Effective-field-theory predictions of the muon-deuteron capture rate

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We quantify the theoretical uncertainties of chiral effective-field-theory predictions of the muon-deuteron capture rate. Theoretical error estimates of this low-energy process are important for a reliable interpretation of forthcoming experimental results by the MuSun Collaboration. Specifically, we estimate the three dominant sources of uncertainties that impact theoretical calculations of this rate: those resulting from uncertainties in the pool of fit data used to constrain the coupling constants in the nuclear interaction, those due to the truncation of the effective field theory, and those due to uncertainties in the axial radius of the nucleon. For the capture rate into the $^1S_0$ channel, we find an uncertainty of approximately 4.6 s^{-1} due to the truncation in the effective field theory and an uncertainty of 3.9 s^{-1} due to the uncertainty in the axial radius of the nucleon, both of which are similar in size to the targeted experimental precision of a few percent.

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I. INTRODUCTION

Effective field theories (EFTs) have become a widely used tool in particle and nuclear physics. They are used to obtain a systematic low-energy expansion of observables when a separation of scales is present in a given problem. In particular, chiral EFT had a transformative effect on low-energy nuclear theory [1–3] by providing a clear path towards a nuclear Hamiltonian that can describe the properties of atomic nuclei to high accuracy. Within this framework, nucleons and pions are the degrees of freedom used to build up the nuclear potential that is used to describe the spectra of nuclei. The expansion parameter Q of chiral EFT is given by $\max(m_\pi/\Lambda_b, q/\Lambda_b)$, where m_π denotes the pion mass, q a low momentum scale, and Λ_b denotes the breakdown scale of the theory, which is expected to be comparable to the lightest degree of freedom not taken into account in the theory. An additional advantage over previous approaches to the internuclear potential is that EFT also provides clear guidance on how to construct the coupling to external sources. Indeed, the electroweak current is also calculated order-by-order in a low-energy expansion in chiral EFT and thus shares a large number of low-energy constants (LECs) with the nuclear potential. Thus, chiral dynamics constrains the form of the nuclear currents significantly.

Uncertainty quantification of theoretical calculations is particularly important in the nuclear electroweak sector where observables that are very challenging, or even impossible, to measure experimentally serve as input to astrophysical models. Fortunately, uncertainty quantification was one of the initial promises of EFT calculations. However, it should be pointed out that there remain several open questions on the meaning and understanding of renormalization group invariance of chiral EFT [4–6] and therefore also the interpretation of truncation errors.

In this paper we build on recent progress in uncertainty quantification for EFTs [7–11] and present new results for the different sources of theoretical uncertainties in the EFT description of muon capture on the deuteron, i.e., the process

$$\mu^- + d \to \nu_\mu + n + n. \tag{1}$$

Currently, the MuSun Collaboration is performing an experiment at the Paul Scherrer Institut to measure the rate of this reaction to percentage precision [12]. This will be the first precise measurement of a weak nuclear process in the two-nucleon (NN) system, and the aim is to determine the LEC c_D that parametrizes the strength of the short-distance part of the axial two-body current as well as the one-pion-exchange contact term in the leading three-nucleon (NNN) interaction in EFT approaches to nuclear forces and currents.

Muon capture on the deuteron has long been expected to provide understanding of the electroweak nuclear operator (see Ref. [13] and references therein). A first chiral EFT calculation of muon capture into the neutron-neutron (nn) singlet S wave was carried out by Ando *et al.* [14]. More recently, more complete calculations of the muon capture rate were carried out in Refs. [15–17].

Here, we focus on the three dominant sources of uncertainties of an EFT calculation of the capture rate: those resulting from uncertainties in the nucleon-nucleon scattering database, those due to the truncation of the EFT and those due to uncertainties in the nucleon axial form factor. We will focus on capture from the S-wave doublet state of the muonic deuterium atom to the singlet S-wave state of the nn system,

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$\Gamma_D^{^1S_0}$, which is the only contribution to Γ_D that is relevant to the contact part of the axial current. Furthermore, this part can be extracted by subtracting from Γ_D the higher partial wave contributions calculated in Refs. [15–17]. While these contributions have theoretical uncertainties of their own, they are not sensitive to physics at range shorter than that of pion exchange at the chiral order we are operating at.

