Band mixing and structure of ^{106,108}Pd

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(Received 24 September 2018; published 3 December 2018)

A simple two-state mixing model has been applied to members of the two lowest 0^+ rotational bands in ^{106,108}Pd. Unique solutions are found for the 0^+ and 2^+ mixing in both cases, and for the $E2$ matrix elements connecting the basis states. The lower basis-state band is found to be somewhat more collective than the second one. Results indicate that ^{108}Pd is slightly more collective than ^{106}Pd. In ^{106}Pd, The 2^+ wave functions from the fit are used to compute matrix elements for $4 \leftrightarrow 2$ transitions, one of which is unknown.

DOI: 10.1103/PhysRevC.98.064303

I. INTRODUCTION AND HISTORY

Several even Cd nuclei ($Z = 48$) have long been thought of as good examples of harmonic vibrators. But, just two protons away, in even-A Pd nuclei ($Z = 46$), the situation (even among the lowest states) is much more complicated. Robinson *et al.* [1] performed an early investigation of Coulomb excitation of ^{106,108,110}Pd. They observed in each nucleus a triplet of states of probable $J^\pi = 0^+$, 2^+, and 4^+ at the approximate energy expectation for a two-phonon triplet. They found that the $B(E2)$ values for transitions from these states to the first 2^+ states were consistently smaller than the prediction of a quadrupole-phonon model. For ^{104,106,108,110}Pd, I demonstrated that the separate proton and neutron contributions to the strengths of the $0_1 \rightarrow 2_1$ transitions could be understood in a very simple model [2].

Svensson *et al.* [3] performed a Coulomb-excitation study of ^{106,108}Pd and concluded that vibrational degrees of freedom are important for the description of the low-spin level structure of the Pd nuclei, but that they cannot account for several $E2$ transition matrix elements. Gürdal *et al.* [4] measured g factors of the first 4^+ and first two 2^+ states of ^{106}Pd and concluded that the excitation energies and g factors are consistent with the simple vibrational model, but $E2$ properties are not.

Delion and Suhonen [5] applied a microscopic description of low-lying two-phonon states to ^{106}Pd and ^{108}Pd. Model predictions of energies and some ratios of $B(E2)$'s were in reasonable agreement with experimental results, but some energies and $E2$ ratios disagreed. The authors attributed these disagreements to "the fact that the considered Pd isotopes have a moderate quadrupole deformation." Zhang and Liu [6] applied the transitional dynamical symmetry $E(5)$ to ^{108}Pd and concluded it was an empirical example of possible $E(5)$ symmetry. Zhang *et al.* [7] suggested that the low-lying levels of ^{108}Pd may be dominated by this five-dimensional Euclidean dynamical symmetry.

Coello Pérez and Papenbrock [8] developed an effective field theory (EFT) for nuclear vibrations and applied it to several Ni, Ru, Pd, Cd, and Te nuclei. They introduced a method for the quantification of theoretical uncertainties. They suggested that ^{106}Pd can be viewed as an anharmonic quadrupole vibrator at the two-phonon level.

Prados-Estevez *et al.* [9] used the $(n, n\gamma)$ reaction to study excited states in ^{106}Pd and organized into bands those states that exhibited collective $E2$ properties. They collected information for transitions (including $E2$ strengths) from all known positive-parity states below about 2.4 MeV.

In ^{106}Pd, the presence of $E0$ transitions with large $\rho^2(E0)$ values suggested the presence of shape coexistence [10]. The low-lying level structure of both ^{106}Pd and ^{108}Pd are complicated, but hints of rotational bands are evident. I have built on the work of others to apply a simple band-mixing model to these two nuclei.

