Isospin mixing and Coulomb mixing in ground states of even-even nuclei

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In this work, the Coulomb mixing and the isospin mixing in the ground states of even-even nuclei are evaluated in perturbation theory. The calculation of the isospin mixing is performed by using the connection to isovector monopole resonance properties. The uncertainty in the results that depends on different choices of the Skyrme interactions is shown. While Coulomb mixing turns out to be large in the ground states of heavy nuclei, isospin mixing is very small.

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I. INTRODUCTION

The best known part of the nuclear Hamiltonian is the Coulomb interaction between protons. The Coulomb interaction is charge asymmetric. As a consequence, isospin breaking is dominated by the Coulomb interaction. The parent state $|\pi\rangle$ of the nucleus with isospin T and $T_z = T$ contains the admixtures of states with isospin $T + 1$,

$$|\pi\rangle = \left(1 - \sum_\alpha \varepsilon_\alpha^2\right)^{1/2} |T, T; 0\rangle + \sum_\alpha \varepsilon_\alpha |T + 1, T; \alpha\rangle. \tag{1}$$

The total probability $\varepsilon^2 = \sum_\alpha \varepsilon_\alpha^2$ is the isospin mixing. In first-order perturbation theory, the expression for isospin mixing is defined as

$$\varepsilon_{T+1}^2 = \sum_{\alpha \neq 0} \frac{\left|\langle T, T; 0|V_C^{(\mathrm{IV})}|T + 1, T; \alpha\rangle\right|^2}{(E_\alpha - E_0)^2}, \tag{2}$$

where $|T, T; 0\rangle$ denotes the ground state (g.s.) at the energy E_0, and α are the various quantum numbers needed to specify the states $|\alpha\rangle$ at their energy E_α. The Coulomb interaction V_C can be rewritten in terms of isoscalar, isovector, and isotensor parts, but only the isovector part $V_C^{(\mathrm{IV})}$ is kept because the isoscalar part does not contribute to (2) and the isotensor part is small because of the long-range nature of the Coulomb interaction. Note that one needs to indicate $T + 1$ because an isovector operator excites not only $|T + 1, T; \alpha\rangle$, but also $|T, T; \alpha\rangle$. If the $|T, T; \alpha\rangle$ are also taken into account, the admixture is usually much larger. This kind of mixing was defined as the Coulomb mixing [1]:

$$\varepsilon_C^2 = \sum_{\alpha \neq 0} \frac{\left|\langle 0|V_C^{(\mathrm{IV})}|\alpha\rangle\right|^2}{(E_\alpha - E_0)^2}, \tag{3}$$

where the states $|\alpha\rangle$ now include both T and $T + 1$ excitations. Therefore, the Coulomb mixing represents the total change induced by the Coulomb force in the wave function of the ground state. This mixing is more general than just isospin mixing because in many instances the effects of the Coulomb force do not lead necessarily to large components that differ in the isospin quantum numbers. In nuclei with $N = Z$, the isospin mixing and the Coulomb mixing are the same, $\varepsilon_C^2 = \varepsilon_{T+1}^2$, as in these nuclei the isovector excitations have isospin $T = 1$ only. In a $N > Z$ nucleus, $\varepsilon_C^2 > \varepsilon_{T+1}^2$ as now there are $|T, T; \alpha\rangle$ and $|T + 1, T; \alpha\rangle$ contributing.

In Refs. [1–5] and references therein, it was shown how the isospin mixing is connected to the notion of the isovector monopole (IVM) resonance that is defined by the operator

$$Q_0^{(\mathrm{IV})} = \sum_i r_i^2 t_z(i), \tag{4}$$

where t_z is the z component of the isospin operator. Attempts were made to observe the IVM resonance experimentally by using different probes [6–9] because it plays an important role in many isospin processes (see Ref. [1] and references therein). Isospin mixing, that is the nonconservation of isospin quantum number, is a good example.

In the past, there have been a few calculations of the isospin mixing, performed just after the Skyrme interactions had become a widely used tool. The old SIII and SIV interactions [10] have been employed in Ref. [1]. The subject has been left aside for a while, and it has become popular again after the start of the new discoveries of exotic nuclei. Proton-rich (or neutron-deficient) isotopes have been either speculated, or shown to be, ideal cases in which the isospin mixing can become quite large: see, e.g., Ref. [11], or Ref. [12] in which also deformed systems have been considered. In these works, typically one or two Skyrme parameter sets have been considered.