We follow two approaches. (i) We use a family of 42 potentials at order Q^3 that have been fitted at seven different regulator cutoffs Λ in the range 450–600 MeV to six different $T_{\rm lab}$ ranges in the NN scattering database. The LECs in this family of $NN + NNN$ interactions were simultaneously fitted to pion-nucleon (πN) and selected NN scattering data, the energies and charge radii of ^{2,3}H and ^{3}He, the quadrupole moment of ^{2}H, as well as the comparative β-decay half-life of ^{3}H. A simple momentum-dependent error term with EFT-like scaling was included in the fits to scattering data, and all 42 potentials reproduce the pool of fit data equally well, see Ref. [7] for details. Clearly, calculating the muon-capture rate with this family of interactions probes an important component of the total theoretical uncertainty. (ii) We also use a set of chiral interactions with regulator cutoff $\Lambda = 500$ MeV at orders Q^0, Q^2, Q^3 with the subleading πN couplings c_1, c_3, c_4 according to the precise Roy-Steiner analysis presented in Refs. [8,18]. The corresponding NN contact potentials of this set of interactions are constrained to reproduce the NN phase shifts of the Granada PWA [19] up to 200 MeV laboratory scattering energy as well as the binding energy and radius of the deuteron. This second class of interactions enables us to parametrize $\Gamma_D^{^1S_0}$ in terms of only one LEC, either $\hat{d}_R$ or c_D, which are related by $\hat{d}_R = -\frac{m}{4g_A\Lambda_b}c_D + \frac{1}{3}\hat{c}_3 + \frac{2}{3}\hat{c}_4 + \frac{1}{6}$, where $\hat{c}_i \equiv c_i m$ and m is the nucleon mass [15,20–23].[1] Indeed, after extracting the LEC c_E of the leading NNN contact from the energy and radius of ^{3}H and ^{3}He, the three-nucleon force is completely predicted up to order Q^3 by $\Gamma_D^{^1S_0}$.

In the following we show that the capture rates extracted from approaches (i) and (ii) agree with each other. Furthermore, we discuss the relative size of the uncertainties of our predictions that arise from the aforementioned sources, and their implications for the interpretation of the impending experimental MuSun results.

II. THE $^1S_0$ CAPTURE RATE

At nuclear energies, the charge-changing weak interaction Hamiltonian $\hat{H}_W$ can be written in terms of the leptonic and the nuclear weak current operators as

$$\hat{H}_W = \frac{G_V}{\sqrt{2}} \int d^3x [j_\alpha(\mathbf{x})J^\alpha(\mathbf{x}) + \text{h.c.}], \tag{2}$$

where G_V is the vector coupling constant which is related to the Fermi coupling constant G_F and the Cabibbo mixing angle θ_C by $G_V = G_F \cos\theta_C$, and "h.c." stands for the Hermitian conjugate of the preceding term. The matrix element of the leptonic weak current operator j^α is $l^\alpha\, e^{-i\mathbf{q}\cdot\mathbf{x}}$, where l^α is the Dirac current of the leptons. The matrix element for the process in Eq. (1) can then be written as

$$T_{fi} = \frac{G_V}{\sqrt{2}}\,\phi_{1S}(\mathbf{0}) \sum_{s_\mu s_d} \left\langle \frac{1}{2}s_\mu,\ 1s_d \middle| \left(\frac{1}{2}1\right)\frac{1}{2}\,s_{\mu d} \right\rangle l^\alpha(h, s_\mu)$$
$$\times \langle \psi_{nn}|J_\alpha(\mathbf{q})|\psi_d; s_d\rangle, \tag{3}$$

where $\phi_{1S}(\mathbf{0}) = [\alpha m_\mu m_d/(m_\mu + m_d)]^{3/2}/\pi^{1/2}$ is the ground-state wave function of the muonic deuterium atom at the origin, and $|\psi_{nn}\rangle$ and $|\psi_d; s_d\rangle$ are, respectively, the states of the nn system and that of the spin-polarized deuteron with projection s_d. Here, we have ignored the quartet channel of muonic deuterium and only coupled the muon and the deuteron spins to 1/2. For capture into the $^1S_0$ singlet nn state with relative momentum p, the differential doublet capture rate is given by