II. CALCULATIONS AND RESULTS

Energies of low-lying states in ^{106}Pd are plotted vs $J(J+1)$ in Fig. 1. The hint of two $K = 0$ and one $K = 2$ rotational bands is clear. Also evident is the presence of a 0^+, 2^+, 4^+ triplet near the energy expected for a two-phonon triplet. The two 0^+ bands in ^{106}Pd and ^{108}Pd are compared in Fig. 2. From energies alone, ^{108}Pd would appear to be slightly more collective than ^{106}Pd. This feature is also evident in the $B(E2)$'s (Fig. 3). In both nuclei, the experimenters [3,9] have identified the third 2^+ state as the one to be associated with the excited 0^+ band, with the second one being the $K = 2$ band head. Experimental matrix elements for $0 \leftrightarrow 2$ transitions in both nuclei are listed in Table I and plotted in Fig. 4, and those for $2 \leftrightarrow 4$ transitions in ^{106}Pd are listed in Table II.

I have applied a simple two-state mixing model to the transitions in these two apparent 0^+ bands in each nucleus. This model has been applied previously to several nuclei in which evidence exists for coexistence of different structures at low excitation energy. These include ^{72}Ge [11–13], ^{98}Sr [14], and ^{152}Sm [15]. This approach attempts to extract maximum information with a minimum number of assumptions. Most other mixing procedures force some particular structure onto the basis-state bands, but I do not.

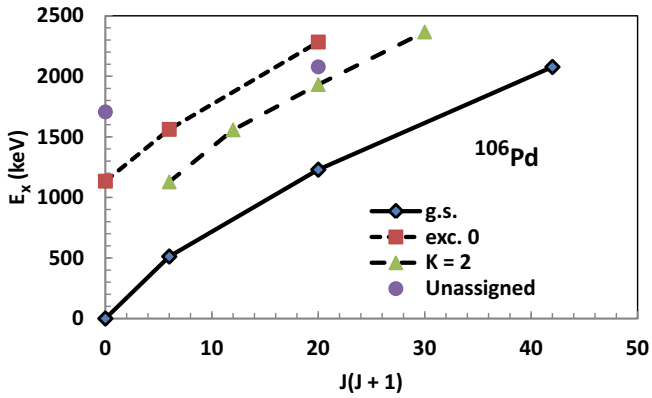

FIG. 1. Energies of low-lying states in ^{106}Pd, plotted vs $J(J+1)$. Apparent rotational bands are connected by lines.

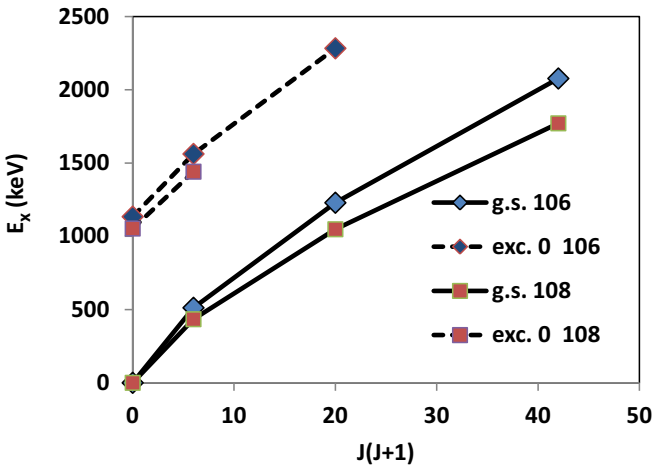

FIG. 2. Comparison of two 0^+ bands in ^{106,108}Pd.

