of the covariant Lagrangian of Table III. The results are summarized in Table IV. They are presented as linear combinations of the nonrelativistic Lagrangian O_i defined in Table V. One can easily check that there are 24 linear independent terms, consistent with the well-known $2 + 7 + 15$ nonrelativistic terms shown in Table V in the Appendix.

IV. SUMMARY

We have constructed a complete set of relativistic NN contact Lagrangian terms up to order $\mathcal{O}(q^4)$. Using the EOM to eliminate the redundant terms, we find only 4 terms of $\mathcal{O}(q^0)$, 13 terms of $\mathcal{O}(q^2)$, and 23 terms of $\mathcal{O}(q^4)$. We compared with previous studies and identified the differences and the reasoning behind them. In addition, we checked that by performing $1/m_N$ expansions one can recover the corresponding nonrelativistic Lagrangian, which has 24 terms up to $\mathcal{O}(q^4)$. The covariant Lagrangian constructed in the present work can be of use in building covariant nuclear forces as well as studying relativistic corrections.

It should be emphasized that the completeness and minimality of the set of covariant Lagrangian terms derived in the present work should be understood with respect to the power counting rules we chose and the choice we made regarding how to use the equation of motion to remove redundant terms and to limit the number of available terms. More precisely, they may better be referred to as an economical set of covariant Lagrangian terms that can recover those of the heavy

[5]For a related discussion in the NN sector, see, e.g., Ref. [51].

[6]Following Ref. [41], its antiparticle component has been dropped for simplicity.

TABLE IV. The nonrelativistic expressions corresponding to the contact interactions of Table III.