$$\frac{d\Gamma_D^{^1S_0}}{dp} = \frac{1}{2\pi^3}p^2E_\nu^2\left(1 - \frac{E_\nu}{m_\mu + m_d}\right)\overline{|T_{fi}|^2}, \tag{4}$$

where the spin-averaged squared matrix element $\overline{|T_{fi}|^2}$ can be obtained from Eq. (3) by averaging over the spin projections $s_{\mu d}$ of the muonic deuterium atom and summing over neutrino helicities h, which gives

$$\overline{|T_{fi}|^2} = \tfrac{1}{6}G_V^2\,\phi_{1S}^2(\mathbf{0})\,|\sqrt{2}\langle\psi_{nn}|J^1(\mathbf{q}) - i\,J^2(\mathbf{q})|\psi_d; 1\rangle$$
$$- \langle\psi_{nn}|J^0(\mathbf{q}) + J^3(\mathbf{q})|\psi_d; 0\rangle|^2. \tag{5}$$

The neutrino energy is $E_\nu = \frac{1}{2m_{\mu d}}[m_{\mu d}^2 - 4(m_n^2 + p^2)]$, where $m_{\mu d}$ and m_n are the masses of the muonic deuterium atom and the neutron, respectively. The integrated capture rate $\Gamma_D^{^1S_0}$ can be obtained by integrating Eq. (4) with respect to p between the limits 0 and $p_{\max} = (m_{\mu d}^2/4 - m_n^2)^{1/2}$.

III. WEAK CURRENTS

The expressions for the charge-changing nuclear electroweak currents, $J^\alpha \equiv V_{1\mathrm{B}}^\alpha + A_{1\mathrm{B}}^\alpha + V_{2\mathrm{B}}^\alpha + A_{2\mathrm{B}}^\alpha$, have been derived in chiral effective field theory in Refs. [24–27]. We take into account operators that give nonvanishing contributions to Eq. (1) up to $\mathcal{O}(Q^3)$ in the chiral expansion. The nuclear wave functions are also consistently calculated up to the same order. In both, the current operators and the wave function, we count Q/m as $\mathcal{O}(Q^2)$ [7,28]. The Gamow-Teller operator,

$$\mathbf{A}_{1\mathrm{B}}^{\mathrm{GT}}(\mathbf{q}) = -F_A(q^2)\sum_i e^{-i\mathbf{q}\cdot\mathbf{r}_i}\tau_i^-\boldsymbol{\sigma}_i, \tag{6}$$

enters at $\mathcal{O}(Q^0)$. Here, F_A is the axial form-factor which is a function of the four-vector inner product $q^2 = m_\mu(m_\mu - 2E_\nu)$. We use $F_A(q^2) = g_A(1 + r_A^2q^2/6)$, where r_A is the axial radius of the nucleon. This truncation is consistent with the chiral order to which we work in this paper, and with both dipole and z parametrizations of the axial form-factor [29].

[1] An error in the coefficient of c_D in Ref. [20] was recently corrected by Ref. [23]. The LECs of Ref. [7] have been reoptimized with a corrected relation between the LECs c_D and $\hat{d}_R$ [23]. The new values [33] are used throughout in this work.

The pseudoscalar operator [26],

$$A^0_{1B}(\mathbf{q}) = -g_A \sum_i e^{-i\mathbf{q}\cdot\mathbf{r}_i}\tau_i^- \frac{\boldsymbol{\sigma}_i\cdot\bar{\mathbf{p}}_i}{m}, \tag{7}$$

