$0\nu\beta\beta$ and $2\nu\beta\beta$ nuclear matrix elements evaluated in closure approximation, neutrino potentials and SU(4) symmetry

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The intimate relation between the Gamow-Teller part of the matrix element $M^{0\nu}_{GT}$ and the $2\nu\beta\beta$ closure matrix element $M^{2\nu}_{cl}$ is explained and explored. If the corresponding radial dependence $C^{2\nu}_{cl}(r)$ would be known, $M^{0\nu}$ corresponding to any mechanism responsible for the $0\nu\beta\beta$ decay can be obtained as a simple integral. However, the $M^{2\nu}_{cl}$ values, and therefore also the functions $C^{2\nu}_{cl}(r)$, sensitively depend not only on the properties of the first few 1^+ states but also of higher-lying 1^+ states in the intermediate odd-odd nuclei. We show that the β^- and β^+ amplitudes of such states typically have opposite relative signs, and their contributions reduce severally the $M^{2\nu}_{cl}$ values. We suggest that demanding that $M^{2\nu}_{cl} = 0$ is a sensible alternative way, within the QRPA method, of determining the amount of renormalization of isoscalar particle-particle interaction strength $g^{T=0}_{pp}$. Using such prescription, the matrix elements $M^{0\nu}$ are evaluated; their values are not very different ($\leqslant 20\%$) from the usual QRPA values when $g^{T=0}_{pp}$ is related to the known $2\nu\beta\beta$ half-lives. We note that vanishing values of $M^{2\nu}_{cl}$ are signs of a partial restoration of the spin-isospin SU(4) symmetry.

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I. INTRODUCTION

Neutrinos are the only known elementary particles that may be Majorana fermions, i.e., identical with their antiparticles. They are also very light, suggesting that the origin of their mass could be different from the origin of mass of all other fermions that are much heavier and charged, supporting such hypothesis. Study of the neutrinoless double beta decay ($0\nu\beta\beta$), the transition among certain even-even nuclei when two neutrons bound in the ground state are transformed into two bound protons and two electrons with nothing else emitted, is the most straightforward test whether neutrinos are indeed Majorana fermions. Obviously, observing such decay would mean that the lepton number is not a conserved quantity as required by the standard model.

There is an intense worldwide effort to search for the $0\nu\beta\beta$ decay. No signal has been observed so far, but impressive half-life limits of more than 10^{25}–10^{26} years have been achieved in several experiments on several target nuclei. Larger, and even more sophisticated experiments are being developed and/or planned. Search for the $0\nu\beta\beta$ decay is at the forefront of the present-day nuclear and particle physics.

While observation of the $0\nu\beta\beta$ decay would constitute a proof that neutrinos are massive Majorana fermions [1], it is obviously desirable to be able to relate the observed half-life to some beyond the standard model particle physics theory. To do that, however, requires understanding of the nuclear structure issues involved in the $(Z, A)_{g.s.} \rightarrow (Z+2, A)_{g.s.} + 2e^-$ transition. The problem at hand is the evaluation of the corresponding nuclear matrix elements. This is a longstanding issue, with a plethora of papers devoted to this subject. A recent review [2] summarizes the present status.

Here we explore in more detail the relation between the nuclear matrix elements of the $0\nu\beta\beta$ decay and of the allowed and experimentally observed $2\nu\beta\beta$ decay, treated, however, in the closure approximation. This is a continuation and expansion of the earlier paper [3]. We concentrate primarily on the expression of these matrix elements as functions of the relative distance r between the two neutrons that are transformed into the two protons in the $\beta\beta$ decay. Naturally, we keep in mind that the closure approximation is not applicable for the $2\nu\beta\beta$ mode of the $\beta\beta$ decay.

Very generally, the observable $0\nu\beta\beta$ decay rate is expressed as a product of three factors

$$\frac{1}{T_{1/2}} = G^{0\nu}(Z, E_0)(M^{0\nu})^2\phi^2, \quad (1)$$

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where $G^{0\nu}(Z, E_0)$ is the calculable phase space factor that in this case also includes all necessary fundamental constants, and that depends on the nuclear charge Z and on the decay endpoint energy E_0. $M^{0\nu}$ is the nuclear matrix element that depends, among other things, on the particle physics mechanism responsible for the $0\nu\beta\beta$ decay, as does the phase-space factor $G^{0\nu}(Z, E_0)$. And by ϕ we symbolically denote the corresponding particle physics parameter that we would like to extract from experiment.

For any mechanism responsible for the decay, the matrix element $M^{0\nu}$ consists of three parts, Fermi, Gamow-Teller, and tensor

$$M^{0\nu} = M^{0\nu}_{GT} - \frac{M^{0\nu}_F}{g_A^2} + M^{0\nu}_T, \tag{2}$$

where g_A is the nucleon axial current coupling constant. And, in turn, the GT part, evaluated in the closure approximation, is

$$M^{0\nu}_{GT} = \langle f|\Sigma_{i,j}\vec{\sigma}_i\cdot\vec{\sigma}_j\tau_i^+\tau_j^+ H_{GT}(r_{ij}, \bar{E})|i\rangle. \tag{3}$$

The Fermi part, again in closure, is given by an analogous formula

$$M^{0\nu}_F = \langle f|\Sigma_{i,j}\tau_i^+\tau_j^+ H_F(r_{ij}, \bar{E})|i\rangle. \tag{4}$$

And the tensor part is

$$M^{0\nu}_T = \langle f|\Sigma_{ij}[3(\vec{\sigma}_i\cdot\hat{r}_{ij})(\vec{\sigma}_i\cdot\hat{r}_{ij}) - \vec{\sigma}_i\cdot\vec{\sigma}_j]\tau_i^+\tau_j^+ H_T(r_{ij}, \bar{E})|i\rangle. \tag{5}$$

Here $|i\rangle$, $|f\rangle$ are the ground-state wave functions of the initial and final nuclei. $H_{GT}(r_{ij}, \bar{E})$, $H_F(r_{ij}, \bar{E})$ and $H_T(r_{ij}, \bar{E})$ are the neutrino potentials that depend on the relative distance r_{ij} of the two nucleons. The sum is over all nucleons in the nucleus. We discuss the validity of the closure approximation for the $0\nu\beta\beta$ mode in the next section.

The paper is organized as follows. After this introduction, in the next section the so-called neutrino potentials are described, and their dependence on the distance r between the decaying neutrons. Next, the two neutrino ($2\nu\beta\beta$) decay matrix elements in closure approximation and their relation to the $0\nu\beta\beta$-decay matrix elements are discussed. In the following section advantages of the LS coupling scheme are described and symmetry consideration are applied. In Sec. V the $0\nu\beta\beta$ matrix elements, based on previous considerations, are evaluated and their values are compared to the previously published ones. The partial restoration of the spin-isospin symmetry SU(4) is also discussed there. Finally, Sec. VI (Summary) concludes the paper.

II. NEUTRINO POTENTIALS

Neutrino potentials in Eqs. (3), (4), and (5) are typically defined as integrals over the momentum transfer q. They cannot be expressed by an analytic formula as functions of the internucleon distance r_{ij}. In the following we will concentrate on the standard scenario, where the $0\nu\beta\beta$ decay is associated with the exchange of light Majorana neutrinos. In that case the particle parameter ϕ in Eq. (1) is the effective neutrino mass

$$m_{\beta\beta} = \left|\sum_{i=1}^{3}|U_{ei}|^2 e^{\mathrm{i}\alpha_i} m_i\right|, \tag{6}$$

where U_{ei} are the, generally complex, matrix elements of the first row of the PMNS neutrino mixing matrix with phases α_i, and m_i are the masses of the corresponding mass eigenstates neutrinos. The present values of the mixing angles and mass squared differences Δm^2_{ij} are listed, e.g., in Ref. [4].

For this mechanism, the dimensionless neutrino potential for the $K = GT$, F, and T parts is

$$H_K(r_{12}, \bar{E}) = f^2_{\rm src}(r_{12})\frac{2}{\pi g_A^2}R\int_0^\infty f_K(qr_{12})\frac{h_K(q^2)qdq}{q+\bar{E}}, \tag{7}$$

here R is the nuclear radius added to make the potential dimensionless. The functions $f_{F,GT}(qr_{12}) = j_0(qr_{12})$ and $f_T(qr_{12}) = -j_2(qr_{12})$ are spherical Bessel functions. The functions $h_K(q^2)$ are defined in Ref. [5]. The potentials depend rather weakly on average nuclear excitation energy $\bar{E}$. The function $f_{\rm src}(r_{12})$ represents the effect of two-nucleon short-range correlations. In the following we use the $f_{\rm src}(r_{12})$ derived in Ref. [6]. The phase-space factors for this mechanism are listed, e.g., in Ref. [7].

