Isospin-symmetry breaking corrections for the description of triplet energy differences

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The charge-independence breaking of the nuclear interaction is analyzed by means of energy differences among analog states in $T = 1$ isobaric multiplets. Data on triplet energy differences in the sd, pf, and pfg shells, i.e., $18 \leqslant A \leqslant 66$, are reproduced with very good accuracy by large-scale shell-model calculations taking into account, aside from the Coulomb interaction, a single isotensor schematic interaction of monopole-pairing type. It is shown that the effect on the triplet energy differences of this isospin-breaking interaction is of the same magnitude as the Coulomb one. Moreover, its strength is the same for every single-particle orbital of the considered model space.

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I. INTRODUCTION

Isospin symmetry is the earliest fundamental symmetry discovered in nuclear physics. It was introduced by Heisenberg in 1932, just after the discovery of the neutron by Chadwick, and generalized by Wigner a few years later. Indeed, the nucleon-nucleon interaction is, to good approximation, charge symmetric and charge independent. The former implies that the interaction between two protons is the same as between two neutrons ($V_{pp} = V_{nn}$), while the latter states that the average of these two should equal the proton-neutron interaction ($V_{pp} + V_{nn} = 2V_{pn}$). In the nuclear environment, the Coulomb field breaks charge symmetry, giving rise to differences of several MeV in the binding energy between isobaric nuclei. It is now well known that also the nuclear interaction breaks, to some extent, the isospin symmetry. The most modern realistic interactions have different strengths for the three $T = 1$ nucleon-nucleon channels. However, the use of these interactions in the theoretical description of differences in binding energies—the so-called Coulomb displacement energies (CDEs)—or differences of excitation energy between analog states in isobaric multiplets have failed so far to reproduce the experimental data; see for example Gadea *et al.* [1] and, most recently, Ormand *et al.* [2].

The study of differences in excitation energy between analog states in isobaric multiplets allows us to analyze isospin-breaking effects as a function of angular momentum. For example, when looking at mirror nuclei, with interchanged number of protons and neutrons, charge-symmetry breaking may be investigated. These energy differences, called mirror energy differences (MED) have been accurately described in the $f_{7/2}$ shell by state-of-the-art shell-model calculations including a range of electromagnetic phenomena as well as a schematic isospin-breaking interaction, deduced empirically from the $T = 1$, $A = 42$ mirror pair [3,4]. Without the inclusion of this latter term, the match to experimental MED data is quite poor.

The MED have a strong sensitivity to nuclear structure properties, which in turn has allowed a detailed interpretation of MED in terms of nuclear structure phenomena along the yrast line such as the alignment of nucleons along rotational bands, the evolution of the nuclear radius with angular momentum and the identification of pure single-particle excitations [4]. In a recent systematic study of mirror nuclei in the $f_{7/2}$ shell [5], a full set of effective isovector ($V_{pp} - V_{nn}$) isospin-nonconserving (INC) matrix elements were extracted by fitting the shell model to all experimental MED in the shell. This demonstrated that there was a very strong angular-momentum dependence of the INC nucleon-nucleon interaction, with the main observation that, for two $f_{7/2}$ particles coupled to $J = 0$, the effective INC term required was around 100 keV lower (more attractive) than the $J \neq 0$ terms. This is a significant effect because the J dependence of the electromagnetic interaction itself has a similar magnitude.

The above discussion, and much of the analysis published to date, focuses on mirror nuclei, and hence extracts information on charge-asymmetric effects. Charge *independence* can be studied through the behavior of $T = 1$ isobaric triplets—i.e., the analog states with isospin $T = 1$ in the three nuclei with $T_z = 0, \pm 1$ (the odd-odd $N = Z$ nucleus and its two neighboring even-even systems of the same A). The differences in excitation energy of analog states are usually written as triplet energy differences (TED) of the form:

$$\mathrm{TED}(I) = E^*_I(T_z = -1) + E^*_I(T_z = +1) - 2E^*_I(T_z = 0), \tag{1}$$

where I is the angular momentum of the state and $T_z = (N - Z)/2$ is the projection of the isospin T on the z axis. Here, the excitation energies of the states E^*_I are referred to the ground state or to the lowest state of the same T in each nucleus. As

can be seen from the form of Eq. (1), the TED is strongly connected to the isotensor interaction $V_{pp} + V_{nn} - 2V_{pn}$.

A simple and consistent method for the description of MED and TED in the shell model framework for nuclei in the $f_{7/2}$ shell was developed for the first time in Ref. [3], and it was further advanced and applied systematically in the review article of Ref. [4]. This work showed that, by using a single parametrization, the available experimental data could be very well described. In those studies, the isospin-symmetry breaking term was considered taking into account the contribution arising from the $f_{7/2}$ shell *only*. This was sensible since the $f_{7/2}$ orbital largely dominates the wave function of the analog states under consideration.

In recent years, however, experimental studies have been extended from nuclei in the $f_{7/2}$ shell to other mass regions due to the progress in experimental techniques and the use of radioactive beams. Mirror nuclei and $T = 1$ triplets have now been studied in detail in the sd, upper-pf, and pfg shells—see, for example, Refs. [6–10]. In these other regions, there is no longer a dominance of a single shell and so all the orbitals should be considered on the same footing. It has, to date, not yet been demonstrated that there is a consistent method for inclusion of the INC interactions across all regions and involving all shells. A first step in making a connection between an *effective* INC interaction in the shell-model approach and a true charge-dependent component of the nucleon-nucleon would be to investigate the dependence of INC interactions on mass region and configuration.