where $\bar{\mathbf{p}}_i = (\mathbf{p}_i + \mathbf{p}'_i)/2 = \mathbf{p}_i + \mathbf{q}/2$ is the average of the momenta of the nucleons before and after coupling with the leptons, only appears at $\mathcal{O}(Q^2)$. Additionally both Eqs. (6) and (7) also include an induced-pseudoscalar contribution [14] that gives $A^\alpha_{1B}(\mathbf{q}) \to A^\alpha_{1B}(\mathbf{q}) + q^\alpha q_\beta A^\beta_{1B}(\mathbf{q})/(m_\pi^2 - q^2)$. The one-body vector operator appears at $\mathcal{O}(Q^2)$ and consists of the so-called convection current and the weak-magnetism terms,

$$\mathbf{V}_{1B}(\mathbf{q}) = \sum_i e^{-i\mathbf{q}\cdot\mathbf{r}_i}\tau_i^- \frac{1}{m}\left(\bar{\mathbf{p}} + i\frac{\mu_V}{2}\mathbf{q}\times\boldsymbol{\sigma}_i\right), \tag{8}$$

where μ_V is the nucleon isovector magnetic moment, whose value is 4.706. In Eqs. (7) and (8), and also in the two-body currents discussed below, we have used the zero four-momentum transfer values for the axial and electromagnetic form factors since their q^2 dependences are higher order in the EFT expansion.

The axial two-body operators, which enter at $\mathcal{O}(Q^3)$, can be written as [14,26]

$$A^\alpha_{2B}(\mathbf{q}) = \hat{A}^\alpha_{2B}(\mathbf{q}) + \frac{q^\alpha\left[q_\beta \hat{A}^\beta_{2B}(\mathbf{q}) + \hat{A}^{\rm PS}_{2B}(\mathbf{q})\right]}{m_\pi^2 - q^2}, \tag{9}$$

where

$$\begin{aligned}\hat{A}^0_{2B}(\mathbf{q}) = & -i\frac{g_A}{4f_\pi^2}\tau_\times^-\left[\frac{\boldsymbol{\sigma}_1\cdot\mathbf{k}_1}{m_\pi^2 - k_1^2} - \frac{\boldsymbol{\sigma}_2\cdot\mathbf{k}_2}{m_\pi^2 - k_2^2}\right] \\ & + \frac{2g_A}{mf_\pi^2}\left(\hat{c}_2 + \hat{c}_3 - \frac{g_A^2}{8}\right)\sum_i \tau_i^- \frac{\boldsymbol{\sigma}_i\cdot\mathbf{k}_i k_i^0}{m_\pi^2 - k_i^2},\end{aligned} \tag{10}$$

$$\begin{aligned}\hat{\mathbf{A}}_{2B}(\mathbf{q}) = \frac{g_A}{2mf_\pi^2}\Bigg\{ & \frac{\boldsymbol{\sigma}_2\cdot\mathbf{k}_2}{m_\pi^2 - k_2^2}\left[\frac{i}{2}\tau_\times^-\bar{\mathbf{p}}_1 + 4\hat{c}_3\tau_2^-\mathbf{k}_2\right. \\ & \left. + \left(\hat{c}_4 + \frac{1}{4}\right)\tau_\times^-\boldsymbol{\sigma}_1\times\mathbf{k}_2 + \frac{\mu_V}{4}\tau_\times^-\boldsymbol{\sigma}_1\times\mathbf{q}\right] \\ & + 2\hat{d}_1\sum_i \tau_i^-\boldsymbol{\sigma}_i + \hat{d}_2\tau_\times^-\boldsymbol{\sigma}_\times + (1\leftrightarrow 2)\Bigg\},\end{aligned} \tag{11}$$

and

$$\hat{A}^{\rm PS}_{2B}(\mathbf{q}) = \frac{4g_A m_\pi^2}{mf_\pi^2}\hat{c}_1\left[\tau_2^-\frac{\boldsymbol{\sigma}_2\cdot\mathbf{k}_2}{m_\pi^2 - k_2^2} + (1\leftrightarrow 2)\right]. \tag{12}$$

