Surveying exotic pentaquarks with the typical $QQqq\bar{q}$ configuration

Qin-Song Zhou,[1,2] Kan Chen,[1,2] Xiang Liu,[1,2,*] Yan-Rui Liu,[3,†] and Shi-Lin Zhu[4,5,6,‡]

[1]*School of Physical Science and Technology, Lanzhou University, Lanzhou 730000, China*

[2]*Research Center for Hadron and CSR Physics, Lanzhou University and Institute of Modern Physics of CAS, Lanzhou 730000, China*

[3]*School of Physics and Key Laboratory of Particle Physics and Particle Irradiation (MOE), Shandong University, Jinan 250100, China*

[4]*School of Physics and State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, China*

[5]*Collaborative Innovation Center of Quantum Matter, Beijing 100871, China*

[6]*Center of High Energy Physics, Peking University, Beijing 100871, China*

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As a hot issue, exploring exotic pentaquarks is full of challenges and opportunities for both theorist and experimentalist. In this work, we focus on a type of pentaquark with the $QQqq\bar{q}$ ($Q = b, c$; $q = u, d, s$) configuration, where their mass spectrum is estimated systematically. In particular, our result indicates that there may exist some stable or narrow exotic pentaquark states. Obviously, our study may provides valuable information for further experimental search for the $QQqq\bar{q}$ pentaquarks. With the running of large Hadron collider beauty (LHCb) and forthcoming Belle II, we have a reason to believe that these predictions presented here can be tested.

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I. INTRODUCTION

Since the proposal of the quark model [1,2], it is popular to identify multiquark states from both theoretical and experimental sides. Many exotic XYZ states observed by experiments in recent years [3–15] are considered as possible tetraquark candidates [16–24]. With one more quark component, intriguing pentaquark states were also studied in various colliders. Although the subsequent experiments [25] did not confirm the light Θ^+ pentaquark with component $uudd\bar{s}$ as claimed by the LEPS Collaboration [26], the large Hadron collider beauty (LHCb) experiment brought us new findings in the heavy quark realm in 2015 [27]. Two hidden-charm pentaquark-like resonances $P_c(4380)$ and $P_c(4450)$ are extracted in the $J/\psi p$ invariant mass distribution of the Λ_b^0 decay into $J/\Psi K^- p$. This observation stimulated further studies on pentaquark states [20,28,29]. In this paper, we pay attention to the $QQqq\bar{q}$ systems, where $Q = b, c$ and $q = u, d, s$, and roughly estimate the masses of such pentaquark states.

In the quark model, the doubly charmed baryon Ξ_{cc} ($J^P = \frac{1}{2}^+$ or $\frac{3}{2}^+$) is in a 20-plet representation of the flavor SU(4) classification [30]. Although its study started 40 years ago [31], its existence has been confirmed very recently [32–34]. The confirmation from LHCb motivates further theoretical studies on the possible stable T_{QQ} ($QQ\bar{q}\bar{q}$) states, which had been predicted in various models. Both the Ξ_{cc} baryon and the T_{QQ} meson contain a heavy diquark. Now we would like to add one more light quark component and discuss the spectra of the doubly heavy pentaquarks within a simple model. The so-called heavy diquark-antiquark symmetry was used to relate the mass splittings of QQq and $QQ\bar{q}\bar{q}$ in Ref. [35]. We hope that the present investigation can also be helpful for further study on such a symmetry in multiquark systems.

Compared to the QQq baryon, the $QQqq\bar{q}$ pentaquark state should be heavier. However, the complicated interactions within multiquark systems may lower the mass, which probably makes it difficult to distinguish experimentally a conventional baryon from a pentaquark baryon just from the mass consideration. One example for this feature is the five newly observed Ω_c states [36,37]. They can be accommodated in both $3q$ configuration [38–46] and $5q$ configuration [47–53] and much more measurements are needed to resolve their nature. As a theoretical prediction, the basic features for the pentaquark spectra may be useful for us to understand possible structures of heavy quark hadrons.

For the doubly heavy five-quark systems, we have a compact $QQqq\bar{q}$ configuration and two baryon-meson molecule configurations, $(QQq)(q\bar{q})$ and $(Qqq)(Q\bar{q})$. As for the latter molecule configuration, there are theoretical studies in the meson exchange methods [54–56]. Here, we discuss the mass splittings of the compact $QQqq\bar{q}$ pentaquark states by considering the color-magnetic interactions between quarks and estimate their rough positions. It is still an open question how to distinguish the two configurations. For example, if we compare the prediction for the Λ-type hidden charm state in the molecule picture [57] and the estimation for the mass of the lowest $c\bar{c}uds$ compact pentaquark [58], one gets consistent results. However, the numbers of possible states in these two pictures are different. The present study should

*xiangliu@lzu.edu.cn
†yrliu@sdu.edu.cn
‡zhusl@pku.edu.cn

be useful in looking for genuine pentaquark states rather than molecules.

This paper is organized as follows. In Sec. II, we construct the flavor $\otimes$ color $\otimes$ spin wave functions for the $QQqq\bar{q}$ pentaquark states. In Sec. III, the relevant Hamiltonians for various systems are presented. In Sec. IV, we give numerical results and discuss the mass spectra of the pentaquark states and their strong decay channels. Finally, we present a summary in Sec. V.

II. COLOR-MAGNETIC INTERACTION AND WAVE FUNCTIONS

The few-body problem is difficult to deal with and there are scarce dynamical studies on pentaquark systems without substructure assumptions [59,60]. To understand systematically the basic features for the properties of multiquark states, as the first step, we here adopt a color-magnetic model and mainly focus on the mass splittings of the S-wave pentaquark states. For the pentaquark masses, we just present some estimations. Their accurate values need further dynamical calculations. For the ground-state hadrons with the same quark content, e.g., Δ and N, their mass splitting is mainly determined by the color-magnetic interaction (CMI). The Hamiltonian in this model reads

$$H = \sum_i m_i + H_{\mathrm{CM}},$$
$$H_{\mathrm{CM}} = -\sum_{i<j} C_{ij}\vec{\lambda}_i\cdot\vec{\lambda}_j\vec{\sigma}_i\cdot\vec{\sigma}_j = -\sum_{i<j} C_{ij}\lambda_i^a\lambda_j^a\sigma_i^b\sigma_j^b, \tag{1}$$

where λ_i^a $(a = 1, \ldots, 8)$ are the Gell-Mann matrices for the ith quark and σ_j^b $(b = 1, 2, 3)$ are the Pauli matrices for the jth quark. For antiquarks, the $\vec{\lambda}_i$ is replaced with $-\vec{\lambda}_i^*$. The effective mass m_i for the ith quark includes the constituent quark mass and contributions from color-electric interactions and color confinements. The effective coupling constants C_{ij} depend on the quark masses and the ground-state spatial wave functions.

The model is an oversimplified one of the realistic quark interactions. We may check its relation with the leading-order Hamiltonian in nonrelativistic approximation in Ref. [31] (ignore the electromagnetic part):

$$\hat{H} = L(\vec{r}_1,\vec{r}_2,\ldots) + \sum_i\left(m_{0i} + \frac{\vec{p}_i}{2m_{0i}}\right) + \frac{1}{4}\sum_{i>j}\alpha_s\vec{\lambda}_i\cdot\vec{\lambda}_j S_{ij}. \tag{2}$$

Here, L is responsible for quark binding and $\vec{r}_i$, $\vec{p}_i$, and m_{0i} are the position, momentum, and mass of the ith quark, respectively. S_{ij} has the form

$$S_{ij} = \frac{1}{|\vec{r}|} - \frac{1}{2m_{0i}m_{0j}}\left(\frac{\vec{p}_i\cdot\vec{p}_j}{|\vec{r}|} + \frac{\vec{r}\cdot(\vec{r}\cdot\vec{p}_i)\vec{p}_j}{|\vec{r}|^3}\right) - \frac{\pi}{2}\delta^3(\vec{r})\left(\frac{1}{m_{0i}^2} + \frac{1}{m_{0j}^2} + \frac{4\vec{\sigma}_i\cdot\vec{\sigma}_j}{3m_{0i}m_{0j}}\right)$$
$$- \frac{1}{4|\vec{r}|^3}\left\{\frac{\vec{r}\times\vec{p}_i\cdot\vec{\sigma}_i}{m_{0i}^2} - \frac{\vec{r}\times\vec{p}_j\cdot\vec{\sigma}_j}{m_{0j}^2} + \frac{1}{m_{0i}m_{0j}}\left[2\vec{r}\times\vec{p}_i\cdot\vec{\sigma}_j - 2\vec{r}\times\vec{p}_j\cdot\vec{\sigma}_i - \vec{\sigma}_i\cdot\vec{\sigma}_j + 3\frac{(\vec{\sigma}_i\cdot\vec{r})(\vec{\sigma}_j\cdot\vec{r})}{|\vec{r}|^2}\right]\right\}, \tag{3}$$

where $\vec{r} = \vec{r}_i - \vec{r}_j$. For S-wave hadrons, the last two lines (spin-orbit and tensor parts) have vanishing contributions. By calculating the average value with the orbital wave function Ψ_0 $(L = 0)$, one may write the Hamiltonian as

$$\begin{aligned} H &= \langle\Psi_0|\hat{H}|\Psi_0\rangle \\ &= \left\{\langle\Psi_0|\left[L(\vec{r}_1,\vec{r}_2,\ldots) + \sum_i\left(m_{0i} + \frac{\vec{p}_i}{2m_{0i}}\right)\right]|\Psi_0\rangle + \frac{1}{4}\sum_{i>j}\vec{\lambda}_i\cdot\vec{\lambda}_j\langle\Psi_0|\alpha_s\left[\frac{1}{|\vec{r}|} - \left(\frac{1}{m_{0i}^2} + \frac{1}{m_{0j}^2}\right)\frac{\pi}{2}\delta^3(\vec{r})\right.\right. \\ &\quad \left.\left. - \frac{1}{2m_{0i}m_{0j}}\left(\frac{\vec{p}_i\cdot\vec{p}_j}{|\vec{r}|} + \frac{\vec{r}\cdot(\vec{r}\cdot\vec{p}_i)\vec{p}_j}{|\vec{r}|^3}\right)\right]|\Psi_0\rangle\right\} - \sum_{i>j}\frac{\pi}{6}\langle\Psi_0|\alpha_s\delta^3(\vec{r})|\Psi_0\rangle\frac{\vec{\lambda}_i\cdot\vec{\lambda}_j\vec{\sigma}_i\cdot\vec{\sigma}_j}{m_{0i}m_{0j}} \\ &\equiv M_0 - \sum_{i>j} C_{ij}\vec{\lambda}_i\cdot\vec{\lambda}_j\vec{\sigma}_i\cdot\vec{\sigma}_j. \end{aligned} \tag{4}$$

For states with the same quark content, M_0 is a constant and it can be expressed as the summation of effective quark masses $M_0 = \sum_i m_i$. Then the model Hamiltonian we will use is obtained. In principle, the values of m_i and C_{ij} should be different for various systems. However, it is difficult to exactly calculate these parameters for a given system without knowing the spatial wave function. In the present study, they will be extracted from the masses of conventional hadrons. That is to say, we use the assumption that quark-quark interactions are the same for various systems. This assumption certainly leads to uncertainties on hadron masses. The uncertainty cause by m_i does not allow us to give accurate pentaquark masses while the uncertainty in coupling parameters has smaller effects and the mass splittings should be more reliable. In order to reduce the uncertainties and obtain more appropriate estimations, we will try to use an alternative form of the mass formula. Whether this manipulation gives results close to realistic masses or not can be tested in future measurements.

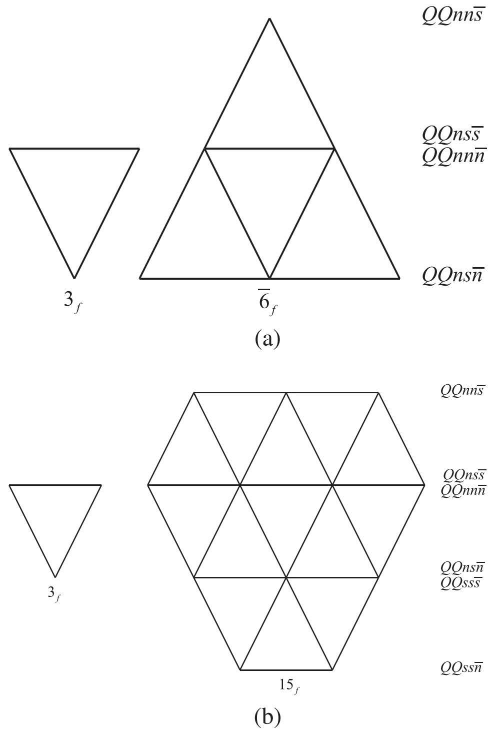


FIG. 1. $SU(3)_f$ weight diagrams for the $QQqq\bar{q}$ pentaquark states. (a) The two light quarks belong to $\bar{3}_f$ and (b) The two light quarks belong to 6_f.

Obviously, we can calculate the color-magnetic matrix elements and investigate the mass spectra for the $QQqq\bar{q}$ systems if the wave functions were constructed. Now we move on to the construction of the flavor-color-spin wave function of a system, which is a direct product of $SU(3)_f$ flavor wave function, $SU(3)_c$ color wave function, and $SU(2)_s$ spin wave function. We construct these wave functions separately and then combine them together by noticing the possible constraint from the Pauli principle. We will use the diquark-diquark-antiquark bases to construct the wave function. In principle, the selection of wave function bases is irrelevant with the final results since we will diagonalize the Hamiltonian in this CMI model. Here, the notation "diquark" only means two quarks and it does not mean a compact substructure.

In flavor space, the heavy quarks are treated as $SU(3)_f$ singlet states and the light diquark may be in the flavor antisymmetric $\bar{3}_f$ or symmetric 6_f representation. For the case of the antisymmetric (symmetric) light diquark, the representations of the pentaquarks are $\bar{6}_f$ and 3_f (3_f and 15_f). We plot the $SU(3)_f$ weight diagrams for the $QQqq\bar{q}$ systems in Fig. 1. The explicit wave functions are similar to the $qq\bar{q}\bar{Q}$ tetraquark states presented in Ref. [61]. Because of the unequal quark masses, we consider $SU(3)_f$ symmetry breaking and the flavor mixing among different representations occurs. The resulting systems we consider are $Q_1Q_2nn\bar{n}$, $Q_1Q_2nn\bar{s}$, $Q_1Q_2ns\bar{n}$, $Q_1Q_2ns\bar{s}$, $Q_1Q_2ss\bar{n}$, and $Q_1Q_2ss\bar{s}$, where n represents u or d.

