Role of angle-dependent phase rotations of reaction amplitudes in η photoproduction on protons

A. Švarc,[1,*] H. Osmanović,[2] M. Hadžimehmedović,[2] R. Omerović,[2] J. Stahov,[2] M. Gorchteyn,[3] V. Kashevarov,[3] K. Nikonov,[3] M. Ostrick,[3] and L. Tiator[3]

[1]*Rudjer Bošković Institute, Bijenička Cesta 54, P.O. Box 180, 10002 Zagreb, Croatia*
[2]*University of Tuzla, Faculty of Natural Sciences and Mathematics, Univerzitetska 4, 75000 Tuzla, Bosnia and Herzegovina*
[3]*Institut für Kernphysik, Johannes Gutenberg-Universität Mainz, D-55099 Mainz, Germany*

(Received 20 July 2018; published 19 October 2018)

It was recently proved that the invariance of observables with respect to angle-dependent phase rotations of reaction amplitudes mixes multipoles, changing also their relative strength [A. Švarc *et al.*, Phys. Rev. C **97**, 054611 (2018)]. Modern partial wave analyses (PWAs) in η photoproduction on protons in the center-of-mass energy range $1.5 \leqslant W \leqslant 2.0$ GeV, either energy-dependent (ED) or single-energy (SE) ones, do not take this effect into consideration. It is commonly accepted that all PWAs give very similar results for the $E0+$ multipole, but notable differences in this and all other partial waves still remain. In this paper we demonstrate that once these phase rotations are properly taken into account, all ED and SE partial wave analysis in η photoproduction become almost identical for the dominant $E0+$ multipole, and the agreement among all other multipoles becomes much better. We also show that measured observables are almost equally well reproduced in all PWAs, and the remaining differences among multipoles can be attributed solely to the difference in predictions for unmeasured observables. So, new measurements are needed.

DOI: 10.1103/PhysRevC.98.045206

I. INTRODUCTION

In recent years, a wealth of new high-precision experimental data dominantly on photo- and electroproduction has been measured at various facilities including JLab, MAMI, LEPS, SLAC, and GRAAL for a number of observables with the goal of better understanding the spectrum of N^* and Δ^* resonances. Several coupled-channel models for photo- and electroproduction have been developed (BG [1], KSU [2], JuBo [3], Mainz-MAID [4], and SAID [5]), with the aim of including the plethora of new data into one, unified overall scheme. The number of channels varied from more than seven in most models to only two (η-N and η'-N) in Ref. [4]. A single-energy (SE), single-channel method for η photoproduction based on achieving the continuity of the solution by imposing fixed-t analyticity was recently added to these theoretical efforts [6]. As a result, a number of equivalent sets of highest partial waves for η photoproduction were generated. Now, after decades of research, it is commonly accepted that all partial wave analyses (PWAs) give very similar results for the $E0+$ multipole, but notable differences in this and all other partial waves still remain. These differences were mostly attributed to the difference in model assumptions (number of resonances, dynamics, background treatment, etc.) and in databases used to constrain the free model parameters (data selection, weighting, interpolations, data binning, etc.), and no one suspected that there might be another, fundamental reason why all these calculations disagree at least for the dominant multipoles. Hereafter we show that such a reason exists, and that it lies in the inadequate treatment of continuum-ambiguity effects which manifest themselves as angle-dependent phase rotations of reaction amplitudes [7].

II. FORMALISM

Since all observables in meson production processes are given in term of bilinears of one reaction amplitude with the complex conjugate of another one, they are invariant with respect to the energy- and angle-dependent phase rotation. This invariance is called continuum ambiguity [8–10]. We formalize it in the following way: The observables in single-channel reactions are given as a sum of products involving one amplitude (helicity, transversity, etc.) with the complex conjugate of another one, so that the general form of any observable is given as $\mathcal{O} = f(H_k \cdot H_l^*)$, where f is a known, well-defined real function. The direct consequence is that any observable is invariant with respect to the following simultaneous phase transformation of all amplitudes:

$$H_k(W,\theta) \to \tilde{H}_k(W,\theta) = e^{i\,\phi(W,\theta)} H_k(W,\theta) \quad \text{for all } k = 1,\dots,n, \tag{1}$$

where k is the index of the amplitude, n is the number of spin degrees of freedom ($n = 1$ for the one-dimensional model, $n = 2$ for π-N elastic scattering, and $n = 4$ for pseudoscalar meson photoproduction), $\phi(W,\theta)$ is an arbitrary real function

*Corresponding author: alfred.svarc@irb.hr

which is the same for all contributing amplitudes, and W and θ are the center-of-mass energy and scattering angle, respectively. Without any further physics constraints like unitarity, this real function $\phi(W,\theta)$ is free, and there exist an infinite number of equivalent solutions which give exactly the same set of observables. The invariance with respect to energy-dependent (ED) phase rotation has been investigated a lot, and synchronizing phases were introduced and analyzed on the level of partial waves [11–14]. These rotations can be handled without any problem. However, almost no attention has been paid to the situation when the arbitrary phase function is angle dependent. This possibility was mentioned in [15–18] where the effect was established, but the discussion was not followed through. The whole deduction chain for understanding the full role of angle-dependent phase rotations in continuum ambiguity was finally presented in [7].