However, the exchange of light Majorana neutrinos is not the only way $0\nu\beta\beta$ decay can occur. Many particle physics models that contain so far unobserved new particles at the ~TeV mass scale also contain $\Delta L = 2$ higher-dimension operators, changing the total lepton number L by two units, that could lead to the $0\nu\beta\beta$ decay with a rate comparable to the rate associated with the light Majorana neutrino exchange. These models also explain why neutrinos are so light. Moreover, some of their predictions can be confirmed (or rejected) at the LHC or beyond. Examples of these models are the left-right symmetric model (LRSM) or the R-parity violating supersymmetry. In them, heavy ($M \gg M_p$, M_p is the proton mass) particles are exchanged between the two neutrons that are transformed into the two protons. There is a large variety of neutrino potentials corresponding to such mechanism of $0\nu\beta\beta$ decay. A list of them, and of the corresponding phase-space factors, can be found, e.g., in Ref. [8]. For a complete description of the $0\nu\beta\beta$ decay it would be, therefore, necessary to evaluate ~20 different nuclear matrix elements. We show below how this task could be substantially simplified.

The matrix elements defined in the Eqs. (3), (4), and (5) are evaluated in the closure approximation. In that case only the wave functions of the initial and final ground states are needed. The validity of this approximation can be tested in QRPA, where the summation over the intermediate states is easily implemented as done in Ref. [3]. There it was shown that the closure approximation typically results in matrix elements that are at most 10% smaller than those obtained by explicitly summing over the intermediate virtual states. The dependence on the assumed average energy $\bar{E}$ is weak; it makes little difference if $\bar{E}$ is varied between 0 and 12 MeV.

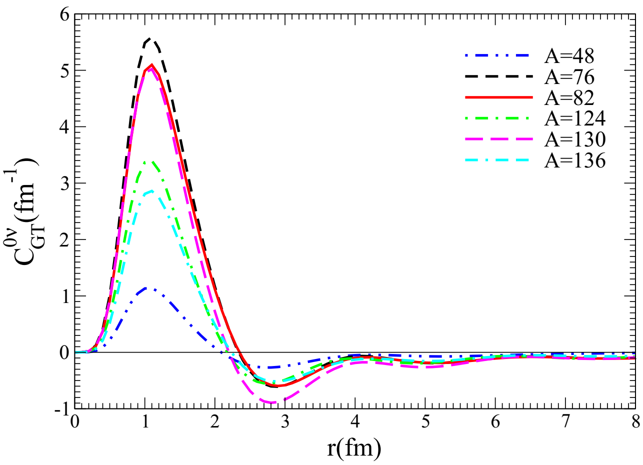

FIG. 1. Functions $C^{0\nu}_{GT}(r)$ evaluated in the QRPA for a number of $0\nu\beta\beta$ candidate nuclei.

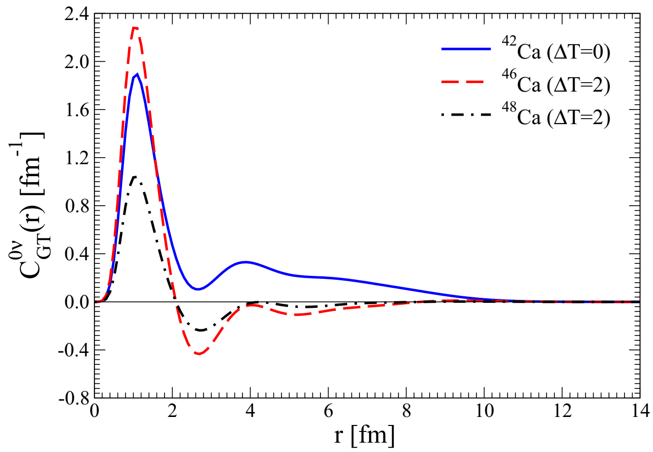

FIG. 2. Functions $C^{0\nu}_{GT}(r)$ evaluated in the QRPA for several Ca isotopes. ^{48}Ca is a real $\beta\beta$ decay candidate. It decays into ^{48}Ti and the isospin T changes in the decay by two units ($\Delta T = 2$). The other two Ca isotopes cannot $\beta\beta$ decay; nevertheless the corresponding matrix elements can be evaluated. The transition ^{42}Ca $\rightarrow$ ^{42}Ti connects mirror nuclei, the isospin does not change, $\Delta T = 0$.

Similar conclusion was reached using the nuclear shell model (see Ref. [9] and references therein).

Better insight into the structure of matrix elements can be gained by explicitly considering their dependence on the distance r between the two neutrons that are transformed into two protons in the decay. Thus we define the function $C^{0\nu}_{GT}(r)$ (and analogous ones for M_F and M_T) as

$$C^{0\nu}_{GT}(r) = \langle f|\Sigma_{i,j}\vec{\sigma}_i\cdot\vec{\sigma}_j\tau_i^+\tau_j^+\delta(r-r_{ij})H(r_{ij},\bar{E})|i\rangle. \quad (8)$$

This function is, obviously, normalized as

$$M^{0\nu}_{GT} = \int_0^\infty C^{0\nu}_{GT}(r)dr. \quad (9)$$

In other words, knowledge of $C^{0\nu}_{GT}(r)$ makes the evaluation of $M^{0\nu}_{GT}$ trivial. The function $C(r)$ was first introduced in Ref. [5].

As one can see in Fig. 1 the function $C^{0\nu}_{GT}(r)$ consists primarily of a peak with the maximum at 1.0–1.2 fm and a node at 2–2.5 fm. The negative tail past this node contributes relatively little to the integral over r and hence to the value of $M^{0\nu}_{GT}$. The shape of the function $C^{0\nu}_{GT}(r)$ is almost the same for all $0\nu\beta\beta$-decay candidates. The magnitude of the matrix element $M^{0\nu}_{GT}$ is determined, essentially, by the value of the peak maximum, which can be related, among other things, to the pairing properties of the involved nuclei.

This characteristic behavior of the function $C^{0\nu}_{GT}(r)$ repeats itself when it is evaluated instead in the nuclear shell model; same peak, same node, little effect of the tail past the node [10]. The same function was also evaluated in Ref. [11] for the hypothetical decay ^{10}He $\rightarrow$ ^{10}Be using the *ab initio* variational Monte Carlo method. The function $C^{0\nu}_{GT}(r)$ has, again even in this case, qualitatively similar shape with a similar peak and same node, but the negative tail appears to be somewhat more pronounced. We might conclude that, at least qualitatively, the shape of $C^{0\nu}_{GT}(r)$ is universal; it does not depend on the method used to calculate it, even though the methods mentioned here, QRPA, nuclear shell model, or the *ab initio* variational Monte Carlo are vastly different in the way the ground-state wave functions $|i\rangle$ and $|f\rangle$ are evaluated.

In all $\beta\beta$-decay candidate nuclei the isospin T of the initial nucleus is different, by two units, from the isospin of the final nucleus; thus $\Delta T = 2$. To study theoretically nuclear matrix element evaluation it is not necessary to consider only the $\beta\beta$ transitions allowed by the energy conservation rules. Thus, transitions within an isospin multiplet ($\Delta T = 0$), such as ^{42}Ca $\rightarrow$ ^{42}Ti or ^{6}He $\rightarrow$ ^{6} Be can be, and are, considered. The corresponding radial dependence $C^{0\nu}_{GT}(r)$ is different in that case. There is no node, the function remain positive over the whole r range. For QRPA this is illustrated in Fig. 2. Again, in the *ab initio* evaluation [11] for the hypothetical transition ^{6}He $\rightarrow$ ^{6}Be that feature is there as well, even though the shape of the curve is rather different than for the ^{42}Ca case. The fact that the functions $C^{0\nu}_{GT}(r)$ are quite different when $\Delta T = 2$ and $\Delta T = 0$ cases are considered, suggests that it is not obvious whether the experience obtained from the latter cases in light nuclei can be easily generalized to the decays of real $0\nu\beta\beta$-decay candidate nuclei, which are all $\Delta T = 2$.

The radial functions $C^{0\nu}_{F}(r)$ and $C^{0\nu}_{T}(r)$ corresponding to the Fermi, Eq. (4), and tensor, Eq. (5), matrix elements are obtained in an analogous way. A typical example is shown in Fig. 3. The function $C^{0\nu}_{F}(r)$ has very similar shape as $C^{0\nu}_{GT}(r)$, but has opposite sign [see, however, the sign in Eq. (2)]. The relation of $C^{0\nu}_{F}(r)$ and $C^{0\nu}_{GT}(r)$ will be discussed in detail in Sec. IV.

III. $2\nu\beta\beta$ MATRIX ELEMENTS IN CLOSURE APPROXIMATION

It would be clearly desirable to find a relation between the $0\nu\beta\beta$ matrix elements and another quantity that does not depend on the unknown fundamental physics and that, in an ideal case, is open to experiment. Here we wish to make a step in that direction.

