Nuclear structure studies of double-charge-exchange Gamow-Teller strength

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(Received 28 August 2018; published 4 December 2018)

The double Gamow-Teller strength distributions in even-A calcium isotopes were calculated using the nuclear shell model by applying the single Gamow-Teller operator two times sequentially on the ground state of the parent nucleus. The number of intermediate states actually contributing to the results was determined. The sum rules for the double Gamow-Teller operator in the full calculation were approximately fulfilled. In the case that the symmetry is restored approximately by introducing degeneracies of the f levels, and the p levels in the fp-model space, the agreement with the sum rules was very close.

DOI: 10.1103/PhysRevC.98.064301

I. INTRODUCTION

The double-charge-exchange (DCX) processes are a promising tool for the study of nuclear structure and in particular for the study of two-body correlations in nuclei. In the 1980s, DCX reactions using pion beams produced in the three meson factories at LAMPF, TRIUMF, and SIN were performed successfully. Studies at lower pion energies ($E \leqslant 50$ MeV) have indeed produced clear signals of nucleon-nucleon correlations [1–5], which were successfully explained by the theoretical studies [6,7]. The pion DCX experiments excited the double isobaric analog states (DIAS) [8,9]. At higher pion energies ($E > 300$ MeV), the studies discovered the giant dipole resonances (GDR) built on the IAS [10–13], and double giant dipole resonances (DGDR) [14–17] (see Refs. [10,14] for definitions).

At present, there is a renewed interest in DCX reactions, to a large extent due to the extensive studies of double-beta ($\beta\beta$) decays, both the decay in which two neutrinos are emitted ($2\nu\beta\beta$) and neutrinoless double-beta decay ($0\nu\beta\beta$). In DCX and $\beta\beta$ decay, two nucleons are involved. The pion, however, interacts weakly with states involving the spin, and the pion DCX reactions do not excite the states involving the spin, such as the double Gamow-Teller (DGT) state. The DGT strength is the essential part of the $\beta\beta$ decay transitions. It was suggested in the past that one could probe the DGT state and hopefully the $0\nu\beta\beta$ decay using DCX reactions with light ions [18,19].

At present, DCX reactions are performed using light ions. There is a large program called NUMEN in Catania, where reactions with ^{18}O and ^{18}Ne have been performed [20]. The hope is that such studies might shed some light on the nature of the nuclear matrix element of the $\beta\beta$ decay and serve as a "calibration" for the size of this matrix element. These DCX studies might also provide new interesting information about nuclear structure. One of the outstanding resonances relevant to the $0\nu\beta\beta$ decay is the double Gamow-Teller (DGT) resonance suggested in the past [18,21]. At RIKEN, there is a DCX program using ion beams for the purpose of observing DGT states and other nuclear structure properties [22]. At Osaka University, new DCX reactions with light ions were used to excite the double-charge-exchange state and compare to the pion DCX reaction results. One additional peak appeared in the cross section, suggesting that it is a DGT resonance [23].

The DGT strength distributions in even-A neon isotopes were discussed in Ref. [24], and recently the calculation of DGT strength for ^{48}Ca was performed in Ref. [25]. In the present paper, the DGT transition strength distributions in even-A calcium isotopes are calculated in the full fp-model space using the nuclear shell model code NUSHELLX@MSU [26,27]. The *single* Gamow-Teller operator is applied two times sequentially on the ground state of the parent nucleus to obtain the DGT strength. This method is different from the method used in Refs. [24,25].

The properties of the DGT distribution are examined, and limiting cases when the SU(4) symmetry holds or when the spin orbit-orbit coupling is put to zero are studied. DGT sum rules were derived in Refs. [24,28,29] and recently discussed in Ref. [30]. The DGT sum rules were used here as a tool to assess whether in our numerical calculations most of the DGT strength is found.

II. METHOD OF CALCULATION

The nuclear shell model wave functions of the initial ground state, $J = 1^+$ intermediate states, and $J = 0^+, 2^+$ final states were obtained using the shell model code NUSHELLX@MSU [26,27] with the FPD6 [31] and KB3G [32] interactions, in the complete fp-model space. The maximum number of intermediate states is 1000. Table I shows the total number of final states that are possible in Ti isotopes. If the total number of final states was larger than 5000, the calculations were done up to 5000 final states. As one will see later, that is enough to exhaust almost the total strength.

After all wave functions were obtained, the single GT operator was applied two times sequentially. First, all transitions from the parent nucleus 0^+ to all 1^+ intermediate states were calculated and then all transitions from 1^+ intermediate states to each 0^+ or 2^+ in the final nucleus were calculated. Figure 1

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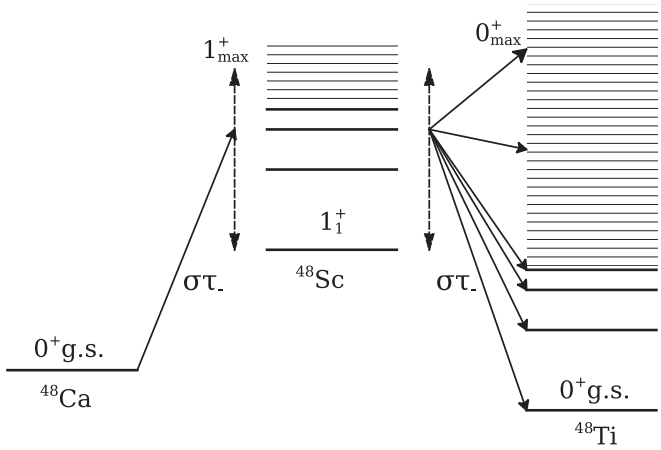

FIG. 1. Illustration of the method of calculation described in the text. The notations 1^+_{max} and 0^+_{max} are the corresponding cutoffs.

illustrates the method of calculation. The single GT operator is denoted as

$$\boldsymbol{Y}_\pm = \sum_{i=1}^{A} \boldsymbol{\sigma} t_\pm(i), \quad t_\pm = t_x \pm i t_y, \tag{1}$$

with $t_- n = p$ and $t_+ p = n$, where $2t_x$ and $2t_y$ are the Pauli isospin operators and $\boldsymbol{\sigma}$ is Pauli spin operator. Then the single GT transition amplitude from the initial state $|i\rangle$ to the final state $|f\rangle$ is

$$M(\mathrm{GT}_\pm; i \to f) = \frac{\langle f||\boldsymbol{Y}_\pm||i\rangle}{\sqrt{2J_i+1}}, \tag{2}$$

and the GT transition strength given by

$$B(\mathrm{GT}_\pm) = |M(\mathrm{GT}_\pm; i \to f)|^2 \tag{3}$$

obeys the $3(N-Z)$ sum rule

$$\sum_f B(\mathrm{GT}_-) - \sum_f B(\mathrm{GT}_+) = S_{\mathrm{GT}_-} - S_{\mathrm{GT}_+} = 3(N-Z), \tag{4}$$

TABLE I. The total number of final states in the fp-model space and f-model space, which includes the $f_{7/2}$ and the $f_{5/2}$ orbits only.