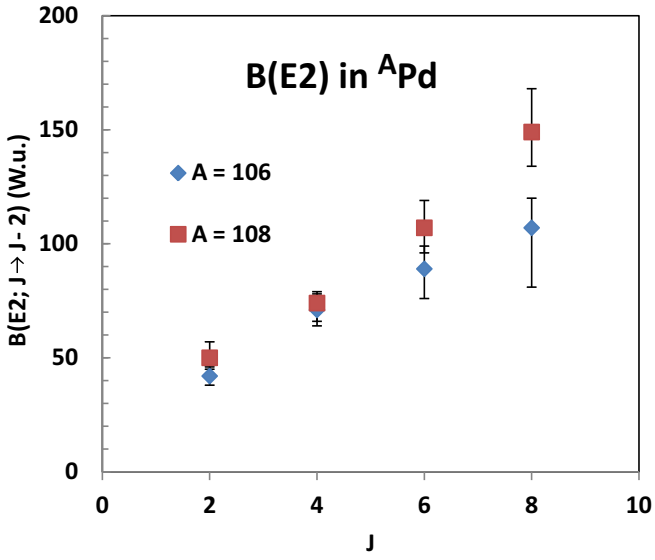

FIG. 3. $E2$ strengths for lowest $J \rightarrow J$ -2 transitions in ^{106,108}Pd.

TABLE I. Experimental $E2$ transition matrix elements connecting relevant 0^+ and 2^+ states in ^{106}Pd and ^{108}Pd [3,9].

Label	i	f	M (W.u.)$^{1/2}$	
			^{106}Pd	^{108}Pd
M0	2_1	0_1	14.88(25)	$15.81^{+1.11}_{-0.79}$
M1	0_2	2_1	5.92(68)	7.21(42)
M2	2_3	0_1	$\pm 0.857^{+0.073}_{-0.058}$	$\pm 0.689^{+0.036}_{-0.054}$
M3	2_3	0_2	$13.8^{+2.4}_{-2.0}$	$17.18^{+1.16}_{-0.87}$

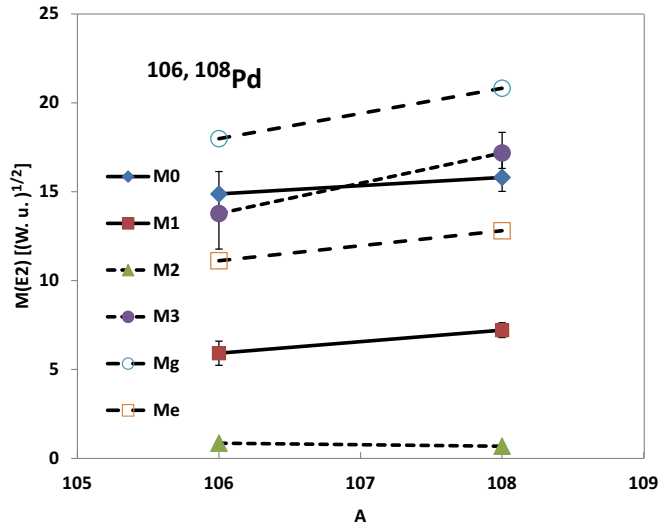

FIG. 4. Experimental $0 \leftrightarrow 2$ $E2$ transition matrix elements in ^{106,108}Pd and values of M_g and M_e that result from the fits.

TABLE II. Experimental $E2$ transition matrix elements connecting relevant 2^+ and 4^+ states in ^{106}Pd and fit results.

Label	i	f	M (W.u.)$^{1/2}$ exp	M (W.u.)$^{1/2}$ fit[a]
M′0	4_1	2_1	26.2(19)	24.65
M′1	2_3	4_1	$7.3^{+1.8}_{-1.5}$	9
M′2	4_4	2_1	unknown	6.4
M′3	4_4	2_3	$11.6^{+6.2}_{-4.3}$	18

[a]Using 2^+ wave functions from $0 \leftrightarrow 2$ fitting.

TABLE III. Results of fitting for $0 \leftrightarrow 2$ transitions.

Nucl.	a	b	A	B	M_g	M_e	M_e/M_g
^{106}Pd	0.702	0.712	0.815	0.579	18.0	11.1	0.618
^{108}Pd	0.566	0.824	0.715	0.699	20.8	12.8	0.615