If one would skip the neutrino potential $H(r_{ij}, \bar{E})$ in Eq. (3) the resulting matrix element is just the matrix element

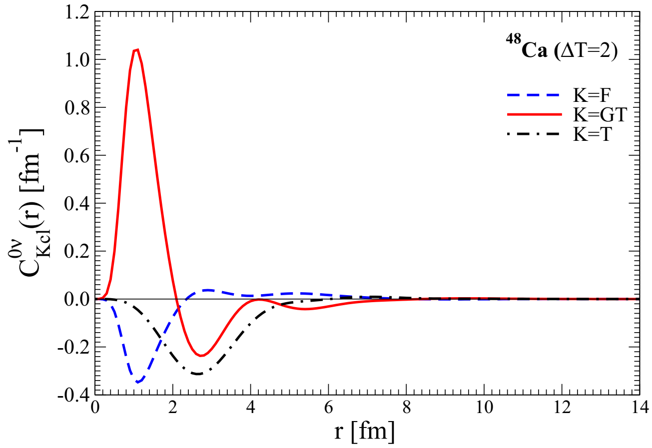

FIG. 3. Functions $C_I^{0\nu}(r)$ evaluated in the QRPA for a the $I =$ Gamow-Teller, Fermi, and tensor matrix elements for ^{48}Ca $0\nu\beta\beta$ decay.

corresponding to the allowed $2\nu\beta\beta$ mode of decay evaluated, however, in the closure approximation. The half-lives of $2\nu\beta\beta$ decay have been experimentally determined for most candidate nuclei. They are related to the matrix elements by

$$\frac{1}{T_{1/2}^{2\nu}} = G^{2\nu}(Z, E_0)(M^{2\nu})^2, \tag{10}$$

where $G^{2\nu}(Z, E_0)$ is the calculable phase-space factor that in this case includes all necessary fundamental constants, including the factor g_A^4. The $2\nu\beta\beta$ matrix element, in turn, is

$$M^{2\nu} = \Sigma_m \frac{\langle f||\sigma\tau^+||m\rangle\langle m||\sigma\tau^+||i\rangle}{E_m - (M_i + M_f)/2}, \tag{11}$$

where the summation extends over all 1^+ virtual intermediate states. The presence of the energy denominators in Eq. (11) is essential, it reduces the dependence on the poorly known higher-lying 1^+ states. Thus, if the $2\nu\beta\beta$ half-life is known experimentally, the values of $M^{2\nu}$ can be extracted. (Actually, keeping in mind a possible renormalization, i.e., quenching, of the g_A value in complex nuclei, the quantity $g_A^2 M^{2\nu}$ can be extracted from the experimental half-life value.)

Evaluation of the $2\nu\beta\beta$ closure matrix element

$$\begin{aligned} M_{GTcl}^{2\nu} &= \langle f|\Sigma_{i,j}\vec{\sigma}_i \cdot \vec{\sigma}_j \tau_i^+\tau_j^+|i\rangle \\ &= \Sigma_m\langle f||\sigma\tau^+||m\rangle\langle m||\sigma\tau^+||i\rangle \end{aligned} \tag{12}$$

implicitly requires the knowledge of all 1^+ intermediate states and the GT amplitudes connecting them to the initial and final ground states. The expression (12) is a product of amplitudes corresponding to the β^- strength of the initial nucleus and the β^+ strength of the final one. The total strengths are connected by the Ikeda sum rule $S(\beta^-) - S(\beta^+) = 3(N - Z)$ which is automatically fulfilled in QRPA and in NSM when the model space involves both spin-orbit partners of all single-particle states. In Fig. 4 the radial dependence of these strengths, i.e., the $C(r)$ functions corresponding to $\langle i|\Sigma_{ij}\tau_i^+\tau_j^-\sigma_i \cdot \sigma_j|i\rangle$, i.e., the $S(\beta^-)$, and $\langle f|\Sigma_{ij}\tau_i^-\tau_j^+\sigma_i \cdot \sigma_j|f\rangle$, i.e., the $S(\beta^+)$, are shown for the case of ^{76}Ge and ^{76}Se. Note not only the different scales of the two panels, but also the substantial cancellation between the $r \leqslant 2.5$ fm and $r > 2.5$ fm in the β^+ case. The $S(\beta^+)$ strength is suppressed because the β^+ operator connects states that belong to different isospin multiplets.

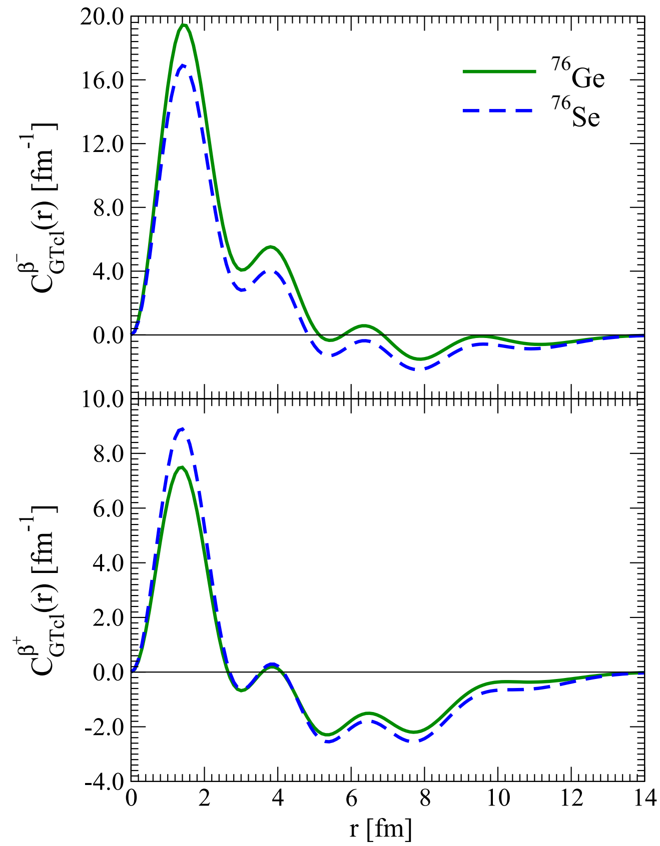

FIG. 4. Functions $C(r)$ corresponding to the total strengths $S(\beta^-)$ and $S(\beta^+)$ for the initial nucleus ^{76}Ge and for the final nucleus ^{76}Se.

While the total strengths represent sums over positive contributions from all 1^+ states in the corresponding odd-odd nuclei, the $M^{2\nu}$ (11) and $M_{GTcl}^{2\nu}$ (12) matrix elements both depend on the signs of the two amplitudes involved in the product and thus have both positive and negative contributions. In fact, the calculations suggest that, as a function of the 1^+ excitation energy, the contributions are positive at first, but above 5–10 MeV negative contributions turn the resulting values of both $M^{2\nu}$ and $M_{GTcl}^{2\nu}$ sharply down as illustrated in Fig. 5. That behavior seems to be universal. Not only qualitatively similar curve are obtained in QRPA for essentially all $\beta\beta$-decay candidate nuclei, but very similar plot was obtained for ^{48}Ca within the nuclear shell model [12].

In this context it is worthwhile to discuss the so-called single-state dominance (SSD) (or low-lying states dominance) often invoked in the analysis of the $2\nu\beta\beta$ decay [13,14]. The staircase plot for $M^{2\nu}$ evaluated within QRPA as seen in the top panel of Fig. 5 have the drop at higher energies that is not as steep as in the case of $M_{GTcl}^{2\nu}$; its magnitude is reduced by the energy denominators.

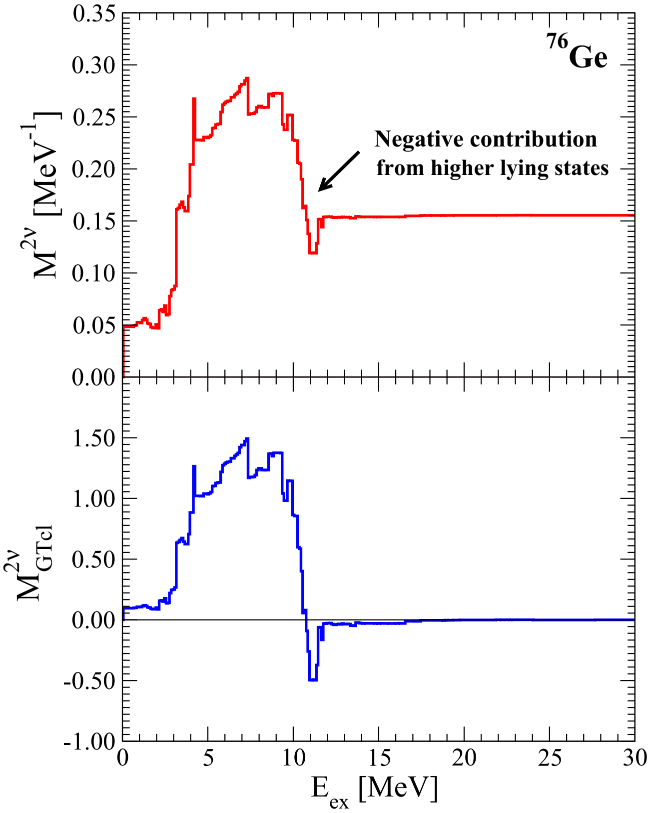

FIG. 5. Cumulative contributions to the $M^{2\nu}$ (11) and $M^{2\nu}_{GTcl}$ (12) as a function of the intermediate state excitation energy. This is for the case of ^{76}Ge.