J_f	^{42}Ti		^{44}Ti		^{46}Ti		^{48}Ti	
	0^+	2^+	0^+	2^+	0^+	2^+	0^+	2^+
fp-space	4	8	158	596	2343	9884	14177	61953
f-space	2	1	29	99	180	741	446	1899

where the $\sum_f$ means summing over all eigenstates of $J_f T_f$. Because the fp-model space is limited, only the valence neutrons participate in the calculation for calcium isotopes. Therefore, we have $S(\mathrm{GT}_+) = 0$.

The dimensionless DGT transition amplitude is defined as

$$M(\mathrm{DGT}_\pm) = \sum_n M(\mathrm{GT}_\pm; i \to n) M(\mathrm{GT}_\pm; n \to f), \tag{5}$$

where n are the intermediate states. Note that this is a coherent sum. Finally, the DGT strength is given by

$$B(\mathrm{DGT}_\pm) = |M(\mathrm{DGT}_\pm)|^2, \tag{6}$$

or in more detail

$$B(\mathrm{DGT}_-; i \to n \to f) = \frac{1}{2J_i+1} \left| \sum_n \langle f|| \sum_i \boldsymbol{\sigma}_i t_-(i) ||n\rangle \langle n|| \sum_j \boldsymbol{\sigma}_j t_-(j) ||i\rangle \right|^2. \tag{7}$$

Note that $B(\mathrm{DGT}_-; i \to n \to f)$ depends on J_f with $J_f = 0$ and 2 only. The matrix element in the case of $J_f = 1$ vanishes because the DGT operator changes sign under the interchange of coordinates of two particles.

The DGT sum rule for $J_f = 0$ is given in Refs. [28,29], and for $J_f = 0, 2$ the sum rules are given in Refs. [24,30]. In summary, the sum rules in our cases $[S(\mathrm{GT}_+) = 0]$ for DGT

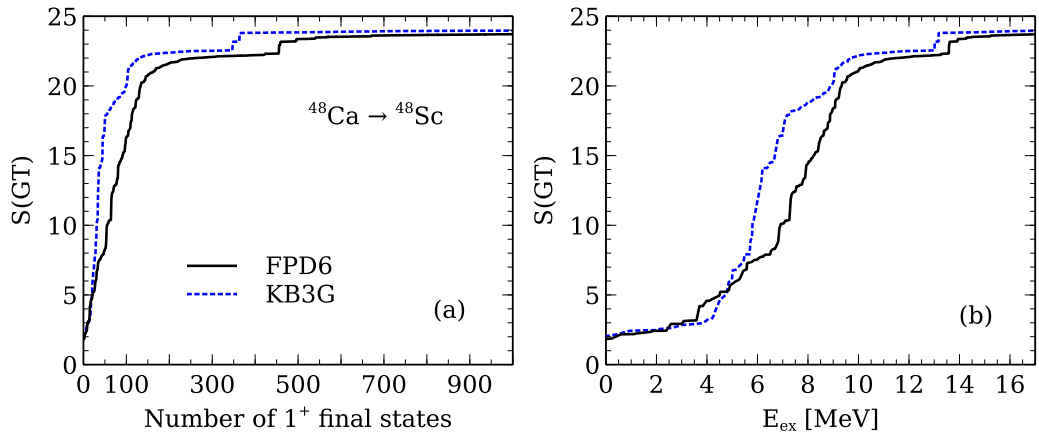

FIG. 2. The cumulative sum of the single Gamow-Teller strength $B(\mathrm{GT}_-)$ as a function of the number of 1^+ states (a) and excitation energies (b) of the intermediate nucleus ^{48}Sc. The calculation used FPD6 and KB3G interactions in the fp-model space.

TABLE II. The properties of DGT transition using FPD6 and KB3G interactions in the complete fp-model space. S(DGT) is the DGT total strength with the FPD6 interaction. $B_1 = B(\mathrm{DGT}; 0^+ \rightarrow 0_1^+, 2_1^+)$ is the reduced transition strength from the g.s. ($J = 0^+$) of the parent nucleus to the first 0_1^+ (g.s.) or the first 2_1^+ state of the final nucleus. $\overline{E}$ (MeV) is the average energy [Eq. (9)].

Initial nucleus	^{42}Ca		^{44}Ca		^{46}Ca		^{48}Ca
J_f	0^+	2^+	0^+	2^+	0^+	2^+	0^+
Sum rule	$\leqslant 36$	$\geqslant 0$	$\leqslant 120$	$\geqslant 240$	$\leqslant 252$	$\geqslant 720$	$\leqslant 432$
S(DGT) (FPD6)	28.1	19.5	102.0	284.0	223.7	752.6	385.0
S(DGT) (KB3G)	28.5	18.8	103.0	282.6	224.5	783.5	395.4
B_1 (FPD6)	16.172	6.117	0.654	0.000	0.201	0.017	0.109
B_1 (KB3G)	17.010	5.942	0.895	0.119	0.182	0.057	0.072
$\overline{E}$ (FPD6)	6.1	4.8	16.3	13.2	21.2	18.0	24.6
$\overline{E}$ (KB3G)	6.1	5.5	14.7	12.2	19.0	16.9	21.9

operators are

$$S_{\mathrm{DGT}}^{J_f=0} = 6(N-Z)(N-Z+1) - 2\Delta,$$
$$S_{\mathrm{DGT}}^{J_f=2} = 30(N-Z)(N-Z-2) + 5\Delta, \qquad (8)$$

where $\Delta = \sqrt{2}\langle 0|[\boldsymbol{Y}_+ \times \boldsymbol{Y}_-]^{(1)} \cdot \boldsymbol{\Sigma} - \boldsymbol{\Sigma} \cdot [\boldsymbol{Y}_- \times \boldsymbol{Y}_+]^{(1)}|0\rangle$, with $\boldsymbol{\Sigma} = \sum_i \boldsymbol{\sigma}(i)$. There is a factor of 3 difference between our work and the work in Refs. [24,25] because the spin operator is not projected in our calculation. The first terms of the sum rules depend only on N and Z, and the second terms (2Δ or 5Δ) need to be calculated separately. The sign of the second term makes the first term the upper limit for $J_f = 0^+$ and lower limit for $J_f = 2^+$.