In color space, the Young diagrams tell us that the pentaquark systems have three color singlets. Then we have three color wave functions. The direct product for the representations can be written as

$$\begin{aligned}
&(3_c \otimes 3_c) \otimes (3_c \otimes 3_c) \otimes \bar{3}_c \\
&= (\bar{3}_c \oplus 6_c) \otimes (\bar{3}_c \oplus 6_c) \otimes \bar{3}_c \\
&= (\bar{3}_c \otimes \bar{3}_c \otimes \bar{3}_c) \oplus (\bar{3}_c \otimes 6_c \otimes \bar{3}_c) \oplus (6_c \otimes \bar{3}_c \otimes \bar{3}_c).
\end{aligned} \tag{5}$$

In the last line, the representations in the parentheses are for the heavy diquark, light diquark, and antiquark, respectively. Then the color-singlet wave functions can be constructed as

$$\begin{aligned}
\phi^{AA} &= [(Q_1Q_2)^{\bar{3}_c}(q_3q_4)^{\bar{3}_c}\bar{q}], \\
\phi^{AS} &= [(Q_1Q_2)^{\bar{3}_c}(q_3q_4)^{6_c}\bar{q}], \\
\phi^{SA} &= [(Q_1Q_2)^{6_c}(q_3q_4)^{\bar{3}_c}\bar{q}],
\end{aligned} \tag{6}$$

where A (S) means antisymmetric (symmetric) for the diquarks. Explicitly, we have

$$\begin{aligned}
\phi^{AA} = \frac{1}{2\sqrt{6}}[&(rbbg - rbgb + brgb - brbg + gbrb - gbbr \\
&+ bgbr - bgrb)\bar{b} + (rbrg - rbgr + brgr - brrg \\
&+ grrb - grbr + rgbr - rgrb)\bar{r} + (gbrg - gbgr \\
&+ bggr - bgrg + grgb - grbg + rgbg - rggb)\bar{g}],
\end{aligned} \tag{7}$$

$$\begin{aligned}
\phi^{AS} = \frac{1}{4\sqrt{3}}[&(2rgbb - 2grbb - rbgb - rbbg + brgb \\
&+ brbg + gbrb + gbbr - bgrb - bgbr)\bar{b} + (2gbrr \\
&- 2bgrr - rbrg - rbgr + brrg + brgr \\
&- grrb - grbr + rgrb + rgbr)\bar{r} + (2brgg \\
&- 2rbgg + gbrg + gbgr - bgrg - bggrg - grgb \\
&- grbg + rggb + rgbg)\bar{g}],
\end{aligned} \tag{8}$$

$$\begin{aligned}
\phi^{SA} = \frac{1}{4\sqrt{3}}[&(2bbgr - 2bbrg + gbrb - gbbr + bgrb \\
&- bgbr - rbgb + rbbg - brgb + brbg)\bar{b} \\
&+ (2rrbg - 2rrgb + rgrb - rgbr + grrb \\
&- grbr + rbgr - rbrg + brgr - brrg)\bar{r} \\
&+ (2ggrb - 2ggbr - rggb + rgbg - grgb + grbg \\
&+ gbgr - gbrg + bggr - bgrg)\bar{g}].
\end{aligned} \tag{9}$$

The spin wave functions for the pentaquark states are

$$\chi^{SS}: \begin{cases}
\chi_1 = [(Q_1Q_2)_1(q_3q_4)_1\bar{q}]_2^{\frac{5}{2}}, \\
\chi_2 = [(Q_1Q_2)_1(q_3q_4)_1\bar{q}]_2^{\frac{3}{2}}, \\
\chi_3 = [(Q_1Q_2)_1(q_3q_4)_1\bar{q}]_1^{\frac{3}{2}}, \\
\chi_4 = [(Q_1Q_2)_1(q_3q_4)_1\bar{q}]_1^{\frac{1}{2}}, \\
\chi_5 = [(Q_1Q_2)_1(q_3q_4)_1\bar{q}]_0^{\frac{1}{2}},
\end{cases} \tag{10}$$

TABLE I. Defined variables to simplify the CMI expressions.

Variable	Definition	Variable	Definition
α	$C_{12}+C_{34}$	β	$C_{13}+C_{14}+C_{23}+C_{24}$
λ	$C_{15}+C_{25}$	γ	$C_{13}+C_{14}-C_{23}-C_{24}$
μ	$C_{15}-C_{25}$	δ	$C_{13}-C_{14}+C_{23}-C_{24}$
ν	$C_{35}+C_{45}$	η	$C_{13}-C_{14}-C_{23}+C_{24}$
ρ	$C_{35}-C_{45}$		
θ	$C_{12}-3C_{34}$		
τ	$3C_{12}-C_{34}$		

$$\chi^{SA}:\begin{cases}\chi_6=[(Q_1Q_2)_1(q_3q_4)_0\bar{q}]_1^{\frac{3}{2}},\\ \chi_7=[(Q_1Q_2)_1(q_3q_4)_0\bar{q}]_1^{\frac{1}{2}},\end{cases} \tag{11}$$

$$\chi^{AS}:\begin{cases}\chi_8=[(Q_1Q_2)_0(q_3q_4)_1\bar{q}]_1^{\frac{3}{2}},\\ \chi_9=[(Q_1Q_2)_0(q_3q_4)_1\bar{q}]_1^{\frac{1}{2}},\end{cases} \tag{12}$$

$$\chi^{AA}:\chi_{10}=[(Q_1Q_2)_0(q_3q_4)_0\bar{q}]_0^{\frac{1}{2}}. \tag{13}$$

Here in the symbol $[(Q_1Q_2)_{\rm spin}(q_3q_4)_{\rm spin}\bar{q}]_j^{\rm total\ spin}$, j is the total spin of the first four quarks. The superscript SA of χ means that the first two quarks are symmetric and the second two quarks are antisymmetric. Other superscripts are understood similarly.

Considering the Pauli principle, we obtain twelve types of total wave functions: $[\phi^{AA}\otimes\chi^{SS}]\delta_{34}^A$, $[\phi^{AA}\otimes\chi^{SA}]\delta_{34}^S$, $[\phi^{AA}\otimes\chi^{AS}]\delta_{12}\delta_{34}^A$, $[\phi^{AA}\otimes\chi^{AA}]\delta_{12}\delta_{34}^S$, $[\phi^{AS}\otimes\chi^{SS}]\delta_{34}^S$, $[\phi^{AS}\otimes\chi^{SA}]\delta_{34}^A$, $[\phi^{AS}\otimes\chi^{AS}]\delta_{12}\delta_{34}^S$, $[\phi^{AS}\otimes\chi^{AA}]\delta_{12}\delta_{34}^A$, $[\phi^{SA}\otimes\chi^{SS}]\delta_{12}\delta_{34}^A$, $[\phi^{SA}\otimes\chi^{SA}]\delta_{12}\delta_{34}^S$, $[\phi^{SA}\otimes\chi^{AS}]\delta_{34}^A$, and $[\phi^{SA}\otimes\chi^{AA}]\delta_{34}^S$. Here, $\delta_{12}=0$ when the first two quarks are identical, or else $\delta_{12}=1$. When the two light quarks are antisymmetric (symmetric) in the flavor space, $\delta_{34}^A=0$ ($\delta_{34}^S=0$), or else $\delta_{34}^A=1$ ($\delta_{34}^S=1$). Then the considered pentaquark states are categorized into six classes:

(1) the $(ccnn)^{I=1}\bar{q}$, $(bbnn)^{I=1}\bar{q}$, $(ccss)\bar{q}$, and $(bbss)\bar{q}$ states with $\delta_{12}=\delta_{34}^S=0$;
(2) the $(ccnn)^{I=0}\bar{q}$ and $(bbnn)^{I=0}\bar{q}$ states with $\delta_{12}=\delta_{34}^A=0$;
(3) the $(bcnn)^{I=1}\bar{q}$ and $(bcss)\bar{q}$ states with $\delta_{12}=1$ and $\delta_{34}^S=0$;
(4) the $(bcnn)^{I=0}\bar{q}$ states with $\delta_{12}=1$ and $\delta_{34}^A=0$;
(5) the $(ccns)\bar{q}$ and $(bbns)\bar{q}$ states with $\delta_{12}=0$ and $\delta_{34}^S=\delta_{34}^A=1$; and
(6) the $(bcns)\bar{q}$ states with $\delta_{12}=\delta_{34}^A=\delta_{34}^S=1$.

In the following discussions, we also use the notation $[(Q_1Q_2)_{\rm spin}^{\rm color}(q_3q_4)_{\rm spin}^{\rm color}\bar{q}]_j^{\rm total\ spin}$ to denote the total wave function.

III. THE HAMILTONIAN EXPRESSIONS

With the constructed wave functions, we calculate color-magnetic matrix elements on various bases. In this section, we present the obtained Hamiltonians in the matrix form. To simplify the expressions, we use the variables defined in Table I.

A. $(ccnn)^{I=1}\bar{q}$, $(bbnn)^{I=1}\bar{q}$, $(ccss)\bar{q}$, and $(bbss)\bar{q}$ states in the first class

Three types of basis vectors are involved in calculating the relevant matrix elements: $[\phi^{AA}\otimes\chi^{SS}]\delta_{34}^A$, $[\phi^{AS}\otimes\chi^{SA}]\delta_{34}^A$, and $[\phi^{SA}\otimes\chi^{AS}]\delta_{34}^A$.

For the $J^P=\frac{5}{2}^-$ states, there is only one basis vector $[(QQ)_1^{\bar{3}}(q_3q_4)_1^{\bar{3}}\bar{q}]_2^{\frac{5}{2}}$. The obtained Hamiltonian is

$$\langle H_{\rm CM}\rangle_{J=\frac{5}{2}}=\tfrac{2}{3}(4\alpha+\beta+2\lambda+2\nu). \tag{14}$$

For the $J^P=\frac{3}{2}^-$ states, we have four basis vectors, $[(QQ)_1^{\bar{3}}(q_3q_4)_1^{\bar{3}}\bar{q}]_2^{\frac{3}{2}}$, $[(QQ)_1^{\bar{3}}(q_3q_4)_1^{\bar{3}}\bar{q}]_1^{\frac{3}{2}}$, $[(QQ)_1^{\bar{3}}(q_3q_4)_0^{6}\bar{q}]_1^{\frac{3}{2}}$, and $[(QQ)_0^{6}(q_3q_4)_1^{\bar{3}}\bar{q}]_1^{\frac{3}{2}}$. The resulting Hamiltonian is

$$\langle H_{\rm CM}\rangle_{J=\frac{3}{2}}=\frac{2}{3}\begin{pmatrix}4\alpha+\beta-3(\lambda+\nu) & \sqrt{5}(\nu-\lambda) & 3\sqrt{5}\nu & 3\sqrt{5}\lambda\\ \sqrt{5}(\nu-\lambda) & 4\alpha-\beta+\lambda+\nu & 3(\beta-\nu) & 3(\lambda-\beta)\\ 3\sqrt{5}\nu & 3(\beta-\nu) & \frac{1}{2}(9\alpha-\theta)-\lambda & -\frac{3}{2}\beta\\ 3\sqrt{5}\lambda & 3(\lambda-\beta) & -\frac{3}{2}\beta & \frac{1}{2}(9\alpha+\tau)-\nu\end{pmatrix}. \tag{15}$$

For the $J^P=\frac{1}{2}^-$ states, the basis vectors are $[(QQ)_1^{\bar{3}}(q_3q_4)_1^{\bar{3}}\bar{q}]_1^{\frac{1}{2}}$, $[(QQ)_1^{\bar{3}}(q_3q_4)_0^{6}\bar{q}]_1^{\frac{1}{2}}$, $[(QQ)_0^{6}(q_3q_4)_1^{\bar{3}}\bar{q}]_1^{\frac{1}{2}}$, and $[(QQ)_1^{\bar{3}}(q_3q_4)_1^{\bar{3}}\bar{q}]_0^{\frac{1}{2}}$ and the Hamiltonian reads

$$\langle H_{\rm CM}\rangle_{J=\frac{1}{2}}=\frac{2}{3}\begin{pmatrix}4\alpha-\beta-2(\lambda+\nu) & 3(\beta+2\nu) & -3(\beta+2\lambda) & 2\sqrt{2}(\nu-\lambda)\\ 3(\beta+2\nu) & \frac{1}{2}(9\alpha-\theta)+2\lambda & -\frac{3}{2}\beta & -3\sqrt{2}\nu\\ -3(\beta+2\lambda) & -\frac{3}{2}\beta & \frac{1}{2}(9\alpha+\tau)+2\nu & -3\sqrt{2}\lambda\\ 2\sqrt{2}(\nu-\lambda) & -3\sqrt{2}\nu & -3\sqrt{2}\lambda & 4\alpha-2\beta\end{pmatrix}. \tag{16}$$

B. $(ccnn)^{I=0}\bar{q}$ and $(bbnn)^{I=0}\bar{q}$ states in the second class

In this case, we also have three types of basis vectors to consider: $[\phi^{AA}\chi^{SA}]\delta_{34}^S$, $[\phi^{AS}\chi^{SS}]\delta_{34}^S$, and $[\phi^{SA}\chi^{AA}]\delta_{34}^S$.

For the $J^P=\frac{5}{2}^-$ states, the involved basis vector is $[(QQ)_1^{\bar{3}}(nn)_1^6\bar{q}]_2^{\frac{5}{2}}$ and the obtained Hamiltonian is

$$\langle H_{\rm CM}\rangle_{J=\frac{5}{2}}=\frac{1}{3}(3\tau-\alpha+5\beta-2\lambda+10\nu). \tag{17}$$

For the $J^P=\frac{3}{2}^-$ states, there are three basis vectors: $[(QQ)_1^{\bar{3}}(nn)_1^6\bar{q}]_2^{\frac{3}{2}}$, $[(QQ)_1^{\bar{3}}(nn)_1^6\bar{q}]_1^{\frac{3}{2}}$, and $[(QQ)_1^{\bar{3}}(nn)_0^{\bar{3}}\bar{q}]_1^{\frac{3}{2}}$. We can get the following Hamiltonian:

$$\langle H_{\rm CM}\rangle_{J=\frac{3}{2}}=\frac{1}{3}\begin{pmatrix}3\tau-\alpha+5\beta+3\lambda-15\nu & \sqrt{5}(\lambda+5\nu) & 6\sqrt{5}\nu\\ \sqrt{5}(\lambda+5\nu) & 3\tau-\alpha-5\beta-\lambda+5\nu & 6(\beta-\nu)\\ 6\sqrt{5}\nu & 6(\beta-\nu) & 4(2\theta+\lambda)\end{pmatrix}. \tag{18}$$

For the $J^P=\frac{1}{2}^-$ states, we have four basis vectors: $[(QQ)_1^{\bar{3}}(nn)_1^6\bar{q}]_1^{\frac{1}{2}}$, $[(QQ)_1^{\bar{3}}(nn)_0^{\bar{3}}\bar{q}]_1^{\frac{1}{2}}$, $[(QQ)_0^6(nn)_0^{\bar{3}}\bar{q}]_0^{\frac{1}{2}}$, and $[(QQ)_1^{\bar{3}}(nn)_1^6\bar{q}]_0^{\frac{1}{2}}$. Then the Hamiltonian

$$\langle H_{\rm CM}\rangle_{J=\frac{1}{2}}=\frac{1}{3}\begin{pmatrix}3\tau-\alpha-5\beta+2\lambda-10\nu & 6(\beta+2\nu) & 0 & 2\sqrt{2}(\lambda+5\nu)\\ 6(\beta+2\nu) & 8(\theta-\lambda) & 6\sqrt{6}\lambda & -6\sqrt{2}\nu\\ 0 & 6\sqrt{6}\lambda & 3(3\theta+\alpha) & 3\sqrt{3}\beta\\ 2\sqrt{2}(\lambda+5\nu) & -6\sqrt{2}\nu & 3\sqrt{3}\beta & 3\tau-\alpha-10\beta\end{pmatrix} \tag{19}$$

can be obtained.