$\widetilde{O}_1$	$O_S + \frac{1}{4m^2}(O_1 - 2O_2 + 2O_3) + \frac{1}{16m^4}(3O_8 - 4O_9 + O_{10} + O_{11} + 4O_{12} - 2O_{13} - O_{16} + O_{17} - O_{21} - O_{22})$
$\widetilde{O}_2$	$O_S + \frac{1}{4m^2}(4O_2 - 6O_3 + O_4 + 2O_5 + O_6 + 2O_7) + \frac{1}{16m^4}(8O_9 - O_{10} + O_{11} - 12O_{12} - 2O_{13} + 3O_{14} + 4O_{15} + O_{16} + O_{17} + 3O_{18} + 4O_{19} - O_{22})$
$\widetilde{O}_3$	$-O_T + \frac{1}{4m^2}(-2O_3 - O_4 + 2O_5 - O_6 + 6O_7) + \frac{1}{16m^4}(O_{10} - O_{11} - 4O_{12} + 2O_{13} - 3O_{14} + 4O_{15} - O_{16} - O_{17} - 3O_{18} + 12O_{19} - 2O_{22})$
$\widetilde{O}_4$	$2O_T + \frac{1}{4m^2}(2O_1 + 4O_2 - 12O_3 + 8O_5 - 2O_6 + 12O_7) + \frac{1}{16m^4}(6O_8 + 8O_9 + 2O_{10} + 2O_{11} - 24O_{12} - 4O_{13} + 16O_{15} - 2O_{16} + 2O_{17} - 6O_{18} + 24O_{19} - 4O_{21} + 4O_{22})$
$\widetilde{O}_5$	$-\frac{1}{4m^2}(O_6 + 2O_7) - \frac{1}{16m^4}(3O_{18} + 4O_{19} + O_{21})$
$\widetilde{O}_6$	$\frac{1}{4m^2}(-4O_6 + 8O_7) + \frac{1}{16m^4}(-4O_{18} + 4O_{21})$
$\widetilde{O}_7$	$\frac{1}{4m^2}(O_1 + 2O_2 - 8O_3 - 4O_4 + 8O_5 - 4O_6 + 8O_7) + \frac{1}{16m^4}(-4O_{10} + 4O_{11} + 2O_{12} - 12O_{13} - 4O_{14} + 8O_{17} - 4O_{18} - 4O_{21} - 8O_{22})$
$\widetilde{O}_8$	$\frac{1}{4m^2}(O_1 + 2O_2 - 4O_3) + \frac{1}{16m^4}(-2O_{12} + 4O_{13} + 4O_{16} - 4O_{17} + 4O_{21} + 4O_{22})$
$\widetilde{O}_9$	$\frac{1}{4m^2}(-O_4 - 2O_5 - O_6 - 2O_7) + \frac{1}{16m^4}(-O_8 - 4O_9 - 2O_{10} - 4O_{11} + 6O_{12} + 12O_{13} - O_{14} - 4O_{15} + 2O_{16} - 8O_{17} - O_{18} - 4O_{19} + 5O_{21} + 8O_{22})$
$\widetilde{O}_{10}$	$\frac{1}{4m^2}(-O_1 - 2O_2) + \frac{1}{16m^4}(-2O_8 + 4O_{11} - 2O_{12} - 4O_{13})$
$\widetilde{O}_{11}$	$\frac{1}{4m^2}(-O_1 - 2O_2) + \frac{1}{16m^4}(-O_8 - 4O_9 + 2O_{10} - 8O_{11} + 6O_{12} + 12O_{13} - O_{14} - 4O_{15} - 2O_{16} - 4O_{17} - O_{18} - 2O_{19} - 2O_{20} - O_{21} - 4O_{22})$
$\widetilde{O}_{12}$	$\frac{1}{4m^2}(O_4 + 2O_5) + \frac{1}{16m^4}(2O_{12} + 4O_{13} + 2O_{14} - 4O_{17} + O_{18} - 6O_{19} + 2O_{20} + O_{21} - 12O_{22})$
$\widetilde{O}_{13}$	$\frac{1}{4m^2}(-2O_4 - 4O_5) + \frac{1}{16m^4}(-2O_8 - 8O_9 - 4O_{10} - 8O_{11} + 12O_{12} + 24O_{13} - 2O_{14} - 8O_{15} + 4O_{16} - 16O_{17} + 2O_{18} - 12O_{19} + 4O_{20} + 2O_{21} - 24O_{22})$
$\widetilde{O}_{14}$	$\frac{1}{4m^2}(-3O_1 + 2O_2) + \frac{1}{16m^4}(-4O_8 + 8O_9 - 4O_{10} - 4O_{11} - 6O_{12} + 4O_{13})$
$\widetilde{O}_{15}$	$\frac{1}{4m^2}(-3O_1 + 2O_2) + \frac{1}{16m^4}(-O_8 - 12O_9 + 2O_{10} + 8O_{11} + 18O_{12} - 12O_{13} - 3O_{14} - 4O_{15} - 6O_{16} + 4O_{17} - 3O_{18} - 6O_{19} + 2O_{20} - 3O_{21} + 4O_{22})$