In this article, we present the results of a systematic and consistent TED analysis across all data available on $T = 1$ triplets in the sd shell, the $f_{7/2}$ shell, and the upper-pf and pfg regions. In Sec. II we present the shell-model approach used, and in Sec. III we discuss the extraction of effective isotensor INC terms through a fit of the model to data in the $f_{7/2}$ shell. In Sec. IV we present a full set of results for all triplets studied, with a discussion in Sec. V, and we draw conclusions in Sec. VI.

II. SHELL-MODEL APPROACH TO TRIPLET ENERGY DIFFERENCES

The theoretical approach adopted here is an extension to other nuclear shells of the one developed in Refs. [3,4] for nuclei in the $f_{7/2}$ shell.

If the nuclear interaction is fully independent of charge, the TED should arise solely from the isotensor contribution of electromagnetic contributions. Thus, conversely, if the electromagnetic effects can be reliably determined, the study of TED has the potential to shed light on the charge-dependence of the nuclear interaction. Indeed, TED, due to the way they are constructed, do not have large contributions from *monopole* electromagnetic effects, driven by nuclear-structure properties such as changes of nuclear radius as a function of the angular momentum or single-particle excitations. Indeed, these monopole effects can give a significant contribution to, and even dominate, the MED. TED can therefore be considered as a much more "transparent" observable to shed light on INC phenomena of *multipole* origin.

It is well known from scattering data that the nucleon-nucleon interaction has a charge dependence—with the np interaction known to be about 2%–3% stronger than the average of the nn and pp interactions (see, e.g., Ref. [11]). Indeed, in the $f_{7/2}$ shell, it was shown that the electromagnetic effects discussed above were *not* sufficient to account for the experimental TED [1,3]. Zuker *et al.* [3] therefore extracted an (empirical) isotensor INC interaction from the TED of the $A = 42$, $T = 1$ triplet, having accounted for the Coulomb multipole effect described above. This yielded a schematic isotensor INC interaction $(V_{pp} + V_{nn} - 2V_{pn})$ of approximately +100 keV for the $J = 0$ coupling of two $f_{7/2}$ nucleons, with zero value for the other angular-momentum couplings. Crucially, this introduces an effective J dependence of the INC interaction which, in turn, strongly affects the predicted TED—which are uniquely sensitive to the relative J dependence, rather than absolute values, of the isotensor matrix elements. This inclusion of an INC isotensor term of monopole pairing type ($J = 0$) was successful in the $f_{7/2}$ shell, overall. In this work, we consider the same strength for all subshells i in the $J = 0$ channel. Following the notation of Ref. [3], the isotensor term $V_B^{(2)}$ for orbital i can be written as

$$V_{Bi}^{(2)} = V_{Bi}^{\pi\pi,J=0} + V_{Bi}^{\nu\nu,J=0} - 2V_{Bi}^{\pi\nu,J=0} = +100\ \text{keV}. \quad (2)$$

The TED are thus obtained in first-order perturbation theory within the shell-model framework by the following expression:

$$\text{TED}(I) = \Delta_T\langle V_C^{(2)}(I)\rangle + \Delta_T\langle V_B^{(2)}(I)\rangle, \quad (3)$$

where Δ_T indicates that the differences of the expectation values are obtained as in Eq. (1). The first term in Eq. (3) represents the contribution from the Coulomb interaction. The Coulomb matrix elements are obtained in the harmonic-oscillator basis within the relevant model space. The second term represents the INC contribution due to the inclusion of the schematic interaction of Eq. (2). We use a value of +100 keV for the isotensor $V_B^{(2)}(J = 0)$ matrix element, in all the analysis that follows, in all orbits. This value is justified further in Sec. III.

III. EXTRACTING EMPIRICAL ISOSPIN NONCONSERVING MATRIX ELEMENTS IN THE $f_{7/2}$ SHELL

In this section, effective INC matrix elements, $V_B^{(2)}(J)$, are extracted for the $f_{7/2}$ shell. This is done by fitting the shell-model predictions for TED to the experimental data, allowing the INC matrix elements to vary freely. This analysis is performed to establish how the effective INC matrix elements vary as a function of angular-momentum coupling, J. An equivalent analysis, but for MED values, was published in Ref. [5]. To do this, the TED for a given set of analog states, α, are determined by using the method described in Sec. II, accounting for the electromagnetic effects only; i.e., the first term in Eq. (3). The second term in Eq. (3) is then determined

TABLE I. The isotensor INC matrix elements, $V_B^{(2)}(J)$, for $f_{7/2}$ pairs, extracted from the fits across the whole $f_{7/2}$ shell—see text for details. The final columns indicate the final rms deviation between the data and the model (using the fit parameters) compared with the calculations assuming $V_B^{(2)} = 0$. See text for details.