III. RESULTS AND DISCUSSIONS

It is well known that the *single* GT strength is quenched (see Ref. [33]). In the shell model calculations, GT strength is fragmented. This is demonstrated in the case of ^{48}Ca in Fig. 2 as an example. The results were obtained with two different interactions that are often used in the fp-model space: FPD6 and KB3G interactions. Within the range of about 17 MeV excitation energy, $S(\mathrm{GT}_-)$ is approximately 24, exhausting the $3(N-Z)$ sum rule [$S(\mathrm{GT}_+) = 0$ in our calculations]. The cumulative sum of the single GT strength $S(\mathrm{GT}_-)$ as a function of the number of ^{48}Sc states is shown in Fig. 2. One can expect that there are about 500 intermediate 1^+ states in ^{48}Sc that actually contribute to the final results of the DGT strength, although the total number of 1^+ states in this nucleus is many thousands.

TABLE III. $B(\mathrm{DGT}; 0^+ \rightarrow 0^+)$ for ^{42}Ca using FPD6 and KB3G interactions in the complete fp-model space. E_{ex} (MeV) is the excited energy of ^{42}Ti.

FPD6			KB3G		
E_{ex}	B(DGT)	S(DGT)	E_{ex}	B(DGT)	S(DGT)
0.0	16.172	16.172	0.0	17.010	17.010
6.0	0.442	16.614	5.7	0.281	17.291
10.9	0.782	17.396	11.3	0.120	17.411
14.9	10.692	28.088	15.4	11.085	28.496

For the study of the DGT resonance, first, we calculated the sum rule using it as a tool to asses whether in our numerical calculations most of the DGT strength is found. While the first terms in Eq. (8) depend only on N and Z (see Table II), the second terms that contain Δ need to be calculated numerically. In Ref. [29], the Δ was related to the magnetic dipole transition $S(M1)$. For direct comparison with the work in Ref. [29], the Δ is extracted by subtracting from our numerical total sum the first term. Besides, Table I in Ref. [29] and Table I in Ref. [30] gave the values of the sum rules for even-A isotopes including calcium isotopes. Our results given in Table II are in agreement with them (Note that there is a factor of 3 difference between our work and Ref. [30]). It means we exhaust all the DGT strength in the study. Obviously, the total DGT strength does not depend on the choice of interaction.

Because all the strengths are obtained, we can show not only the values of the total sum but also the cumulative sums of the DGT strength. The cumulative sums are given in Fig. 3 for ^{44}Ca, Fig. 4 for ^{46}Ca, and Fig. 5 for ^{48}Ca. In these figures, the solid lines are the shell model calculations described in Sec. II using the FPD6 interaction. They are denoted as "FPD6

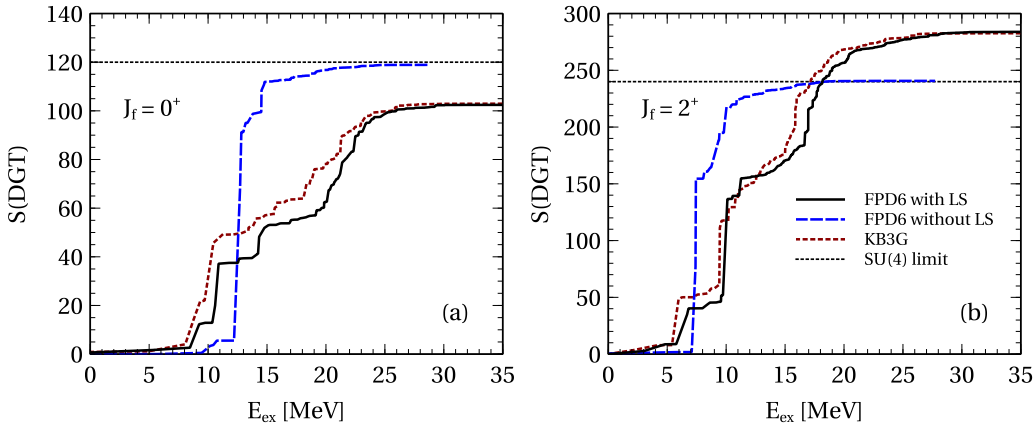

FIG. 3. The cumulative sum of the $B(\mathrm{DGT}; 0^+ \rightarrow 0^+)$ (a) and $B(\mathrm{DGT}; 0^+ \rightarrow 2^+)$ (b) in ^{44}Ca.

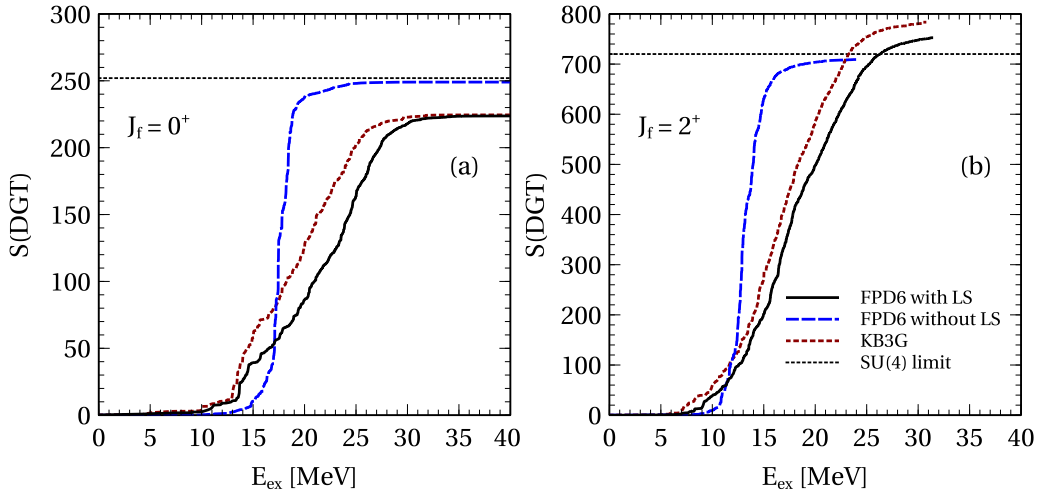

FIG. 4. The same as Fig. 3, but for ^{46}Ca.