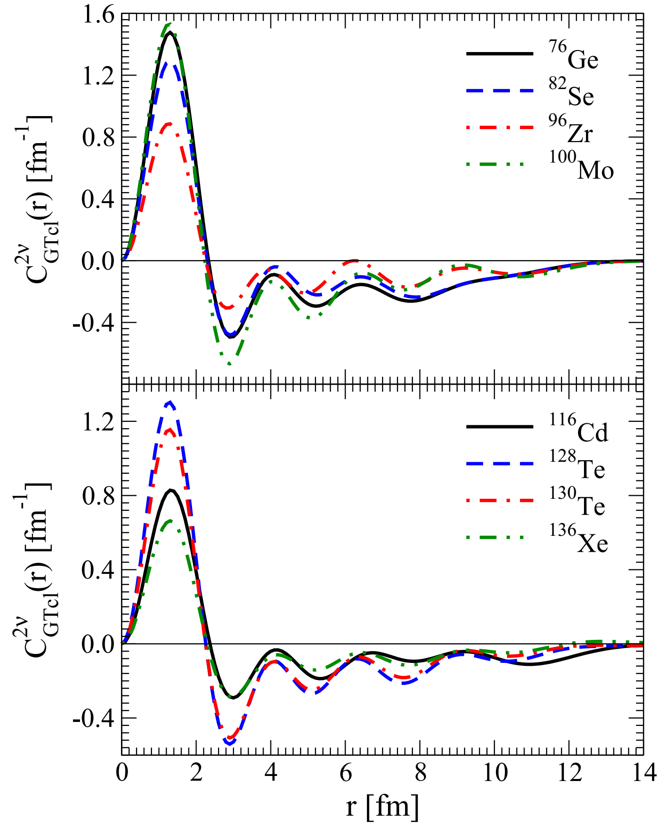

FIG. 6. Functions $C^{2\nu}_{cl}(r)$ for several $\beta\beta$ candidate nuclei evaluated within the QRPA.

The contributions to $M^{2\nu}$ are positive at first, followed at energies $\geqslant$5 MeV by several negative ones. Due to this, the true value of $M^{2\nu}$ (0.14 MeV^{-1} in the case of ^{76}Ge, assuming $g_A = 1.269$) is reached twice as a function of the excitation energy, once at relatively low E_{exc} and then again at its asymptotic value. This is a typical situation encountered in most $2\nu\beta\beta$-decay candidate nuclei. In the charge exchange experiments, e.g., in Ref. [15], the GT strength exciting several low-lying 1^+ states is determined in both the β^- and β^+ directions. Assuming that all contributions to the $M^{2\nu}$ from these states are positive, one usually soon reaches a value that is close to the experimental one. That is considered as indication of the validity of the low-lying states dominance hypothesis. The single (or low-lying) state dominance is also invoked in Refs. [16,17] where also a good agreement with the experimental $M^{2\nu}$ matrix element was reached. However, according to our evaluation, some more positive contributions to the $M^{2\nu}$ in such a case are missed, as well as negative contributions from the higher-lying 1^+ states. Thus, the low-lying states, while giving by themselves the correct (or almost correct) value of $M^{2\nu}$, miss other contributions which, in particular, are decisively important for the closure matrix element $M^{2\nu}_{GTcl}$.

It would be clearly desirable to confirm, or reject, the behavior illustrated in Fig. 5. In particular, to check that the β^+ amplitudes above ~5 MeV are nonvanishing and that their contribution to $M^{2\nu}$ is indeed negative.

The single-state dominance (SSD) in the $2\nu\beta\beta$ decay can be tested by observing the two- and single-electron spectra [18], in particular at low electron energies. This was done, for example, in the case of ^{82}Se in Ref. [20], indicating its validity. Does it really mean that only low-lying intermediate states contribute to the $M^{2\nu}$ and $M^{2\nu}_{GTcl}$? As was shown in Ref. [19], the deviation of the electron spectrum from the standard form can be described by the Taylor expansion of the energy denominators when the phase-space factors are evaluated. The leading correction, called $\xi^{2\nu}_{31}$ there, contains the third power of the energy denominator in the expression analogous to (11). Thus, the quantity $\xi^{2\nu}_{31}$ is dominated by the low-lying states and insensitive to the higher-lying ones. [In the case of higher states dominance (HSD) this quantity is practically zero. But, its absolute value depends on the position of the lowest 1^+ state of the intermediate nucleus and on the Q value of the process.] The indication of SSD validity, such as those in Ref. [20], does not mean that there are no higher-lying contributions, and in particular a significant cancellations in the $M^{2\nu}_{GTcl}$.

The radial dependence $C^{2\nu}_{cl}(r)$ corresponding to the $2\nu\beta\beta$ closure matrix element (12) can be obtained, again, by inserting the Dirac δ function in between the brackets. Note that while the closure matrix element (12) itself depends only on the 1^+ intermediate states, presence of the δ function means that all multipoles participate. In Fig. 6 we show the resulting radial function for a number of nuclei. The peak at

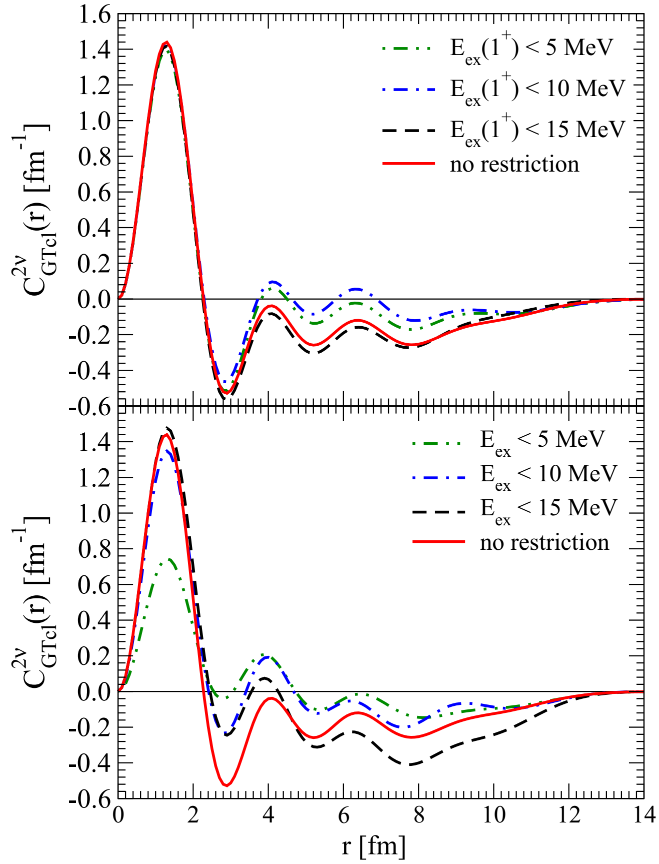

FIG. 7. Dependence of the $C^{2\nu}_{GTcl}(r)$ on the cutoff in the 1^+ excitation energy (top panel) and all J^π excitation energies (bottom panel) evaluated for ^{76}Ge decay.

$r \leqslant 2.5$ fm is almost fully compensated by the negative tail at larger r values. The actual value of $M^{2\nu}_{GTcl}$, while always small, depends sensitively on the input parameters (isovector and isoscalar pairing coupling constants).

It is important to add properly the contribution of all J^π states when evaluating $M^{2\nu}_{GTcl}$. In Fig. 7 we show how the corresponding $C^{2\nu}_{cl}(r)$ depends on the possible energy cutoff of 1^+ states in the top panel and on the cutoff of all J^π states in the bottom panel. The negative tail becomes deeper, and thus the magnitude of $M^{2\nu}_{GTcl}$ becomes smaller as more excited states are included. Thus, when the $M^{2\nu}_{GTcl}$ is evaluated in the shell model using incomplete oscillator shells, with missing spin-orbit partners, as done, e.g., in Ref. [21] for the $\beta\beta$ candidate nuclei (except ^{48}Ca), the results might be uncertain.