Extracted $V_B^{(2)}$ parameters					
Matrix elements $V_B^{(2)}$ (keV)				RMS deviation	
$J=0$	$J=2$	$J=4$	$J=6$	Fit	No V_B
One-parameter fit					
98(11)				33	90
Full fit					
53(160)	−37(178)	−55(170)	−82(157)	31	90
Full fits: centroid-subtracted					
113(18)	23(29)	5(24)	−21(22)	31	90

in a perturbative approach by

$$\Delta_T\langle V_B^{(2)}(I)\rangle(\alpha) = \sum_{J=0,2,4,6} \Delta c_B^J(\alpha) V_B^{(2),J}, \tag{4}$$

where $c_B^J(\alpha)$ is the expectation value of the operator

$$\left[(a^\dagger a^\dagger)^{T=1,J}_{\pi f_{7/2}}(aa)^{T=1,J}_{\pi f_{7/2}}\right],$$

which provides a coefficient that "counts" $T=1$ pairs of $f_{7/2}$ protons coupled to angular momentum J. $\Delta c_B^J(\alpha)$ is then determined relative to the ground state by

$$\Delta c_B^J(\alpha) = c_B^J(\alpha)_{T_z=-1} + c_B^J(\alpha)_{T_z=+1} - 2c_B^J(\alpha)_{T_z=0}. \tag{5}$$

This method creates an isotensor effect through adding an additional interaction $V_B^{(2)}$ in the pp channel for that J coupling. (An equivalent result would be obtained by adding the same interaction in the nn channel, or $-V_B^{(2)}/2$ in the np channel.) Here, $V_B^{(2)}$ was restricted to the $f_{7/2}$ shell, since the wave functions are dominated by $(f_{7/2})^n$ configurations. The four $V_B^{(2),J}$ two-body matrix elements in the $f_{7/2}$ shell ($J=0, 2, 4, 6$) were allowed to vary freely until the resulting theoretical TED provided the closest fit to the experimental values for all data points in the shell taken together.

The ANTOINE shell-model code was used [13,14], with the KB3G interaction [15] in the full pf valence space, with no restrictions on the total number of excitations from $f_{7/2}$ to the higher-lying pf orbits. The data points fitted corresponded to the 16 known TED values for the $A=42$, 46, 50 and 54 $T=1$ triplets. The coefficients $V_B^{(2),J}$ were then allowed to vary freely, for the $f_{7/2}$ shell only, and the best fit was obtained. To estimate errors in the fit parameters, the theoretical error was first obtained by requiring that $\sqrt{2\chi^2} \approx \sqrt{2n_d - 1}$ where n_d is the number of degrees of freedom, and it dominates over any experimental errors. This is the same approach as used in Ref. [5].

The results of the fit are shown in Table I. The first row shows the result obtained when only $V_B^{(2),J=0}$ is allowed to vary, keeping the other J terms fixed at zero. It is clear that a large positive $J=0$ term of around +100 keV appears,

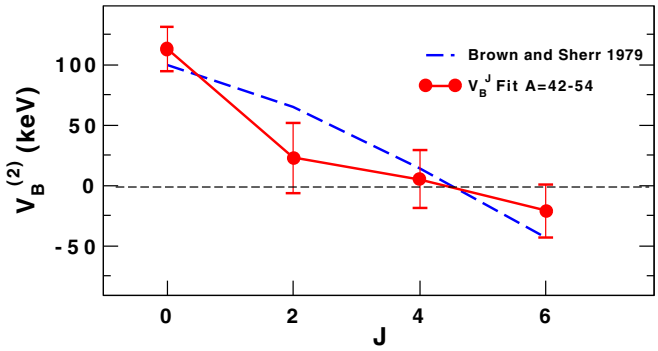

FIG. 1. Data points: The isotensor INC matrix elements, $V_B^{(2)}(J)$, for $f_{7/2}$ pairs, extracted from the fits across the whole $f_{7/2}$ shell—see final row of Table I. Dashed line (blue) shows results for the same parameters extracted from Brown and Sherr [12] who fit the Coulomb displacement energies to a single-$f_{7/2}$-shell-model calculation.

which reduces very significantly the rms deviation of the theory from the data—see the final two columns of Table I. The next row shows the results of the fit when all four $V_B^{(2),J}$ matrix elements are varied. A strong J dependence is seen, although notably the errors on the fit parameters are now an order of magnitude larger. This is due to the fact that the TED are very sensitive to the J dependence of the multipole matrix elements and largely *insensitive* to the absolute values. This can be seen from the correlation matrix obtained for the four-parameter fit, for which all off-diagonal correlations are positive and >0.96. To account for this, one can subtract the monopole contribution (centroid) from the extracted matrix elements, which for any two-body multipole interaction with matrix elements, V^J, can be written as

$$V^{\text{cent}} = \frac{\sum_J (2J+1)V^J}{\sum_J (2J+1)}. \tag{6}$$

The final row of Table I shows the monopole-subtracted values. The errors on the monopole-subtracted parameters are now much reduced because of the smaller degree of correlation between the parameters. This is because, once the centroid is subtracted, the results rely on the J dependence of the parameters, to which the TED are very sensitive. The final column shows that allowing all four terms to vary rather than just the $J=0$ term does not improve the fit to any significant degree. This demonstrates clearly that an effective INC interaction can be well described with a positive $J=0$ isotensor matrix element of +100 keV, with the other matrix elements set at zero to yield the required J dependence. Figure 1 shows the results of the full fit (final row of Table I) as a function of J.