Starting with Eq. (1), we focus on resonance properties of amplitudes $H_k(W,\theta)$. Since resonances are identified with poles of the partial-wave (or multipole) amplitudes, we must analyze the influence of the continuum ambiguity not upon helicity or transversity amplitudes, but upon their partial-wave decompositions. To streamline the study we introduce partial waves in a version simplified with respect to the form found in, for instance, Ref. [12]:

$$A(W,\theta)=\sum_{\ell=0}^{\infty}(2\ell+1)A_\ell(W)P_\ell(\cos\theta), \tag{2}$$

where $A(W,\theta)$ is a generic notation for any amplitude $H_k(W,\theta)$, $k=1,\dots,n$. The complete set of observables remains unchanged when we make the following transformation:

$$A(W,\theta)\to\tilde{A}(W,\theta)=e^{i\phi(W,\theta)}\sum_{\ell=0}^{\infty}(2\ell+1)A_\ell(W)P_\ell(\cos\theta),$$

$$\tilde{A}(W,\theta)=\sum_{\ell=0}^{\infty}(2\ell+1)\tilde{A}_\ell(W)P_\ell(\cos\theta). \tag{3}$$

We are interested in the rotated partial wave amplitudes $\tilde{A}_\ell(W)$, defined by Eq. (3), and are free to introduce the Legendre decomposition of an exponential function as

$$e^{i\,\phi(W,\theta)}=\sum_{\ell=0}^{\infty}L_\ell(W)P_\ell(\cos\theta). \tag{4}$$

After some manipulation of the product $P_\ell(x)P_k(x)$ (see Refs. [19,20] for details of the summation rearrangement) we obtain

$$\tilde{A}_\ell(W)=\sum_{\ell'=0}^{\infty}L_{\ell'}(W)\sum_{m=|\ell'-\ell|}^{\ell'+\ell}\langle\ell',0;\ell,0|m,0\rangle^2\,A_m(W), \tag{5}$$

where $\langle\ell',0;\ell,0|m,0\rangle$ is a standard Clebsch-Gordan coefficient. A similar relation was also derived in Ref. [16].

III. RESULTS AND DISCUSSION

To get a better insight into the mechanism of multipole mixing, let us expand Eq. (5) in terms of phase-rotation Legendre coefficients $L_{\ell'}(W)$, and demonstrate that angle-dependent phase invariance mixes multipoles:

$$\begin{aligned}
\tilde{A}_0(W)&=L_0(W)A_0(W)+L_1(W)A_1(W)+L_2(W)A_2(W)+\cdots,\\
\tilde{A}_1(W)&=L_0(W)A_1(W)+L_1(W)\left[\tfrac{1}{3}A_0(W)+\tfrac{2}{3}A_2(W)\right]+L_2(W)\left[\tfrac{2}{5}A_1(W)+\tfrac{3}{5}A_3(W)\right]+\cdots,\\
\tilde{A}_2(W)&=L_0(W)A_2(W)+L_1(W)\left[\tfrac{2}{5}A_1(W)+\tfrac{3}{5}A_3(W)\right]+L_2(W)\left[\tfrac{1}{5}A_0(W)+\tfrac{2}{7}A_2(W)+\tfrac{18}{35}A_4(W)\right]+\cdots.\\
&\vdots
\end{aligned} \tag{6}$$

We cite here the conclusion given in Ref. [7], in which the message essential for this paper is explicitly given:

> The consequence of Eqs. (5) and (6) is that angular-dependent phase rotations mix multipoles. Without fixing the free continuum ambiguity phase $\phi(W,\theta)$, the partial-wave decomposition $A_\ell(W)$ defined in Eq. (2) is non-unique. Partial waves get mixed, and identification of resonance quantum numbers might be changed. To compare different partial-wave analyses, it is essential to match continuum ambiguity phase; otherwise the mixing of multipoles is yet another, uncontrolled, source of systematic errors. Observe that this phase rotation does not create new pole positions, but just reshuffles the existing ones among several partial waves.

This is a starting point of our further analysis. We also observe that continuum-ambiguity invariance is discussed at the level of amplitudes, and can be applied to any choice of reaction amplitudes; in the following we apply our analysis to one such possible choice: helicity amplitudes.

We begin by comparing the dominant $E0+$ multipole for η photoproduction for the EtaMAID2018 solution of the Mainz EtaMAID model [4], the Bonn-Gatchina model [1,21], the Kent State University model [2,22], the Jülich-Bonn model [3,23], and three solutions[1] from the only SE analysis based on a fixed-t constraint [6]; see Fig. 1. We take the results of those models directly as we get them from original publications, and without paying any attention to the reaction amplitude phase. The shape is fairly similar, and the sign difference between coupled-channel models on one side and EtaMAID and SE solutions on the other can be attributed to the initial assumptions of the model. However, the discrepancies are

[1]Solutions I and II are Solutions II and III of Ref. [6] respectively, and the new yet unpublished solution which is obtained using the same formalism, but in which multipoles from Ref. [4] are used for the initial and first constraining solution, is denoted as Solution III.