From the way the functions $C^{0\nu}_{GT}(r)$ and $C^{2\nu}_{cl}(r)$ were constructed, it immediate follows that they are related by

$$C^{0\nu}_{GT}(r) = H(r, \bar{E}) \cdot C^{2\nu}_{cl}(r), \tag{13}$$

as already pointed out in Ref. [3]. Therefore, if $C^{2\nu}_{cl}(r)$ were known, the $C^{0\nu}_{GT}(r)$ can be easily constructed and hence also the 0ν matrix element $M^{0\nu}_{GT}$. The analogous procedure can be followed, of course, also for $M^{0\nu}_{F}$ and $M^{0\nu}_{T}$. But Eq. (13) is much more general. Knowing $C^{2\nu}_{cl}(r)$ makes it possible to evaluate the corresponding matrix element for any neutrino potential $H_{GT}(r, \bar{E})$ like all of those listed in Ref. [8]. That represents, no doubt, a significant practical simplification.

For example, one of the short-range nuclear matrix elements [see Ref. [8], Eq. (20d)] involving the heavy neutrino exchange is characterized by the neutrino potential

$$H_{GTN}(r) = \frac{2R}{\pi m_e m_p} f^2_{src}(r) \int g_A(q^2) j_0(qr) q^2 dq. \tag{14}$$

The corresponding matrix element is therefore simply

$$M_{GTN} = \int H_{GTN}(r) \cdot C^{2\nu}_{cl}(r). \tag{15}$$

The same procedure can be used for any GT-type nuclear matrix elements.

IV. USING THE LS COUPLING SCHEME

From the discussion above it is clear that the determination of the correct value of the 2ν closure matrix element $M^{2\nu}_{GTcl}$ and its radial dependence function $C^{2\nu}_{cl}(r)$ is of primary importance. Insight into this issue can be gained by considering the LS coupling scheme.

Let us divide the $M^{2\nu}_{GTcl}$ and $M^{2\nu}_{Fcl}$ into two parts, corresponding to the $S = 0$ and $S = 1$, where S is the spin of the two decaying neutrons (or spin of the created protons) in their center-of mass system. The corresponding expression is rather complex so we leave it to the Appendix. Having the decomposition of the $M^{2\nu}_{GTcl}$ and its corresponding radial dependence $C^{2\nu}_{cl}(r)$ into their spin components, we can establish a relation between the GT and F parts.

$$\begin{aligned} M^{2\nu}_{Fcl} &= (\delta_{S1} + \delta_{S0}) \times \langle s1s2 : S \parallel O_{F,GT} \parallel s1s2 : S \rangle \\ M^{2\nu}_{GTcl} &= (\delta_{S1} - 3\delta_{S0}) \times \langle s1s2 : S \parallel O_{F,GT} \parallel s1s2 : S \rangle. \end{aligned} \tag{16}$$

Therefore, for the closure matrix elements

$$M^{2\nu}_{GT,S=0} = -3 \times M^{2\nu}_{F,S=0} \quad M^{2\nu}_{GT,S=1} = M^{2\nu}_{F,S=1}. \tag{17}$$

These are exact relations. The radial functions $C^{2\nu}_{F,GT,cl}(r)(S)$ obey them as well.

Example of this separation are shown in Fig. 8. Clearly, the $S = 0$ represents the main part, its amplitude is everywhere dominating over the $S = 1$ component. Note that the standard like nucleon pairing supports the dominance of the $S = 0$ component.

Isospin is a good quantum number in nuclei, $T = (N - Z)/2$ in the ground states; the admixtures of higher values of T is negligible for our purposes. From this it immediately follows that $M^{2\nu}_{Fcl} = 0$. That relation is obeyed automatically in the nuclear shell model where isospin is a good quantum number by construction. In QRPA, however, the isospin is, generally, not conserved. It was shown in Ref. [22] that partial restoration of the isospin symmetry, and validity of the $M^{2\nu}_{Fcl} = 0$, can be achieved within the QRPA by choosing the isospin symmetry for the $T = 1$ nucleon-nucleon interaction, i.e., by choosing the same strength for the neutron-neutron and proton-proton pairing force treated within the BCS method, and the isovector neutron-proton interaction treated by the

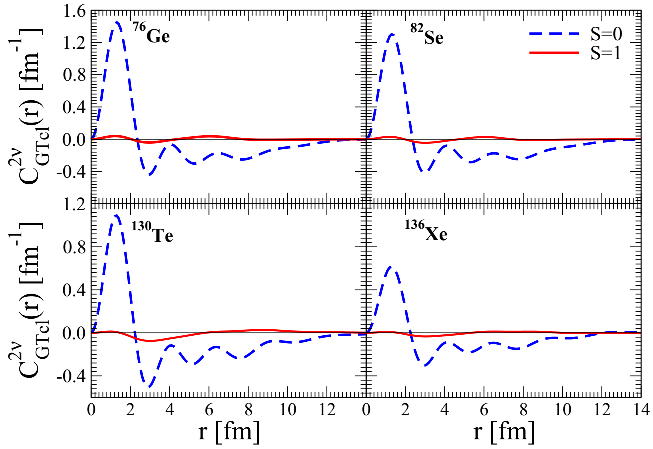

FIG. 8. The functions $C^{2\nu}_{GTcl}(r)$ separated into the spin $S = 0$ and $S = 1$ components, shown for several $\beta\beta$ decay candidate nuclei. The functions were evaluated requiring that $M^{2\nu}_{Fcl} = 0$ using the isospin conservation. The values of $M^{2\nu}_{GTcl} \neq 0$ were obtained by choosing the renormalization $g^{T=0}_{pp}$ from the usual condition that the half-life of the $2\nu\beta\beta$ decay is correctly reproduced by the QRPA.

QRPA equations of motion. (In practice, the five effective coupling constants, corresponding to $T = 1$, are close to each other, but not exactly equal since the renormalization of the pairing strength couplings $d^{i,f}_{nn}$ and $d^{i,f}_{pp}$ are adjusted to reproduce the corresponding neutron and proton gaps and the neutron-proton isovector coupling renormalization $g^{T=1}_{pp}$ is chosen to reproduce the $M^{2\nu}_{Fcl} = 0$ relation.) The values of these parameters are shown in Table I

$M^{2\nu}_{Fcl} = 0$ follows from the isospin conservation and implies that $M_{Fcl}(S = 0) = -M_{Fcl}(S = 1)$, [see Eq. (17)] but both could be, in principle, large in absolute value. However, the plots in Fig. 8 suggest that the $C^{2\nu}_{GTcl}(S = 1) = C^{2\nu}_{Fcl}(S = 1)$ [see again Eq. (17)] are negligibly small for all radii r; hence the integrals $M_{Fcl}(S = 0) = -M_{GTcl}(S = 0)/3$ should be both negligibly small as well. This is in agreement with the discussion in the preceding section, where we saw that the $M^{2\nu}_{GTcl}$ values are numerically close to zero, actually oscillating between the positive and negative values for different nuclei, and depending sensitively on the properties of the poorly known higher-lying 1^+ states. We conclude, therefore, that demanding that the $M^{2\nu}_{cl}$ vanishes is a reasonable assumption that reflects better physics of the problem. Once the $M^{2\nu}_{GTcl}$ and $M^{2\nu}_{Fcl}$ have been fixed, the corresponding radial function $C^{2\nu}_{cl}(r)$ can be obtained, and from them, using Eq. (13), the values of $M^{0\nu}_{GT}$ and $M^{0\nu}_{F}$ follow. The results are described and discussed in the following section.

Vanishing of the $M^{2\nu}_{GTcl}$, based on the discussion in the preceding paragraph is, at the same time, one of the requirements of the spin-isospin symmetry group SU(4). In practice we can fulfill the relation $M^{2\nu}_{GTcl} = 0$ by adjustment of the renormalization of the isoscalar neutron-proton coupling strength $g^{T=0}_{pp}$. As we effectively restored the isospin symmetry by the proper choice of the $g^{T=1}_{pp}$, choosing the $g^{T=0}_{pp}$. so that $M^{2\nu}_{cl} = 0$, corresponds to the partial restoration of the spin-isospin symmetry SU(4).

Note that, obviously, choosing the renormalization parameter $g^{T=0}_{pp}$ so that $M^{2\nu}_{GTcl} = 0$ is quite different from the approach of Ref. [21] where the proportionality between the $M^{0\nu}_{GT}$ and $M^{2\nu}_{cl}$ evaluated in the shell model is proposed. According to our QRPA results, the shell model, with its restricted single-particle basis, misses important negative contributions to the closure matrix elements $M^{2\nu}_{cl}$.