Our goal is to look more systematically than has done in the past at the dependence of the values of the isospin mixing on the chosen Skyrme set, as well as at the variation of the

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IVM properties. Many modern Skyrme sets have been, in fact, introduced after the works of the 1980s and 1990s that we have just quoted; nonetheless, the systematic investigation we plan to illustrate here is not available in the literature. Meanwhile, attention has been devoted instead to calculating isospin mixing through projection schemes [13,14] and/or by including proton-neutron mixing and general isospin breaking forces (cf. [15] and references therein).

With this main idea in mind, the outline of the paper is as follows. The next section describes the method of calculation. We basically connect the isospin mixing to the IVM resonance. We present results obtained by using 12 Skyrme parameter sets, including SIII [10], SGII [16], SKM* [17], SkP, SkI2 [18], SLy4 [19], SkO, SkO' [20], LNS [21], SK255 [22], BSk17 [23], and SAMi [24]. This demonstrates how much the results depend on the choice of different Skyrme interactions. We restrict our discussion to even-even nuclei. The isospin mixing and Coulomb mixing were calculated for $N = Z$ including ^{40}Ca, ^{56}Ni, and ^{100}Sn, and $N > Z$ nuclei including ^{48}Ca, ^{78}Ni, ^{90}Zr, ^{120}Sn, and ^{208}Pb.

II. METHOD OF CALCULATION

The two-body Coulomb potential is given as

$$V_C = \frac{1}{2}\sum_{i,j}^{A}\frac{e^2}{|\boldsymbol{r}_i - \boldsymbol{r}_j|}\left(\frac{1}{2} - t_z(i)\right)\left(\frac{1}{2} - t_z(j)\right), \quad (5)$$

where t_z is the z component of the nucleon-isospin operator, and its eigenvalue is $+\frac{1}{2}$ for neutron and $-\frac{1}{2}$ for proton. In the Hartree-Fock (HF) calculation, one has the one-body Coulomb potential that can be used in Eqs. (2) and (3). A simplification is to approximate V_C^{IV} using a homogeneous density distribution,

$$V_C^{(\mathrm{IV})} = -\frac{Ze^2}{2R^3}\sum_{i=1}^{A}\left(3R^2 - r_i^2\right)t_z(i), \quad (6)$$

for $r \leqslant R$. In this case, the isospin mixing becomes

$$\varepsilon_{T+1}^2 = \left(\frac{Ze^2}{2R^3}\right)^2 \sum_{\alpha\neq 0}\frac{\left|\langle 0|Q_0^{(\mathrm{IV})}|T+1;\alpha\rangle\right|^2}{(E_\alpha - E_0)^2}, \quad (7)$$

where $Q_0^{(\mathrm{IV})}$ is the z component of the IVM operator and $R = r_0 A^{1/3}$. It is pointed out that in this approximation, the results are, of course, affected by the choice of the value r_0.

In Eq. (4), the operator $Q_0^{(\mathrm{IV})}$ is part of a single-particle isovector operator $Q_\mu^{(\mathrm{IV})}$ with $\mu = 0, \pm 1$

$$Q_\mu^{(\mathrm{IV})} = \sum_i r_i^2 t_\mu(i), \quad (8)$$

where

$$t_{-1} = +\frac{t_x - it_y}{\sqrt{2}}; \quad t_{+1} = -\frac{t_x + it_y}{\sqrt{2}}; \quad \text{and} \quad t_0 = t_z. \quad (9)$$

When the isovector operator $Q_0^{(\mathrm{IV})}$ is applied in the parent nucleus with $N > Z$, both $|T, T;\alpha\rangle$ and $|T+1, T;\alpha\rangle$ are excited and the isospin of these states cannot be determined when performing complicated HF-RPA calculations involving many orbits and thus many particle-hole (p-h) states. We should mention here that the states constructed of 1p-1h component only do not have good isospin, and in order to have good isospin one has to include certain class of 2p-2h components [1,5]. These components are small and usually are not included. Their effect on the calculation of isospin mixing as performed here is very small.