While information on the isotensor interaction has been extracted here from excitation energies only, it is also possible to gain information from the nuclear binding energies through studies of Coulomb displacement energies (CDEs). This was done in the $f_{7/2}$ shell by Brown and Sherr [12], in which a single-j shell-model calculation was fit to experimental CDEs by allowing the J-dependent $V_{pp} - V_{nn}$ and $V_{np} - V_{nn}$ interactions to vary. From their resulting parameters in that work, we have extracted $V_{pp} + V_{nn} - 2V_{np}$ and subtracted the

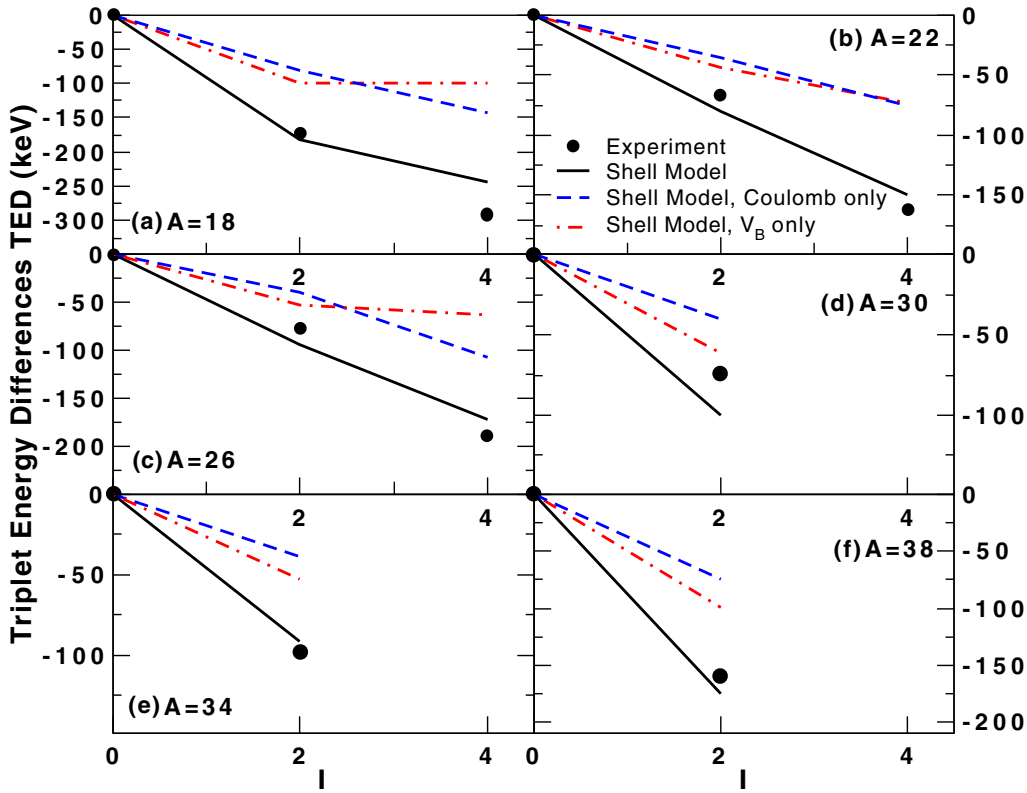

FIG. 2. Comparison of experimental triplet energy differences (TED) in the *sd* shell to the predictions of the shell model using the USD interaction—see text for details. Dots (black) show the experimental data. Solid line (black) shows the total TED calculation from the shell model. Dashed line (blue) shows the contribution to the calculated TED from the Coulomb two-body interaction alone. Dot-dashed line (red) shows the contribution to the calculated TED from the INC ($V_B^{(2)}$) interaction alone. The black line is the sum of the blue (dashed) and the red (dot-dashed) lines.

monopole centroid. This is shown by the dashed line in Fig. 1. The results are essentially consistent, and both approaches yield a strong and similar J dependence, despite there being relatively little overlap in the data used.

Based on the fit results here, a single INC matrix element of $+100$ keV for $J = 0$ couplings (only) is appropriate for TED calculations in the $f_{7/2}$ shell, and is consistent with the value originally estimated (and used) by Zuker *et al.* [3], and also with the analysis of Ref. [12]. This value is therefore adopted for all orbitals and is used for the results in the following section.

IV. COMPARISON OF EXPERIMENTAL AND THEORETICAL TRIPLET ENERGY DIFFERENCES

Using the shell-model methodology outlined in Sec. II, based on Eqs. (2) and (3), we have calculated the triplet energy differences of nuclei in the mass range $A = 18$–66 where experimental values are available. All the calculations have been performed by using the shell-model code ANTOINE [13,14]. Above $A = 66$, heavier nuclei become quite deformed and the inclusion of the gds orbitals becomes necessary, which is unfeasible at the moment due to computational capability.

For nuclei in the *sd* shell, the USD interaction [16,17] has been adopted in the full *sd* valence space. The $+100$ keV $V_B^{(2)}$ term has been added into each of the $d_{5/2}$, $s_{1/2}$, and $d_{3/2}$ orbitals. Six triplets have been calculated with $A = 18$, 22, 26, 30, 34, and 38, and the results are shown in Fig. 2. The total calculation [Eq. (3)] is plotted (solid line) and compared with the experimental TED (circles). Overall, the agreement is very good. Plotted separately for each triplet are the TED calculation assuming only the electromagnetic effects included in this shell-model description [i.e., the first term in Eq. (3)]; see the blue dashed line. It is clear that, in each case, this is insufficient by about a factor of two to account for the data. The red dot-dashed line shows the calculated contribution to the TED resulting from (only) the isotensor INC interaction, $V_B^{(2)}$. Overall, the inclusion of the $V_B^{(2)}$ term corrects the underestimation of the TED magnitude across the whole shell. A feature is that the INC and Coulomb contributions to the TED are, in this analysis, similar in trend and magnitude. This last point may be understood by recalling that the TED are sensitive to the J dependence of the isotensor matrix elements. Both the Coulomb and INC matrix elements have (coincidentally) a similar J dependence—reducing by about 100 keV as particles recouple from $J = 0$ to $J = J_{\max}$ (where $J_{\max}$ is the maximal alignment for the orbital).