C. $(cbnn)^{I=1}\bar{q}$ and $(cbss)\bar{q}$ states in the third class

Now, one does not need to consider the constraint for the heavy diquark from the Pauli principle and we then have six types of basis vectors, $[\phi^{AA}\chi^{SS}]\delta_{34}^A$, $[\phi^{AA}\chi^{AS}]\delta_{12}\delta_{34}^A$, $[\phi^{AS}\chi^{SA}]\delta_{34}^A$, $[\phi^{AS}\chi^{AA}]\delta_{12}\delta_{34}^A$, $[\phi^{SA}\chi^{SS}]\delta_{12}\delta_{34}^A$, and $[\phi^{SA}\chi^{AS}]\delta_{34}^A$.

For the $J^P=\frac{5}{2}^-$ states, two basis vectors, $[(cb)_1^{\bar{3}}(q_3q_4)_1^{\bar{3}}\bar{q}]_2^{\frac{5}{2}}$ and $[(cb)_1^6(q_3q_4)_1^{\bar{3}}\bar{q}]_2^{\frac{5}{2}}$, are involved and the obtained Hamiltonian is

$$\langle H_{\rm CM}\rangle_{J=\frac{5}{2}}=\frac{1}{3}\begin{pmatrix}2(4\alpha+\beta+2\lambda+2\nu) & 3\sqrt{2}(\gamma-2\mu)\\ 3\sqrt{2}(\gamma-2\mu) & 5\beta+10\lambda-2\nu-\alpha-3\theta\end{pmatrix}. \tag{20}$$

For the $J^P=\frac{3}{2}^-$ states, there are seven basis vectors, $[(cb)_1^{\bar{3}}(q_3q_4)_1^{\bar{3}}\bar{q}]_2^{\frac{3}{2}}$, $[(cb)_1^{\bar{3}}(q_3q_4)_1^{\bar{3}}\bar{q}]_1^{\frac{3}{2}}$, $[(cb)_0^{\bar{3}}(q_3q_4)_1^{\bar{3}}\bar{q}]_1^{\frac{3}{2}}$, $[(cb)_1^{\bar{3}}(q_3q_4)_0^6\bar{q}]_1^{\frac{3}{2}}$, $[(cb)_1^6(q_3q_4)_1^{\bar{3}}\bar{q}]_2^{\frac{3}{2}}$, $[(cb)_1^6(q_3q_4)_1^{\bar{3}}\bar{q}]_1^{\frac{3}{2}}$, and $[(cb)_0^6(q_3q_4)_1^{\bar{3}}\bar{q}]_1^{\frac{3}{2}}$. One obtains the Hamiltonian as follows:

$$\langle H_{\rm CM}\rangle_{J=\frac{3}{2}}=\frac{2}{3}\begin{pmatrix}\begin{pmatrix}4\alpha+\beta\\-3(\lambda+\nu)\end{pmatrix} & \sqrt{5}(\nu-\lambda) & -\sqrt{10}\mu & 3\sqrt{5}\nu & \frac{3}{\sqrt{2}}(\gamma+3\mu) & \frac{3\sqrt{10}}{2}\mu & 3\sqrt{5}\lambda\\ \sqrt{5}(\nu-\lambda) & \begin{pmatrix}4\alpha-\beta\\+\lambda+\nu\end{pmatrix} & -\sqrt{2}(\gamma+\mu) & 3(\beta-\nu) & \frac{3\sqrt{10}}{2}\mu & -\frac{3}{\sqrt{2}}(\gamma+\mu) & 3(\lambda-\beta)\\ -\sqrt{10}\mu & -\sqrt{2}(\gamma+\nu) & 2(\nu-2\tau) & \frac{3}{\sqrt{2}}\gamma & 3\sqrt{5}\lambda & 3(\lambda-\beta) & 0\\ 3\sqrt{5}\nu & 3(\beta-\nu) & \frac{3}{\sqrt{2}}\gamma & \frac{1}{2}(9\alpha-\theta)-\lambda & 0 & -\frac{3}{\sqrt{2}}\gamma & -\frac{3}{2}\beta\\ \frac{3}{\sqrt{3}}(\gamma+3\mu) & \frac{3\sqrt{10}}{2}\mu & 3\sqrt{5}\lambda & 0 & \frac{1}{2}\begin{pmatrix}5\beta-15\lambda+\\3\nu-\alpha-3\theta\end{pmatrix} & -\frac{\sqrt{5}}{2}(5\lambda+\nu) & -\frac{5\sqrt{10}}{2}\mu\\ \sqrt{5}(\nu-\lambda) & -\frac{3}{\sqrt{2}}(\gamma+\mu) & 3(\lambda-\beta) & -\frac{3}{\sqrt{2}}\gamma & -\frac{\sqrt{5}}{2}(5\lambda+\nu) & \frac{1}{2}\begin{pmatrix}5\lambda-\alpha-3\theta\\-5\beta-\nu\end{pmatrix} & -\frac{5}{\sqrt{2}}(\gamma+\mu)\\ 3\sqrt{5}\lambda & 3(\lambda-\beta) & 0 & -\frac{3}{2}\beta & -\frac{5\sqrt{10}}{2}\mu & -\frac{5}{\sqrt{2}}(\gamma+\mu) & \frac{1}{2}(9\alpha+\tau)-\nu\end{pmatrix}. \tag{21}$$

For the $J^P=\frac{1}{2}^-$ states, eight basis vectors are involved, $[(cb)_1^{\bar{3}}(q_3q_4)_1^{\bar{3}}\bar{q}]_1^{\frac{1}{2}}$, $[(cb)_1^{\bar{3}}(q_3q_4)_1^{\bar{3}}\bar{q}]_0^{\frac{1}{2}}$, $[(cb)_0^{\bar{3}}(q_3q_4)_1^{\bar{3}}\bar{q}]_1^{\frac{1}{2}}$, $[(cb)_1^{\bar{3}}(q_3q_4)_0^6\bar{q}]_1^{\frac{1}{2}}$, $[(cb)_0^{\bar{3}}(q_3q_4)_0^6\bar{q}]_0^{\frac{1}{2}}$, $[(cb)_1^6(q_3q_4)_1^{\bar{3}}\bar{q}]_1^{\frac{1}{2}}$, $[(cb)_1^6(q_3q_4)_1^{\bar{3}}\bar{q}]_0^{\frac{1}{2}}$, and $[(cb)_0^6(q_3q_4)_1^{\bar{3}}\bar{q}]_1^{\frac{1}{2}}$. The resulting Hamiltonian is

$$\langle H_{\mathrm{CM}}\rangle_{J=\frac{1}{2}} = \frac{2}{3}\begin{pmatrix} \begin{pmatrix}4\alpha-\beta- \\ 2\lambda-2\nu\end{pmatrix} & 2\sqrt{2}(\nu-\lambda) & \sqrt{2}(2\mu-\gamma) & 3(\beta+2\nu) & 0 & \frac{3}{\sqrt{2}}(2\mu-\gamma) & 6\mu & -3(\beta+2\lambda) \\ 2\sqrt{2}(\nu-\lambda) & 2(2\alpha-\beta) & 2\mu & -3\sqrt{2}\nu & -\frac{3\sqrt{6}}{2}\gamma & 6\mu & -3\sqrt{2}\gamma & -3\sqrt{2}\lambda \\ \sqrt{2}(2\mu-\gamma) & -3\sqrt{2}\nu & -4(\tau+\nu) & \frac{3}{\sqrt{2}}\gamma & 3\sqrt{6}\nu & -3(\beta+2\lambda) & -3\sqrt{2}\lambda & 0 \\ 3(\beta+2\nu) & -\frac{3\sqrt{6}}{2}\gamma & \frac{3}{\sqrt{2}}\gamma & \frac{1}{2}(9\alpha-\theta)+2\lambda & \sqrt{3}\mu & -\frac{3}{\sqrt{2}}\gamma & 0 & -\frac{3}{2}\beta \\ 0 & -\frac{3\sqrt{6}}{2}\gamma & 3\sqrt{6}\nu & \sqrt{3}\mu & \frac{3}{2}(\alpha-3\tau) & 0 & \frac{3\sqrt{3}}{2}\beta & 0 \\ \frac{3}{\sqrt{2}}(2\mu-\gamma) & -3\sqrt{2}\gamma & -3(\beta+2\lambda) & -\frac{3}{\sqrt{2}}\gamma & 0 & \begin{pmatrix}-\frac{1}{2}(\alpha+3\theta)- \\ \frac{5}{2}\beta-5\lambda+\nu\end{pmatrix} & -\sqrt{2}(5\lambda+\nu) & \frac{5}{\sqrt{2}}(2\mu-\gamma) \\ 6\mu & -3\sqrt{2}\gamma & -3\sqrt{2}\lambda & 0 & \frac{3\sqrt{3}}{2}\beta & -\sqrt{2}(5\lambda+\nu) & -\frac{1}{2}(\alpha+3\theta)-5\beta & 5\mu \\ -3(\beta+2\lambda) & -3\sqrt{2}\lambda & 0 & -\frac{3}{2}\beta & 0 & \frac{5}{\sqrt{2}}(2\mu-\gamma) & 5\mu & \frac{1}{2}(9\alpha+\tau)+2\nu \end{pmatrix}. \tag{22}$$

D. $(cbnn)^{I=0}\bar{q}$ states in the fourth class

In this case, we also have six types of basis vectors, $[\phi^{AA}\chi^{SA}]\delta^S_{34}$, $[\phi^{AA}\chi^{AA}]\delta_{12}\delta^S_{34}$, $[\phi^{AS}\chi^{SS}]\delta^S_{34}$, $[\phi^{AS}\chi^{AS}]\delta_{12}\delta^S_{34}$, $[\phi^{SA}\chi^{SA}]\delta_{12}\delta^S_{34}$, and $[\phi^{SA}\chi^{AA}]\delta^S_{34}$.

For the $J^P=\frac{5}{2}^-$ states, there is only one basis vector, $[(cb)^{\bar{3}}_1(nn)^6_1\bar{q}]^{\frac{5}{2}}_2$. The obtained Hamiltonian is

$$\langle H_{\mathrm{CM}}\rangle_{J=\frac{5}{2}} = \frac{1}{3}(3\tau-\alpha+5\beta-2\lambda+10\nu). \tag{23}$$

For the $J^P=\frac{3}{2}^-$ states, the involved basis vectors are $[(cb)^{\bar{3}}_1(nn)^{\bar{3}}_0\bar{q}]^{\frac{3}{2}}_1$, $[(cb)^{\bar{3}}_1(nn)^6_1\bar{q}]^{\frac{3}{2}}_2$, $[(cb)^{\bar{3}}_1(nn)^6_1\bar{q}]^{\frac{3}{2}}_1$, $[(cb)^{\bar{3}}_0(nn)^6_1\bar{q}]^{\frac{3}{2}}_1$, and $[(cb)^6_1(nn)^{\bar{3}}_0\bar{q}]^{\frac{3}{2}}_1$. The Hamiltonian can be written as

$$\langle H_{\mathrm{CM}}\rangle_{J=\frac{3}{2}} = \frac{2}{3}\begin{pmatrix} 4\theta+2\lambda & 3\sqrt{5}\nu & 3(\beta-\nu) & \frac{3}{\sqrt{2}}\gamma & -3\sqrt{2}\mu \\ 3\sqrt{5}\nu & \frac{1}{2}\begin{pmatrix}3\tau-\alpha+5\beta \\ +3\lambda-15\nu\end{pmatrix} & \frac{\sqrt{5}}{2}(\lambda+5\nu) & \frac{\sqrt{10}}{2}\mu & 0 \\ 3(\beta-\nu) & \frac{\sqrt{5}}{2}(\lambda+5\nu) & \frac{1}{2}\begin{pmatrix}3\tau-\alpha-5\beta \\ -\lambda+5\nu\end{pmatrix} & \frac{1}{\sqrt{2}}(\mu-5\gamma) & -\frac{3}{\sqrt{2}}\gamma \\ \frac{3}{\sqrt{2}}\gamma & \frac{\sqrt{10}}{2}\mu & \frac{1}{\sqrt{2}}(\mu-5\gamma) & 5\nu-\frac{1}{2}(9\alpha+5\tau) & -\frac{3}{2}\beta \\ -3\sqrt{2}\mu & 0 & -\frac{3}{\sqrt{2}}\gamma & -\frac{3}{2}\beta & 5\lambda+\frac{1}{2}(5\theta-9\alpha) \end{pmatrix}. \tag{24}$$

For the $J^P=\frac{1}{2}^-$ states, we have seven basis vectors, $[(cb)^{\bar{3}}_1(nn)^{\bar{3}}_0\bar{q}]^{\frac{1}{2}}_1$, $[(cb)^{\bar{3}}_0(nn)^{\bar{3}}_0\bar{q}]^{\frac{1}{2}}_0$, $[(cb)^{\bar{3}}_1(nn)^6_1\bar{q}]^{\frac{1}{2}}_1$, $[(cb)^{\bar{3}}_1(nn)^6_1\bar{q}]^{\frac{1}{2}}_0$, $[(cb)^{\bar{3}}_0(nn)^6_1\bar{q}]^{\frac{1}{2}}_1$, $[(cb)^6_1(nn)^{\bar{3}}_0\bar{q}]^{\frac{1}{2}}_1$, and $[(cb)^6_0(nn)^{\bar{3}}_0\bar{q}]^{\frac{1}{2}}_0$. The obtained Hamiltonian reads

$$\langle H_{\mathrm{CM}}\rangle_{J=\frac{1}{2}} = \frac{2}{3}\begin{pmatrix} 4(\theta-\lambda) & -2\sqrt{3}\mu & 3(\beta+2\nu) & -3\sqrt{2}\nu & \frac{3}{\sqrt{2}}\gamma & 6\sqrt{2}\mu & 3\sqrt{6}\lambda \\ -2\sqrt{3}\mu & -12\alpha & 0 & -\frac{3\sqrt{6}}{2}\gamma & 3\sqrt{6}\nu & 3\sqrt{6}\lambda & 0 \\ 3(\beta+2\nu) & 0 & \frac{1}{2}\begin{pmatrix}3\tau-\alpha-5\beta \\ +2\lambda-10\nu\end{pmatrix} & \sqrt{2}(\lambda+5\nu) & -\frac{1}{\sqrt{2}}(5\gamma+2\mu) & -\frac{3}{\sqrt{2}}\gamma & 0 \\ -3\sqrt{2}\nu & -\frac{3\sqrt{6}}{2}\gamma & \sqrt{2}(\lambda+5\nu) & \frac{1}{2}(3\tau-\alpha)-5\beta & -\mu & 0 & \frac{3\sqrt{3}}{2}\beta \\ \frac{3}{\sqrt{2}}\gamma & 3\sqrt{6}\nu & -\frac{1}{\sqrt{2}}(5\gamma+2\mu) & -\mu & -\frac{1}{2}(9\alpha+5\tau)-10\nu & -\frac{3}{2}\beta & 0 \\ 6\sqrt{2}\mu & 3\sqrt{6}\lambda & -\frac{3}{\sqrt{2}}\gamma & 0 & -\frac{3}{2}\beta & \frac{1}{2}(5\theta-9\alpha)-10\lambda & -5\sqrt{3}\mu \\ 3\sqrt{6}\lambda & 0 & 0 & \frac{3\sqrt{3}}{2}\beta & 0 & -5\sqrt{3}\mu & \frac{3}{2}(3\theta+\alpha) \end{pmatrix}. \tag{25}$$

E. $(ccns)\bar{q}$ and $(bbns)\bar{q}$ states in the fifth class

In this case, again we have six types of basis vectors, $[\phi^{AA}\chi^{SS}]\delta^A_{34}$, $[\phi^{AA}\chi^{SA}]\delta^S_{34}$, $[\phi^{AS}\chi^{SS}]\delta^S_{34}$, $[\phi^{AS}\chi^{SA}]\delta^A_{34}$, $[\phi^{SA}\chi^{AS}]\delta^A_{34}$, and $[\phi^{SA}\chi^{AA}]\delta^S_{34}$.