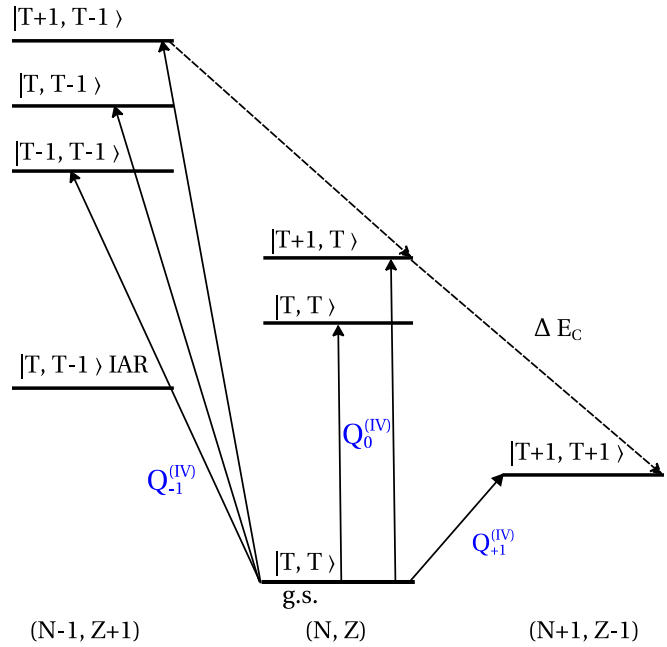

FIG. 1. Isovector states in nuclei with $N > Z$. A single-particle isovector operator $Q_\mu^{(\mathrm{IV})}$, has three components, $Q_{-1}^{(\mathrm{IV})}$, $Q_0^{(\mathrm{IV})}$, and $Q_{+1}^{(\mathrm{IV})}$. In the parent nucleus, $Q_0^{(\mathrm{IV})}$ excites $|T+1, T;\alpha\rangle$ and $|T, T;\alpha\rangle$. In the analog nucleus $(N-1, Z+1)$, $Q_{-1}^{(\mathrm{IV})}$ excites $|T+1, T-1;\alpha\rangle$, $|T, T-1;\alpha\rangle$, and $|T-1, T-1;\alpha\rangle$. In the nucleus $(N+1, Z-1)$, $Q_{+1}^{(\mathrm{IV})}$ excites $|T+1, T+1;\alpha\rangle$ only.

A technique based on the properties of isovector states in nuclei with $N > Z$ (see Fig. 1) is used to determine separately $|T+1, T;\alpha\rangle$ for the sum in Eq. (7). First, the calculation using operator $Q_{+1}^{(\mathrm{IV})}$

$$Q_{+1}^{(\mathrm{IV})} = \sum_{i=1}^{A} r_i^2 t_{+1}(i) \quad (10)$$

that excites only $|T+1, T+1;\alpha\rangle$ in the nucleus $(N+1, Z-1)$ was done. After that, $|T+1, T;\alpha\rangle$ states in the parent nucleus were obtained by using the fact that their energies $E_0(T+1;\alpha)$ differ from the energies of $|T+1, T+1;\alpha\rangle$ in the nucleus $(N+1, Z-1)$ by one Coulomb displacement energy (CDE), ΔE_C, i.e.,

$$E_0(T+1;\alpha) - E_{+1}(T+1;\alpha) = \Delta E_C. \quad (11)$$

We used the notations E_μ $(\mu = 0, \pm 1)$ for the energies in the three nuclei in Fig. 1. The CDE, ΔE_C, can be obtained from the Skyrme-HF calculation. The transition strengths to various isospin components T' of the $Q_\mu^{(\mathrm{IV})}$ matrix elements are given by the Wigner-Eckart theorem:

$$\begin{aligned} S_{T'}^{(\mu)}(\alpha) &= \left|\langle T, T; 0|Q_\mu^{(\mathrm{IV})}|\alpha; T', T+\mu\rangle\right|^2 \\ &= |\langle TT1\mu|T'T+\mu\rangle|^2 \cdot |\langle T;0||Q^{(\mathrm{IV})}||\alpha;T'\rangle|^2. \end{aligned} \quad (12)$$

The expression $S_{T'} = \sum_\alpha |\langle 0||Q^{(\mathrm{IV})}||\alpha; T'\rangle|^2$ is the total reduced transition strengths. With $\langle TT10|T+1\ T\rangle^2 = 1/(T+1)$ we find:

$$\varepsilon^2_{T+1} = \sum_{\alpha\neq 0} \frac{S^{(+1)}_{T+1}(\alpha)}{(E_\alpha - E_0)^2}, \tag{13}$$

with

$$S^{(+1)}_{T+1}(\alpha) = \frac{1}{T+1}|\langle T;0||Q^{(\mathrm{IV})}||\alpha; T+1\rangle|^2, \tag{14}$$

and $E_\alpha = E_0(T+1;\alpha) = E_{+1}(T+1;\alpha) + \Delta E_C$.