In the $f_{7/2}$ shell, calculations were performed with the KB3G interaction [15] in the full pf valence space, with no restrictions on the total number of excitations across the $A = 56$ shell gap to the higher-lying pf orbits. Again, the $+100$ keV $V_B^{(2)}$ term has been added into each of the $f_{7/2}$,

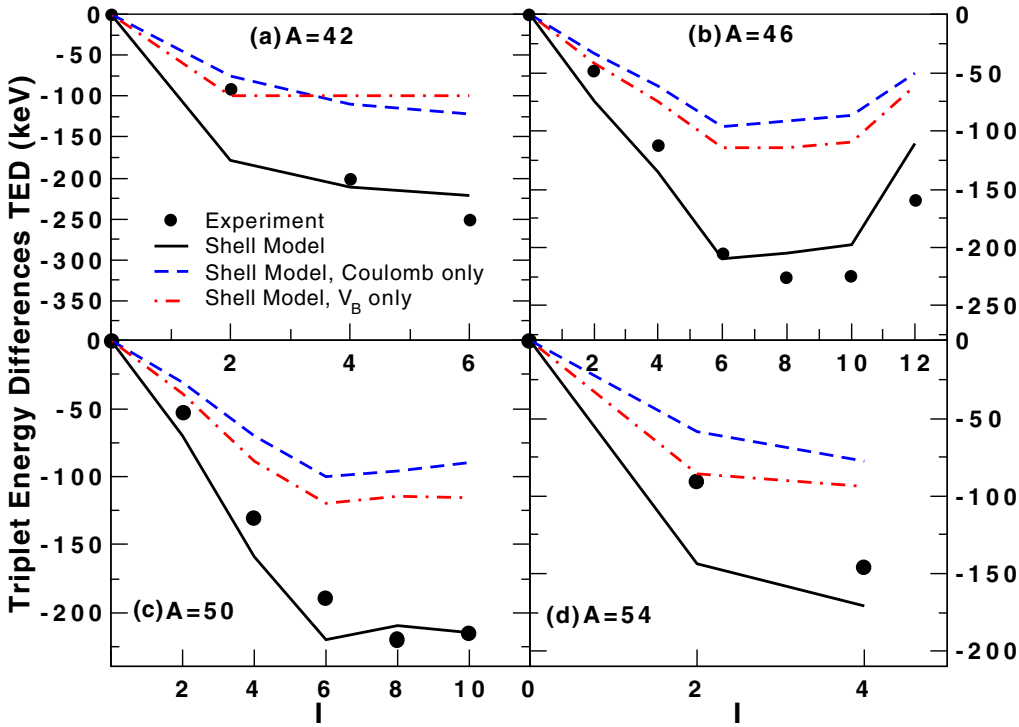

FIG. 3. Comparison of experimental triplet energy differences (TED) in the $f_{7/2}$ shell to the predictions of the shell model using the KB3G interaction—see caption of Fig. 2 for further details.

$p_{3/2}$, $f_{5/2}$ and $p_{1/2}$ orbitals. The results are shown in Fig. 3. Again, the overall agreement with the data is, in general, good—once both the electromagnetic and INC contributions are included and, again, both are clearly required. The exceptions to this are the first-excited 2^+ states in the $A = 42$ and 54 triplets. For $A = 42$, the assumption of a closed ^{40}Ca shell is a poor one, especially for the 2^+, and so the structure of this state may be poorly described. For $A = 54$, the reason for the poor description of the 2^+ state is not clear—the case of $A = 54$ will be addressed in more detail in the later discussion.

For the $A = 58$ and 62 triplets, the GXPF1A interaction [18–20] has been used; see Fig. 4. For heavier nuclei we have used the JUN45 [21] interaction in the pfg space—see below. Above ^{56}Ni, it is necessary, due to computational limitations, to restrict the number of excitations from the $f_{7/2}$ shell to the higher pf orbits—for $A = 58$ this was limited to eight particle-hole excitations from $f_{7/2}$ and, for $A = 62$, to six excitations. The analysis for $A = 58$ is made possible by the recent data on the yrast states of ^{58}Zn [9]. The agreement between the data and the calculation [see Fig. 4(b)] is excellent and would fail completely without the INC interaction included. For $A = 62$, there is some experimental uncertainty on the precise location of the $T = 1$ yrast states of $T_z = 0$ ^{62}Ga and $T_z = -1$ ^{62}Ge. $T = 1$ 2^+ and 4^+ states were suggested by Rudolph *et al.* [22] for ^{62}Ge following the tentative observation of two γ rays of 964 and 1321 keV, assumed to be the decays of the 2^+ and 4^+ states. These assignments await experimental confirmation. For $N = Z$ ^{62}Ga, only candidates for the $T = 1$ 2^+ have been identified. An $I = 2$ state at 1015 keV, first observed by Rudolph *et al.* [23], has been identified as the $T = 1$ 2^+ state on the basis of its similarity to the energy of the analog states in the other members of the multiplet. However, Henry *et al.* [24] subsequently identified a state at 977 keV which they suggested as an alternative candidate for the $T = 1$ 2^+ state, on the basis of the knockout methodology used in that work. In Fig. 4(c) two TED values for the 2^+ state are shown, where the two possible candidates described above have been used for ^{62}Ga. We cannot yet distinguish experimentally between these possibilities, so we simply note that one of these (the TED based on the 977 keV state from Ref. [24]) matches well with the calculations, and the other does not. As with other mass regions, the TED calculation provides a good agreement with the data only when the INC interaction is included.