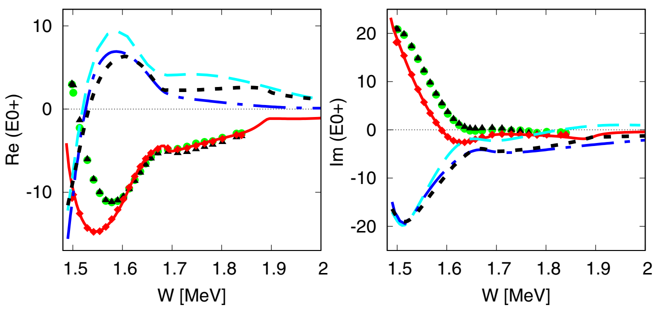

FIG. 1. Comparison of $E0+$ η photoproduction multipoles for the Mainz EtaMAID model [4] (red, full line), Bonn-Gatchina calculation [21] (black, short-dashed line), Kent State University calculation [22] (cyan, long-dashed line), Jülich-Bonn model [23] (blue, dash-dotted line), and three solutions from SE fixed-t analysis [6] (discrete symbols).

notable, and important. These figures are well known, and very recently shown in Fig. 4 of Ref. [2].[2]

As a second step, we perform the synchronization of phases among all models at the level of helicity amplitudes by introducing the following phase rotation:

$$\tilde{H}_k^{\mathrm{MD}}(W,\theta) = H_k^{\mathrm{MD}}(W,\theta)\, e^{i\,\Phi_{H_1}^{\mathrm{BG}}(W,\theta) - i\,\Phi_{H_1}^{\mathrm{MD}}(W,\theta)}, \quad k = 1,\dots,4, \tag{7}$$

where MD is the generic notation for Mainz-MAID, Bonn-Gatchina, Kent State University, Jülich-Bonn, and three fixed-t SE solutions, and $\Phi_{H_1}^{\mathrm{BG}}(W,\theta)$ is the phase of helicity amplitude $H_1(W,\theta)$ of the Bonn-Gatchina model. In this way we have practically replaced different phases of $H_1(W,\theta)$ amplitude of all models with only one phase, and this phase is arbitrarily (our convention) chosen to be the one from the Bonn-Gatchina model. Then we multiplied remaining three helicity amplitudes in all models with the same phase factor leaving the set of observables unchanged, and finally compared the rotated multipoles. So, the Bonn-Gatchina model results stay untouched, as the rotating phase for this model is 1, and the overall energy and angle-dependent phases of all other models are changed.

So, let us summarize the procedure:

(1) We reconstructed all four helicity amplitudes for all seven models from obtained multipoles.
(2) We applied the phase rotation defined by Eq. (7) to all four helicity amplitudes of all seven models
(3) We made a partial wave decomposition of rotated sets of amplitudes
(4) We show the final result for the rotated $E0+$ multipole in Fig. 2.

We stress that we could have taken the phase from any other model, and we could have decided to replace the phase of any out of three remaining helicity amplitudes

[2]Some small differences can be seen when one compares Fig. 1 of this publication and Fig. 4 of Ref. [2], but this is due to the different version of solutions used.

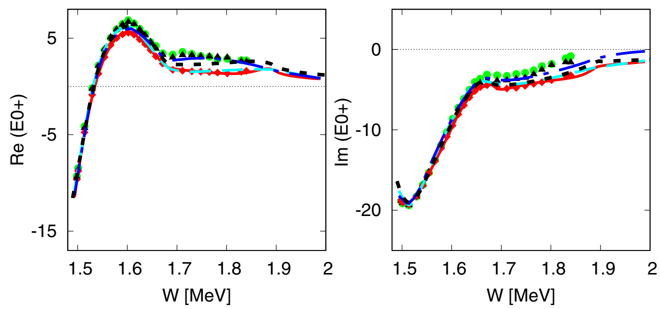

FIG. 2. Comparison of $E0+$ η photoproduction multipoles after the phase rotation defined with Eq. (7) for the Mainz EtaMAID model [4], Bonn-Gatchina calculation [21], Kent State University calculation [22], Jülich-Bonn model [23], and three solutions from SE fixed-t analysis [6]. The notation is the same as in Fig. 1.

$H_2(W,\theta)$–$H_4(W,\theta)$. The conclusion would be the same, but the figure would just have the different phase. As we claimed, the disagreement among all solutions for the $E0+$ multipole practically disappeared for energies $W_{CM} < 1650$ MeV, and it is significantly improved at higher energies.