The μ_V term in Eq. (11) and the pion pole contribution given by the second term in Eq. (9) were ignored by Ref. [26] in their proton-proton fusion calculation but were included by Ref. [14] since they are non-negligible for the muon capture process. In these equations, $\mathbf{k}_i = \mathbf{p}'_i - \mathbf{p}_i$, $\tau_\times^- = (\tau_1\times\tau_2)^x - i(\tau_1\times\tau_2)^y$, $\boldsymbol{\sigma}_\times = \boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2$, and f_π is the pion decay constant. The linear combination $g_A\Lambda_b(\hat{d}_1 + 2\hat{d}_2) = c_D$ is conventionally used to combine the $\hat{d}_1$ and $\hat{d}_2$ terms, which are rendered redundant by the Pauli principle [26]. The LECs c_i in the pion-exchange current also appear in πN and NN interactions and in the long-range part of the NNN interaction, whereas c_D (or $\hat{d}_R$) simultaneously parametrizes both the strength of the short-range part of the meson-exchange axial currents and that of the intermediate-range part of the NNN interaction. The vector part of the two-body current is given by the sum of the so-called seagull and pion-in-flight terms [14],

$$\begin{aligned}\mathbf{V}_{2B}(\mathbf{q}) = -i\tau_\times^-\frac{g_A^2}{4f_\pi^2}\Bigg[& \frac{\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2\cdot\mathbf{k}_2}{m_\pi^2 - k_2^2} - \frac{\boldsymbol{\sigma}_2\boldsymbol{\sigma}_1\cdot\mathbf{k}_1}{m_\pi^2 - k_1^2} \\ & + \frac{\boldsymbol{\sigma}_1\cdot\mathbf{k}_1}{m_\pi^2 - k_1^2}\frac{\boldsymbol{\sigma}_2\cdot\mathbf{k}_2}{m_\pi^2 - k_2^2}(\mathbf{k}_2 - \mathbf{k}_1)\Bigg].\end{aligned} \tag{13}$$

The two-body vector charge operator, V^0_{2B}, is suppressed by an additional factor of the chiral EFT expansion parameter.

IV. COVARIANCE ANALYSIS

The covariance matrices provided in Ref. [7] offer a straightforward handle on the statistical uncertainties in the integrated and differential muon-capture rate stemming from the experimental uncertainties in the fit data. The Jacobians of of $\Gamma_D^{^1S_0}$ with respect to relevant LECs were computed in a simple finite difference scheme and derivatives could be reliably extracted using splines. For the nuclear wave functions we do not allow any variation in the axial coupling constant g_A. We start from $g_A = 1.276$ [30] which after renormalization to account for the Goldberger-Treiman discrepancy is matched to the empirically determined πN coupling strength $g^2_{\pi NN}/4\pi = 13.7$ [31]. This value for g_A is slightly larger than the most recently adopted Particle Data Group (PDG) value $g_A = 1.2723(23)$ but in fair agreement with the value $g_A = 1.2749(9)$ employed by Hill *et al.* [32].

It is sufficient to use the first-order statistical methods described in Ref. [7]. From this we can establish that the uncertainty in $\Gamma_D^{^1S_0}$ due to uncertainties in the determination of the LECs at Q^3 from experimental data is very small and certainly not of any primary concern. We find that the typical size of the statistical uncertainties in $\Gamma_D^{^1S_0}$ is 0.5 s^{-1}. The sensitivity to different truncations of the NN scattering database is of similar size, while variations of the regulator cutoff is up to five times larger, see Fig. 1.

Based on the covariance analysis, variations of the regulator cutoff, and the pool of fit data for extracting the LECs we obtain a conservative estimate for the model uncertainty at order Q^3 in chiral EFT. Using a weighted average of the results shown in Fig. 1, we find

$$\Gamma_D^{^1S_0} = 252.4^{+1.5}_{-2.1}\ \mathrm{s}^{-1}. \tag{14}$$

V. CORRELATION WITH THE ASTROPHYSICAL PROTON-PROTON S FACTOR

Using the chiral interactions at orders Q^3 with the πN LECs, $c_{1,2,3,4} = (-0.74, 1.81, -3.61, 2.44)\ \mathrm{GeV}^{-1}$, determined in a Roy-Steiner analysis [18], we can analyze the relation between different observables via a variation of the short-distance LEC c_D. For example, in Fig. 2 we trace out the correlation between the proton-proton (pp) S_{pp} factor at zero energy and the muon-capture rate $\Gamma_D^{^1S_0}$. Different points on the black line in this figure only differ in the values of the LEC c_D and c_E that reproduce the binding energies and