The choice of where to switch from the KB3G to the GXPF1A interaction is not especially well defined. Currently, we have only used GXPF1A for $A = 58$ and 62, with KB3G and JUN45 used for lighter and heavier nuclei, respectively. For the sake of completeness, we have calculated the TED for the $A = 54$ triplet also with GXPF1A and plotted the results in Fig. 4(a). These results should be compared with those of Fig. 3(d), and clearly the results do not change significantly with the interaction.

Finally, we perform a calculation here for the $A = 66$ triplet using the JUN45 interaction [21]. This interaction allows the inclusion of the $g_{9/2}$ orbital, which is expected to become increasingly important above $A = 64$, but comes at the expense of closing the $f_{7/2}$ orbital. The valence space is therefore $p_{3/2}$, $f_{5/2}$, $p_{1/2}$, $g_{9/2}$ and, again a +100 keV $V_B^{(2)}$ term has been included in all orbits. The result, Fig. 5, follows the same pattern—the agreement with the model is

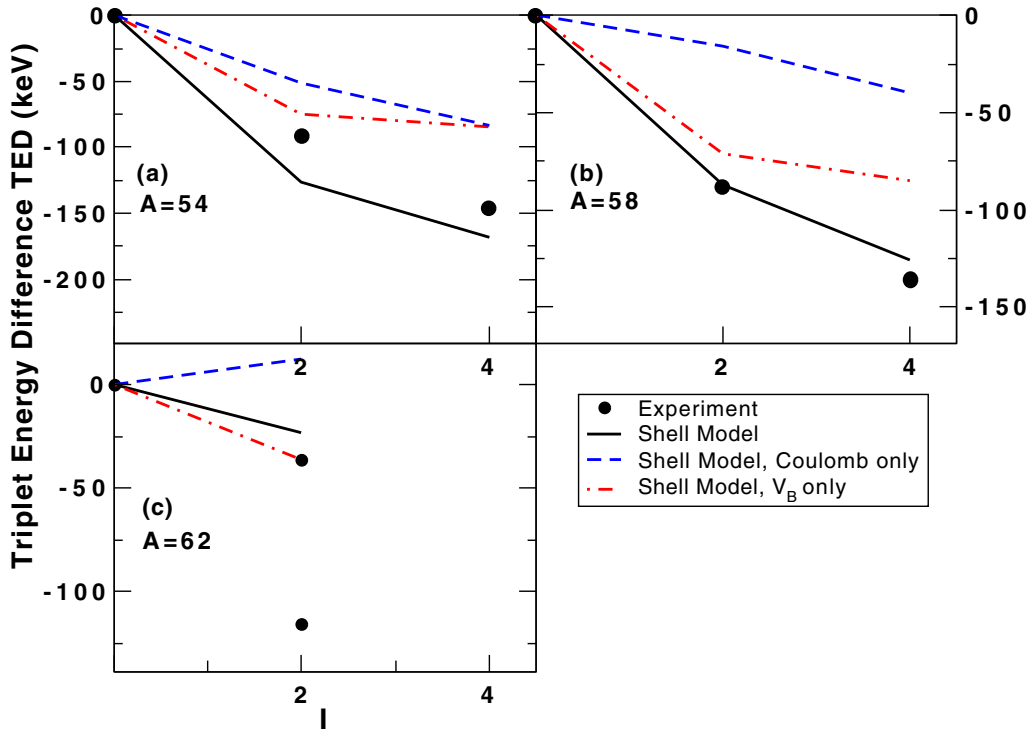

FIG. 4. Comparison of experimental triplet energy differences (TED) in the upper pf shell to the predictions of the shell model using the GXPF1A interaction—see caption of Fig. 2 for further details.

again excellent but requires the inclusion of the INC term. A very similar calculation for $A = 66$ has been performed by Kaneko and collaborators in Ref. [25] again using JUN45 and with an isotensor INC of +100 keV for $J = 0$ couplings, and similar conclusions were drawn. In that work, the same approach was used to predict TED for $A = 66$, 70, 74, and 78. A comparison with the data for $A = 70$ and 74 (the latter of which is the heaviest measured triplet) leads, largely, to the same conclusion about the required INC terms in the shell model. However, as one proceeds towards the well-deformed region above $A = 68$, the $p_{3/2}$, $f_{5/2}$, $p_{1/2}$, $g_{9/2}$ space of the JUN45 interaction becomes increasingly inadequate, since the inclusion of the upper ds orbitals will be required to generate sufficient deformation and collectivity [26]. Hence, we have placed an upper limit here of $A = 66$ for our analysis.