It is useful to recall the isospin properties of the IVM resonance [25] related to the calculation performed here. The total transition strength of $Q^{(\mathrm{IV})}$ is expressed in terms of three reduced transition strengths $S_{T'}$

$$m_\mu(0) = \sum_{T'} \langle TT1\mu|T'T+\mu\rangle^2 S_{T'}, \tag{15}$$

and if $\overline{E}_\mu(T')$ is the centroid energy of states of isospin T' excited by $Q^{(\mathrm{IV})}_\mu$, we also have

$$m_\mu(1) = \sum_{T'} \langle TT1\mu|T'T+\mu\rangle^2 S_{T'}\overline{E}_\mu(T'). \tag{16}$$

Using expressions (15) and (16), we can obtain $S^{(0)}_T$ and $E_0(T)$ and determine the isospin energy splitting $\Delta\overline{E}_+$

$$\Delta\overline{E}_+ = \overline{E}_0(T+1) - \overline{E}_0(T) \tag{17}$$

that relates to the symmetry potential V_1 defined by the expression

$$V_1 = \frac{A}{T+1}\Delta\overline{E}_+. \tag{18}$$

In practice, the sum in Eq. (3) for the calculation of the Coulomb mixing was obtained from the HF-RPA code of Ref. [26]. The sum in Eq. (2) for the isospin mixing was calculated using the HF-RPA code including the charge-exchange mode (HF-pnRPA) [27].

III. RESULTS AND DISCUSSION

Although the actual HF Coulomb potential $V^{(\mathrm{IV})}_C$ can be used directly as the probing operator, in our calculation the IVM operator $Q^{(\mathrm{IV})}_0$ is also utilized to have the connection between the Coulomb mixing, the isospin mixing, and the properties of the IVM resonance. As mentioned above, when the IVM operator is used, the results are affected by the choice of r_0. In Fig. 2, the distribution of IVM strength evaluated using the $Q^{(\mathrm{IV})}_0$ [Fig. 2(a)] and Coulomb strength evaluated using the $V^{(\mathrm{IV})}_C$ [Fig. 2(b)] are shown for ^{208}Pb. We can see in Fig. 2 their close similarity. This is the reason why one can use the ratio η between the total strength of Coulomb distribution and that of the IVM distribution:

$$\eta = \frac{\sum\langle 0|V^{(\mathrm{IV})}_C|\alpha\rangle}{\sum\langle 0|Q^{(\mathrm{IV})}_0|\alpha\rangle} \tag{19}$$

instead of the factor $(\frac{Ze^2}{2R^3})^2$ in Eq. (7). Therefore, the uncertainty from the value of r_0 is avoided. In addition, it was found

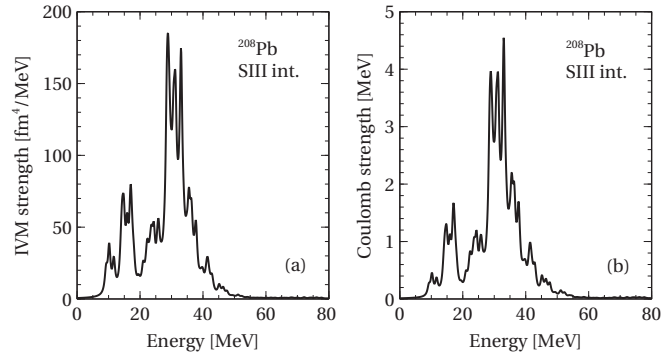

FIG. 2. The distribution of IVM strength (a) and Coulomb strength (b) in ^{208}Pb. The discrete RPA peaks for both operators were smoothed by using the same Lorentzian averaging with the width of 1 MeV.

that the ratio η is close to the value of the factor in Eq. (7) if $r_0 = 1.25$ fm. In the calculation of the Coulomb strength above, the $V^{(\mathrm{IV})}_C$ contains not only the inside part ($r \leqslant R$) but also the outside part ($r > R$). From the similarity shown in Fig. 2, we can conclude that the outside part ($r > R$) does not contribute much to the result.

Table I shows the values of isospin mixing (or Coulomb mixing) for $N = Z$ nuclei including ^{40}Ca, ^{56}Ni, and ^{100}Sn. In this case, the Coulomb mixing and isospin mixing are the same because there are only $T = 1$ states. The difference between two different operators is very small, and it allows us to use the $Q^{(\mathrm{IV})}_0$ with the ratio η instead of $V^{(\mathrm{IV})}_C$. In general, the difference in the value of the isospin mixing between different Skyrme interactions is usually not large. In $N > Z$ nuclei, the SKO interaction gives isospin mixing a factor of 2 smaller than the other Skyrme interactions in some nuclei.