with LS." The results using the KB3G interaction are also shown as dotted lines. The long-dash lines are the calculation with the FPD6 interaction in the SU(4) limit. The SU(4) limit in our work is restored approximately by making the $f_{5/2}$, $f_{7/2}$ and the $p_{1/2}$, $p_{3/2}$ degenerate following Ref. [34]. It means there is no spin-orbit coupling and therefore they were denoted as "FPD6 without LS." We want to show that in the SU(4) limit, the cumulative sums approach the horizontal lines [denoted as the "SU(4) limit'] that represent the values of the terms that depend only on N and Z in Eq. (8). This is in agreement with the fact that Δ vanishes in the SU(4) limit according to Ref. [35]. In the cases of ^{44}Ca (Fig. 3), the sum rules are fully exhausted because all intermediate states and all final states can be taken into account. In the cases of ^{46}Ca with $J_f = 2^+$, and ^{48}Ca (Figs. 4 and 5), the cumulative sums are still increasing. For the case of the DGT transition to the 2^+ state in ^{48}Ca, we choose not to do the calculation because the total number of final states is too large. The result is not convergent using the standard NUSHELLX@MSU code [36].

Most of the sum rule is satisfied, and therefore the entire distributions of DGT strength can now be discussed. We remind the reader that Ref. [24] showed the entire DGT distributions for even-A neon isotopes, and recently Ref. [25] showed the result for ^{48}Ca for the first time. For the lightest nucleus, ^{42}Ca, the DGT distributions with FPD6 and KB3G interactions are shown in Tables III and IV. The difference between the results of the two interactions is not large.

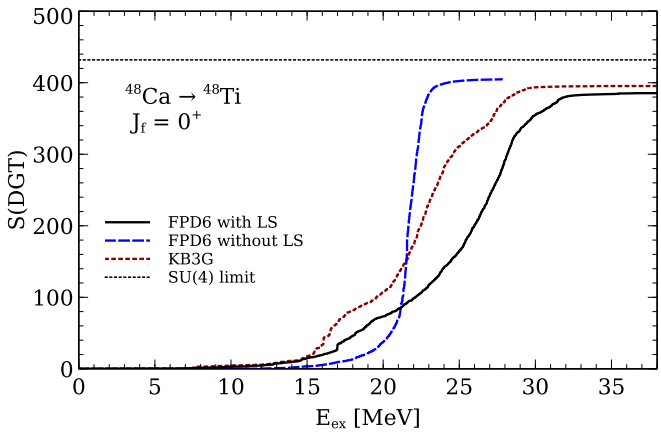

FIG. 5. The cumulative sum of the $B(\mathrm{DGT}; 0^+ \rightarrow 0^+)$ in ^{48}Ca.

In the case $B(\mathrm{DGT}; 0^+ \rightarrow 0^+)$ of ^{42}Ca, the reduced transition strength from the g.s. of the parent nucleus to the first 0^+ (g.s.) of the final nucleus is large because the g.s. of ^{42}Ti is the DIAS of the g.s. of ^{42}Ca. Moreover, in the SU(4) limit the g.s. of ^{42}Ti absorbs all the DGT strength (36), following Refs. [28,29].

The DGT distributions are drawn in Figs. 6–10. They contain inserts which show the DGT strength in the low-lying states of ^{44,46,48}Ti. B_1 is a very tiny fraction of the total strength. For example, the strength in the ground state of ^{48}Ti is only 3×10^{-4} of the total strength (see Table II). This strength enters in the calculation of the $\beta\beta$ decay. In Ref. [25], it is pointed out that a very good linear correlation between the DGT transition to the ground state of the final nucleus and the $0\nu\beta\beta$ decay matrix element exists.

When the strengths are spread by using the same Lorentzian averaging with the width of 1 MeV to simulate the experimental energy resolution, the results show that the DGT distributions are not single-peaked. The distributions have at least two peaks and in some nuclei as many as four major peaks. We remind the reader that the single GT resonances

TABLE IV. The same as Table III, but for $B(\mathrm{DGT}; 0^+ \rightarrow 2^+)$.

FPD6			KB3G		
E_{ex}	B(DGT)	S(DGT)	E_{ex}	B(DGT)	S(DGT)
0.0	6.117	6.117	0.0	5.942	5.943
2.3	1.536	7.653	2.6	0.520	6.463
5.1	0.125	7.778	5.2	0.009	6.472
6.6	5.523	13.301	7.2	1.355	7.827
7.3	4.916	18.217	7.8	9.679	17.506
9.7	0.071	18.288	10.1	0.188	17.694
11.6	0.039	18.327	11.9	0.017	17.711
14.2	1.148	19.475	14.2	1.047	18.757

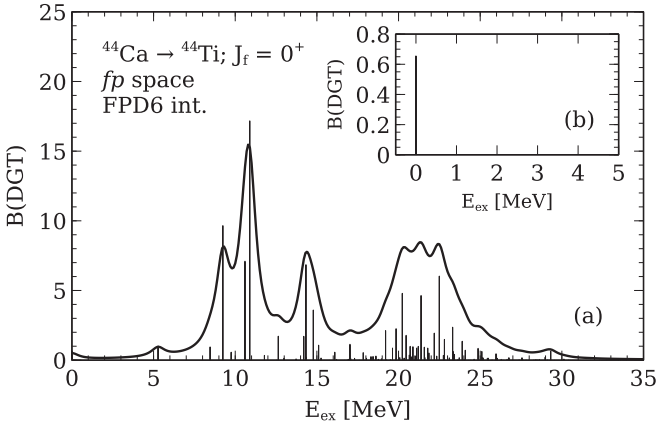

FIG. 6. $B(\mathrm{DGT}; 0^+ \to 0^+)$ for ^{44}Ca as the function of the excitation energy of the final nucleus ^{44}Ti (a). The DGT transitions to low-lying states are shown in (b). The strengths were spread by using Lorentzian averaging with the width of 1 MeV.