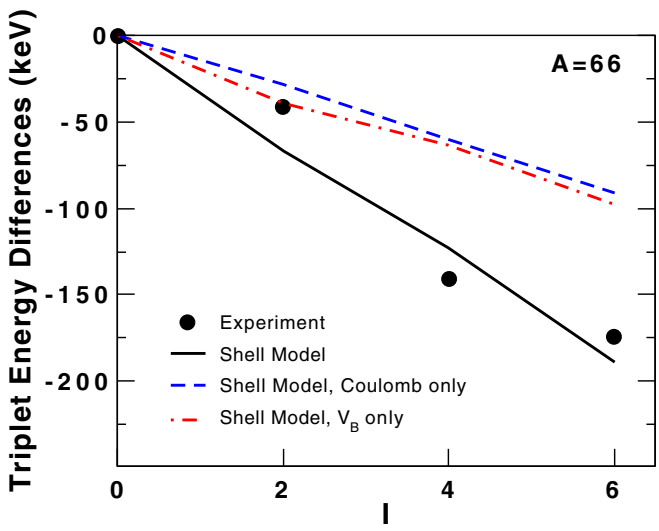

FIG. 5. Comparison of experimental $A = 66$ triplet energy differences (TED) to the predictions of the shell model using the JUN45 interaction—see caption of Fig. 2 for further details.

V. DISCUSSION

The analysis presented in the previous section demonstrates that the inclusion of the electromagnetic effects, through the shell-model method described in Sec. III, is insufficient in just about all cases to account for the experimental TED values. We have shown that an isotensor INC interaction $V_B^{(2)}(J = 0)$ of +100 keV is required. This picture is found across all the mass regions studied to date, irrespective of mass, valence space [i.e., dominant orbital(s)], proximity to shell closures, or degree of deformation or collectivity. The $V_B^{(2)}(J = 0)$ term we have established in this analysis for $A = 18$ to 66 is also consistent with that used by Kaneko *et al.* [25] to explain the data of the $A = 70$ and 74 $T = 1$ triplets [7,8].

It is clear from our analysis that these INC interactions are required in all orbitals in the valence space. This can be seen from Fig. 6 which shows the contribution to the TED arising from the inclusion of the $V_B^{(2)}$ interaction in each orbital. Five plots are presented in Fig. 6, for the TED calculations of $A = 26$, 30, 50, 58 and 66—covering all of the valence spaces and interactions used in this analysis. The contribution to the TED obtained through the inclusion of the $V_B^{(2)}$ term in each

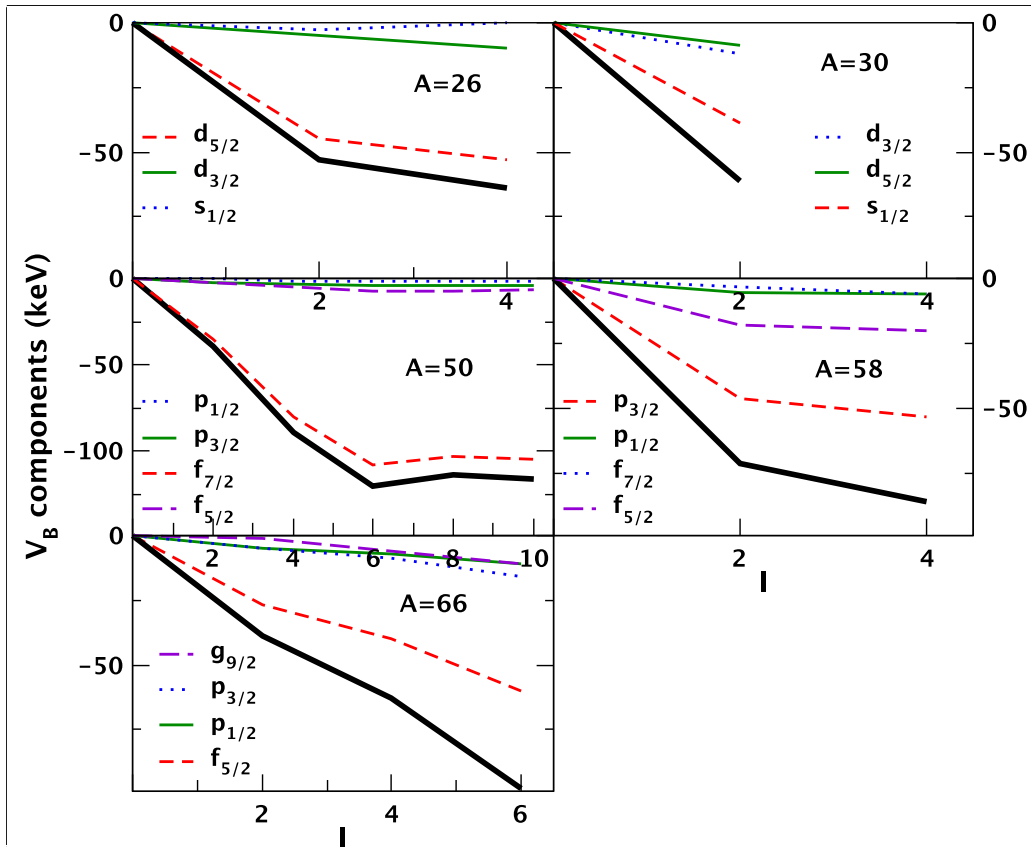

FIG. 6. The contribution to the calculated TED from the INC ($V_B^{(2)}$) interaction, determined for each orbital individually for selected examples. For each panel the sum of the single-orbital contributions (full thick black line) corresponds to the dot-dashed (red) curve in Figs. 2 to 5.

specific orbital is presented. In each case, it is clear that there is usually one dominant orbital (the one located at the Fermi surface, naturally)—in these cases the $s_{1/2}$, $d_{5/2}$, $f_{7/2}$, $p_{3/2}$, and $f_{5/2}$ orbitals, respectively. However, it is clear that the contributions from other orbitals can be significant.