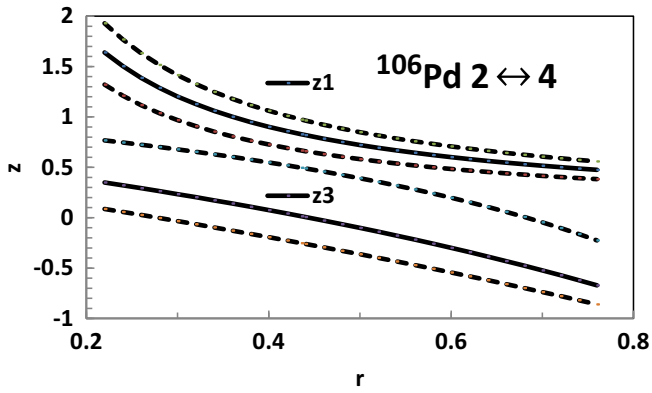

FIG. 5. Plot of $z = \mathrm{D/C}$ vs $r = \mathrm{M'_e/M'_g}$ for $2 \leftrightarrow 4$ transitions in ^{106}Pd, with uncertainty bands indicated by dashed curves.

I introduce two bands, g and e, with 0^+, 2^+, and 4^+ basis-state wave functions 0_g, 0_e; 2_g, 2_e; and 4_g, 4_e, respectively. I write

$0_1 = \mathrm{a}\,0_g + \mathrm{b}\,0_e$, $0_2 = \mathrm{b}\,0_g - \mathrm{a}\,0_e$,
$2_1 = \mathrm{A}\,2_g + \mathrm{B}\,2_e$, $2_3 = \mathrm{B}\,2_g - \mathrm{A}\,2_e$; and
$4_1 = \mathrm{C}\,4_g + \mathrm{D}\,4_e$, $4_4 = \mathrm{D}\,4_g - \mathrm{C}\,4_e$. (The authors [9] placed the fourth 4^+ state in the excited 0^+ band.)

I define $\mathrm{M_g} = \langle 0_g|\mathrm{E2}|2_g\rangle$, $\mathrm{M_e} = \langle 0_e|\mathrm{E2}|2_e\rangle$; $\mathrm{M'_g} = \langle 2_g|\mathrm{E2}|4_g\rangle$, $\mathrm{M'_e} = \langle 2_e|\mathrm{E2}|4_e\rangle$.

Furthermore, I assume the g states are not connected to the e states by the $E2$ operator. No other assumptions are necessary. Thus, $\mathrm{M0} = \mathrm{aAM_g} + \mathrm{bBM_e}$, and similarly for the other matrix elements. Whenever all four of the relevant $E2$ transition matrix elements are available, the solution of a mixing fit is unique.

Results for the $0 \leftrightarrow 2$ fits are listed in Table III. Inspection of the experimental matrix elements for ^{106}Pd in Table I indicates that the two 0^+ states are almost maximally mixed. The sum of M^2 for the g.s. to the first and third 2^+ states is 222(8) W.u., and for the second 0^+ state this sum is 225^{+65}_{-55} W.u. This expectation is borne out in the wave-function amplitudes in Table III. Similar remarks hold for the 2^+ states in ^{108}Pd.

The ratios $\mathrm{M_e/M_g}$ are remarkably constant in the two nuclei. The absolute values of $\mathrm{M_g}$ and $\mathrm{M_e}$ also demonstrate the earlier remark that ^{108}Pd is slightly more collective than ^{106}Pd.

For the $2 \leftrightarrow 4$ transitions, insufficient information is available to enable a full fit. However, as can be seen in Table II, for ^{106}Pd, three of the four relevant matrix elements are known. Given 2^+ wave functions from the fit above, these three matrix elements should be enough to determine the 4^+ wave functions and the $2 \leftrightarrow 4$ basis-state matrix elements $\mathrm{M'_g}$ and $\mathrm{M'_e}$. But, it turns out that no such fit exists for the central values of the $2 \leftrightarrow 4$ matrix elements, if the fit is required to have the 2^+ wave functions from above. So, I have sought a fit with slightly altered values of these data.

TABLE IV. Results of fitting $2 \leftrightarrow 4$ transitions in ^{106}Pd.