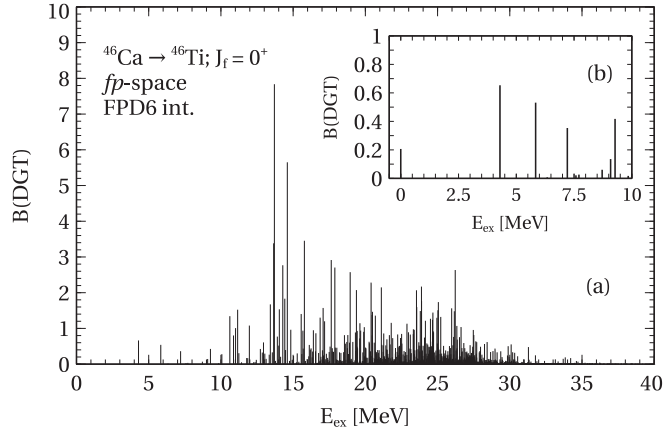

FIG. 8. $B(\mathrm{DGT}; 0^+ \to 0^+)$ in ^{46}Ca (a). The DGT transitions to low-lying states are shown in (b).

have at least two peaks [37]. Figsures 11–13 show the dependence of the DGT distributions on the number of intermediate states. We can see that about 100 intermediate states actually contribute to final results in the cases of ^{44}Ca (Figs. 11 and 12). Although the total number of intermediate states in heavier isotopes, including ^{48}Ca, is many thousands, about 500 intermediate states actually contribute to final results (Fig. 13). The sum rule is useful to determine this number. (We remind the reader that the number of intermediate states involved in the calculations for $0\nu\beta\beta$ decay is smaller; see Ref. [38]). Figure 14 shows the DGT transitions to $J_f = 0^+$ together with the transition to $J_f = 2^+$ in ^{44}Ca and ^{46}Ca. As one can see, the DGT transitions to $J_f = 2^+$ are larger than the transitions to $J_f = 0^+$.

The centroids (average energies) of the DGT distributions defined by

$$\overline{E} = \frac{\sum_f E_f B_f(\mathrm{DGT}_-)}{\sum_f B_f(\mathrm{DGT}_-)} \tag{9}$$

are given in Table II. In ^{46}Ti, with the FPD6 interaction, for example, the average energy for the $J_f = 0^+$ is $\overline{E} = 21.2$ MeV and for the $J_f = 2^+$ it is lower, $\overline{E} = 18.0$ MeV. In ^{48}Ti the average energy $J_f = 0^+$ is $\overline{E} = 24.6$ MeV. In Ref. [25], a simple relation between the average energy of the ^{48}Ca DGT giant resonance and the $0\nu\beta\beta$ decay nuclear matrix element was pointed out. The authors conclude that the uncertainties due to the nuclear interaction in the calculation of the DGT distribution in ^{48}Ca are relatively under control. Figures 15–17 show the DGT distributions calculated with FPD6 and KB3G interactions. We see that the distributions and the average energies (see Table II) using FPD6 and KB3G are similar. Our calculated distribution for ^{48}Ca is in agreement (when the factor of 3 is taken into account) with the recent result using the same KB3G interaction but a different method. As one can see in Fig. 17, the DGT giant resonance in ^{48}Ca is at the energy around 20–30 MeV. In a recent paper [23] the experimental results for the double-charge-exchange reaction ^{56}Fe(^{11}B, ^{11}Li) were presented. In this reaction, several resonances were excited, in agreement with the pion DCX studies. There is a peak at 25 MeV excitation, that the authors indicated could be the DGT resonance.

In addition, the numerical calculations in the f-model space (including the $f_{7/2}$ and the $f_{5/2}$ orbits only) using the same Hamiltonian are given in Fig. 18. For ^{42}Ca, there are two

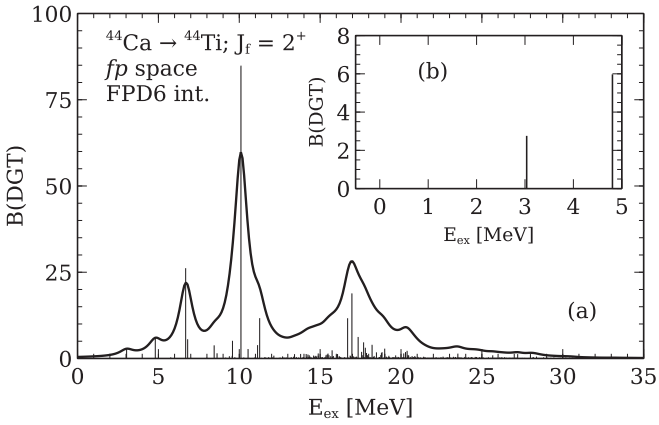

FIG. 7. The same as Fig. 6, but for $B(\mathrm{DGT}; 0^+ \to 2^+)$.

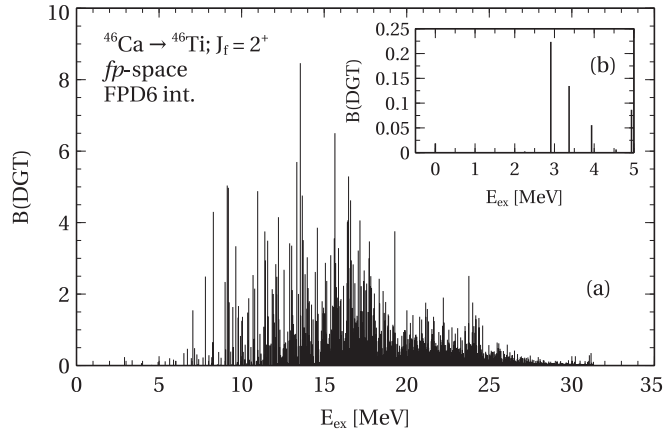

FIG. 9. The same as Fig. 8, but for $B(\mathrm{DGT}; 0^+ \to 2^+)$.