$\widetilde{O}_{16}$	$\frac{1}{4m^2}(3O_4 - 2O_5) + \frac{1}{16m^4}(6O_{12} - 4O_{13} + 4O_{14} - 8O_{15} + 4O_{16} + 4O_{17} + 3O_{18} - 18O_{19} - 2O_{20} + 3O_{21} + 12O_{22})$
$\widetilde{O}_{17}$	$\frac{1}{4m^2}(-6O_4 + 4O_5) + \frac{1}{16m^4}(-6O_8 - 8O_9 - 12O_{10} + 8O_{11} + 36O_{12} - 24O_{13} - 2O_{14} - 24O_{15} + 4O_{16} + 16O_{17} + 6O_{18} - 36O_{19} - 4O_{20} + 6O_{21} + 24O_{22})$
$\widetilde{O}_{18}$	$\frac{1}{16m^4}(O_{18} + 2O_{19} + 2O_{20} + O_{21} + 4O_{22})$
$\widetilde{O}_{19}$	$\frac{1}{16m^4}(4O_{18} - 8O_{19} + 8O_{20} + 4O_{21} - 16O_{22})$
$\widetilde{O}_{20}$	$\frac{1}{16m^4}(O_8 + 4O_9 + 2O_{10} + 4O_{11} - 8O_{12} - 16O_{13} - 4O_{14} - 8O_{16} + 16O_{17} - 4O_{18} + 8O_{19} - 8O_{20} - 4O_{21} + 16O_{22})$
$\widetilde{O}_{21}$	$\frac{1}{16m^4}(-O_8 - 4O_9 - 2O_{10} - 4O_{11} + 4O_{12} + 8O_{13})$
$\widetilde{O}_{22}$	$\frac{1}{16m^4}(O_{14} + 4O_{15} + 2O_{16} + 4O_{17} + O_{18} + 2O_{19} + 2O_{20} + O_{21} + 4O_{22})$
$\widetilde{O}_{23}$	$\frac{1}{16m^4}(-O_8 - 4O_9 - 2O_{10} - 4O_{11} + 8O_{12} + 16O_{13} + 16O_{16} - 16O_{17} + 16O_{21} + 16O_{22})$
$\widetilde{O}_{24}$	$\frac{1}{16m^4}(O_8 + 4O_9 + 2O_{10} + 4O_{11})$
$\widetilde{O}_{25}$	$\frac{1}{16m^4}(O_8 + 4O_9 + 2O_{10} + 4O_{11})$
$\widetilde{O}_{26}$	$\frac{1}{16m^4}(-O_{14} - 4O_{15} - 2O_{16} - 4O_{17})$
$\widetilde{O}_{27}$	$\frac{1}{16m^4}(2O_{14} + 8O_{15} + 4O_{16} + 8O_{17})$
$\widetilde{O}_{28}$	$\frac{1}{16m^4}(3O_{18} + 6O_{19} - 2O_{20} + 3O_{21} - 4O_{22})$
$\widetilde{O}_{29}$	$\frac{1}{16m^4}(12O_{18} - 24O_{19} - 8O_{20} + 12O_{21} + 16O_{22})$
$\widetilde{O}_{30}$	$\frac{1}{16m^4}(-3O_8 - 4O_9 - 6O_{10} + 4O_{11} + 24O_{12} - 16O_{13} + 12O_{14} - 32O_{15} + 24O_{16} + 16O_{17} + 12O_{18} - 24O_{19} - 8O_{20} + 12O_{21} + 16O_{22})$
$\widetilde{O}_{31}$	$\frac{1}{16m^4}(-O_8 - 4O_9 - 2O_{10} - 4O_{11} + 4O_{12} + 8O_{13})$
$\widetilde{O}_{32}$	$\frac{1}{16m^4}(3O_{14} + 4O_{15} + 6O_{16} - 4O_{17} + 3O_{18} + 6O_{19} - 2O_{20} + 3O_{21} - 4O_{22})$
$\widetilde{O}_{33}$	$\frac{1}{16m^4}(3O_8 + 4O_9 + 6O_{10} - 4O_{11})$
$\widetilde{O}_{34}$	$\frac{1}{16m^4}(3O_8 + 4O_9 + 6O_{10} - 4O_{11})$
$\widetilde{O}_{35}$	$\frac{1}{16m^4}(-3O_{14} - 4O_{15} - 6O_{16} + 4O_{17})$
$\widetilde{O}_{36}$	$\frac{1}{16m^4}(6O_{14} + 8O_{15} + 12O_{16} - 8O_{17})$
$\widetilde{O}_{37}$	$\frac{1}{16m^4}(9O_8 - 12O_9 + 18O_{10} + 4O_{11})$
$\widetilde{O}_{38}$	$\frac{1}{16m^4}(9O_8 - 12O_9 + 18O_{10} + 4O_{11})$
$\widetilde{O}_{39}$	$\frac{1}{16m^4}(-9O_{14} + 12O_{15} - 18O_{16} - 4O_{17})$
$\widetilde{O}_{40}$	$\frac{1}{16m^4}(18O_{14} - 24O_{15} + 36O_{16} + 8O_{17})$