We should also discuss whether it is allowed to touch the phase of reaction amplitudes obtained in coupled-channel calculations. Namely, continuum ambiguity (invariance with respect to the phase rotation) is the consequence of the loss of unitarity. Once the unitarity is restored, continuum ambiguity should disappear. However, the main aim of coupled-channel (CC) models is to restore the unitarity, so the phase ambiguity should be automatically eliminated. Or, a direct consequence should be that all phases of CC ED calculations should be the same, and the phase rotation defined in Eq. (7) should be equal to 1. On the other hand, in Fig. 2 we do see that disturbing differences for the $E0+$ multipole among all models have disappeared after we applied our phase-rotation synchronization. This means that the differences seen in Fig. 1 were the consequence of the mismatch of phases of reaction amplitudes, and that they were not generated by differences either in model assumptions or in databases chosen. After the phase rotation, the dominant $E0+$ multipoles shown in Fig. 2 are up to ≈1650 MeV practically identical for all models and all three SE solutions. This shows that all models have

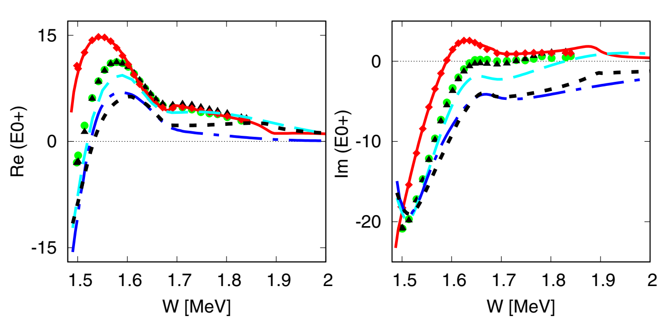

FIG. 3. Comparison of $E0+$ η photoproduction multipoles of the Kent State University calculation [22], Bonn-Gatchina calculation [21], Jülich-Bonn model [23], Mainz EtaMAID model [4], and three solutions from SE fixed-t analysis [6] after the latter four were multiplied with −1. The notation is the same as in Fig. 1.

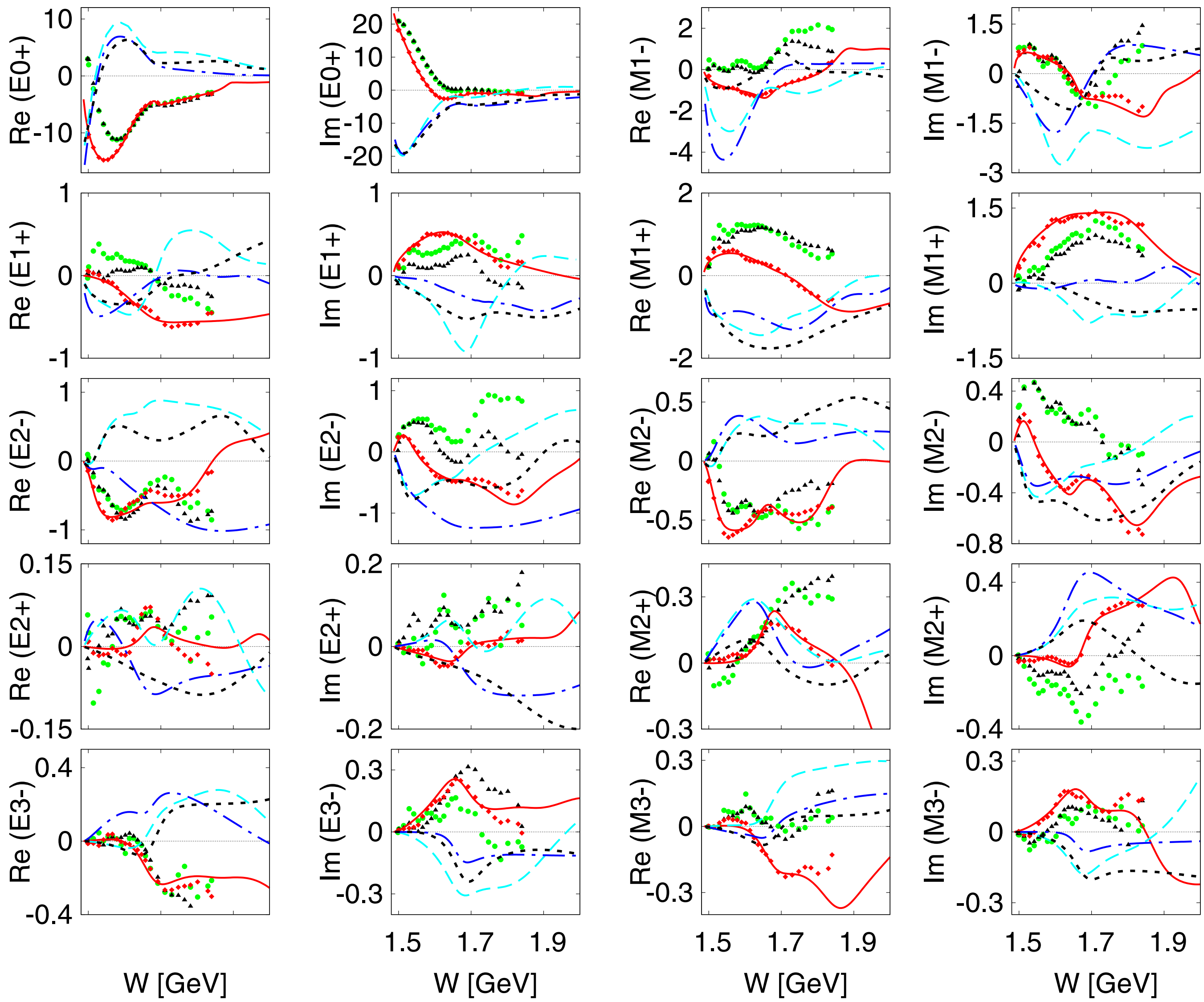

FIG. 4. Comparison of all η photoproduction multipoles for the Kent State University model [22], Bonn-Gatchina model [21], Jülich-Bonn model [23], Mainz EtaMAID model [4], and three solutions from SE fixed-t analysis [6]. The notation is the same as in Fig. 1.