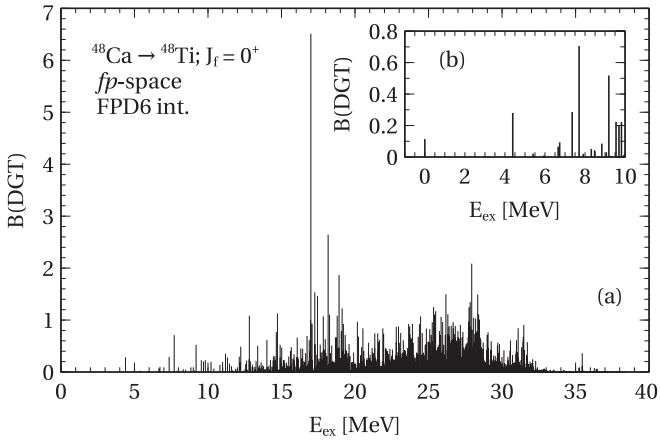

FIG. 10. The same as Fig. 8, but for ^{48}Ca.

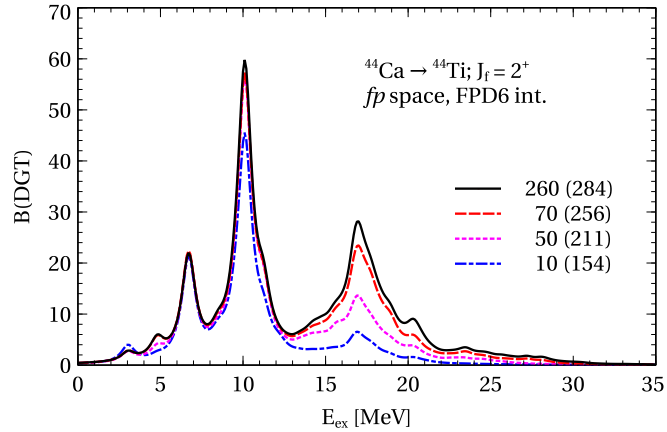

FIG. 12. The same as Fig. 11, but for $B(\mathrm{DGT}; 0^+ \rightarrow 2^+)$.

0^+ DGT states, at the excitation energies 0.0 and 18.3 MeV. Their strengths are 11.178 and 13.791, respectively. There is one 2^+ DGT state at 0.0 MeV and its strength is 8.658. Note that the sum rules do not depend on the model space. In the f-model space, we obtained exactly the sum rules even for the case of the DGT transitions to the 2^+ states in ^{48}Ca because the calculation can be done without any limitation, as now the total number of possible final states is strongly reduced (see Table II). The DGT distributions in the f-model space are much more concentrated. In Refs. [6,7], the analytical calculations in the limited f-model space provided valuable information for the studies of the DIAS. Therefore, we can expect that a similar calculation for the DGT strength will be useful to determine the structure of DGT strength distribution and where the DGT strength is concentrated.

Finally, we comment on the question of quenching of GT strength, sometimes phrased as the renormalization of the GT operator. The single GT strength is reduced considerably, 30–40% quenched. The origins of this quenching are still a puzzle. Two basic mechanisms were introduced. The quenching in charge exchange reactions is with respect to strength on the main peaks of the GT resonance. It is suggested that the rest of the strength is strongly fragmented and spread out at energies several tens of MeV above the main peaks. The nuclear force mixes the one-particle–one-hone (1p-1h) configurations that make up the GT state with 2p-2h configurations, leading to fragmentation of strength [39]. There are experimental attempts to locate this strength but the results so far are not conclusive. The second possibility is that the missing GT strength is due to the coupling of the GT state to the internal excitation of the nucleon, namely to the Δ resonance, removing the strength to a very high energy of about 300 MeV excitation [40]. Of course, there is the possibility that both mechanisms could contribute. The quenching of the GT strength would affect the DGT strength as well and probably to a larger extent. The $\beta\beta$ decay, as already mentioned, is directly related to the DGT strength. The transition operator of the $2\nu\beta\beta$ is the same as the DGT operator. In the case of the $0\nu\beta\beta$ the transition operators have in addition to the DGT operator also an r dependence. The quenching will affect both types of $\beta\beta$ decay transitions. However, the effect on the $0\nu\beta\beta$ may be different than for the $2\nu\beta\beta$, depending on which mechanism of quenching is the dominant one. This aspect, however, is beyond the scope of the present shell model calculation.

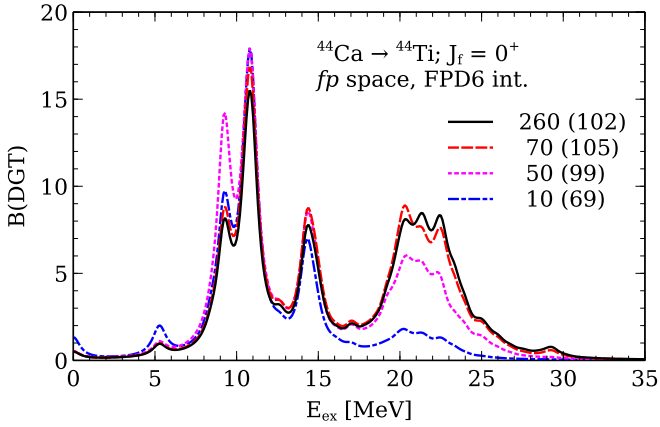

FIG. 11. The dependence on the number of intermediate states of $B(\mathrm{DGT}; 0^+ \rightarrow 0^+)$ in ^{44}Ca. The numbers in parentheses are the corresponding total strengths.