TABLE V. $\mathcal{O}(q^0)$, $\mathcal{O}(q^2)$, and $\mathcal{O}(q^4)$ nonrelativistic NN contact Lagrangian terms. The left (right) arrow on $\boldsymbol{\nabla}$ indicates that the derivative acts on the left (right) nucleon field.

O_S	$(N^\dagger N)(N^\dagger N)$	O_{11}	$(N^\dagger \overrightarrow{\boldsymbol{\nabla}} \cdot \overleftarrow{\boldsymbol{\nabla}} N)(N^\dagger \overrightarrow{\boldsymbol{\nabla}} \cdot \overleftarrow{\boldsymbol{\nabla}} N)$
O_T	$(N^\dagger \boldsymbol{\sigma} N) \cdot (N^\dagger \boldsymbol{\sigma} N)$	O_{12}	$i\,(N^\dagger \boldsymbol{\sigma} \cdot \overrightarrow{\boldsymbol{\nabla}} \times \overleftarrow{\boldsymbol{\nabla}} N)(N^\dagger \overrightarrow{\boldsymbol{\nabla}}^2 N) + \text{H.c.}$
O_1	$(N^\dagger N)(N^\dagger \overrightarrow{\boldsymbol{\nabla}}^2 N) + \text{H.c.}$	O_{13}	$i\,(N^\dagger \boldsymbol{\sigma} \cdot \overrightarrow{\boldsymbol{\nabla}} \times \overleftarrow{\boldsymbol{\nabla}} N)(N^\dagger \overrightarrow{\boldsymbol{\nabla}} \cdot \overleftarrow{\boldsymbol{\nabla}} N)$
O_2	$(N^\dagger N)(N^\dagger \overrightarrow{\boldsymbol{\nabla}} \cdot \overleftarrow{\boldsymbol{\nabla}} N)$	O_{14}	$(N^\dagger \sigma^j N)(N^\dagger \sigma^j \overrightarrow{\boldsymbol{\nabla}}^4 N) + \text{H.c.}$
O_3	$i\,(N^\dagger \boldsymbol{\sigma} N) \cdot (N^\dagger \overrightarrow{\boldsymbol{\nabla}} \times \overleftarrow{\boldsymbol{\nabla}} N)$	O_{15}	$(N^\dagger \sigma^j \overrightarrow{\boldsymbol{\nabla}} \cdot \overleftarrow{\boldsymbol{\nabla}} N)(N^\dagger \sigma^j \overrightarrow{\boldsymbol{\nabla}}^2 N) + \text{H.c.}$
O_4	$(N^\dagger \sigma^j N)(N^\dagger \sigma^j \overrightarrow{\boldsymbol{\nabla}}^2 N) + \text{H.c.}$	O_{16}	$(N^\dagger \sigma^j \overrightarrow{\boldsymbol{\nabla}}^2 N)(N^\dagger \sigma^j \overleftarrow{\boldsymbol{\nabla}}^2 N)$
O_5	$(N^\dagger \sigma^j N)(N^\dagger \sigma^j \overrightarrow{\boldsymbol{\nabla}} \cdot \overleftarrow{\boldsymbol{\nabla}} N)$	O_{17}	$(N^\dagger \sigma^j \overrightarrow{\boldsymbol{\nabla}} \cdot \overleftarrow{\boldsymbol{\nabla}} N)(N^\dagger \sigma^j \overrightarrow{\boldsymbol{\nabla}} \cdot \overleftarrow{\boldsymbol{\nabla}} N)$
O_6	$(N^\dagger \boldsymbol{\sigma} \cdot \overrightarrow{\boldsymbol{\nabla}} N)(N^\dagger \boldsymbol{\sigma} \cdot \overrightarrow{\boldsymbol{\nabla}} N) + \text{H.c.}$	O_{18}	$(N^\dagger \boldsymbol{\sigma} \cdot \overrightarrow{\boldsymbol{\nabla}} N)(N^\dagger \boldsymbol{\sigma} \cdot \overrightarrow{\boldsymbol{\nabla}}\, \overrightarrow{\boldsymbol{\nabla}}^2 N) + \text{H.c.}$