C	D	$\mathrm{M'_g}$	$\mathrm{M'_e}$	$\mathrm{M'_e/M'_g}$
0.852	0.524	29.8	13.0	0.436

TABLE V. Mixing matrix elements in ^{106,108}Pd.

J	V (keV)	
	^{106}Pd	^{108}Pd
0	567	491
2	495	503
4	471	

I define dimensionless ratios $m_1 = \mathrm{M'1/M'0}$, $m_3 = \mathrm{M'3/M'0}$, $z = \mathrm{D/C}$, $y = \mathrm{B/A}$, and $r = \mathrm{M'_e/M'_g}$. Then, one has two equations in two unknowns, z and r. If a solution is found, $\mathrm{M'_g}$ and $\mathrm{M'_e}$ can then be computed from y, z, r, and M'0. The dependences of z on r for $\mathrm{m_1}$ and $\mathrm{m_3}$ are plotted in Fig. 5. It can be seen that the lines do not cross, and they do not even intersect at the 1 σ level. Furthermore, they move approximately in the same direction, meaning a wide range of near solutions exists. The closest approach of the two uncertainty bands is at $r = 0.436$. The matrix elements for that near solution are listed in the last column of Table II, and the fit parameters are given in Table IV. This solution has $\mathrm{M'_e/M'_g} = 0.436$, which is considerably different from the ratio $\mathrm{M_e/M_g} = 0.618$ for the $0 \leftrightarrow 2$ case. These difficulties may indicate that the 4^+ mixing is not a two-state problem. Attempts to reduce the uncertainties in these transition matrix elements would be welcome.

The spacings of the physical states can be combined with the mixing amplitudes to obtain the matrix elements responsible for the mixing. Thus, in each nucleus, $\mathrm{V_0} = \mathrm{ab}\Delta\mathrm{E_0}$, and $\mathrm{V_2} = \mathrm{AB}\Delta\mathrm{E_2}$. Results are listed in Table V. The similarity of the four values is striking. The value of $\mathrm{V_4}$ in ^{106}Pd is also similar. Now, with these V's, the physical energy spacings can be used to obtain the energies of the unmixed basis states. These are plotted in Fig. 6. As expected, the slopes of the g bands in the two nuclei are very similar, as are the two e bands;

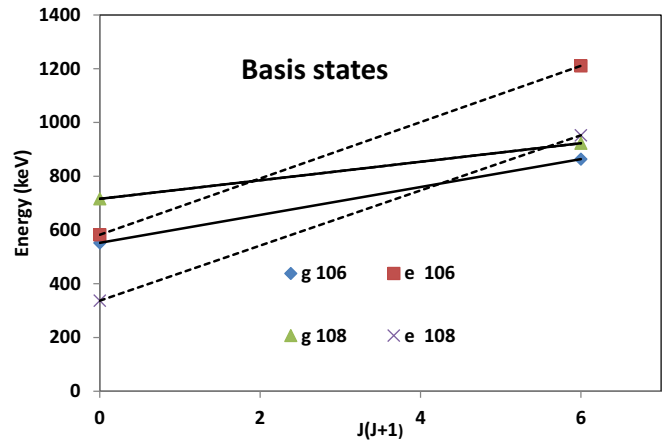

FIG. 6. Plot of basis-state energies derived from mixing amplitudes and energies of physical states.

and the g band is more collective as evidenced by the values of M_g and M_e.

III. SUMMARY

A simple two-state mixing model has been applied to members of the two lowest 0^+ rotational bands in ^{106,108}Pd. The fits provide the 0^+ and 2^+ mixing in both cases, and the $E2$ matrix elements connecting the basis states. The first two 0^+ states in ^{106}Pd are almost maximally mixed, as are the first and third 2^+ states in ^{108}Pd. Results indicate that ^{108}Pd is slightly more collective than ^{106}Pd. In both nuclei, the lower basis-state band is found to be somewhat more collective than the second one. In ^{106}Pd, The 2^+ wave functions from the fit are used to compute matrix elements for $4 \leftrightarrow 2$ transitions, one of which is unknown.

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