The Coulomb potential is kept in our HF and RPA calculation. One can argue that the Coulomb potential should not be included in this calculation. The code [26] we use allows us easily to include or exclude the Coulomb potential consis-

TABLE I. Isospin mixing (or Coulomb mixing) (%) of $N = Z$ nuclei including ^{40}Ca, ^{56}Ni, and ^{100}Sn. The Coulomb potential is included in the HF-RPA calculation.

No.	Int.	^{40}Ca		^{56}Ni		^{100}Sn	
		$Q^{(\mathrm{IV})}_0$	$V^{(\mathrm{IV})}_C$	$Q^{(\mathrm{IV})}_0$	$V^{(\mathrm{IV})}_C$	$Q^{(\mathrm{IV})}_0$	$V^{(\mathrm{IV})}_C$
1	SIII	0.96	0.68	1.55	1.22	5.44	4.54
2	SGII	1.09	0.79	1.85	1.46	6.58	5.51
3	SKM*	1.12	0.78	1.82	1.42	6.44	5.34
4	SkP	1.15	0.81	2.00	1.56	6.82	5.69
5	SkI2	0.87	0.62	1.43	1.11	5.34	4.46
6	SLy4	1.05	0.77	1.78	1.43	6.17	5.27
7	SKO	0.90	0.62	1.31	0.98	5.04	4.13
8	SKO'	1.06	0.73	1.45	1.13	5.64	4.68
9	LNS	1.15	0.81	1.90	1.49	6.64	5.47
10	SK255	0.89	0.62	1.55	1.17	5.37	4.40
11	BSK17	1.02	0.70	1.54	1.23	5.68	4.75
12	SAMi0	1.01	0.74	1.73	1.36	6.13	5.15

TABLE II. Isospin mixing (or Coulomb mixing) (%) of $N = Z$ nuclei including ^{40}Ca, ^{56}Ni, and ^{100}Sn. The Coulomb potential is excluded in the HF-RPA calculation.

No.	Int.	^{40}Ca		^{56}Ni		^{100}Sn	
		$Q_0^{(\mathrm{IV})}$	$V_C^{(\mathrm{IV})}$	$Q_0^{(\mathrm{IV})}$	$V_C^{(\mathrm{IV})}$	$Q_0^{(\mathrm{IV})}$	$V_C^{(\mathrm{IV})}$
1	SIII	1.06	0.80	1.69	1.42	5.54	4.94
2	SGII	1.21	0.94	2.01	1.68	6.70	5.97
3	SKM*	1.23	0.93	1.97	1.64	6.53	5.76
4	SLy4	1.17	0.92	1.95	1.67	6.40	5.79
5	SAMi0	1.12	0.88	1.86	1.55	6.21	5.53

tently (in both HF and RPA). We find that this uncertainty, in this case, is not large even in $^{100}_{50}$Sn (see Tables I and II). We prefer to use the results of the HF and RPA included Coulomb potential as inputs into the calculation of isospin and Coulomb mixing because they are more realistic and can be compared to experiment.

In the HF calculation for nuclei with $N \neq Z$ a very small (a fraction of a percent) isospin mixing component appears even when the Coulomb interaction is put to zero [1,28,29]. In the present approach, the HF wave functions serve as the basis for the RPA calculations. The RPA restores the isospin symmetry [28,30] and the above spurious isospin mixing causes negligible uncertainties in the numbers ε^2_{T+1}, which are calculated in perturbation theory.

In the case of nuclei that have $N > Z$, the Coulomb mixing and isospin mixing are different (see Table III). Among $N > Z$ nuclei, ^{78}Ni is an interesting nucleus because $T = 11$ is large while $Z = 20$ is relatively small. As one expects, the isospin mixing is strongly reduced by the factor $1/(T + 1)$. In most nuclei, the Coulomb potential can be treated in perturbation theory. When Z becomes large and the Coulomb potential becomes very strong such as in the case of oganesson ($^{302}_{118}$Og), the perturbative approach in the calculation of the Coulomb mixing is not correct. However, the isospin mixing in $^{302}_{118}$Og ($T = 33$) is still small, around 2%, because of the factor $1/(T + 1)$. It is useful to remind that the uncertainties caused by other sources besides the Coulomb interaction are expected to be an order of magnitude smaller, thus they are smaller than the difference between results obtained with different choices of the Skyrme interactions.