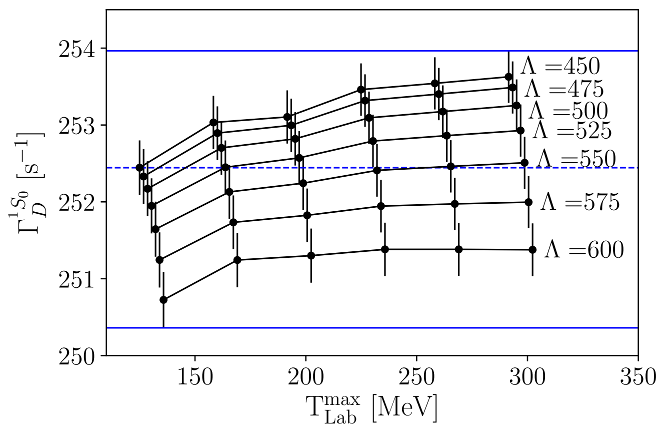

FIG. 1. Distribution of central values for the muon-capture rate $\Gamma_D^{^1S_0}$ when using the family of 42 chiral EFT potentials at order Q^3 from Ref. [7]. The vertical bars indicate the respective statistical uncertainty propagated from the underlying uncertainties in the LECs. Calculations with identical regulator cutoffs Λ but different truncations $T_{\rm Lab}^{\rm max}$ in the NN scattering database are connected with a line to guide the eye. The weighted average of all calculations and conservative error limits are indicated with dashed and solid lines, respectively. The numerical values of the combined model error is given in Eq. (14).

radii of ^{3}H and ^{3}He while the two-body interaction remain unchanged.

As in Ref. [20], we can also use the triton binding energy and β-decay half-life, corresponding to a reduced matrix element of the $J = 1$ electric multipole of the axial-vector current $|\langle^3{\rm He}||E_1^A||^3{\rm H}\rangle| = 0.6848 \pm 0.0011$, to fix c_D and c_E, and thus also the muon-capture rate and the pp fusion S factor, see Fig. 2 (dashed lines). With $c_D = -0.39$ and $c_E = -0.44$, we find that $S_{pp}(0) = 4.058 \times 10^{-23}$ MeV fm^2, which is consistent with our previously published result [34], and a muon-capture rate

$$\Gamma_D^{^1S_0} = 252.8 \pm 4.6 \pm 3.9 \text{ s}^{-1}. \tag{15}$$

These results are not sensitive to variations of the tritium β-decay matrix element within the range of uncertainty quoted above. The first uncertainty in the above expression for $\Gamma_D^{^1S_0}$ estimates the effect of truncating the chiral EFT expansion at order Q^3. The second uncertainty indicates the sensitivity to variations of the axial radius within the error budget $r_A^2 = 0.46(22)$ fm^2 [29]. The truncation error is extracted by following the method discussed in Ref. [9]. In brief, we calculate the capture rate at the lower orders Q^0 and Q^2, in the currents as well as the wave functions, and express the results as an expansion of the form $\Gamma_D^{^1S_0} = \Gamma_{\rm LO}^{^1S_0} \sum_{n=0}^{3} c_n (p/\Lambda_b)^n$, where we assume that the breakdown scale of theory is $\Lambda_b = 500$ MeV and the inherent momentum p of the problem is provided by the soft scale of chiral EFT, i.e., $p = m_\pi$. Note that the maximum of the momentum-differential doublet-capture rate in Eq. (4) occurs at a momentum scale $p \sim 25$ MeV. We obtain an estimate for the EFT truncation error by calculating $(p/\Lambda_b)^4 \max(|c_0|, |c_2|, |c_3|)$. The order-by-order capture rates with a c_D that reproduces the comparative inverse β-decay half-life of triton are (186.3, 247.3, 252.8) s^{-1} at orders (Q^0, Q^2, Q^3), respectively. We find that the uncertainty estimate resulting from an analysis of the EFT truncation is comparable to the error induced by the imprecise value of the axial radius. In turn, both of these errors are twice as large as the uncertainty related to the cutoff variation of the chiral potential and truncations in the pool of fit data.