SU(4) symmetry in nuclei is broken mainly by the mean field, in particular by the spin-orbit splitting. Yet, as far as the GT response is concerned, many requirements of that symmetry are actually present. The GT strength is concentrated in the giant resonance, the β decays connecting low-lying states in heavier nuclei, forbidden under SU(4), have $\log(f\tau)$ values $\sim$5, while the superallowed β decays, which do not violate SU(4), have $\log(f\tau)$ values $\sim$3. And the ground state to ground state $2\nu\beta\beta$ decay exhausts only about 10^{-4} fraction of the sum rule. Thus, it is perhaps natural to demand, following the QRPA calculations described here, that one of the SU(4) symmetry features, namely that $M^{2\nu}_{GTcl} = 0$ is obeyed.

TABLE I. Renormalization parameters of the pairing interaction $d^{i,f}_{p,n}$ (i: initial nucleus; f: final nucleus; p: protons; n: neutrons) adjusted to reproduce experimental pairing gaps. Renormalization parameters of the isovector $g^{T=1}_{pp}$ and isoscalar $g^{T=0}_{pp}$ particle-particle interactions of the residual Hamiltonian adjusted to reproduce, respectively, $M^{2\nu}_{\rm Fcl} = 0$ and $M^{2\nu}_{\rm GTcl} = 0$, an effective restoration of the isospin SU(2) and spin-isospin SU(4). The corresponding values of the $2\nu\beta\beta$-decay Fermi $M^{2\nu}_{\rm F}$ and Gamow-Teller $M^{2\nu}_{\rm GT} \times q^2$ matrix elements, where $q = 0.712$ is the effective quenching factor, $g^{\rm eff}_{\rm A} = q \times g^{\rm free}_{\rm A} = 0.904$. In the last column are the experimentally determined matrix elements $M^{2\nu}_{\rm exp}$ for unquenched $g_{\rm A}$.

Nucleus	d^i_{pp}	d^f_{pp}	d^i_{nn}	d^f_{nn}	$g^{T=1}_{pp}$	$g^{T=0}_{pp}$	$M^{2\nu}_F$ (MeV^{-1})	$M^{2\nu}_{\rm GT} \times q^2$ (MeV^{-1})	$M^{2\nu}_{\rm exp}$ (MeV^{-1})
^{48}Ca	–	1.069	–	0.982	1.028	0.745	−0.003	0.019	0.046
^{76}Ge	0.922	0.960	1.053	1.085	1.021	0.733	0.003	0.077	0.136
^{82}Se	0.861	0.921	1.063	1.108	1.016	0.737	0.001	0.071	0.100
^{96}Zr	0.910	0.984	0.752	0.938	0.961	0.739	0.001	0.162	0.097
^{100}Mo	1.000	1.021	0.926	0.953	0.985	0.799	−0.001	0.306	0.251
^{116}Cd	0.998	–	0.934	0.890	0.892	0.877	−0.000	0.059	0.136
^{128}Te	0.816	0.857	0.889	0.918	0.965	0.741	0.017	0.076	0.052
^{130}Te	0.847	0.922	0.971	1.011	0.963	0.737	0.016	0.065	0.037
^{136}Xe	0.782	0.885	–	0.926	0.910	0.685	0.014	0.036	0.022

The relation between SU(4) symmetry and $2\nu\beta\beta$ decay has been invoked repeatedly in many publications, starting with Ref. [23]. Recently, in Ref. [24] partial restoration of the symmetry, somewhat different than the one employed here, was used in order to evaluate the matrix elements of both the $2\nu\beta\beta$ and $0\nu\beta\beta$ decay. The $M^{2\nu}$ evaluated in Ref. [24] are similar, but further from the experimental values, than those in our Table I.

Since we know the experimental values of the $2\nu\beta\beta$ matrix elements $M^{2\nu}$, it is legitimate to ask whether the fact that they do not vanish can be compatible with our assumption that the closure matrix elements $M^{2\nu}_{cl}$ vanish. Clearly, if $\bar{E}_{av}$ is the properly averaged energy denominator, then

$$\bar{E}_{av} \times M^{2\nu} = M^{2\nu}_{GTcl} \tag{18}$$

must be obeyed. If the right-hand side of this equation is vanishing, then one of the factors on the left-hand side must vanish as well. In our case it must be the average energy $\bar{E}_{av}$ reflecting the fact that in both $M^{2\nu}$ and $M^{2\nu}_{GTcl}$ are both positive and negative contributions to the corresponding sums [by treating the negative sign in the numerator of (11) as negative denominator].

In our approach the parameter $g^{T=0}_{pp}$ is fixed by the requirement that $M^{2\nu}_{GTcl} = 0$, it is thus straightforward to evaluate, within QRPA, the $M^{2\nu}$ and compare them with their experimental values derived from the observed $2\nu\beta\beta$ half-lives. In agreement with the idea of g_A quenching, the calculated matrix elements are typically larger than the experimental values. That discrepancy can be, at least in part, remedied by choosing the effective g_A value, $g^{\rm eff}_A = q \times g^{\rm free}_A$. (Even somewhat better agreement is achieved by assuming that $g^{\rm eff}_A$ scales like $1/A^{1/2}$. We do not see any obvious justification for such a dependence, and use $g^{\rm eff}_A$ independent of A.) Taking the average ratio of the calculated and experimental matrix elements, we arrive at $q = 0.712$. The resulting quenched calculated matrix elements are compared with the experimental ones in Table I. The agreement is only within a factor of ∼2, reflecting the known strong sensitivity of $M^{2\nu}$ on the $g^{T=0}_{pp}$ values.

V. $0\nu\beta\beta$ MATRIX ELEMENTS AND PARTIAL SU(4) SYMMETRY RESTORATION.

The matrix elements $M^{2\nu}$ of the $2\nu\beta\beta$ decay involve only 1^+ virtual intermediate states. Within the QRPA they

TABLE II. The NMEs associated with light neutrino mass mechanism of the $0\nu\beta\beta$ decay calculated within the proton-neutron QRPA using two ways of fixing the strengths of residual interactions in the nuclear Hamiltonian: i) $g^{T=1}_{pp}$ and $g^{T=0}_{pp}$ are adjusted to reproduce $M^{2\nu}_F = 0$ and the experimental $2\nu\beta\beta$ half-life, respectively ($T^{2\nu}_{1/2}$); ii) $g^{T=1}_{pp}$ and $g^{T=0}_{pp}$ are adjusted to reproduce $M^{2\nu}_{Fcl} = 0$ and $M^{2\nu}_{GTcl} = 0$ - an effective restoration of the isospin SU(2) and spin-isospin SU(4) symmetry. In (i) and (ii) the sum over all virtual excitations is explicitly performed. The partial Fermi, Gamow-Teller, tensor, and full $0\nu\beta\beta$-decay NMEs are presented for $S = 0$ and $S = 1$ channels and for the sum of them. Unquenched value of axial-vector coupling constant ($g_A = 1.269$), Argonne two-nucleon short-range correlations and $\bar{E} = 8$ MeV are considered.

Nucl.	par.	$S=0$				$S=1$				full NME			
		M_F	M_{GT}	M_T	$M^{0\nu}$	M_F	M_{GT}	M_T	$M^{0\nu}$	M_F	M_{GT}	M_T	$M^{0\nu}$
^{48}Ca	$T^{2\nu}_{1/2}$	−0.253	0.659	0.00	0.816	−0.027	−0.021	−0.156	−0.161	−0.280	0.638	−0.156	0.656
	SU(4)	−0.285	0.748	0.00	0.925	0.006	0.009	−0.158	−0.153	−0.280	0.757	−0.158	0.773
^{76}Ge	$T^{2\nu}_{1/2}$	−1.719	4.482	0.00	5.550	0.111	0.102	−0.588	−0.554	−1.608	4.584	−0.588	4.995
	SU(4)	−1.705	4.443	0.00	5.502	0.097	0.089	−0.588	−0.559	−1.570	4.455	−0.583	4.846
^{82}Se	$T^{2\nu}_{1/2}$	−1.537	3.995	0.00	4.949	0.037	0.035	−0.544	−0.532	−1.500	4.029	−0.544	4.417
	SU(4)	−1.587	4.133	0.00	5.119	0.089	0.082	−0.540	−0.513	−1.499	4.216	−0.540	4.606
^{94}Zr	SU(4)	−1.171	3.066	0.00	3.793	−0.066	−0.050	−0.392	−0.401	−1.237	3.016	−0.392	3.392
^{96}Zr	$T^{2\nu}_{1/2}$	−0.916	2.359	0.00	2.928	−0.272	−0.242	−0.420	−0.494	−1.188	2.117	−0.420	2.435
	SU(4)	−1.174	3.069	0.00	3.798	−0.008	−0.001	−0.405	−0.401	−1.182	3.068	−0.405	3.396
^{100}Mo	$T^{2\nu}_{1/2}$	−1,799	4.658	0.00	5.775	−0.410	−0.362	−0.707	−0.814	−2.209	4.296	−0.707	4.961
	SU(4)	−2.038	5.327	0.00	6.592	−0.168	−0.136	−0.692	−0.724	−2.206	5.191	−0.692	5.868
^{110}Pd	SU(4)	−1.961	5.115	0.00	6.332	−0.174	−0.145	−0.607	−0.643	−2.135	4.970	−0.607	5.689
^{116}Cd	$T^{2\nu}_{1/2}$	−1.280	3.328	0.00	4.123	0.274	−0.235	−0.290	−0.355	−1.554	3.093	−0.290	3.768
	SU(4)	−1.272	3.305	0.00	4.095	−0.283	−0.243	−0.291	−0.358	−1.555	3.062	−0.291	3.737
^{124}Sn	SU(4)	−1.096	2.862	0.00	3.543	0.032	0.031	−0.347	−0.336	−1.064	2.894	−0.347	3.207
^{128}Te	$T^{2\nu}_{1/2}$	−1.638	4.248	0.00	5.265	−0.146	−0.125	−0.604	−0.638	−1.784	4.122	−0.604	4.626
	SU(4)	−1.839	4.784	0.00	5.923	−0.044	−0.033	−0.588	−0.594	−1.878	4.751	−0.588	5.329
^{130}Te	$T^{2\nu}_{1/2}$	−1.411	3.655	0.00	4.531	−0.162	−0.140	−0.554	−0.593	−1.573	3.515	−0.554	3.939
	SU(4)	−1.616	4.215	0.00	5.219	−0.053	−0.042	−0.536	−0.545	−1.669	4.173	−0.536	4.673
^{134}Xe	SU(4)	−1.598	4.163	0.00	5.156	−0.044	−0.034	−0.498	−0.504	−1.642	4.129	−0.498	4.652
^{136}Xe	$T^{2\nu}_{1/2}$	−0.780	2.009	0.00	2.493	−0.035	−0.028	−0.285	−0.291	−0.815	1.980	−0.285	2.202
	SU(4)	−0.927	2.410	0.00	2.985	0.022	0.022	−0.274	−0.266	−0.905	2.432	−0.274	2.720