included a sufficient amount of physics to get the unique S-wave result, but this also automatically confirms that the unitarity in CC calculations is not perfect,[3] and that it can only be achieved up to a certain approximation. It is understandable that since all channels can never be included in a realistic calculation, the treatment of unitarity can vary from one calculation to another. So, unitarity is only approximately restored, and the phase is only approximately obtained. This explains the fairly good agreement between ED calculations with many channels (Bonn-Gatchina, Jülich-Bonn, and Kent State University) in Fig. 1, while the discrepancy with the calculation where only two channels are included (η and η') is significant.

[3]Otherwise, the phases would be identical, and the phase rotation would have no effect.

It is important to stress that it might seem that Mainz EtaMAID and all three SE solutions only differ up to an overall sign from the remaining three ED calculations.[4] It is not so. We show in Fig. 3 that multiplying by -1 does not give any major improvement at all. On the other hand, synchronizing the phase on the level of helicity amplitudes, shown in Fig. 2, solves the problem.

[4]This sign is an isospin convention, and is -1 for MAID/SAID and $+1$ for BnGa/JuBo/KSU. The phase of three SE solutions is similar to the Mainz EtaMAID solution. The reason for that lies in the mechanism of fixed-t constraining. The fixed-t method is a sophisticated way of fixing the free phase, and it is done by constraining it to the "MAID type" models. So all three solutions also notably deviate from CC ED calculations, and resemble Mainz EtaMAID type models very much.