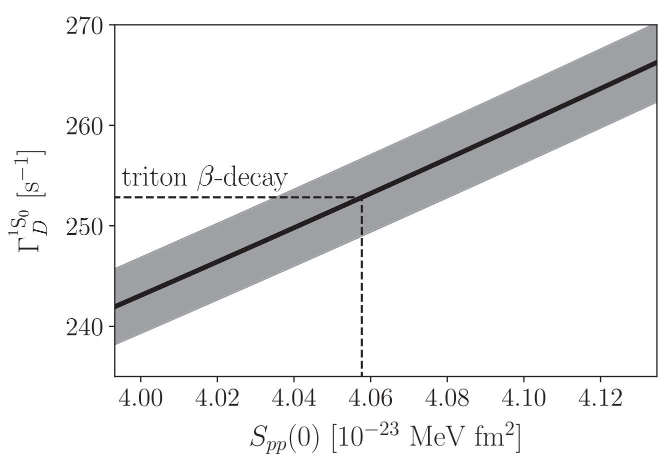

FIG. 2. The muon-capture rate $\Gamma_D^{^1S_0}$ as a function of the pp fusion S_{pp} factor at zero energy parameterized by the axial current LEC $c_D \in [-3.6, +3.6]$ at order Q^3 using the Roy-Steiner based interaction. The grey band indicates the uncertainty in the muon-capture rate due to the uncertainty in the axial radius of the nucleon $r_A^2 = 0.46(22)$ fm^2. The dashed lines indicates the values for $S_{pp}(0)$ and Γ_D when the experimental value for the triton β-decay half-life is used to determine the LEC c_D.

VI. CONCLUSION

We have analyzed uncertainties in calculations for the muon-capture rate using two classes of interactions: (i) order Q^3 interactions constructed as described above and in Ref. [7], and (ii) a set of interactions at order Q^0, Q^2, Q^3, whose πN couplings c_1, c_3, and c_4 were taken from Ref. [18].

The analysis carried out in Refs. [8,18] has reduced the uncertainties in the πN LECs significantly. This leaves c_D as the only undetermined LEC in the weak axial two-body current. We demonstrated that this leads to linear correlations between electroweak observables in the two-nucleon sector that involve phase-shift equivalent NN interactions.

We focused on the singlet S-wave nn channel, which is the only channel sensitive to the weak axial two-body contact current. Our results for muon capture and the associated uncertainties are shown in Eqs. (14) and (15). These uncertainty estimates are rooted in the description of the strong-interaction part of the calculation. We also emphasize the importance of the additional ∼1.5% uncertainty due to the uncertainty in the nucleon axial radius. We note that the central value we obtain is in excellent agreement with a prior chiral EFT calculation [15] even though our error estimate is larger because we perform a more rigourous treatment of uncertainties.

Using the result for muon capture into the single S wave from Eq. (15) and the results from Ref. [15] for muon capture into higher partial waves, we can obtain an estimate of

397.8 s^{-1} for the total capture rate, Γ_D. We expect that higher accuracy can be obtained for capture into higher partial waves since these are less sensitive to the axial two-body current. However, we refrain from giving a total uncertainty for this capture rate.

We have also studied the correlation of the capture rate with other NN observables. In agreement with previous work [35], we find that the capture rate depends only weakly on the nn scattering length a_{nn} provided that it is negative. However, the capture rate would be significantly smaller if a_{nn} was positive due to the existence of a shallow dineutron.

In the future, we will carry out a complete uncertainty analysis for pp fusion and muon capture on the deuteron, including the effect of higher partial waves. This analysis will provide a full picture on the uncertainties and correlations of electroweak processes in the NN sector. We emphasize that the axial radius r_A is a significant source of uncertainty in our analysis. Future improvements in experimental precision and lattice QCD results [36] will lead to important insights into how the nuclear Hamiltonian correlates various electroweak observables.

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