sensitively depend on the magnitude of the isoscalar neutron-proton interaction, conventionally denoted as $g_{pp}^{T=0}$. On the other hand, matrix elements $M^{0\nu}$ of the $0\nu\beta\beta$ decay contain many multipoles of the intermediate states. Among them the 1^+, or GT, is particularly sensitive to the $g_{pp}^{T=0}$; other multipoles are less dependent to its magnitude. That led to the practice [25,26], commonly used in QRPA now, to adjust the $g_{pp}^{T=0}$ so that the experimental half-life $T_{1/2}^{2\nu}$ is correctly reproduced. That way the most sensitive multipole contributing to $M^{0\nu}$ has been tied to the experimentally determined quantity. (Also, it turns out that with this adjustment, the magnitude of $M^{0\nu}$ becomes essentially independent on the size of the single-particle basis included.)

As explained above, in this work we propose to use the condition $M_{GTcl}^{2\nu} = 0$, i.e., partial restoration of the SU(4) symmetry, to adjust the value of the renormalization parameter $g_{pp}^{T=0}$. We are particularly interested to check how sensitive the $M^{0\nu}$ values are to this change. The matrix elements $M^{0\nu}$ evaluated by these two alternative methods are compared in Table II together with the corresponding partial values M_F, M_{GT}, and M_T separated into the spin $S = 0$ and $S = 1$ components. Few candidate nuclei (^{94}Zr, ^{110}Pd, ^{124}Sn, and ^{134}Xe), where the 2ν decay has not been observed as yet, are also included in Table II. All entries there were obtained when the sum over the virtual intermediate states was explicitly evaluated. When the closure approximation is used together with the SU(4) adjustment, the results are similar, with the final $M^{0\nu}$ values about 10% smaller, similar to the previous experience described above. Typically, the contributions of the spin $S = 1$ component to the M_F and M_{GT} are indeed negligible. However, the tensor mart, M_T gets its value only from $S = 1$; it constitutes about 10% of the total $M^{0\nu}$ value.

Adjusting $g_{pp}^{T=0}$ to the condition of partial restoration of the SU(4) symmetry means that the 2ν matrix elements (and,

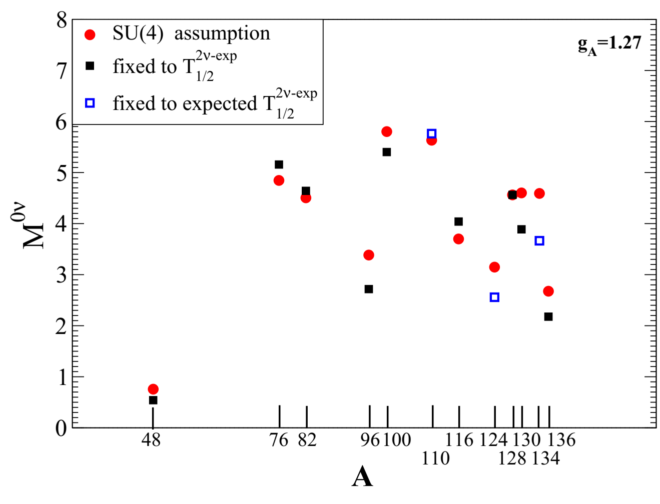

FIG. 9. $M^{0\nu}$ evaluated with $g_{pp}^{T=0}$ adjusted such that $M_{cl}^{2\nu} = 0$, i.e., partial restoration of the SU(4) symmetry (red circles), or so that the $2\nu\beta\beta$ decay experimental half-lives are correctly reproduced (black squares). For several candidate nuclei (^{94}Zr, ^{110}Pd, ^{124}Sn, and ^{134}Xe) instead the expected $2\nu\beta\beta$ half-lives were used for the adjustment of the $g_{pp}^{T=0}$ (empty blue squares).

naturally, the half-lives $T_{1/2}^{2\nu}$) are not any longer tied to their experimental values. The theoretical values of $M^{2\nu}$ are only in qualitative agreement with experiment, as we saw in the previous section. However, remarkably, the new adjustment of $g_{pp}^{T=0}$ causes only relatively small changes in the $M^{0\nu}$ as one could see in Table II. In Fig. 9 the two ways of the $g_{pp}^{T=0}$ adjustment are compared. The largest effect, for ^{130}Te and ^{136}Xe is an increase of $M^{0\nu}$ by $\sim$20%. Note that both variants shown in Fig. 9 were evaluated with $g_A = 1.27$, i.e., without quenching.

VI. SUMMARY

In this work we discuss the importance of dependence of the 0ν and 2ν nuclear matrix elements on the distance r_{ij} between the two neutrons that are transformed in two protons in the double-beta decay. We show that, if this function, $C(r)$, is known for any particular mechanism of the decay, evaluation of the matrix element for any other mechanism is reduced to an integral using Eq. (13).

Further, we show that there is a close relation between the GT part of the $M^{0\nu}$ and the matrix element of the experimentally observed $2\nu\beta\beta$ decay, evaluated, however, in the closure approximation, $M_{cl}^{2\nu}$. Our work does not support the conjecture in Ref. [21] of proportionality between the $M_{GT}^{0\nu}$ and $M_{cl}^{2\nu}$. Instead, we argue that the positive contributions to $M_{cl}^{2\nu}$ from the lower-lying 1^+ intermediate states is essentially fully cancelled by the negative contribution of the higher-lying 1^+ states. We also show that the contribution of the triplet spin $S = 1$ two neutron states is much smaller than the contribution of the singlet $S = 0$ states. (Note that when $M_F^{2\nu} = 0$ the $S = 0$ part is always three times larger that the $S = 1$ part.) From these considerations follows a simple proportionality between the Fermi and GT parts of the $M_{cl}^{2\nu}$.

Based on these consideration we arrive at a new way of adjusting the important QRPA parameter, the renormalization of the isoscalar particle-particle interaction, $g_{pp}^{T=0}$. We propose that its value should be determined from the requirement that $M_{GTcl}^{2\nu} = 0$. Together with $M_{Fcl}^{2\nu} = 0$, following from isospin conservation, these two conditions are equivalent to partial restoration of the spin-isospin SU(4) symmetry.

We then evaluate the true 2ν matrix elements and compare them to the corresponding experimental values. The calculated $M^{2\nu}$ values are mostly larger than the experimental ones, suggesting on average a relatively modest quenching $g_A^{\text{eff}} = 0.712 \times g_A^{\text{free}}$. The agreement between the calculated and experimental values of $M^{2\nu}$ is, however, only qualitative. That is, perhaps, not surprising given the strong dependence of the calculated $M^{2\nu}$ values on the $g_{pp}^{T=0}$.