The approach presented so far in this analysis considers a purely phenomenological INC interaction—we have identified a significant additional interaction, isotensor in nature, which is required to reproduce experimental TED data in a shell-model analysis. This of course points to phenomena which are missing from the model. One such effect is, of course, the true *nuclear* charge dependence of the NN interaction (all the interactions used here are charge independent).

Nucleon-nucleon scattering data yield information on the charge dependence of the nuclear force. Ormand and Brown [11] showed, based on the work of Henley [27], that the observed nn, pp, and np scattering lengths can be interpreted as the interaction V_{np} being about 2%–3% stronger than the average of V_{pp} and V_{nn}. If we now make the (perhaps unrealistic) assumption that the same fractional difference should appear in the residual interaction matrix elements of the shell model, then (based on an average $J = 0$ matrix element of $\sim$1.7 MeV for the main active orbitals in each region), this would correspond to $V_{np} - \frac{V_{pp}+V_{nn}}{2} \approx -40$ keV. This in turn corresponds to an isotensor interaction for $J = 0$ of $\sim$+80 keV—consistent with our above conclusions.

In Ref. [11], Ormand and Brown extracted some isovector and isotensor INC interactions through analysis of displacement energies, rather than TED - again through fitting the shell-model to experimental data. This is a different approach to that presented here, since TED are specifically sensitive to the J-dependence of the INC interaction. Nevertheless, their conclusion was broadly similar—a roughly constant isotensor INC interaction is needed in the *sd* and *pf* shells, and is consistent with the conclusions from scattering lengths.

The above analysis points to the need of considering the inclusion of realistic isospin-symmetry breaking nuclear interactions in the shell model. In terms of energy differences between $T = 1$ triplets, this has been attempted previously. Gadea *et al.* [1] presented the first results on excited states of ^{54}Ni allowing for an analysis of the $T = 1$, $A = 54$ triplet. In their shell-model study of the TED, a charge-dependent interaction based on the AV18 potential [28] was used, which includes both electromagnetic and (nuclear) charge-dependence effects. In this case, the correct trend of the TED with angular momentum was properly reproduced, although the magnitude of the TED was overpredicted by about a factor of two—the experimental TED was well reproduced when the shell-model result was scaled by a factor of 0.45.

In a recent work, Ormand *et al.* [2] produced a theoretical analysis of the isobaric multiplet mass equation (IMME) for $T = 1$ isospin triplets. In this work, binding energy (rather

than excitation energy) was computed and the "c coefficients" predicted for $T = 1$ triplets in the $f_{7/2}$ shell. The c coefficients of the IMME and the TED are connected through TED $= 2\Delta c_I$ where Δc_I is the c coefficient relative the ground state. Hence, the c coefficient has the identical I dependence as the TED, scaled by a factor of two. Ormand *et al.* incorporated three different types of charge-dependent interaction in a shell-model analysis, including renormalization of matrix elements appropriate to the valence space. In their analysis the magnitude of the c coefficients were reproduced fairly well, although underestimated, by using only Coulomb effects. The I dependence of the c coefficients (equivalent to the TED) is not well reproduced when taking into account just Coulomb effects. The inclusion of the charge-dependent components of the nuclear interaction significantly changes the I dependence of the results. However, as with the $A = 54$ case discussed above, the effect appears to be too large, and over-prediction results; see Ref. [2].

VI. CONCLUSIONS

The charge independence of the nuclear interaction has been investigated through the analysis of the triplet energy differences between analog states in $T = 1$ isobaric nuclei along a wide range of masses. It has been shown that an isospin-symmetry breaking interaction needs to be added to the Coulomb interaction to account for the experimental findings. The schematic isotensor ISB interaction, of monopole pairing type, deduced for nuclei in the $f_{7/2}$ shell has been extended here to all single-particle orbits, using the same strength. Its contribution to the TED is as important as the Coulomb contribution and allows us to reproduce data on TED with excellent accuracy. Due to the way TED are obtained—the (double-difference) form of Eq. (1), the TED equation—monopole terms cancel out and only multipole terms contribute to their theoretical description. Indeed, under the assumption that the three members of the multiplet have identical radial and deformation behavior, and that they have identical (analog) wave functions, the monopole electromagnetic effects vanish in the subtraction. This is at odds with what happens in the case of MED, where monopole contributions, related to nuclear structure features, are very important, and can even be dominant. This makes the TED an ideal tool to put in evidence isospin-independence breaking effects. From our analysis the ISB interaction within the shell-model framework has a simple "universal" form and strength all along the $N = Z$ line up to $A = 66$. Although a pure nuclear nature of the deduced INC interaction cannot be claimed, its effect is consistent with the ISB contribution to isospin-dependent effective interactions derived from realistic nucleon-nucleon potentials but its strength is smaller. While further investigations based on realistic interactions in a no-core approach may give a deeper microscopic understanding, the approach proposed in this work demonstrates a high predictive power for triplet energy differences.

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