For the $J^P=\frac{5}{2}^-$ states, the basis vectors are $[(QQ)^{\bar{3}}_1(ns)^{\bar{3}}_1\bar{q}]^{\frac{5}{2}}_2$ and $[(QQ)^{\bar{3}}_1(ns)^{6}_1\bar{q}]^{\frac{5}{2}}_2$ and the Hamiltonian is

$$\langle H_{\rm CM}\rangle_{J=\frac{5}{2}}=\frac{2}{3}\begin{pmatrix}4\alpha+\beta+2\lambda+2\nu & \frac{3}{\sqrt{2}}(\delta-2\rho)\\ \frac{3}{\sqrt{2}}(\delta-2\rho) & \frac{1}{2}(3\tau-\alpha+5\beta-2\lambda+10\nu)\end{pmatrix}. \tag{26}$$

For the $J^P=\frac{3}{2}^-$ states, the involved basis vectors are $[(QQ)^{\bar{3}}_1(ns)^{\bar{3}}_1\bar{q}]^{\frac{3}{2}}_2$, $[(QQ)^{\bar{3}}_1(ns)^{\bar{3}}_1\bar{q}]^{\frac{3}{2}}_1$, $[(QQ)^{\bar{3}}_1(ns)^{\bar{3}}_0\bar{q}]^{\frac{3}{2}}_1$, $[(QQ)^{\bar{3}}_1(ns)^{6}_1\bar{q}]^{\frac{3}{2}}_2$, $[(QQ)^{\bar{3}}_1(ns)^{6}_1\bar{q}]^{\frac{3}{2}}_1$, $[(QQ)^{\bar{3}}_1(ns)^{6}_0\bar{q}]^{\frac{3}{2}}_1$, and $[(QQ)^{6}_0(ns)^{\bar{3}}_1\bar{q}]^{\frac{3}{2}}_1$. Then one can get

$$\langle H_{\rm CM}\rangle_{J=\frac{3}{2}}=\frac{2}{3}\begin{pmatrix}\begin{pmatrix}4\alpha+\beta-\\3(\lambda+\nu)\end{pmatrix} & \sqrt{5}(\nu-\lambda) & -\sqrt{10}\rho & \frac{3}{\sqrt{2}}(\delta+3\rho) & -\frac{3\sqrt{10}}{2}\rho & 3\sqrt{5}\nu & 3\sqrt{5}\lambda\\ \sqrt{5}(\nu-\lambda) & \begin{pmatrix}4\alpha-\beta\\+\lambda+\nu\end{pmatrix} & \sqrt{2}(\delta+\rho) & -\frac{3\sqrt{10}}{2}\rho & -\frac{3}{\sqrt{2}}(\delta+\rho) & 3(\beta-\nu) & 3(\lambda-\beta)\\ -\sqrt{10}\rho & \sqrt{2}(\delta+\rho) & 4\theta+2\lambda & 3\sqrt{5}\nu & 3(\beta-\nu) & 0 & \frac{3}{\sqrt{2}}\delta\\ \frac{3}{\sqrt{2}}(\delta+3\rho) & -\frac{3\sqrt{10}}{2}\rho & 3\sqrt{5}\nu & \frac{1}{2}\begin{pmatrix}3\tau-\alpha+5\beta\\+3\lambda-15\nu\end{pmatrix} & \frac{\sqrt{5}}{2}(\lambda+5\nu) & -\frac{5\sqrt{10}}{2}\rho & 0\\ -\frac{3\sqrt{10}}{2}\rho & -\frac{3}{\sqrt{2}}(\delta+\rho) & 3(\beta-\nu) & \frac{\sqrt{5}}{2}(\lambda+5\nu) & \frac{1}{2}\begin{pmatrix}3\tau-\alpha-\lambda\\-5\beta+5\nu\end{pmatrix} & \frac{5}{\sqrt{2}}(\delta+\rho) & \frac{3}{\sqrt{2}}\delta\\ 3\sqrt{5}\nu & 3(\beta-\nu) & 0 & -\frac{5\sqrt{10}}{2}\rho & \frac{5}{\sqrt{2}}(\delta+\rho) & \frac{1}{2}(9\alpha-\theta)-\lambda & -\frac{3}{2}\beta\\ 3\sqrt{5}\lambda & 3(\lambda-\beta) & \frac{3}{\sqrt{2}}\delta & 0 & \frac{3}{\sqrt{2}}\delta & -\frac{3}{2}\beta & \frac{1}{2}(9\alpha+\tau)-\nu\end{pmatrix}. \tag{27}$$

For the $J^P=\frac{1}{2}^-$ states, the basis vectors are $[(QQ)^{\bar{3}}_1(ns)^{\bar{3}}_1\bar{q}]^{\frac{1}{2}}_1$, $[(QQ)^{\bar{3}}_1(ns)^{\bar{3}}_1\bar{q}]^{\frac{1}{2}}_0$, $[(QQ)^{\bar{3}}_1(ns)^{\bar{3}}_0\bar{q}]^{\frac{1}{2}}_1$, $[(QQ)^{\bar{3}}_1(ns)^{6}_1\bar{q}]^{\frac{1}{2}}_1$, $[(QQ)^{\bar{3}}_1(ns)^{6}_1\bar{q}]^{\frac{1}{2}}_0$, $[(QQ)^{\bar{3}}_1(ns)^{6}_0\bar{q}]^{\frac{1}{2}}_1$, $[(QQ)^{6}_0(ns)^{\bar{3}}_1\bar{q}]^{\frac{1}{2}}_1$, and $[(QQ)^{6}_0(ns)^{\bar{3}}_0\bar{q}]^{\frac{1}{2}}_0$. The derived Hamiltonian reads

$$\langle H_{\rm CM}\rangle_{J=\frac{1}{2}}=\frac{2}{3}\begin{pmatrix}\begin{pmatrix}4\alpha-\beta\\-2\lambda-2\nu\end{pmatrix} & 2\sqrt{2}(\nu-\lambda) & \sqrt{2}(\delta-2\rho) & \frac{3}{\sqrt{2}}(2\rho-\delta) & -6\rho & 3(\beta+2\nu) & -3(\beta+2\lambda) & 0\\ 2\sqrt{2}(\nu-\lambda) & 2(2\alpha-\beta) & 2\rho & -6\rho & -3\sqrt{2}\delta & -3\sqrt{2}\nu & -3\sqrt{2}\lambda & -\frac{3\sqrt{6}}{2}\delta\\ \sqrt{2}(\delta-2\rho) & 2\rho & 4(\theta-\lambda) & 3(\beta+2\nu) & -3\sqrt{2}\nu & 0 & \frac{3}{\sqrt{2}}\delta & 3\sqrt{6}\lambda\\ \frac{3}{\sqrt{2}}(2\rho-\delta) & -6\rho & 3(\beta+2\nu) & \frac{1}{2}\begin{pmatrix}3\tau-\alpha-5\beta\\+2\lambda-10\nu\end{pmatrix} & \sqrt{2}(\lambda+5\nu) & \frac{5}{\sqrt{2}}(\delta-2\rho) & \frac{3}{\sqrt{2}}\delta & 0\\ -6\rho & -3\sqrt{2}\delta & -3\sqrt{2}\nu & \sqrt{2}(\lambda+5\nu) & \frac{1}{2}(3\tau-\alpha)-5\beta & 5\rho & 0 & \frac{3\sqrt{3}}{2}\beta\\ 3(\beta+2\nu) & -3\sqrt{2}\nu & 0 & \frac{5}{\sqrt{2}}(\delta-2\rho) & 5\rho & \frac{1}{2}(9\alpha-\theta)+2\lambda & -\frac{3}{2}\beta & 0\\ -3(\beta+2\lambda) & -3\sqrt{2}\lambda & \frac{3}{\sqrt{2}}\delta & \frac{3}{\sqrt{2}}\delta & 0 & -\frac{3}{2}\beta & \frac{1}{2}(9\alpha+\tau)+2\nu & \sqrt{3}\rho\\ 0 & -\frac{3\sqrt{6}}{2}\delta & 3\sqrt{6}\lambda & 0 & \frac{3\sqrt{3}}{2}\beta & 0 & \sqrt{3}\rho & \frac{3}{2}(3\theta+\alpha)\end{pmatrix}. \tag{28}$$

F. $(cbns)\bar{q}$ states in the sixth class

$$
\langle H_{\rm CM}\rangle_{J=\frac{3}{2}} = \frac{2}{3}
\times\begin{pmatrix}
\begin{pmatrix}4\alpha+\beta\\-3\lambda-3\nu\end{pmatrix} & \sqrt{5}(\nu-\lambda) & -\sqrt{10}\rho & -\sqrt{10}\mu & \frac{3}{\sqrt{2}}(\delta+3\rho) & -\frac{3\sqrt{10}}{2}\rho & 3\sqrt{5}\nu & 0 & \frac{3\sqrt{2}}{2}(\gamma+3\mu) & \frac{3\sqrt{10}}{2}\mu & 0 & 3\sqrt{5}\lambda \\
\sqrt{5}(\nu-\lambda) & \begin{pmatrix}4\alpha-\beta\\+\lambda+\nu\end{pmatrix} & \sqrt{2}(\delta+\rho) & -\sqrt{2}(\gamma+\mu) & -\frac{3\sqrt{10}}{2}\rho & -\frac{3\sqrt{2}}{2}(\delta+\rho) & 3(\beta-\nu) & -3\eta & \frac{3\sqrt{10}}{2}\mu & -\frac{3\sqrt{2}}{2}(\gamma+\mu) & 3\eta & 3(\lambda-\beta) \\
-\sqrt{10}\rho & \sqrt{2}(\delta+\rho) & 4\theta+2\lambda & \eta & 3\sqrt{5}\nu & 3(\beta-\nu) & 0 & \frac{3}{\sqrt{2}}\gamma & 0 & 3\eta & -3\sqrt{2}\mu & \frac{3}{\sqrt{2}}\delta \\
-\sqrt{10}\mu & -\sqrt{2}(\gamma+\mu) & \eta & 2\nu-4\tau & 0 & -3\eta & \frac{3}{\sqrt{2}}\gamma & -3\sqrt{2}\rho & 3\sqrt{5}\lambda & 3(\lambda-\beta) & \frac{3}{\sqrt{2}}\delta & 0 \\
\frac{3}{\sqrt{2}}(\delta+3\rho) & -\frac{3\sqrt{10}}{2}\rho & 3\sqrt{5}\nu & 0 & \frac{1}{2}\begin{pmatrix}3\tau-\alpha\\+5\beta+3\lambda\\-15\nu\end{pmatrix} & \frac{\sqrt{5}}{2}(\lambda+5\nu) & -\frac{5\sqrt{10}}{2}\rho & \frac{\sqrt{10}}{2}\mu & -\frac{3}{2}\eta & 0 & 0 & 0 \\
-\frac{3\sqrt{10}}{2}\rho & -\frac{3\sqrt{2}}{2}(\delta+\rho) & 3(\beta-\nu) & -3\eta & \frac{\sqrt{5}}{2}(\lambda+5\nu) & \frac{1}{2}\begin{pmatrix}3\tau-\alpha\\-5\beta-\lambda\\+5\nu\end{pmatrix} & \frac{5}{\sqrt{2}}(\delta+\rho) & \frac{1}{\sqrt{2}}(\mu-5\gamma) & 0 & \frac{3}{2}\eta & -\frac{3}{\sqrt{2}}\gamma & \frac{3}{\sqrt{2}}\delta \\
3\sqrt{5}\nu & 3(\beta-\nu) & 0 & \frac{3}{\sqrt{2}}\gamma & -\frac{5\sqrt{10}}{2}\rho & \frac{5}{\sqrt{2}}(\delta+\rho) & \frac{9}{2}\alpha-\frac{1}{2}\theta-\lambda & \frac{5}{2}\eta & 0 & -\frac{3}{\sqrt{2}}\gamma & 0 & -\frac{3}{2}\beta \\
0 & -3\eta & \frac{3}{\sqrt{2}}\gamma & -3\sqrt{2}\rho & \frac{\sqrt{10}}{2}\mu & \frac{1}{\sqrt{2}}(\mu-5\gamma) & \frac{5}{2}\eta & 5\nu-\frac{9}{2}\alpha-\frac{5}{2}\tau & 0 & \frac{3}{\sqrt{2}}\delta & -\frac{3}{2}\beta & 0 \\
\frac{3\sqrt{2}}{2}(\gamma+3\mu) & \frac{3\sqrt{10}}{2}\mu & 0 & 3\sqrt{5}\lambda & -\frac{3}{2}\eta & 0 & 0 & 0 & \frac{1}{2}\begin{pmatrix}5\beta-\alpha\\-3\theta+3\nu\\-15\lambda\end{pmatrix} & -\frac{\sqrt{5}}{2}(5\lambda+\nu) & \frac{\sqrt{10}}{2}\rho & -\frac{5\sqrt{10}}{2}\mu \\
\frac{3\sqrt{10}}{2}\mu & -\frac{3\sqrt{2}}{2}(\gamma+\mu) & 3\eta & 3(\lambda-\beta) & 0 & \frac{3}{2}\eta & -\frac{3}{\sqrt{2}}\gamma & \frac{3}{\sqrt{2}}\delta & -\frac{\sqrt{5}}{2}(5\lambda+\nu) & \frac{1}{2}\begin{pmatrix}5\lambda-3\theta\\-\alpha-\nu\\-5\beta\end{pmatrix} & \frac{1}{\sqrt{2}}(5\delta-\rho) & -\frac{5}{\sqrt{2}}(\gamma+\mu) \\
0 & 3\eta & -3\sqrt{2}\mu & \frac{3}{\sqrt{2}}\delta & 0 & -\frac{3}{\sqrt{2}}\gamma & 0 & -\frac{3}{2}\beta & \frac{\sqrt{10}}{2}\rho & \frac{1}{\sqrt{2}}(5\delta-\rho) & 5\lambda+\frac{5}{2}\theta-\frac{9}{2}\alpha & \frac{5}{2}\eta \\
3\sqrt{5}\lambda & 3(\lambda-\beta) & \frac{3}{\sqrt{2}}\delta & 0 & 0 & \frac{3}{\sqrt{2}}\delta & -\frac{3}{2}\beta & 0 & -\frac{5\sqrt{10}}{2}\mu & -\frac{5}{\sqrt{2}}(\gamma+\mu) & \frac{5}{2}\eta & \frac{9}{2}\alpha+\frac{1}{2}\tau-\nu
\end{pmatrix}. \tag{29}
$$