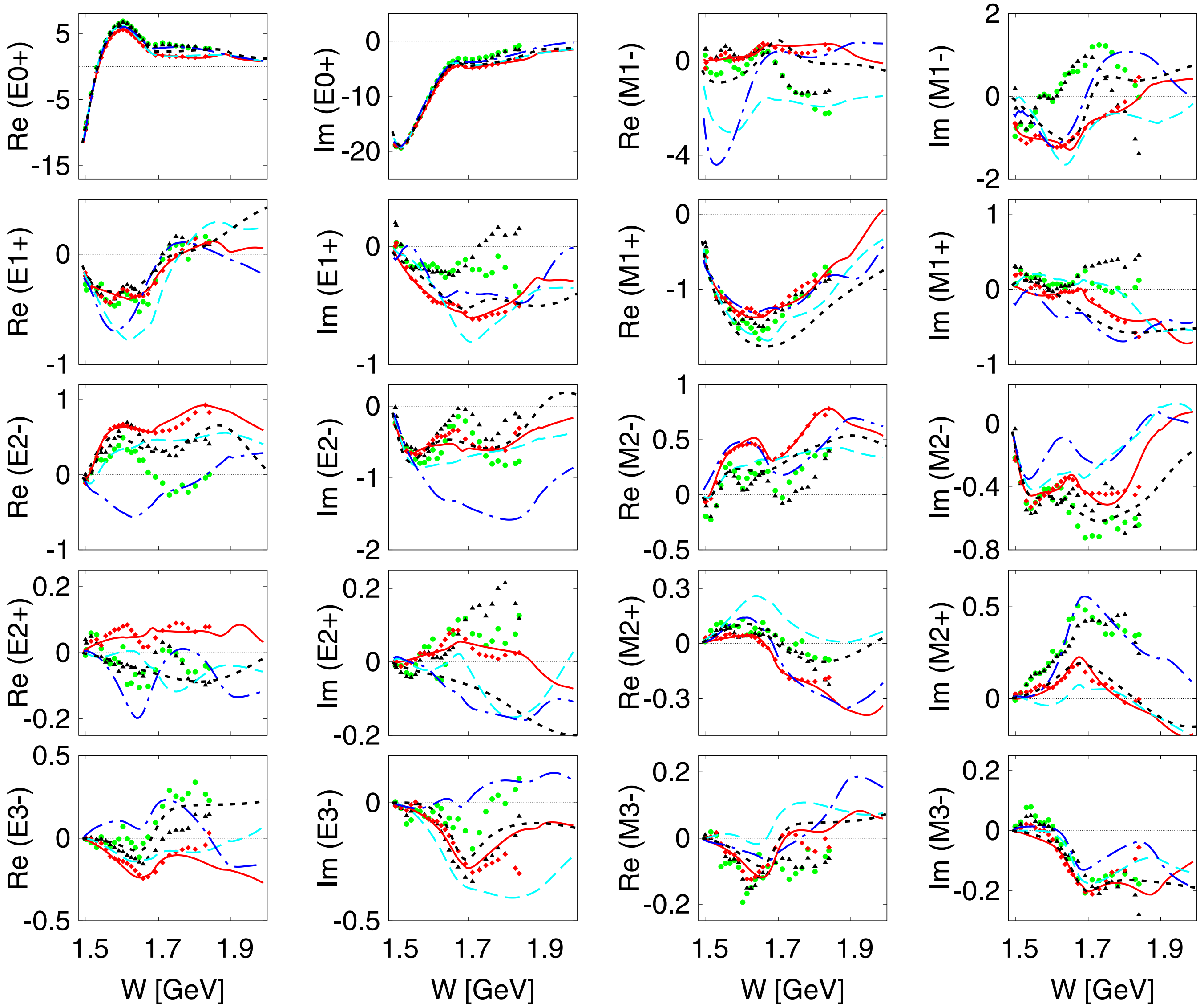

FIG. 5. Comparison of all η photoproduction multipoles after the phase rotation defined with Eq. (7) for the Kent State University model [22], Bonn-Gatchina model [21], Jülich-Bonn model [23], Mainz EtaMAID model [4], and three solutions from SE fixed-t analysis [6]. The notation is the same as in Fig. 1.

Comparison of other multipoles is shown in Figs. 4 and 5. In Fig. 4 we show the comparison of nonrotated multipoles, exactly as they are given in original publications, and in Fig. 5 we display their comparison after the phase rotation defined by Eq. (7). We see that the grouping of solutions after the phase rotation is for some multipoles improved, but no definite consensus can yet be made. So, it seems that we have seven solutions with very similar S-wave results, and which are rather different elsewhere. Consequently, the difference should be visible when we show the prediction for all observables from all seven analyzed solutions. The agreement of all solutions with measured observables should be very similar, and for the unmeasured ones it can be very different. So, in Fig. 6 we show the predictions for 12 measured and unmeasured

TABLE I. Experimental data from A2@MAMI and GRAAL used in our PWA.

Obs.	N	E_{lab} (MeV)	N_E	θ_{cm} (deg)	N_θ	Reference
σ_0	2400	710–1395	120	18–162	20	A2@MAMI (2010, 2017) [27,28]
Σ	150	724–1472	15	40–160	10	GRAAL (2007) [29]
T	144	725–1350	12	24–156	12	A2@MAMI (2014) [30]
F	144	725–1350	12	24–156	12	A2@MAMI (2014) [30]