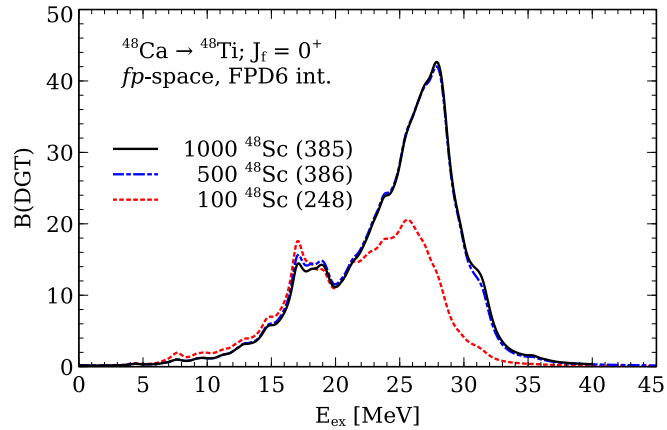

FIG. 13. The same as Fig. 11, but for $B(\mathrm{DGT}; 0^+ \rightarrow 0^+)$ in ^{48}Ca.

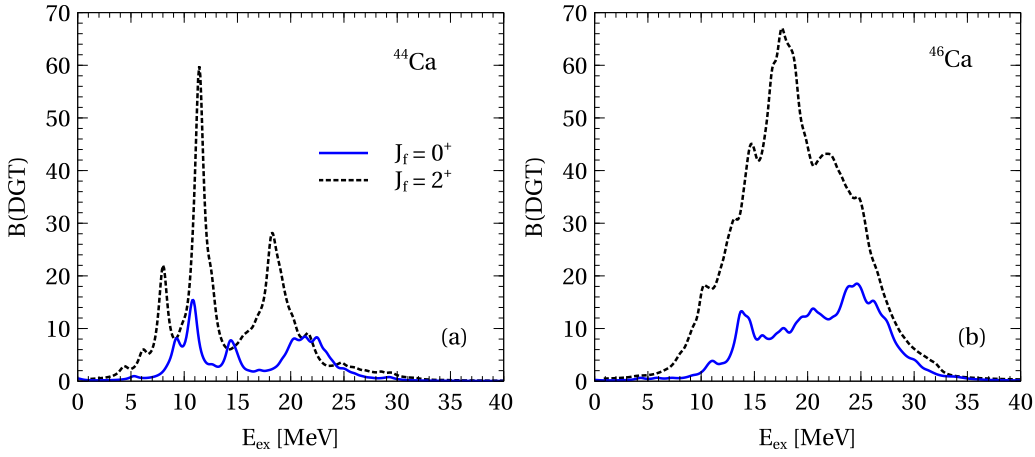

FIG. 14. $B(\mathrm{DGT}; 0^+ \rightarrow 0^+, 2^+)$ in ^{44}Ca (a) and ^{46}Ca (b).

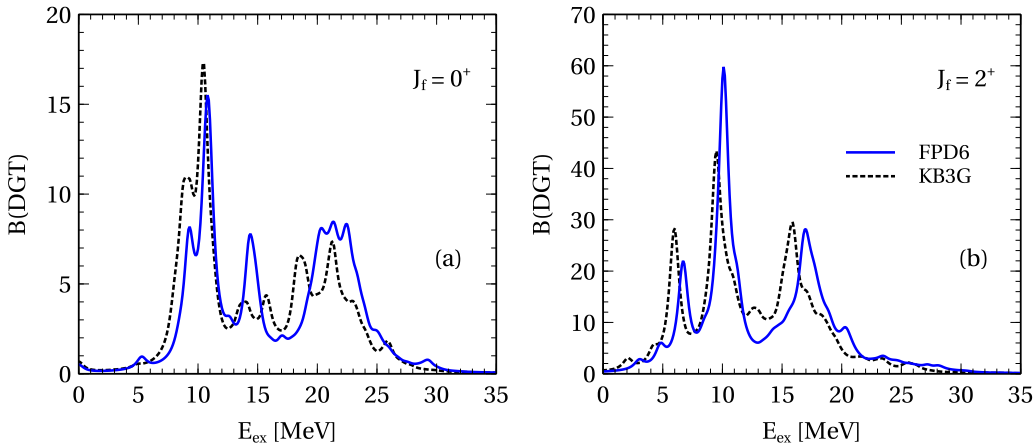

FIG. 15. $B(\mathrm{DGT}; 0^+ \rightarrow 0^+)$ (a) and $B(\mathrm{DGT}; 0^+ \rightarrow 2^+)$ (b) in ^{44}Ca using FPD6 and KB3G interactions.

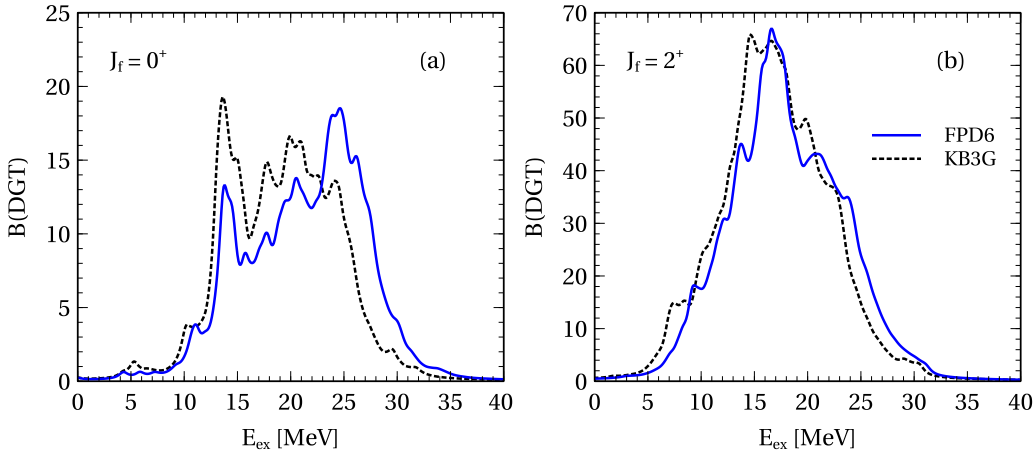

FIG. 16. The same as Fig. 15, but for ^{46}Ca.