$$
\langle H_{\mathrm{CM}}\rangle_{J=\frac{1}{2}} = \frac{2}{3}\times
\begin{pmatrix}
\begin{pmatrix}4\alpha-\beta\\-2\lambda-2\nu\end{pmatrix} & 2\sqrt{2}(\nu-\lambda) & \sqrt{2}(\delta-2\rho) & \sqrt{2}(2\mu-\gamma) & 0 & \frac{3}{\sqrt{2}}(2\rho-\delta) & -6\rho & 3(\beta+2\nu) & -3\eta & 0 & \frac{3}{\sqrt{2}}(2\mu-\gamma) & 6\mu & 3\eta & -3(\beta+2\lambda) & 0 \\
2\sqrt{2}(\nu-\lambda) & 2(2\alpha-\beta) & 2\rho & 2\mu & -\sqrt{3}\eta & -6\rho & -3\sqrt{2}\delta & -3\sqrt{2}\nu & 0 & -\frac{3\sqrt{6}}{2}\gamma & 6\mu & -3\sqrt{2}\gamma & 0 & -3\sqrt{2}\lambda & -\frac{3\sqrt{6}}{2}\delta \\
\sqrt{2}(\delta-2\rho) & 2\rho & 4(\theta-\lambda) & \eta & -2\sqrt{3}\mu & 3(\beta+2\nu) & -3\sqrt{2}\nu & 0 & \frac{3}{\sqrt{2}}\gamma & 0 & 3\eta & 0 & 6\sqrt{2}\mu & \frac{3}{\sqrt{2}}\delta & 3\sqrt{6}\lambda \\
\sqrt{2}(2\mu-\gamma) & 2\mu & \eta & -4(\tau+\nu) & -2\sqrt{3}\rho & -6\eta & 0 & \frac{3}{\sqrt{2}}\gamma & 6\sqrt{2}\rho & 3\sqrt{6}\nu & -3(\beta+2\lambda) & -3\sqrt{2}\lambda & \frac{3}{\sqrt{2}}\delta & 0 & 0 \\
0 & -\sqrt{3}\eta & -2\sqrt{3}\mu & -2\sqrt{3}\rho & -12\alpha & 0 & -\frac{3\sqrt{6}}{2}\gamma & 0 & 3\sqrt{6}\nu & 0 & 0 & -\frac{3\sqrt{6}}{2}\delta & 3\sqrt{6}\lambda & 0 & 0 \\
\frac{3}{\sqrt{2}}(2\rho-\delta) & -6\rho & 3(\beta+2\nu) & -6\eta & 0 & \frac{1}{2}\begin{pmatrix}3\tau-\alpha-\\5\beta+2\lambda\\-10\nu\end{pmatrix} & \sqrt{2}(\lambda+5\nu) & \frac{5}{\sqrt{2}}(\delta-2\rho) & -\frac{1}{\sqrt{2}}(5\gamma-2\mu) & 0 & -\frac{3}{2}\eta & 0 & -\frac{3}{\sqrt{2}}\gamma & \frac{3}{\sqrt{2}}\delta & 0 \\
-6\rho & -3\sqrt{2}\delta & -3\sqrt{2}\nu & 0 & -\frac{3\sqrt{6}}{2}\gamma & \sqrt{2}(\lambda+5\nu) & \frac{3}{2}\tau-\frac{1}{2}\alpha-5\beta & 5\rho & -\lambda & -\frac{5\sqrt{3}}{2}\eta & 0 & 3\eta & 0 & 0 & \frac{3\sqrt{3}}{2}\beta \\
3(\beta+2\nu) & -3\sqrt{2}\nu & 0 & \frac{3}{\sqrt{2}}\gamma & 0 & \frac{5}{\sqrt{2}}(\delta-2\rho) & 5\rho & \frac{9}{2}\alpha-\frac{1}{2}\theta+2\lambda & \frac{5}{2}\eta & \sqrt{3}\mu & -\frac{3}{\sqrt{2}}\gamma & 0 & 0 & -\frac{3}{2}\beta & 0 \\
-3\eta & 0 & \frac{3}{\sqrt{2}}\gamma & 6\sqrt{2}\rho & 3\sqrt{6}\nu & \sqrt{2}\mu-\frac{5}{\sqrt{2}}\gamma & -\lambda & \frac{5}{2}\eta & -\frac{9}{2}\alpha-\frac{5}{2}\tau-10\nu & -5\sqrt{3}\rho & \frac{3}{\sqrt{2}}\delta & 0 & -\frac{3}{2}\beta & 0 & 0 \\
0 & -\frac{3\sqrt{6}}{2}\gamma & 0 & 3\sqrt{6}\nu & 0 & 0 & -\frac{5\sqrt{3}}{2}\eta & \sqrt{3}\mu & -5\sqrt{3}\rho & \frac{3}{2}\alpha-\frac{9}{2}\tau & 0 & \frac{3\sqrt{3}}{2}\beta & 0 & 0 & 0 \\
\frac{3}{\sqrt{2}}(2\mu-\gamma) & 6\mu & 3\eta & -3(\beta+2\lambda) & 0 & -\frac{3}{2}\eta & 0 & -\frac{3}{\sqrt{2}}\gamma & \frac{3}{\sqrt{2}}\delta & 0 & \frac{1}{2}\begin{pmatrix}-3\theta-\alpha\\+\nu-5\beta\\-10\lambda\end{pmatrix} & -\sqrt{2}(5\lambda+\nu) & \sqrt{2}(\frac{5}{2}\delta+\rho) & \frac{5}{\sqrt{2}}(2\mu-\gamma) & 0 \\
6\mu & -3\sqrt{2}\gamma & 0 & -3\sqrt{2}\lambda & -\frac{3\sqrt{6}}{2}\delta & 0 & 3\eta & 0 & 0 & \frac{3\sqrt{3}}{2}\beta & -\sqrt{2}(5\lambda+\nu) & -\frac{3}{2}\theta-\frac{1}{2}\alpha-5\beta & -\rho & 5\mu & -\frac{5\sqrt{3}}{2}\eta \\
3\eta & 0 & 6\sqrt{2}\mu & \frac{3}{\sqrt{2}}\delta & 3\sqrt{6}\lambda & -\frac{3}{\sqrt{2}}\gamma & 0 & 0 & -\frac{3}{2}\beta & 0 & \sqrt{2}(\frac{5}{2}\delta+\rho) & -\rho & \frac{5}{2}\theta-\frac{9}{2}\alpha-10\lambda & \frac{5}{2}\eta & -5\sqrt{3}\mu \\
-3(\beta+2\lambda) & -3\sqrt{2}\lambda & \frac{3}{\sqrt{2}}\delta & 0 & 0 & \frac{3}{\sqrt{2}}\delta & 0 & -\frac{3}{2}\beta & 0 & 0 & \frac{5}{\sqrt{2}}(2\mu-\gamma) & 5\mu & \frac{5}{2}\eta & \frac{9}{2}\alpha+\frac{1}{2}\tau+2\nu & \sqrt{3}\rho \\
0 & -\frac{3\sqrt{6}}{2}\delta & 3\sqrt{6}\lambda & 0 & 0 & 0 & \frac{3\sqrt{3}}{2}\beta & 0 & 0 & 0 & 0 & -\frac{5\sqrt{3}}{2}\eta & -5\sqrt{3}\mu & \sqrt{3}\rho & \frac{3}{2}\alpha+\frac{9}{2}\theta
\end{pmatrix}. \tag{30}
$$

TABLE II. The extracted effective coupling parameters.

Hadron	CMI	Hadron	CMI	Parameter (MeV)
N	$-8C_{nn}$	Δ	$8C_{nn}$	$C_{nn}=18.4$
Σ	$\frac{8}{3}C_{nn}-\frac{32}{3}C_{ns}$	Σ^*	$\frac{8}{3}C_{nn}+\frac{16}{3}C_{ns}$	$C_{ns}=12.4$
Ξ^0	$\frac{8}{3}(C_{ss}-4C_{ns})$	Ξ^{*0}	$\frac{8}{3}(C_{ss}+C_{ns})$	
Ω	$8C_{ss}$			$C_{ss}=6.5$
Λ	$-8C_{nn}$			
π^0	$-16C_{n\bar{n}}$	ρ	$\frac{16}{3}C_{n\bar{n}}$	$C_{n\bar{n}}=30.0$
K	$-16C_{n\bar{s}}$	K^*	$\frac{16}{3}C_{c\bar{s}}$	$C_{n\bar{s}}=18.7$
D	$-16C_{c\bar{n}}$	D^*	$\frac{16}{3}C_{c\bar{n}}$	$C_{c\bar{n}}=6.7$
D_s	$-16C_{c\bar{s}}$	D_s^*	$\frac{16}{3}C_{c\bar{s}}$	$C_{c\bar{s}}=6.7$
B	$-16C_{b\bar{n}}$	B^*	$\frac{16}{3}C_{b\bar{n}}$	$C_{b\bar{n}}=2.1$
B_s	$-16C_{b\bar{s}}$	B_s^*	$\frac{16}{3}C_{b\bar{s}}$	$C_{b\bar{s}}=2.3$
B_c	$-16C_{b\bar{c}}$	B_c^* [63]	$\frac{16}{3}C_{b\bar{c}}$	$C_{b\bar{c}}=3.3$
η_c	$-16C_{c\bar{c}}$	J/ψ	$\frac{16}{3}C_{c\bar{c}}$	$C_{c\bar{c}}=5.3$
η_b	$-16C_{b\bar{b}}$	Υ	$\frac{16}{3}C_{b\bar{b}}$	$C_{b\bar{b}}=2.9$
Σ_c	$\frac{8}{3}C_{nn}-\frac{32}{3}C_{cn}$	Σ_c^*	$\frac{8}{3}C_{nn}+\frac{16}{3}C_{cn}$	$C_{cn}=4.0$
Ξ_c'	$\frac{8}{3}C_{ns}-\frac{16}{3}C_{cn}-\frac{16}{3}C_{cs}$	Ξ_c^*	$\frac{8}{3}C_{ns}+\frac{8}{3}C_{cn}+\frac{8}{3}C_{cs}$	$C_{cs}=4.8$
Σ_b	$\frac{8}{3}C_{nn}-\frac{32}{3}C_{bn}$	Σ_b^*	$\frac{8}{3}C_{nn}+\frac{16}{3}C_{bn}$	$C_{bn}=1.3$
Ξ_b'	$\frac{8}{3}C_{ns}-\frac{16}{3}C_{bn}-\frac{16}{3}C_{bs}$	Ξ_b^*	$\frac{8}{3}C_{ns}+\frac{8}{3}C_{bn}+\frac{8}{3}C_{bs}$	$C_{bs}=1.2$

Since the Pauli principle has no effects in this case, the most basis vectors are involved. There are twelve types of bases, $[\phi^{AA}\chi^{SS}]\delta_{34}^A$, $[\phi^{AA}\chi^{SA}]\delta_{34}^S$, $[\phi^{AA}\chi^{AS}]\delta_{12}\delta_{34}^A$, $[\phi^{AA}\chi^{AA}]\delta_{12}\delta_{34}^S$, $[\phi^{AS}\chi^{SS}]\delta_{34}^S$, $[\phi^{AS}\chi^{SA}]\delta_{34}^A$, $[\phi^{AS}\chi^{AS}]\delta_{12}\delta_{34}^S$, $[\phi^{AS}\chi^{AA}]\delta_{12}\delta_{34}^A$, $[\phi^{SA}\chi^{SS}]\delta_{12}\delta_{34}^A$, $[\phi^{SA}\chi^{SA}]\delta_{12}\delta_{34}^S$, $[\phi^{SA}\chi^{AS}]\delta_{34}^A$, and $[\phi^{SA}\chi^{AA}]\delta_{34}^S$.

For the $J^P=\frac{5}{2}^-$ states, the basis vectors are $[(cb)_1^{\bar{3}}(ns)_1^{\bar{3}}\bar{q}]_2^{\frac{5}{2}}$, $[(cb)_1^{\bar{3}}(ns)_1^{6}\bar{q}]_2^{\frac{5}{2}}$, and $[(cb)_1^{6}(ns)_1^{\bar{3}}\bar{q}]_2^{\frac{5}{2}}$ and the Hamiltonian is

$$\langle H_{\rm CM}\rangle_{J=\frac{5}{2}}=\frac{1}{3}\begin{pmatrix} 2(4\alpha+\beta+2\lambda+2\nu) & 3\sqrt{2}(\delta-2\rho) & 3\sqrt{2}(\gamma-2\mu) \\ 3\sqrt{2}(\delta-2\rho) & 3\tau-\alpha+5\beta-2\lambda+10\nu & -3\eta \\ 3\sqrt{2}(\gamma-2\mu) & -3\eta & 5\beta+10\lambda-2\nu-(\alpha+3\theta) \end{pmatrix}. \quad (31)$$

For the $J^P=\frac{3}{2}^-$ states, the basis vectors are $[(cb)_1^{\bar{3}}(ns)_1^{\bar{3}}\bar{q}]_2^{\frac{3}{2}}$, $[(cb)_1^{\bar{3}}(ns)_1^{\bar{3}}\bar{q}]_1^{\frac{3}{2}}$, $[(cb)_1^{\bar{3}}(ns)_0^{\bar{3}}\bar{q}]_1^{\frac{3}{2}}$, $[(cb)_0^{\bar{3}}(ns)_1^{\bar{3}}\bar{q}]_1^{\frac{3}{2}}$, $[(cb)_1^{\bar{3}}(ns)_1^{6}\bar{q}]_2^{\frac{3}{2}}$, $[(cb)_1^{\bar{3}}(ns)_1^{6}\bar{q}]_1^{\frac{3}{2}}$, $[(cb)_1^{\bar{3}}(ns)_0^{6}\bar{q}]_1^{\frac{3}{2}}$, $[(cb)_0^{\bar{3}}(ns)_1^{6}\bar{q}]_1^{\frac{3}{2}}$, $[(cb)_1^{6}(ns)_1^{\bar{3}}\bar{q}]_2^{\frac{3}{2}}$, $[(cb)_1^{6}(ns)_1^{\bar{3}}\bar{q}]_1^{\frac{3}{2}}$, $[(cb)_1^{6}(ns)_0^{\bar{3}}\bar{q}]_1^{\frac{3}{2}}$, and $[(cb)_0^{6}(ns)_1^{\bar{3}}\bar{q}]_1^{\frac{3}{2}}$. The resulting Hamiltonian is given in Eq. (29).