O_7	$(N^\dagger \boldsymbol{\sigma} \cdot \overrightarrow{\boldsymbol{\nabla}} N)(N^\dagger \boldsymbol{\sigma} \cdot \overleftarrow{\boldsymbol{\nabla}} N)$	O_{19}	$(N^\dagger \boldsymbol{\sigma} \cdot \overrightarrow{\boldsymbol{\nabla}} N)(N^\dagger \boldsymbol{\sigma} \cdot \overleftarrow{\boldsymbol{\nabla}}\, \overrightarrow{\boldsymbol{\nabla}}^2 N) + \text{H.c.}$
O_8	$(N^\dagger N)(N^\dagger \overrightarrow{\boldsymbol{\nabla}}^4 N) + \text{H.c.}$	O_{20}	$(N^\dagger \boldsymbol{\sigma} \cdot \overrightarrow{\boldsymbol{\nabla}} N)(N^\dagger \boldsymbol{\sigma} \cdot \overrightarrow{\boldsymbol{\nabla}}\, \overrightarrow{\boldsymbol{\nabla}} \cdot \overleftarrow{\boldsymbol{\nabla}} N) + \text{H.c.}$
O_9	$(N^\dagger \overrightarrow{\boldsymbol{\nabla}}^2 N)(N^\dagger \overrightarrow{\boldsymbol{\nabla}} \cdot \overleftarrow{\boldsymbol{\nabla}} N) + \text{H.c.}$	O_{21}	$(N^\dagger \boldsymbol{\sigma} \cdot \overrightarrow{\boldsymbol{\nabla}} N)(N^\dagger \boldsymbol{\sigma} \cdot \overrightarrow{\boldsymbol{\nabla}}\, \overleftarrow{\boldsymbol{\nabla}}^2 N) + \text{H.c.}$
O_{10}	$(N^\dagger \overrightarrow{\boldsymbol{\nabla}}^2 N)(N^\dagger \overleftarrow{\boldsymbol{\nabla}}^2 N)$	O_{22}	$(N^\dagger \boldsymbol{\sigma} \cdot \overrightarrow{\boldsymbol{\nabla}} N)(N^\dagger \boldsymbol{\sigma} \cdot \overleftarrow{\boldsymbol{\nabla}}\, \overrightarrow{\boldsymbol{\nabla}} \cdot \overleftarrow{\boldsymbol{\nabla}} N)$

baryon. We hope that the present work can motivate more studies in this direction.

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APPENDIX

In this Appendix, we list the nonrelativistic Lagrangian terms up to order $\mathcal{O}(q^4)$. Compared to the covariant case, the procedures to construct the nonrelativistic contact Lagrangian is simple and straightforward. The basic requirement for a nonrelativistic Lagrangian is that the Lagrangian must be scalar. Because of parity constraints, the nonrelativistic Lagrangian must contain an even number of gradient operators. Because the three-momentum of the nucleon is the only small expansion parameter in the nonrelativistic power counting, to the nth order, one needs to include n gradient operators. Using the criteria listed above, one can easily construct the nonrelativistic Lagrangians up to order $\mathcal{O}(q^4)$. They are summarized in Table V, where N and $N^\dagger$ donate the nucleon field and its Hermitian conjugate, and $\boldsymbol{\nabla}$ refers to the gradient operator. Note that $(N^\dagger \boldsymbol{\nabla} N)(N^\dagger N) = -(N^\dagger N)(N^\dagger \boldsymbol{\nabla} N)$ since we are working in the center-of-mass frame.

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