TABLE IV. Isospin properties of the IVM resonance for ^{48}Ca ($\hbar\omega = 11.28$ MeV). $\overline{E}_0$ is the average energy of the strength distribution. S_T and S_{T+1} are the reduced transition strength to $|T, T; \alpha\rangle$, and $|T + 1, T; \alpha\rangle$, respectively. ΔE_C is the direct term of the CDE. $\Delta\overline{E}_+$ is the energy difference between $|T, T; \alpha\rangle$ and $|T + 1, T; \alpha\rangle$. V_1 is the symmetry potential as defined in the expression (18).

		$\overline{E}_0$	S_T	S_{T+1}	ΔE_C	$\Delta\overline{E}_+$	V_1
1	SIII	34.79	149.27	125.68	7.27	11.06	106.14
2	SGII	32.87	153.12	113.98	7.33	6.88	66.01
3	SKM*	32.54	156.72	125.68	7.23	9.60	92.17
4	SkP	30.00	160.48	123.00	7.23	11.15	107.02
5	SkI2	33.08	141.64	91.56	7.10	7.43	71.34
6	SLy4	30.57	148.43	123.51	7.22	10.19	97.80
7	SKO	32.60	144.58	72.78	6.96	15.84	152.08
8	SKO'	32.32	138.67	81.67	7.14	11.75	112.77
9	LNS	33.14	134.72	102.85	7.52	9.78	93.86
10	SK255	34.65	153.56	100.39	7.08	11.77	112.99
11	BSk17	32.83	139.00	110.69	7.31	11.32	108.63
12	SAMi0	32.28	154.64	109.01	7.20	8.05	77.25

Finally, as mentioned in the text, for the isospin mixing, only the $T + 1$ states are taken into account and the isospin properties of the IVM resonance are useful for the calculation. Therefore, the properties of the IVM resonance are shown in Tables IV–VIII for ^{48}Ca, ^{78}Ni, ^{90}Zr, ^{120}Sn, and ^{208}Pb, respectively. In Tables IV–VIII, the average energy of the transition strength distribution is $\overline{E}_0 = m_0(1)/m_0(0)$. S_T and S_{T+1} are the total transition strength to the $|T, T; \alpha\rangle$ and $|T + 1, T; \alpha\rangle$ states, respectively. ΔE_C is the direct CDE. $\Delta\overline{E}_+$ given by Eq. (17) is the difference in energy between

TABLE III. Coulomb mixing ε^2_C (%) and isospin mixing ε^2_{T+1} (%) of $N > Z$ nuclei including ^{48}Ca ($T = 4$), ^{78}Ni ($T = 11$), ^{90}Zr ($T = 5$), ^{120}Sn ($T = 10$), and ^{208}Pb ($T = 22$).

		^{48}Ca		^{78}Ni		^{90}Zr		^{120}Sn		^{208}Pb	
		ε^2_C	ε^2_{T+1}	ε^2_C	ε^2_{T+1}	ε^2_C	ε^2_{T+1}	ε^2_C	ε^2_{T+1}	ε^2_C	ε^2_{T+1}
1	SIII	1.14	0.10	3.83	0.04	4.13	0.52	10.85	0.23	29.90	0.29
2	SGII	1.33	0.12	4.28	0.05	4.98	0.66	11.66	0.32	34.38	0.41
3	SKM*	1.35	0.12	4.66	0.05	4.91	0.62	12.10	0.30	35.57	0.36
4	SkP	1.52	0.11	5.53	0.04	5.18	0.58	12.41	0.28	38.50	0.32
5	SkI2	1.20	0.09	4.71	0.03	4.13	0.53	11.03	0.25	32.36	0.32
6	SLy4	1.37	0.12	4.45	0.04	4.73	0.56	11.36	0.25	33.64	0.29
7	SKO	1.32	0.05	6.19	0.02	4.02	0.41	10.50	0.23	32.75	0.26
8	SKO'	1.20	0.07	4.79	0.03	4.36	0.49	11.69	0.25	34.64	0.28
9	LNS	1.43	0.12	4.85	0.04	5.10	0.63	12.86	0.29	38.04	0.34
10	SK255	1.24	0.08	4.64	0.03	4.13	0.49	10.84	0.24	31.94	0.28
11	BSk17	1.18	0.10	4.40	0.04	4.30	0.53	11.38	0.23	33.24	0.28
12	SAMi0	1.35	0.11	4.27	0.05	4.72	0.60	10.93	0.30	31.85	0.37

TABLE V. The same as in Table IV, but for ^{78}Ni ($\hbar\omega = 9.60$ MeV).