For the $J^P=\frac{1}{2}^-$ states, the fifteen basis vectors are $[(cb)_1^{\bar{3}}(ns)_1^{\bar{3}}\bar{q}]_1^{\frac{1}{2}}$, $[(cb)_1^{\bar{3}}(ns)_1^{\bar{3}}\bar{q}]_0^{\frac{1}{2}}$, $[(cb)_1^{\bar{3}}(ns)_0^{\bar{3}}\bar{q}]_1^{\frac{1}{2}}$, $[(cb)_0^{\bar{3}}(ns)_1^{\bar{3}}\bar{q}]_1^{\frac{1}{2}}$, $[(cb)_0^{\bar{3}}(ns)_0^{\bar{3}}\bar{q}]_0^{\frac{1}{2}}$, $[(cb)_1^{\bar{3}}(ns)_1^{6}\bar{q}]_1^{\frac{1}{2}}$, $[(cb)_1^{\bar{3}}(ns)_1^{6}\bar{q}]_0^{\frac{1}{2}}$, $[(cb)_1^{\bar{3}}(ns)_0^{6}\bar{q}]_1^{\frac{1}{2}}$, $[(cb)_0^{\bar{3}}(ns)_1^{6}\bar{q}]_1^{\frac{1}{2}}$, $[(cb)_0^{\bar{3}}(ns)_0^{6}\bar{q}]_0^{\frac{1}{2}}$, $[(cb)_1^{6}(ns)_1^{\bar{3}}\bar{q}]_1^{\frac{1}{2}}$, $[(cb)_1^{6}(ns)_1^{\bar{3}}\bar{q}]_0^{\frac{1}{2}}$, $[(cb)_1^{6}(ns)_0^{\bar{3}}\bar{q}]_1^{\frac{1}{2}}$, $[(cb)_0^{6}(ns)_1^{\bar{3}}\bar{q}]_1^{\frac{1}{2}}$, and $[(cb)_0^{6}(ns)_1^{\bar{3}}\bar{q}]_0^{\frac{1}{2}}$. We present the obtained Hamiltonian in Eq. (30).

IV. THE $QQqq\bar{q}$ PENTAQUARK MASS SPECTRA

Now, we determine the values of the seventeen coupling parameters (C_{nn}, C_{ns}, C_{ss}, C_{cn}, C_{bn}, C_{cs}, C_{bs}, C_{bc}, C_{cc}, C_{bb}, $C_{n\bar{n}}$, $C_{s\bar{n}}=C_{n\bar{s}}$, $C_{s\bar{s}}$, $C_{c\bar{n}}$, $C_{b\bar{n}}$, $C_{c\bar{s}}$, and $C_{b\bar{s}}$) and the four effective quark masses (m_n, m_s, m_c, and m_b) in order to estimate the pentaquark masses. The procedure to extract the parameters has been illustrated in Ref. [62]. From the calculated CMI matrix elements for ground-state hadrons and their mass splittings, we can get most values of the coupling parameters, which are shown in Table II. To determine $C_{s\bar{s}}$, one needs the mass of a ground pseudoscalar meson having the same quark content with ϕ. Since there is no such a state, here we adopt approximately $C_{s\bar{s}}=C_{ss}$. Similarly, we use the approximation $C_{QQ}=C_{Q\bar{Q}}$ ($C_{bb}=C_{b\bar{b}}=2.9$ MeV, $C_{cc}=C_{c\bar{c}}=5.3$ MeV, and $C_{bc}=C_{b\bar{c}}=3.3$ MeV) since only one doubly heavy baryon Ξ_{cc} is observed. In Table II, the B_c^* has not been observed yet and we take its mass from a model calculation [63]. The effective quark masses can be extracted from the ground-state baryons after the determination of the coupling parameters, and we present them in Table III.

With these parameters, we can estimate the pentaquark masses in two ways. In the first method, one substitutes the relevant parameters into $M=\sum_i m_i+\langle H_{\rm CM}\rangle$. In the second method, we employ the formula $M=M_{\rm ref}-\langle H_{\rm CM}\rangle_{\rm ref}+\langle H_{\rm CM}\rangle$, where $M_{\rm ref}=M_{\rm baryon}+M_{\rm meson}$ is a reference mass scale and $\langle H_{\rm CM}\rangle_{\rm ref}=\langle H_{\rm CM}\rangle_{\rm baryon}+\langle H_{\rm CM}\rangle_{\rm meson}$. The reference baryon and meson system should have the same

TABLE III. The effective constituent quark masses extracted from conventional baryons.

Mass formula	Quark mass (MeV)
$M_N = 3m_n - 8C_{nn}$	$m_n = 361.8$
$M_\Omega = 3m_s + 8C_{ss}$	$m_s = 540.4$
$M_{\Sigma_c} = \frac{8}{3}C_{nn} - \frac{32}{3}C_{nc} + 2m_n + m_c$ $M_{\Sigma_c^*} = \frac{8}{3}C_{nn} + \frac{16}{3}C_{nc} + 2m_n + m_c$	$m_c = 1724.8$
$M_{\Sigma_b} = \frac{8}{3}C_{nn} - \frac{32}{3}C_{nb} + 2m_n + m_b$ $M_{\Sigma_b^*} = \frac{8}{3}C_{nn} + \frac{16}{3}C_{nb} + 2m_n + m_b$	$m_b = 5052.9$

constituent quarks as the considered system [64]. Although the mass formula in the second method is from that in the first method, one should note the difference in adopting them. When applying the first formula to conventional hadrons, the resulting masses are usually higher than the experimental measurements, which is illustrated in Table IV. This indicates that the simple model does not incorporate attraction sufficiently. As a result, we may treat the pentaquark masses estimated with the first method as theoretical upper limits. In the second method, we use the realistic values rather than the calculated values for the hadron masses of the reference system. The attraction that the model does not incorporate is somehow phenomenologically compensated in this procedure. The estimated masses in the second method should be more reasonable than those in the first method. In the following parts, we will present numerical results obtained in both methods. To understand the decay properties in the following discussions, we will adopt some masses of the not-yet-observed doubly heavy baryons, which were obtained from several theoretical calculations. They are presented in Table V.

A. The $ccnn\bar{q}$, $ccss\bar{q}$, $bbnn\bar{q}$, and $bbss\bar{q}$ pentaquark states

For the $ccnn\bar{q}$ ($q = n, s$) systems, we can use two types of threshold to estimate their masses: (charmed baryon)-(charmed meson) and (doubly charmed baryon)-(light meson). We will use $M_{\Xi_{cc}} = 3621.4$ MeV from the LHCb Collaboration [34] in the latter case. For the $bbnn\bar{q}$ systems, we only use the (bottom baryon)-(bottom meson) threshold since no doubly bottom baryon has been observed. For the $ccss\bar{q}$ and $bbss\bar{q}$ systems, only (heavy baryon)-(heavy meson)-type thresholds are adopted because of the same reason. We present the estimated masses for the $ccnn\bar{q}$, $bbnn\bar{q}$, $ccss\bar{q}$, and $bbss\bar{q}$ pentaquark states in Tables VI, VII, VIII, and IX, respectively. From these tables, it is obvious that different estimation approaches give different masses. The reason is that the model does not involve dynamics and contributions from other terms in the potential are not elaborately considered. For the $ccnn\bar{n}$ and $bbnn\bar{n}$ systems with $I_{nn} = 1$, we get the same spectra for the case of the total isospin $I = \frac{1}{2}$ and $\frac{3}{2}$, which comes from the fact that the color-magnetic interaction for a quark and an antiquark is irrelevant with the isospin.

TABLE IV. Mass differences ($\Delta M = M_{Th.} - M_{Ex.}$) between the calculated values and experimental values for conventional hadrons in units of MeV.

Hadron	ΔM	Hadron	ΔM	Hadron	ΔM	Hadron	ΔM
π	109.5	ρ	107.2	N	0	Δ	0
K	110.6	K^*	105.3	Σ	−12.4	Σ^*	−5.4
ω	99.8	ϕ	96.0	Ξ	9.4	Ξ^*	−7.3
D	112.2	D^*	113.7	Λ	1.1	Ω	0
D_s	189.7	D_s^*	188.7	Σ_c	0	Σ_c^*	0
B	101.6	B^*	101.2	Λ_c	14.7	Ξ_c	58.4
B_s	189.6	B_s^*	190.2	Ξ_c'	35	Ξ_c^*	37.6
η_c	380.9	J/ψ	381.0	Ω_c	76.5	Ω_c^*	82.6
η_b	660.0	Υ	661.0	Σ_b	0	Σ_b^*	0
B_c	450.0			Λ_b	9.7	Ξ_b	62.7
				Ξ_b'	39.8	Ξ_b^*	45.0
				Ω_b	92.1	Ξ_{cc}	161.5

TABLE V. The adopted masses of the not-yet-observed doubly heavy baryons from several methods: RQM (relativized quark model), ECM (extended chromomagnetic model), FH (Feynman-Hellmann mass formulas), and NRM (nonrelativistic potential model).

Baryon	Mass	Theoretical model
Ξ_{bb}	10138	RQM [65]
Ξ_{bb}^*	10169	RQM [65]
Ω_{cc}	3715	RQM [65]
Ω_{cc}^*	3772	RQM [65]
Ω_{bb}	10230	RQM [65]
Ω_{bb}^*	10258	RQM [65]
Ξ_{cb}	6922	ECM [66]
Ξ_{cb}'	6948	ECM [66]
Ξ_{cb}^*	6973	ECM [66]
Ω_{cb}	7011	ECM [66]
Ω_{cb}'	7047	ECM [66]
Ω_{cb}^*	7066	ECM [66]
Ξ_{bb}	10340	FH [67]
Ξ_{bb}^*	10370	FH [67]
Ξ_{bb}	10340	NRM [68]
Ξ_{bb}^*	10367	NRM [68]

Table VI shows us that the pentaquark masses obtained with $\Xi_{cc}\pi$ and $\Xi_{cc}K$ are lower than those with $\Sigma_c D$ and $\Sigma_c D_s$, respectively. This feature is consistent with the observation that more effects contribute to the effective attractions in the former systems, which can be seen from the inequalities $\Delta M_{\Xi_{cc}} + \Delta M_\pi > \Delta M_{\Sigma_c} + \Delta M_D$ and $\Delta M_{\Xi_{cc}} + \Delta M_K > \Delta M_{\Sigma_c} + \Delta M_{D_s}$ according to Table IV. If the adopted model could reproduce all the hadron masses accurately, all the mentioned approaches would give consistent pentaquark masses. At present, we are not sure which type of threshold results in more appropriate pentaquark masses. For a multiquark hadron, the effective attraction is probably not strong and maybe a higher mass is more reasonable. We plot the relative positions for the $ccnn\bar{n}$, $ccnn\bar{s}$, $bbnn\bar{n}$, $bbnn\bar{s}$, $ccss\bar{n}$, $ccss\bar{s}$, $bbss\bar{n}$, and $bbss\bar{s}$ systems in Figs. 2(a)–2(h).

TABLE VI. The estimated masses for the $ccnn\bar{q}$ systems in units of MeV. The values in the third column are obtained with the effective quark masses and are theoretical upper limits. The masses after this column are determined with relevant thresholds.

J^P	Eigenvalue	Mass	$(\Sigma_c D)$	$\Xi_{cc}\pi$
	$ccnn\bar{n}(I_{nn}=1, I=\frac{1}{2},\frac{3}{2})$			
$\frac{5}{2}^-$	171.7	4706.7	4591.3	4436.7
$\frac{3}{2}^-$	288.6	4823.6	4708.2	4553.5
	127.7	4662.7	4547.3	4392.6
	35.6	4570.6	4455.2	4300.5
	−314.3	4220.7	4105.3	3950.6
$\frac{1}{2}^-$	350.5	4885.5	4770.1	4615.4
	197.9	4732.9	4617.5	4462.8
	40.1	4575.1	4459.7	4305.0
	−336.1	4198.9	4083.5	3928.8
	$ccnn\bar{n}$ $(I_{nn}=0, I=\frac{1}{2})$			
$\frac{5}{2}^-$	207.3	4742.3	4626.9	4472.3
$\frac{3}{2}^-$	191.6	4726.6	4611.2	4456.6
	46.5	4581.5	4466.1	4311.5
	−565.2	3969.8	3854.4	3699.7
$\frac{1}{2}^-$	169.4	4704.4	4589.0	4434.3
	43.4	4578.4	4463.0	4308.3
	−134.5	4400.6	4285.2	4130.5
	−665.0	3870.0	3754.6	3600.0

J^P	Eigenvalue	Mass	$(\Sigma_c D_s)$	$(\Xi_{cc} K)$
	$ccnn\bar{s}$ $(I=1)$			
$\frac{5}{2}^-$	141.6	4855.2	4664.6	4584.4
$\frac{3}{2}^-$	205.9	4919.5	4728.9	4648.7
	98.8	4812.4	4621.8	4541.6
	54.9	4768.5	4577.9	4497.7
	−176.8	4536.8	4346.3	4266.0
$\frac{1}{2}^-$	271.5	4985.1	4794.5	4714.3
	162.7	4876.3	4685.7	4605.5
	12.5	4726.1	4535.5	4455.3
	−194.2	4519.4	4328.8	4248.6
	$ccnn\bar{s}$ $(I=0)$			
$\frac{5}{2}^-$	132.0	4845.6	4655.0	4574.8
$\frac{3}{2}^-$	118.8	4832.4	4641.8	4561.6
	−12.1	4701.5	4510.9	4430.7
	−358.4	4355.2	4164.7	4084.4
$\frac{1}{2}^-$	96.7	4810.3	4619.8	4539.5
	−11.0	4702.6	4512.0	4431.8
	−134.2	4579.4	4388.8	4308.6
	−462.8	4250.8	4060.3	3980.0

Here we select the masses obtained with the thresholds of $\Sigma_c D$, $\Sigma_c D_s$, $\Sigma_b \bar{B}$, $\Sigma_b \bar{B}_s$, $\Omega_c D$, $\Omega_c D_s$, $\Omega_b \bar{B}$, and $\Omega_b \bar{B}_s$, respectively. The thresholds relevant with rearrangement decay patterns are also displayed in the figure. The following discussions are based on the assumption that the obtained positions in this figure are all reasonable. For the figures in the other systems, we will also adopt pentaquark masses estimated with higher thresholds. One should note that the figures show only rough positions of the pentaquarks. Their properties may be changed accordingly once the positions for states in a system are determined by an observed pentaquark. However, the mass splittings should not be affected.