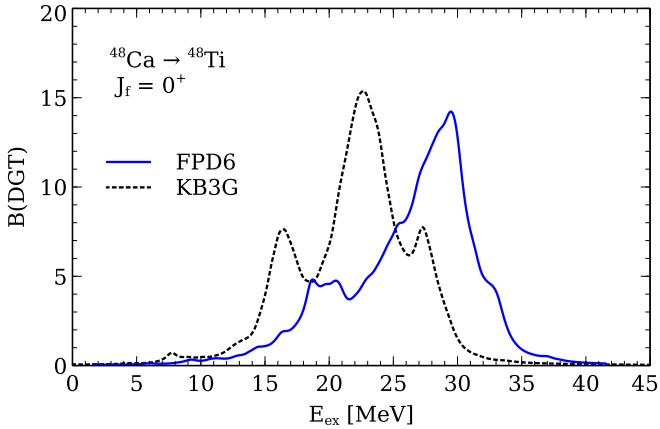

FIG. 17. $B(\mathrm{DGT}; 0^+ \rightarrow 0^+)$ in ^{48}Ca using FPD6 and KB3G interactions. The strengths are spread with the width of 1 MeV. For comparison to Ref. [25] that also calculated the DGT distribution in ^{48}Ca using the KB3G interaction, the strengths in our calculation shown in this figure are divided by the factor of 3 (see text).

IV. CONCLUSIONS

The general features and trends of the DGT sum rules in even-A calcium isotopes are described using numerical results. The properties of the entire distribution of the DGT transitions are discussed. By studying the stronger DGT transitions—in particular, the DGT giant resonance experimentally and theoretically—the calculations of $\beta\beta$-decay nuclear matrix elements can be calibrated to some extent. There is no doubt that the pion DCX is a sensitive tool for probing nuclear structure. Nowadays the ion DCX reactions have been discussed mainly in the context of $0\nu\beta\beta$; however, the ion DCX reaction itself is a new probing tool of nuclear structure, in particular of spin degrees of freedom. The DGT resonance is just one example. Because two nucleons participate in the DCX reactions with pions or heavy ions, one can expect that the nucleon-nucleon interaction and correlations can be probed, including those for nuclei that are far from the stability region. Finally, the problem of quenching of GT strength is of course very important and should be studied in order to improve the understanding of the DGT strength and the $\beta\beta$ decay.

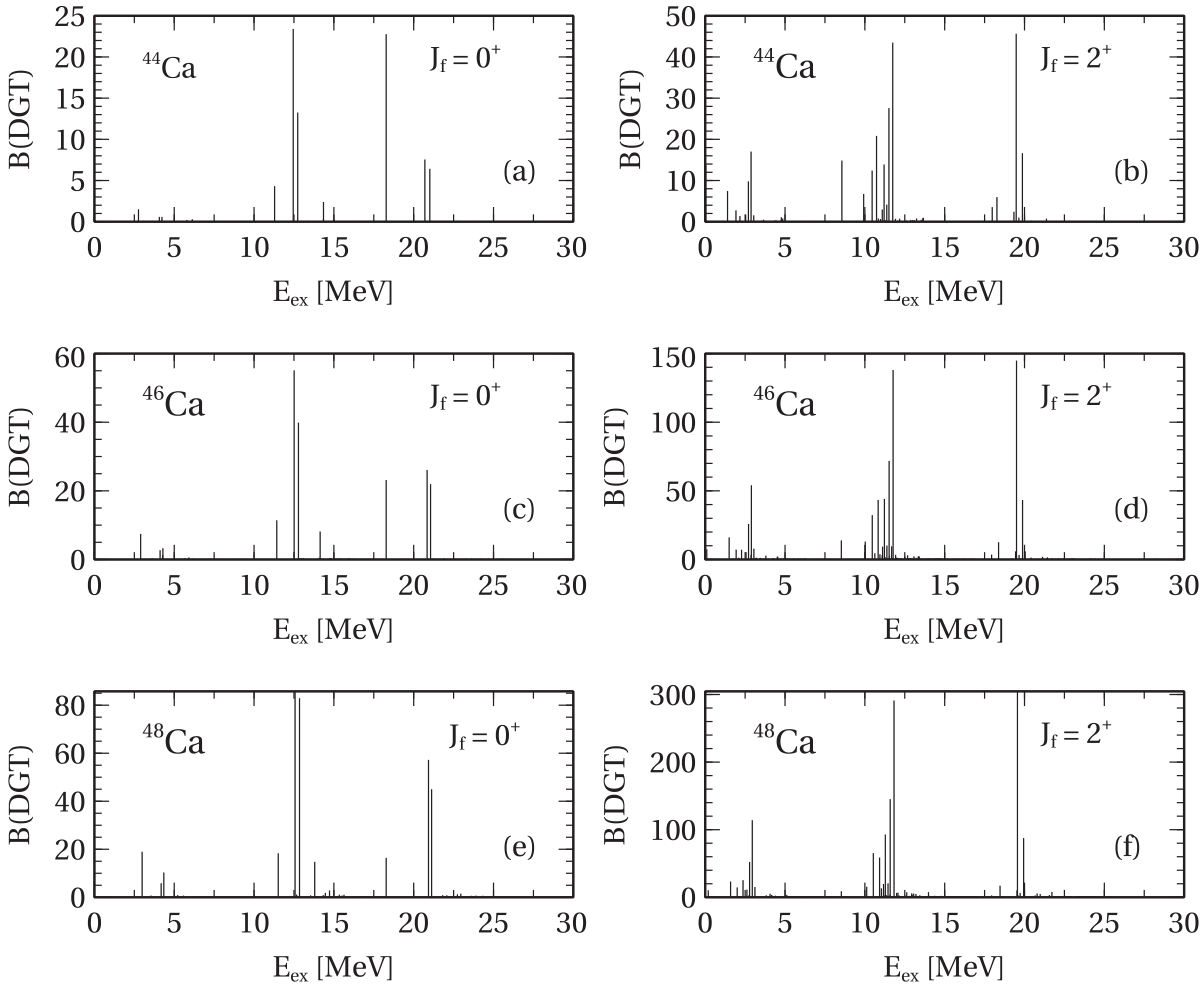

FIG. 18. The DGT distributions in the f-model space (including the $f_{7/2}$ and the $f_{5/2}$ orbits only) in even-A calcium isotopes using the FPD6 interaction.

ACKNOWLEDGMENTS

The authors thank to B. A. Brown and Vladimir Zelevinsky for discussions made possible by the travel grant from the US-Israel Binational Science Foundation (2014024). This work was supported by the US-Israel Binational Science Foundation, Grant 2014024.

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