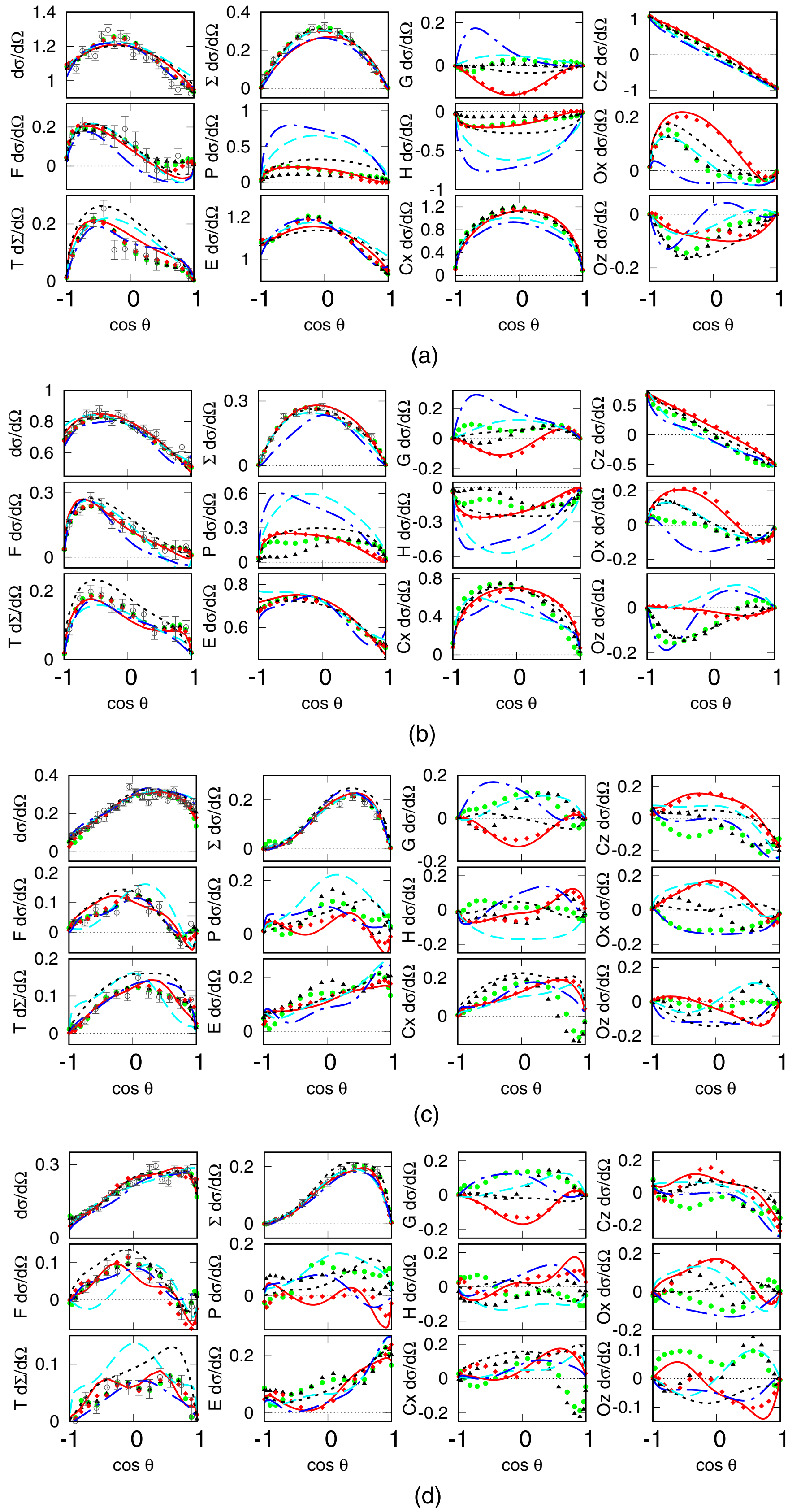

FIG. 6. Panels (a)–(d) show predictions for all seven solutions for 12 measured and unmeasured observables at $W_{CM} = 1554$, 1602, 1765, and 1840 MeV respectively. Experimental data are shown with grey symbols with error bars, and the notation of all seven model is given in Fig. 1.

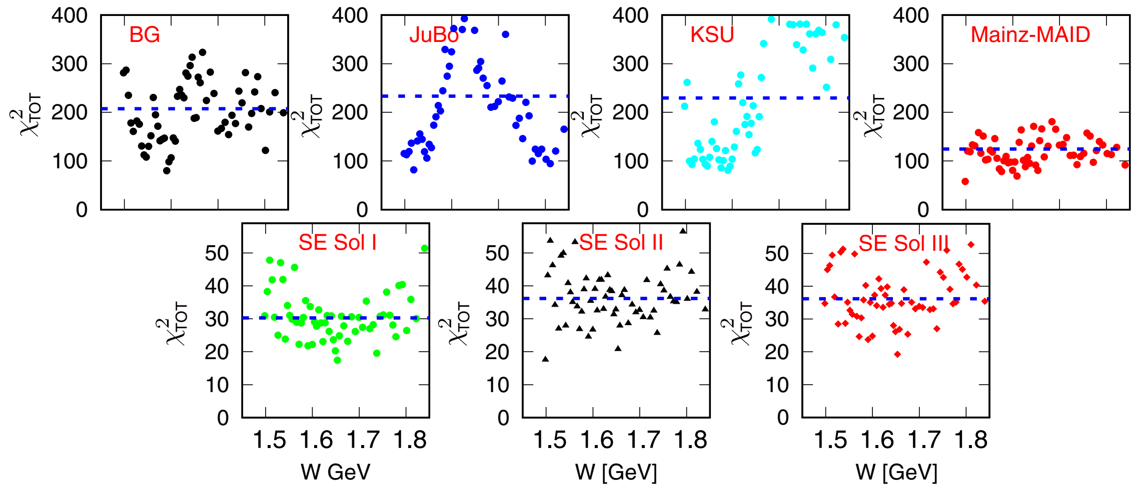

FIG. 7. Energy distribution of $\chi^2_{\rm TOT}$ for all seven solutions. Blue dashed lines indicate the $\chi^2_{\rm TOT}$ average over all energies.

observables at four typical energies of $W_{CM} = 1554$, 1602, 1765, and 1840 MeV for the database of Ref. [6]. We used recent A2@MAMI data for the unpolarized differential cross section σ_0, single target polarization asymmetry T, and double beam-target polarization with circular polarized photons F. In addition we used the GRAAL data for single beam polarization Σ. For details, see Table I. There are additional data from CLAS [24,25] and from CBELSA [26], which we do not use in our analysis. Our center-of-mass energy limit is 1.85 GeV for SE PWA, and 2.0 GeV for ED PWA. At low energy the cross section data from MAMI have much better statistics. At the higher energies we did not analyze, as our fixed-t method becomes more difficult at higher energies. We see that the agreement of all seven solutions with measured observables is good, so the reason why $L \neq 0$ partial waves in all seven solutions shown in Fig. 5 differ has to be found in other, nonmeasured observables which significantly disagree for all model predictions. To quantify this discussion we in Fig. 7 show the energy distribution of $\chi^2_{\rm TOT}$ for all seven solutions for the database described by Table I. One has to be very careful not to confuse our numbers with numbers quoted in original publications, because they are produced with a different database, but the overall trend must be similar. In this analysis, systematic errors are also not explicitly included.

The best agreement with the data is achieved for the all three SE solutions obtained by the fixed-t analysis. This is normal as this is a model-independent, single-channel and single-energy analysis which is made continuous by fixing the phase by imposing fixed-t analyticity. The second best agreement is shown by the Mainz EtaMAID analysis which is a two-channel analysis (η-N and η'-N channels) with more free parameters per analyzed datum than the remaining three ED analyses, so this is not a surprise too. The apparently worst result is shown by BG, JuBo, and KSU ED analyses, but this was to be expected as they fit many more channels at the same time, and some compromise among channels has to be made. Due to the coupling with other channels such as π-N, σ-N, ρ-N, π-Δ, K-Λ, K-Σ, ω-N, the BnGa, JuBo, and KSU analysis have significantly larger $\chi^2_{\rm TOT}$ values in some energy regions. What is surprising is the energy dependence of $\chi^2_{\rm TOT}$ in all three ED coupled-channel models, which still awaits some explanation.