For the $ccnn\bar{n}$ system, the $I_{nn}=0$ states are generally lower than the $I_{nn}=1$ states and the lowest state is around

TABLE VII. The estimated masses for the $bbnn\bar{q}$ systems in units of MeV. The values in the third column are obtained with the effective quark masses and are theoretical upper limits. The masses after this column are determined with relevant thresholds.

J^P	Eigenvalue	Mass	$(\Sigma_b \bar{B})$
	$bbnn\bar{n}(I_{nn}=1, I=\frac{1}{2},\frac{3}{2})$		
$\frac{5}{2}^-$	145.9	11337.1	11234.9
$\frac{3}{2}^-$	291.7	11482.9	11380.7
	131.1	11322.3	11220.1
	21.2	11212.4	11110.2
	−316.8	10874.4	10772.2
$\frac{1}{2}^-$	309.7	11500.9	11398.7
	157.3	11348.5	11246.3
	104.2	11295.4	11193.2
	−326.0	10865.2	10763.0
	$bbnn\bar{n}$ $(I_{nn}=0, I=\frac{1}{2})$		
$\frac{5}{2}^-$	189.1	11380.3	11278.1
$\frac{3}{2}^-$	180.7	11371.9	11269.7
	47.5	11238.7	11136.5
	−592.9	10598.3	10496.1
$\frac{1}{2}^-$	174.2	11365.4	11263.2
	43.2	11234.4	11132.3
	−136.4	11054.8	10952.6
	−624.1	10567.1	10464.9

J^P	Eigenvalue	Mass	$(\Sigma_b \bar{B}_s)$
	$bbnn\bar{s}$ $(I=1)$		
$\frac{5}{2}^-$	116.3	11486.1	11296.0
$\frac{3}{2}^-$	207.2	11577.0	11386.9
	101.2	11471.0	11280.9
	36.5	11406.3	11216.3
	−173.3	11196.5	11006.4
$\frac{1}{2}^-$	226.4	11596.2	11406.1
	128.0	11497.8	11307.8
	71.9	11441.7	11251.6
	−181.2	11188.6	10998.6
	$bbnn\bar{s}$ $(I=0)$		
$\frac{5}{2}^-$	113.5	11483.3	11293.2
$\frac{3}{2}^-$	105.9	11475.7	11285.7
	−8.4	11361.4	11171.4
	−386.1	10983.7	10793.6
$\frac{1}{2}^-$	99.6	11469.4	11279.3
	−12.6	11357.2	11167.1
	−136.6	11233.2	11043.1
	−418.9	10950.9	10760.8

TABLE VIII. The estimated masses for the $ccss\bar{q}$ systems in units of MeV. The values in the third column are obtained with the effective quark masses and are theoretical upper limits. The masses after this column are determined with relevant thresholds.

J^P	Eigenvalue	Mass	$(\Omega_c D)$	J^P	Eigenvalue	Mass	$(\Omega_c D_s)$
	$ccss\bar{n}(I=\frac{1}{2})$				$ccss\bar{s}$ $(I=0)$		
$\frac{5}{2}^-$	112.0	5004.2	4813.1	$\frac{5}{2}^-$	79.5	5150.3	4875.5
$\frac{3}{2}^-$	163.7	5055.9	4864.8	$\frac{3}{2}^-$	85.5	5156.3	4881.5
	62.4	4954.6	4763.5		−82.7	5132.1	4857.3
	26.3	4918.5	4727.4		61.3	5095.5	4820.8
	−212.5	4679.7	4488.6		24.7	4988.1	4713.3
$\frac{1}{2}^-$	240.1	5132.3	4941.2	$\frac{1}{2}^-$	166.8	5237.7	4962.9
	126.6	5018.9	4827.8		77.0	5147.8	4873.0
	−24.7	4867.5	4676.4		−33.3	5037.5	4762.7
	−238.8	4653.4	4462.3		−107.3	4963.5	4688.7

the $\Xi_{cc}\pi$ threshold. This pentaquark is in the mass range of excited Ξ_{cc} states [65]. It is highly probable that an observed excited Ξ_{cc} gets contributions from coupled channel effects. An inverted mass order that the $I_{nn}=0$ state is heavier is observed for the $J=\frac{5}{2}$ states. This feature exists because of the stronger $n\bar{n}$ interaction in the $I_{nn}=0$ state, which can be understood from the comparison between Eqs. (17) and (14). The $bbnn\bar{n}$ system should have similar properties. From the mass distributions in Figs. 2(a) and 2(c), we may guess roughly the mass of Ξ_{bb}, $m_{\Xi_{bb}} \approx 10465 - 135 = 10330$ MeV, a value consistent with Refs. [67,68]. Replacing the antiquark with an $\bar{s}$, we get the spectra of $QQnn\bar{s}$ in Figs. 2(b) and 2(d). The difference from the $QQnn\bar{n}$ case lies only in the interaction strengths between the antiquark and other quarks. The remaining systems are obtained by exchanging s and n. All the lowest states have the quantum numbers $J^P=\frac{1}{2}^-$. In these systems, the $QQnn\bar{s}$, $QQss\bar{n}$, and $I=\frac{3}{2}$ $QQnn\bar{n}$ states are explicitly exotic.

Now we move on to the possible rearrangement decays of the pentaquarks, which may occur through S wave or D wave, depending on the conservation laws. The mass, total angular momentum, isospin, and parity all together determine whether the relevant decay channels are open or not. For convenience, we label in Fig. 2 the spin and isospin of the baryon-meson states in the superscripts and subscripts of their symbols, respectively. From the quantum numbers of the decay product, it is possible to find pentaquark candidates. First, we take a look at the $ccnn\bar{n}$ system. In the case of $I(J^P)=\frac{1}{2}(\frac{5}{2}^-)$, the possible S-wave decay channel is just $\Sigma_c^* D^*$. In the case of $I(J^P)=\frac{1}{2}(\frac{3}{2}^-)$, the possible S-wave channels are $\Sigma_c^* D^*$, $\Sigma_c D^*$, $\Sigma_c^* D$, $\Lambda_c D^*$, $\Xi_{cc}\rho$, and $\Xi_{cc}\omega$. In the case of $I(J^P)=\frac{1}{2}(\frac{1}{2}^-)$, the possible S-wave channels are $\Sigma_c^* D^*$, $\Sigma_c D^*$, $\Sigma_c D$, $\Lambda_c D^*$, $\Lambda_c D$, $\Xi_{cc}\rho$, $\Xi_{cc}\omega$, $\Xi_{cc}\eta$, and $\Xi_{cc}\pi$. More channels will open if one includes the D-wave decay modes. However, only the observation of these decay patterns cannot prove the existence of a pentaquark state consisting of $ccnn\bar{n}$ because the initial state may also be an excited Ξ_{cc}. In this case, the mixing between $3q$ state and $5q$ state is probably important. In the case of $I=\frac{3}{2}$, an observed state would be a good pentaquark candidate. The $J^P=\frac{5}{2}^-$ state with either isospin is probably not a very broad pentaquark. For the $bbnn\bar{n}$ system, the situation is similar to the $ccnn\bar{n}$ system. For the $ccss\bar{s}$ and $bbss\bar{s}$ systems, the identification of a pentaquark state is not so easy. On the contrary, the pentaquark states $ccnn\bar{s}$, $ccss\bar{n}$, $bbnn\bar{s}$, and $bbss\bar{n}$ are easier to identify since the quantum numbers are not allowed for the conventional baryons. For example, if we observed a state in the decay pattern $\Xi_{cc}K$, $\Xi_{cc}K^*$, $\Lambda_c D_s$, $\Lambda_c D_s^*$, $\Sigma_c D_s$, $\Sigma_c D_s^*$, $\Sigma_c^* D_s$, or $\Sigma_c^* D_s^*$, it would be a good candidate for a $ccnn\bar{s}$ pentaquark state. From the diagrams in Fig. 2, the lowest $ccnn\bar{s}$ pentaquark may be stable and the lowest one with $J=\frac{3}{2}$ is

TABLE IX. The estimated masses for the $bbss\bar{q}$ systems in units of MeV. The values in the third column are obtained with the effective quark masses and are theoretical upper limits. The masses after this column are determined with relevant thresholds.

J^P	Eigenvalue	Mass	$(\Omega_b \bar{B})$	J^P	Eigenvalue	Mass	$(\Omega_b \bar{B}_s)$
	$bbss\bar{n}(I=\frac{1}{2})$				$bbss\bar{s}$ $(I=0)$		
$\frac{5}{2}^-$	83.7	11632.1	11438.5	$\frac{5}{2}^-$	51.7	11778.7	11515.2
$\frac{3}{2}^-$	165.6	11714.0	11520.3	$\frac{3}{2}^-$	75.1	11802.1	11538.5
	69.8	11618.2	11424.6		38.8	11765.8	11502.3
	4.6	11553.0	11359.4		22.1	11749.1	11485.6
	−210.4	11338.0	11144.4		−58.4	11668.6	11405.1
$\frac{1}{2}^-$	183.3	11731.7	11538.1	$\frac{1}{2}^-$	95.6	11822.6	11559.1
	94.6	11643.0	11449.4		60.7	11787.7	11524.2
	43.1	11591.5	11397.9		9.4	11736.4	11472.9
	−217.8	11330.6	11136.9		−62.5	11664.5	11400.9

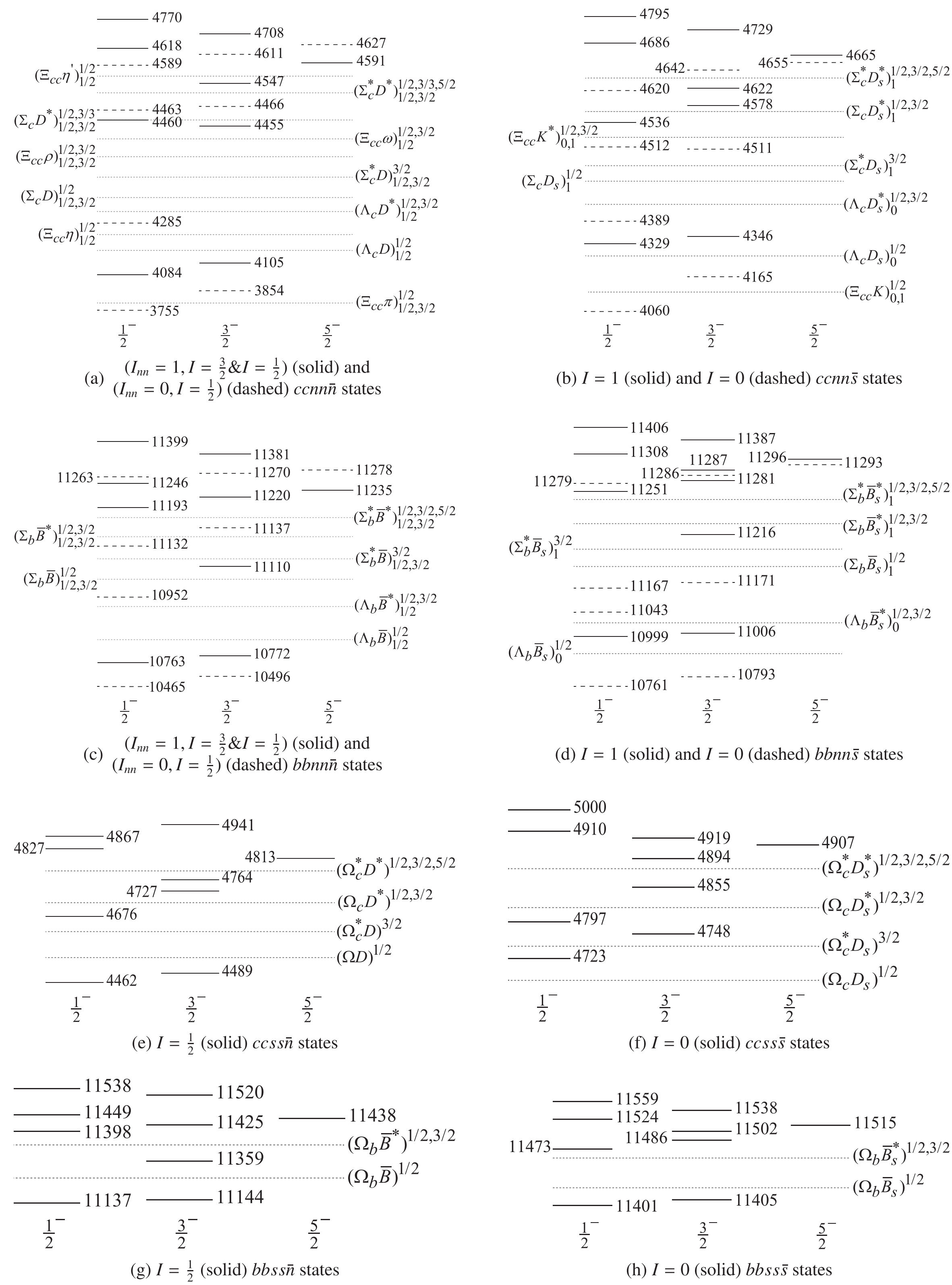


FIG. 2. Relative positions (units: MeV) for the $ccnn\bar{q}$, $bbnn\bar{q}$, $ccss\bar{q}$, and $bbss\bar{q}$ pentaquark states labeled with solid and dashed lines. The dotted lines indicate various baryon-meson thresholds. The $I = \frac{3}{2}$ and $\frac{1}{2}$ $ccnn\bar{n}$ states with $I_{nn} = 1$ have the same mass spectrum and are shown in the diagram (a) with solid lines. The doubly bottom analog is shown in the diagram (c). When the isospin (spin) of an initial pentaquark state is equal to a number in the subscript (superscript) of a baryon-meson state, its decay into that baryon-meson channel through S or D wave is allowed by the isospin (angular momentum) conservation. We have adopted the masses estimated with the reference thresholds of (a) $\Sigma_c D$, (b) $\Sigma_c D_s$, (c) $\Sigma_b \bar{B}$, (d) $\Sigma_b \bar{B}_s$, (e) $\Omega_c D$, (f) $\Omega_c D_s$, (g) $\Omega_b \bar{B}$, and (h) $\Omega_b \bar{B}_s$.

TABLE X. The estimated masses for the $bcnn\bar{q}$ systems in units of MeV. The values in the third column are obtained with the effective quark masses and are theoretical upper limits. The masses after this column are determined with relevant thresholds.