The 0ν matrix elements, corresponding to the standard light Majorana neutrino exchange are evaluated next using the new adjustment of the $g_{pp}^{T=0}$. When they are compared to the the values obtained when $g_{pp}^{T=0}$ is chosen so that the 2ν half-life is correctly reproduced, which was a QRPA standard procedure until now, only relatively modest changes of the $M^{0\nu}$ are obtained. This shows that, within QRPA, the $M^{0\nu}$ values are quite stable. It also represents an alternative way to determine the parameter $g_{pp}^{T=0}$, and through the corresponding function $C_{GTcl}^{2\nu}(r)$ all possible 0ν nuclear matrix elements.

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APPENDIX: *LS* COUPLING SCHEME

In the QRPA the closure matrix element $M_{\rm K}^{\alpha}$ [$\alpha = 0\nu, 2\nu$ and K $= F$ (Fermi), GT (Gamow-Teller), and T (tensor)] can be written as a sum over two neutron (initial nucleus) and two proton (final nucleus) states participating in the two virtual β decays inside nucleus, angular momentum $\mathcal{J}$ to which they are coupled, and angular momentum and parity J^π of the intermediate nucleus as follows:

$$M_{\rm K}^{\alpha} = \sum_{pnp'n'}\sum_{J^\pi \mathcal{J}} (-1)^{j_n+j_{p'}+J+\mathcal{J}}\sqrt{2\mathcal{J}+1}\begin{Bmatrix} j_p & j_n & J \\ j_{n'} & j_{p'} & \mathcal{J}\end{Bmatrix} \times D(p'n', pn; J^\pi) T_K^{\alpha}(pp', nn'; \mathcal{J}), \tag{A1}$$

where

$$D(p'n', pn, J^\pi) = \sum_{J^\pi, k_i, k_f} \langle 0_f^+ \parallel \widetilde{[c_{p'}^+ \tilde{c}_{n'}]}_J \parallel J^\pi k_f\rangle \times \langle J^\pi k_f | J^\pi k_i\rangle \langle J^\pi k_i \parallel [c_p^+ \tilde{c}_n]_J \| 0_i^+\rangle \tag{A2}$$

includes products of reduced matrix elements of one-body densities $c_p^+\tilde{c}_n$ ($\tilde{c}_n$ denotes the time-reversed state) connecting the initial nuclear ground state with the final nuclear ground state through a complete set of states of the intermediate nucleus labeled by their angular momentum and parity, J^π, and indices k_i and k_f. They depend on the BCS coefficients u_i, v_j and on the QRPA vectors X, Y [22]. The coupling (lsj) for each single-proton (-neutron) state is considered, i.e., the individual orbital momentum l_p (l_n) and spin s_p (s_n) is coupled to the total angular momentum j_p (j_n). The nonantisymmetrized two-nucleon matrix element takes the form

$$T_K^{\alpha}(pp', nn'; \mathcal{J}) = \langle p(1)p'(2); \mathcal{J} \| \mathcal{O}_K^{\alpha} \| n(1)n'(2); \mathcal{J}\rangle, \tag{A3}$$

where

$$\mathcal{O}_F^{2\nu} = 1, \quad \mathcal{O}_{GT}^{2\nu} = \sigma_{12}, \quad \mathcal{O}_T^{2\nu} = S_{12}$$
$$\mathcal{O}_K^{0\nu}(r_{12}) = \mathcal{O}_K^{2\nu}\, H_K(r_{12}, \bar{E}) \tag{A4}$$

with $K = F, GT, T$, $S_{12} = 3(\vec{\sigma}_1 \cdot \hat{r}_{12})(\vec{\sigma}_2 \cdot \hat{r}_{12}) - \sigma_{12}$, $\sigma_{12} = \vec{\sigma}_1 \cdot \vec{\sigma}_2$. $\vec{r}_{12} = \vec{r}_1 - \vec{r}_2$, $r_{12} = |\vec{r}_{12}|$, and $\hat{r}_{12} = \vec{r}_{12}/r_{12}$, where $\vec{r}_1$ and $\vec{r}_2$ are coordinates of nucleons undergoing β decay. For the exchange of light Majorana neutrinos, the $0\nu\beta\beta$-decay mechanism we are considering here, the neutrino potentials $H_K(r_{12}, \bar{E})$ are given in Eq. (7)

It practice, the calculation of nonantisymmetrized two-nucleon matrix element in Eq. (A3) is performed in center-of-mass frame by using a harmonic oscillator single-particle basis set. The transformation from jj to LS coupling is used and the Talmi transformation via the Moshinsky transformation brackets is considered. In the case of the $0\nu\beta\beta$-decay two-nucleon matrix elements we obtain

$$\begin{pmatrix} T_F^{0\nu} \\ T_{GT}^{0\nu} \\ T_T^{0\nu}\end{pmatrix}(pp', nn'; \mathcal{J}) = \hat{\mathcal{J}}\hat{j}_n\hat{j}_{n'}\hat{j}_p\hat{j}_{p'}\sum_{SL}(2S+1)(2L+1)\begin{Bmatrix} 1/2 & l_p & j_p \\ 1/2 & l_{p'} & j_{p'} \\ S & L & \mathcal{J}\end{Bmatrix}\begin{Bmatrix} 1/2 & l_n & j_n \\ 1/2 & l_{n'} & j_{n'} \\ S & L & \mathcal{J}\end{Bmatrix}$$
$$\times \sum_{\substack{nln'l' \\ \mathcal{N}L}} \langle nl, \mathcal{N}L, L | n_p l_p, n_{p'} l_{p'}, L\rangle \langle n'l', \mathcal{N}L, L | n_n l_n, n_{n'} l_{n'}, L\rangle \sum_{J'} (2J'+1)$$
$$\times \sqrt{(2l+1)(2l'+1)}\begin{Bmatrix} l & L & \mathcal{L} \\ \mathcal{J} & J' & S\end{Bmatrix}\begin{Bmatrix} l' & L & \mathcal{L} \\ \mathcal{J} & J' & S\end{Bmatrix}\langle nl, S; J' | \begin{pmatrix} (\delta_{S0} + \delta_{S1}) H_F(r_{12}, \bar{E}) \\ (-3\delta_{S0} + \delta_{S1}) H_{GT}(r_{12}, \bar{E}) \\ S_{12} H_T(r_{12}.\bar{E})\end{pmatrix} | n'l', S; J'\rangle. \tag{A5}$$

Here, $\hat{\mathcal{J}} = \sqrt{2\mathcal{J}+1}$ and $\hat{j}_\alpha = \sqrt{2j_\alpha + 1}$ with $\alpha = p,\ p', n$, and n'. We note that in the case of the Fermi and Gamow-Teller transitions there are both $S = 0$ an $S = 1$ contributions, unlike the case of the tensor transition where only $S = 1$ is allowed. Due to the presence of neutrino potentials $H_K(r_{12}, \bar{E})$ ($K = F, GT$, and T) in two-body transition operators there is dominance of the $S = 0$ contribution to $M^{0\nu}$. There is a small difference between the Fermi and Gamow-Teller neutrino potentials due to a different form factor's cutoff and contributions from higher-order terms of the nucleon currents. If they would be equal, and the

$S = 1$ contribution could be neglected, we would end up with

$$M^{0\nu}_{GT} \simeq -3M^{0\nu}_{F}. \tag{A6}$$

The $2\nu\beta\beta$-decay Fermi and Gamow-Teller matrix elements can be decomposed into the $S = 0$ and $S = 1$ contributions as follows [see Eq. (16)]:

$$M^{2\nu}_{GT} = -3M^{2\nu}_{S=0} + M^{2\nu}_{S=1}, \quad M^{2\nu}_{F} = M^{2\nu}_{S=0} + M^{2\nu}_{S=1}. \tag{A7}$$

The corresponding decomposition of the nonantisymmetrized two-nucleon matrix element is given by

$$\begin{pmatrix} T^{2\nu}_{F} \\ T^{2\nu}_{GT} \end{pmatrix}(pp', nn'; \mathcal{J}) = \hat{\mathcal{J}}\hat{j}_n\hat{j}_{n'}\hat{j}_p\hat{j}_{p'} \sum_{SL}(2S+1)(2L+1)\begin{Bmatrix} 1/2 & l_p & j_p \\ 1/2 & l_{p'} & j_{p'} \\ S & L & \mathcal{J} \end{Bmatrix}\begin{Bmatrix} 1/2 & l_n & j_n \\ 1/2 & l_{n'} & j_{n'} \\ S & L & \mathcal{J} \end{Bmatrix}$$
$$\times \delta_{n_p n_{p'}}\delta_{l_p l_{p'}}\delta_{n_n n_{n'}}\delta_{l_n l_{n'}} \times \begin{pmatrix} \delta_{S0} + \delta_{S1} \\ -3\delta_{S0} + \delta_{S1} \end{pmatrix}. \tag{A8}$$

If $M^{2\nu}_{F} = 0$ because of isospin conservation (see Ref. [22]), then $S = 0$ and $S = 1$ contributions are equal in magnitude but opposite in sign.

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