		$\overline{E}_0$	S_T	S_{T+1}	ΔE_C	$\Delta\overline{E}_+$	V_1
1	SIII	30.55	382.04	186.47	8.90	18.19	118.23
2	SGII	28.86	385.18	186.63	8.96	14.34	93.22
3	SKM*	28.47	400.08	175.45	8.85	16.29	105.91
4	SkP	26.14	405.03	165.44	8.88	18.40	119.57
5	SkI2	27.01	387.49	123.12	8.64	19.35	125.79
6	SLy4	27.10	373.59	168.29	8.87	17.50	113.74
7	SKO	25.51	412.48	117.36	8.60	26.73	173.73
8	SKO'	26.77	365.85	112.93	8.80	21.14	137.41
9	LNS	29.24	341.93	121.66	9.18	17.14	111.43
10	SK255	29.37	401.49	120.28	8.64	19.59	127.33
11	BSk17	28.01	364.18	155.15	8.93	19.89	129.29
12	SAMi0	28.96	388.83	196.12	8.87	15.02	97.66

TABLE VII. The same as in Table IV, but for ^{120}Sn ($\hbar\omega = 8.31$ MeV).

		$\overline{E}_0$	S_T	S_{T+1}	ΔE_C	$\Delta\overline{E}_+$	V_1
1	SIII	29.86	804.40	469.01	13.97	8.73	95.20
2	SGII	28.25	798.77	499.43	14.12	6.87	74.98
3	SKM*	28.21	818.03	492.10	13.92	7.51	81.94
4	SkP	26.77	795.35	466.98	14.00	8.39	91.48
5	SkI2	27.78	735.27	402.59	13.73	8.03	87.55
6	SLy4	26.96	756.12	427.30	13.97	8.91	97.18
7	SKO	28.10	695.72	401.01	13.95	8.53	93.08
8	SKO'	27.04	709.41	405.85	13.99	8.98	97.98
9	LNS	28.70	701.41	414.99	14.37	8.49	92.57
10	SK255	30.11	777.41	453.56	13.67	8.43	91.93
11	BSk17	27.87	740.77	404.41	14.03	9.12	99.45
12	SAMi0	28.70	788.05	490.95	14.08	7.21	78.63

$|T, T; \alpha\rangle$ and $|T+1, T; \alpha\rangle$, and V_1 is the symmetry potential defined in Eq. (18). These values can be compared to the work in Ref. [25] where the Green's function method was employed using the SIII Skyrme interaction. The values of $\hbar\omega = 41 \times A^{-1/3}$ are given to describe the $A^{-1/3}$ behavior of the energy of the IVM- $\overline{E}_0$, from our calculation. This behavior was also obtained in the hydrodynamical model or other collective models. Due to the Pauli blocking of the excess neutrons, it is expected that $S_{T+1}/S_T < 1$. Indeed, the p-h excitations involving a proton transformed into a neutron in an excess neutron orbit are forbidden by the Pauli principle. Not so in the excitations represented by S_T. There one can have p-h components in which the proton is placed in the orbits occupied by the excess neutrons. However, it is different in the case of ^{90}Zr. A plausible explanation is in the following. In ^{90}Zr the excess neutrons occupy mostly the $1g_{9/2}$ orbit. There is no $j = 9/2$ orbit below the $1g_{9/2}$ and therefore one cannot occupy this orbit if one constructs a $J^\pi = 0^+$ state. Thus the amount of p-h components for the T, and $T+1$ is the same and there should be little difference between S_T and S_{T+1}.

In the calculation of ΔE_C, only the direct part that contributes more than 90% to the CDE was included. Other effects, such as the exchange term, the finite proton size effect, the neutron-proton mass difference, the electromagnetic spin-orbit, and the vacuum polarization have not been taken into account. These effects are expected to be small, of the order of one or few percents. They have been estimated in Ref. [31] as a function of A and Z. Very recently, a fully self-consistent calculation of these effects has been carried out for ^{208}Pb in Ref. [32] and confirmed the smallness of the effects. This makes the value of the direct CDE quite acceptable for the purpose of our study. It should be also noted that the other isospin breaking forces of strong origin have not been taken into account here. In general, our isospin mixing numbers are lower than in other approaches where the collective effect of the p-h states is neglected (see a discussion in Ref. [1]). It is remarkable that our results (Tables I, III) for isospin mixing are close to the results obtained in Ref. [13]. For example, in ^{40}Ca, the isospin mixing in Ref. [13] is 0.9%. Our numbers are around 1.0%. Our results for ^{48}Ca were close to the result given in Fig. 1 of Ref. [13]. In ^{100}Sn, our result is 5–7%, close to the values given in Fig. 5 of Ref. [13] [in the after rediagonalization (AR) calculation] and also Refs. [11,12].