IV. CONCLUSIONS

As a summary, we state that matching angular dependent phases of all solutions on the level of helicity amplitudes brings all $E0+$ multipoles from all seven analyzed PWA into complete agreement. The differences in other partial waves remain. New measurements are needed to fix higher partial waves.

ACKNOWLEDGMENTS

We want to thank Deborah Rönchen of the Jülich-Bonn Collaboration, Victor Nikonov of the Bonn-Gatchina Collaboration, and Mark Manley of the Kent State University Collaboration for providing us with their most recent partial wave results. This work was supported by the Deutsche Forschungsgemeinschaft (SFB 1044).

[1] A. V. Anisovich, V. Burkert, N. Compton, K. Hicks, F. J. Klein, E. Klempt, V. A. Nikonov, A. M. Sandorfi, A. V. Sarantsev, and U. Thoma, Phys. Rev. C **96**, 055202 (2017), and references therein.
[2] B. C. Hunt and D. M. Manley, arXiv:1804.06031.
[3] D. Rönchen, M. Döring, and U.-G. Meißner, Eur. Phys. J. A **54**, 110 (2018), and references therein.
[4] L. Tiator, M. Gorchteyn, V. L. Kashevarov, K. Nikonov, M. Ostrick, M. Hadžimehmedović, R. Omerović, H. Osmanović, J. Stahov, and A. Švarc, arXiv:1807.04525.
[5] For SAID solutions GE09 and E429 see Ref. [27].
[6] H. Osmanović, M. Hadžimehmedović, R. Omerović, J. Stahov, V. Kashevarov, K. Nikonov, M. Ostrick, L. Tiator, and A. Švarc, Phys. Rev. C **97**, 015207 (2018).

[7] A. Švarc, Y. Wunderlich, H. Osmanović, M. Hadžimehmedović, R. Omerović, J. Stahov, V. Kashevarov, K. Nikonov, M. Ostrick, L. Tiator, and R. Workman, Phys. Rev. C **97**, 054611 (2018).
[8] D. Atkinson, P. W. Johnson, and R. L. Warnock, Commun. Math. Phys. **33**, 221 (1973).
[9] J. E. Bowcock and H. Burkhard, Rep. Prog. Phys. **38**, 1099 (1975).
[10] D. Atkinson and I. S. Stefanescu, Commun. Math. Phys. **101**, 291 (1985).
[11] A. V. Anisovich, R. Beck, E. Klempt, V. A. Nikonov, A. V. Sarantsev, and U. Thoma, Eur. Phys. J. A **48**, 15 (2012).
[12] L. Tiator, D. Drechsel, S. S. Kamalov, and M. Vanderhaeghen, Eur. Phys. J. Spec. Top. **198**, 141 (2011).
[13] A. M. Sandorfi, S. Hoblit, H. Kamano, and T.-S. H. Lee, J. Phys. G: Nucl. Part. Phys. **38**, 053001 (2011).
[14] M. Shrestha and D. M. Manley, Phys. Rev. C **86**, 045204 (2012)
[15] A. S. Omalaenko, Sov. J. Nucl. Phys. **34**, 406 (1981).
[16] J.-P. Dedonder, W. R. Gibbs, and Mutazz Nuseirat, Phys. Rev. C **77**, 044003 (2008).
[17] N. W. Dean and P. Lee, Phys. Rev. D **5**, 2741 (1972).
[18] G. Keaton and R. Workman, Phys. Rev. C **54**, 1437 (1996).
[19] J. Dougall, Glasgow Math. J. **1**, 121 (1953).
[20] Y. Wunderlich, A. Švarc, R. L. Workman, L. Tiator, and R. Beck, Phys. Rev. C **96**, 065202 (2017).
[21] V. Nikonov provided us with tables from Ref. [1].
[22] B. C. Hunt provided us with tables from Ref. [2].
[23] D. Rönchen provided us with tables from Ref. [3].
[24] M. Dugger *et al.* (CLAS Collaboration), Phys. Rev. Lett. **89**, 222002 (2002).
[25] M. Williams *et al.* (CLAS Collaboration), Phys. Rev. C **80**, 045213 (2009).
[26] V. Crede *et al.* (CBELSA/TAPS Collaboration), Phys. Rev. C **80**, 055202 (2009).
[27] E. F. McNicoll *et al.* (Crystal Ball Collaboration at MAMI), Phys. Rev. C **82**, 035208 (2010); **84**, 029901(E) (2011).
[28] V. L. Kashevarov *et al.*, Phys. Rev. Lett. **118**, 212001 (2017).
[29] O. Bartalini *et al.* (GRAAL Collaboration), Eur. Phys. J. A **33**, 169 (2007).
[30] C. S. Akondi *et al.* (A2 Collaboration at MAMI), Phys. Rev. Lett. **113**, 102001 (2014).