TABLE VI. The same as in Table IV, but for ^{90}Zr ($\hbar\omega = 9.15$ MeV).

		$\overline{E}_0$	S_T	S_{T+1}	ΔE_C	$\Delta\overline{E}_+$	V_1
1	SIII	33.53	425.52	430.42	12.22	4.24	63.66
2	SGII	31.45	426.10	444.33	12.38	3.52	52.74
3	SKM*	31.53	435.10	445.68	12.25	3.98	59.64
4	SkP	29.61	431.82	424.44	12.28	5.30	79.56
5	SkI2	31.89	380.95	368.23	12.04	3.12	46.82
6	SLy4	29.96	405.62	405.58	12.25	4.90	73.44
7	SKO	32.08	377.30	293.82	11.85	3.41	51.18
8	SKO'	30.98	375.09	330.47	12.08	3.71	55.64
9	LNS	32.08	371.34	385.09	12.70	4.82	72.28
10	SK255	33.93	412.47	400.59	12.01	4.60	69.04
11	BSk17	31.61	387.62	380.95	12.34	4.32	64.73
12	SAMi0	31.78	424.63	426.62	12.23	3.59	53.78

TABLE VIII. The same as in Table IV, but for ^{208}Pb ($\hbar\omega = 6.92$ MeV).

		$\overline{E}_0$	S_T	S_{T+1}	ΔE_C	$\Delta\overline{E}_+$	V_1
1	SIII	28.19	2081.68	1164.34	19.34	9.87	89.25
2	SGII	26.35	2104.85	1271.02	19.52	7.90	71.45
3	SKM*	26.50	2130.51	1221.69	19.36	8.81	79.69
4	SkP	25.04	2065.16	1111.53	19.46	10.59	95.75
5	SkI2	25.37	1972.33	1012.50	19.00	8.64	78.12
6	SLy4	25.29	1954.69	1025.64	19.39	10.85	98.12
7	SKO	25.28	1918.29	895.07	19.08	9.28	83.90
8	SKO'	24.94	1878.92	928.52	19.28	10.06	91.01
9	LNS	26.85	1839.60	1034.58	19.93	10.29	93.03
10	SK255	27.89	2064.87	1106.52	18.98	10.00	90.46
11	BSk17	26.04	1918.62	986.63	19.50	10.48	94.80
12	SAMi0	26.87	2077.67	1215.95	19.45	8.10	73.26

IV. CONCLUSION

It is well known that the isospin formalism is very useful in nuclear physics, where many examples of isospin symmetry can be found. In particular, this is true for the ground states where isospin mixing does not exceed a few percent. Among the $N = Z$ nuclei studied, ^{100}Sn has the largest isospin admixture, $\varepsilon^2_{T+1} \sim 5\%$ (see also Refs. [11–13]). This is of course due to its large charge, $Z = 50$. As one proceeds to exotic nuclei ($N > Z$) with large numbers of protons and a large excess of neutrons, the isospin mixing remains small because the increase in the strength of the Coulomb field is balanced by the geometrical factor (Clebsch-Gordan coefficient) $1/(T + 1)$ in the denominator. This does not mean that the isospin nonconserving interaction, the Coulomb force, plays a minor role in forming the nucleus, as can be seen in the large Coulomb mixing we calculated. The isospin mixing and the Coulomb mixing do not depend strongly on the choice of the Skyrme interaction. We emphasize again that our results here and in Refs. [1–5] for isospin mixing in the ground states of even-even nuclei, ε^2_{T+1} are considerably smaller than in the very early shell-model calculations [33,34]. This reduced isospin mixing is due to the fact that the repulsive p-h interaction in the isovector channel, the Coulomb strength is shifted in energy from a $2\hbar\omega$ to close to $4\hbar\omega$ depending on the interaction used. Also due to the conservation of the energy-weighted sum rule in the RPA an upper shift in energy means a reduction in the strength. Therefore, the reduction of isospin mixing is more than a factor of four (and possibly closer to a factor of eight) compared to the models in which the p-h strength is left at the unperturbed position of $2\hbar\omega$. Our values for the isospin mixing agree well with the results of isospin-projected HF calculations [13].

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