J^P	Eigenvalue	Mass	$(\Sigma_c\bar{B})$	$(\Sigma_b D)$	J^P	Eigenvalue	Mass	$(\Sigma_c\bar{B}_s)$	$(\Sigma_b D_s)$
		$bcnn\bar{n}(I_{nn}=1, I=\frac{1}{2},\frac{3}{2})$					$bcnn\bar{s}$ $(I=1)$		
$\frac{5}{2}^-$	156.9	7993.0	7917.4	7905.1	$\frac{5}{2}^-$	127.2	8141.9	7978.4	7978.8
	51.4	7887.5	7811.9	7799.5		67.0	8081.7	7918.2	7918.6
$\frac{3}{2}^-$	288.4	8124.5	8048.8	8036.5	$\frac{3}{2}^-$	204.9	8219.6	8056.1	8056.4
	161.1	7997.2	7921.6	7909.3		131.2	8145.9	7982.4	7982.8
	131.4	7967.5	7891.8	7879.5		102.3	8117.0	7953.5	7953.9
	73.0	7909.1	7833.5	7821.2		52.9	8067.6	7904.1	7904.5
	37.6	7873.7	7798.0	7785.7		46.3	8061.0	7897.5	7897.8
	−46.4	7789.7	7714.1	7701.7		−31.2	7983.5	7820.0	7820.3
	−319.6	7516.5	7440.9	7428.6		−181.9	7832.8	7669.3	7669.7
$\frac{1}{2}^-$	330.0	8166.1	8090.5	8078.2	$\frac{1}{2}^-$	248.3	8263.0	8099.5	8099.9
	266.9	8103.0	8027.4	8015.0		186.5	8201.3	8037.7	8038.1
	178.8	8014.9	7939.3	7926.9		146.1	8160.8	7997.3	7997.7
	117.1	7953.2	7877.6	7865.2		85.7	8100.4	7936.9	7937.3
	69.6	7905.7	7830.1	7817.7		39.8	8054.5	7891.0	7891.4
	−68.4	7767.8	7692.1	7679.8		−46.5	7968.2	7804.7	7805.1
	−333.8	7502.3	7426.7	7414.3		−191.3	7823.4	7659.9	7660.3
	−364.2	7471.9	7396.3	7384.0		−228.1	7786.6	7623.1	7623.5
		$bcnn\bar{n}$ $(I_{nn}=1, I=\frac{1}{2})$					$bcnn\bar{s}$ $(I=0)$		
$\frac{5}{2}^-$	196.1	8032.2	7956.6	7944.2	$\frac{5}{2}^-$	120.6	8135.3	7971.8	7972.2
$\frac{3}{2}^-$	183.5	8019.6	7944.0	7931.6	$\frac{3}{2}^-$	109.4	8124.1	7960.6	7961.0
	150.4	7986.5	7910.9	7898.5		75.8	8090.5	7926.9	7927.3
	45.1	7881.2	7805.6	7793.3		−12.4	8002.3	7838.8	7839.2
	−123.0	7713.1	7637.4	7625.1		−122.4	7892.3	7728.8	7729.2
	−581.4	7254.7	7179.1	7166.7		−374.7	7640.0	7476.4	7476.8
$\frac{1}{2}^-$	170.1	8006.2	7930.6	7918.2	$\frac{1}{2}^-$	96.3	8111.0	7947.5	7947.9
	41.9	7878.0	7802.4	7790.1		−13.0	8001.7	7838.1	7838.5
	15.2	7851.3	7775.7	7763.3		−39.9	7974.8	7811.3	7811.7
	−128.6	7707.5	7631.9	7619.6		−129.6	7885.1	7721.6	7721.9
	−221.1	7615.0	7539.4	7527.0		−218.6	7796.1	7632.6	7633.0
	−623.0	7213.1	7137.5	7125.2		−418.0	7596.7	7433.2	7433.6
	−663.8	7172.3	7096.7	7084.3		−462.1	7552.6	7389.1	7389.5

TABLE XI. The estimated masses for the $bcss\bar{q}$ systems in units of MeV. The values in the third column are obtained with the effective quark masses and are theoretical upper limits. The masses after this column are determined with relevant thresholds.

J^P	Eigenvalue	Mass	$(\Omega_c\bar{B})$	$(\Omega_b D)$	J^P	Eigenvalue	Mass	$(\Omega_c\bar{B}_s)$	$(\Omega_b D_s)$
		$bcss\bar{n}(I=\frac{1}{2})$					$bcss\bar{s}$ $(I=0)$		
$\frac{5}{2}^-$	95.9	8289.2	8137.9	8109.8	$\frac{5}{2}^-$	64.0	8435.9	8196.7	8181.3
	37.2	8230.5	8079.2	8051.1		53.7	8425.6	8186.4	8171.1
$\frac{3}{2}^-$	163.8	8357.1	8205.8	8177.7	$\frac{3}{2}^-$	85.2	8457.1	8217.9	8202.6
	100.4	8293.7	8142.4	8114.3		62.2	8434.1	8194.9	8179.6
	70.5	8263.8	8112.5	8084.4		44.9	8416.8	8177.6	8162.2
	24.7	8218.0	8066.7	8038.6		36.4	8408.3	8169.1	8153.7
	9.2	8202.5	8051.2	8023.1		−11.6	8360.3	8121.1	8105.8
	−61.9	8131.4	7980.1	7952.0		−24.9	8347.0	8107.8	8092.5
	−219.4	7974.0	7822.6	7794.5		−105.8	8266.1	8026.8	8011.5
$\frac{1}{2}^-$	211.3	8404.6	8253.3	8225.2	$\frac{1}{2}^-$	129.7	8501.6	8262.4	8247.1
	148.7	8342.0	8190.7	8162.6		89.8	8461.7	8222.5	8207.1
	111.1	8304.4	8153.1	8125.0		49.4	8421.3	8182.1	8166.7
	46.3	8239.6	8088.3	8060.2		10.3	8382.2	8143.0	8127.6
	5.6	8198.9	8047.6	8019.5		−17.5	8354.4	8115.2	8099.8
	−78.9	8114.4	7963.1	7935.0		−31.5	8340.4	8101.2	8085.9
	−232.2	7961.1	7809.8	7781.7		−87.8	8284.1	8044.9	8029.6
	−266.1	7927.2	7775.9	7747.8		−148.4	8223.5	7984.3	7968.9

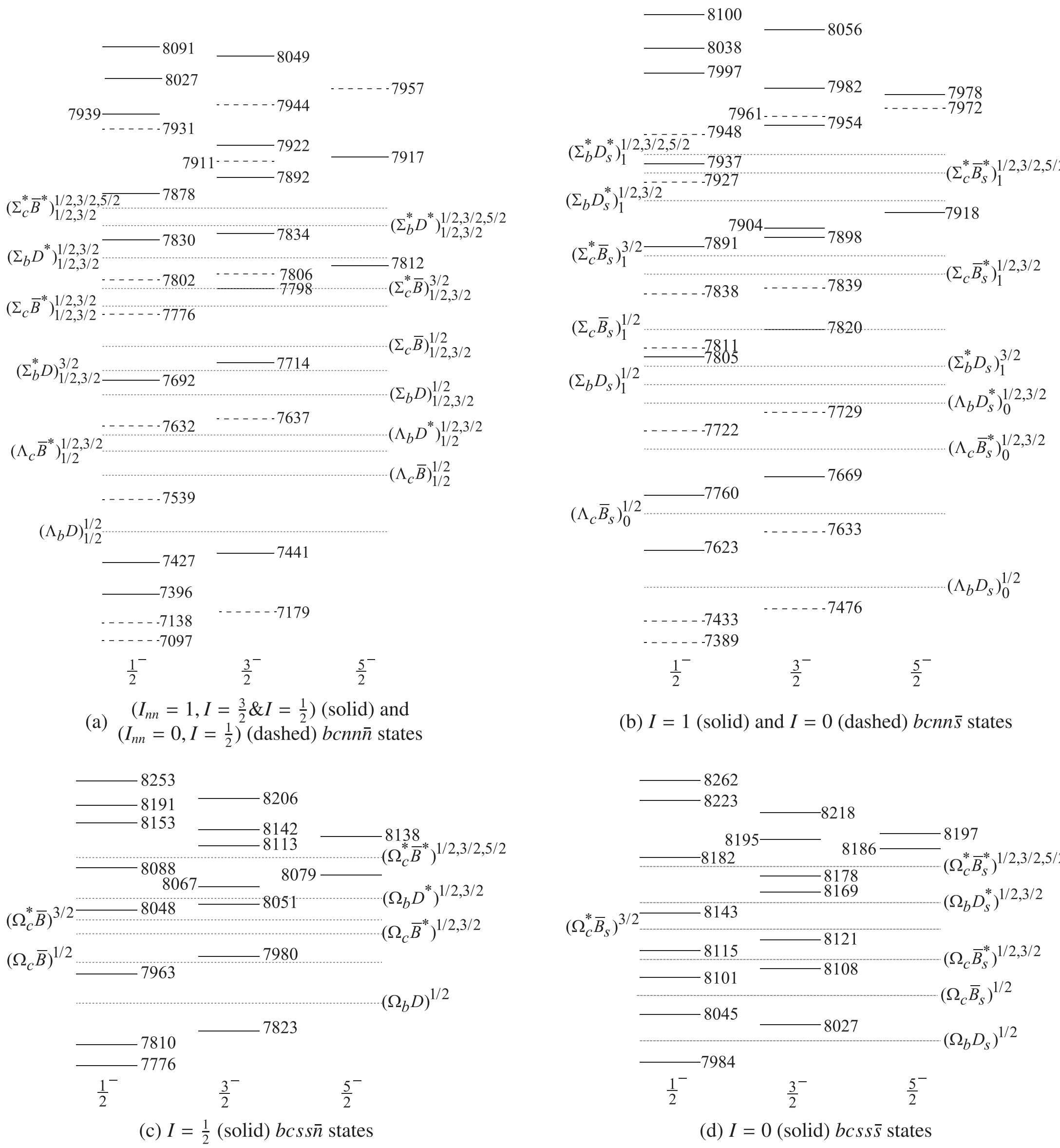


FIG. 3. Relative positions (units: MeV) for the $bcnn\bar{q}$ and $bcss\bar{q}$ pentaquark states labeled with solid and dashed lines. The dotted lines indicate various baryon-meson thresholds. The $I=\frac{3}{2}$ and $\frac{1}{2}$ $bcnn\bar{n}$ states with $I_{nn}=1$ have the same mass spectrum and are shown in the diagram (a) with solid lines. When the isospin (spin) of an initial pentaquark state is equal to a number in the subscript (superscript) of a baryon-meson state, its decay into that baryon-meson channel through S or D wave is allowed by the isospin (angular momentum) conservation. We have adopted the masses estimated with the reference thresholds of (a) $\Sigma_c\bar{B}$, (b) $\Sigma_c\bar{B}_s$, (c) $\Omega_c\bar{B}$, and (d) $\Omega_c\bar{B}_s$.

also relatively stable. Because of the difference in coupling constants, the lowest two $I=0$ $bbnn\bar{s}$ pentaquarks, $J^P=1/2^-$ and $3/2^-$, probably both have strong decay patterns. This can be seen with the values $m_{\Xi_{bb}}=10138$ MeV and $m_{\Xi_{bb}*}=10169$ MeV obtained in Ref. [65]. If such masses are not far from the realistic values, the decay into $\Xi_{bb}K$ or $\Xi^*_{bb}K$ may occur. Once the Ξ_{bb} (Ξ^*_{bb}) state is observed, the search for pentaquark candidates in the $\Xi_{bb}K$ ($\Xi^*_{bb}K$) channel may be performed. For the exotic $ccss\bar{n}$ and $bbss\bar{n}$ pentaquarks, only the isospin $I=1/2$ is allowed. The lowest $J=1/2$ state and the lowest $J=3/2$ state in both systems are lower than the (Qss)-$(Q\bar{n})$-type thresholds and such decay patterns are forbidden. However, one finds that the decay for the $J=1/2$ $(3/2)$ $ccss\bar{n}$ pentaquark into $\Omega_{cc}\bar{K}$ ($\Omega^*_{cc}\bar{K}$) is possible if the mass $m_{\Omega_{cc}}=3715$ MeV ($m_{\Omega^*_{cc}}=3772$ MeV) obtained in Ref. [65] is close to the realistic mass. Similarly, the decay for the $J=1/2$ $(3/2)$ $bbss\bar{n}$ pentaquark into $\Omega_{bb}\bar{K}$ ($\Omega^*_{bb}\bar{K}$) is possible if one checks the threshold with $m_{\Omega_{bb}}=10230$ MeV ($m_{\Omega^*_{bb}}=10258$ MeV). With the (QQs)-$(s\bar{n})$-type channels, the identification of $ccss\bar{n}$ and $bbss\bar{n}$ pentaquarks may be performed in the future measurements.

B. The $bcnn\bar{q}$ and $bcss\bar{q}$ pentaquark states

To estimate the masses of the $bcnn\bar{q}$ and $bcss\bar{q}$ states ($q=n,s$), we can also use two types of thresholds: (charmed

TABLE XII. The estimated masses for the $ccns\bar{q}$ systems in units of MeV. The values in the third column are obtained with the effective quark masses and are theoretical upper limits. The masses after this column are determined with relevant thresholds.

J^P	Eigenvalue	Mass	$(\Xi_c' D)$	$(\Xi_{cc} K)$	J^P	Eigenvalue	Mass	$(\Xi_c' D_s)$	$(\Xi_{cc}\phi)$
		$ccns\bar{n}(I=0,1)$					$ccns\bar{s}$ $(I=\frac{1}{2})$		
$\frac{5}{2}^-$	200.3	4913.9	4764.1	4643.1	$\frac{5}{2}^-$	143.2	5035.4	4810.5	4777.9
	121.7	4835.3	4685.5	4564.6		69.2	4961.4	4736.4	4703.9
$\frac{3}{2}^-$	230.9	4944.5	4794.7	4673.7	$\frac{3}{2}^-$	151.7	5043.9	4818.9	4786.4
	173.6	4887.2	4737.4	4616.4		106.9	4999.1	4774.2	4741.7
	88.2	4801.8	4652.0	4531.0		62.9	4955.1	4730.2	4697.7
	46.5	4760.1	4610.3	4489.3		45.2	4937.4	4712.5	4680.0
	29.5	4743.1	4593.3	4472.3		−13.3	4878.9	4653.9	4621.4
	−231.9	4481.7	4331.9	4210.9		−89.6	4802.6	4577.6	4545.1
	−473.9	4239.7	4089.9	3968.9		−275.6	4616.6	4391.6	4359.1
$\frac{1}{2}^-$	296.1	5009.7	4859.9	4739.0	$\frac{1}{2}^-$	219.1	5111.3	4886.3	4853.8
	169.8	4883.4	4733.6	4612.6		126.4	5018.6	4793.7	4761.2
	140.7	4854.3	4704.5	4583.5		76.2	4968.4	4743.4	4710.9
	48.9	4762.5	4612.7	4491.7		−5.5	4886.7	4661.8	4629.3
	5.3	4718.9	4569.1	4448.1		−14.8	4877.4	4652.5	4620.0
	−89.6	4624.0	4474.2	4353.3		−80.3	4812.0	4587.0	4554.5
	−263.2	4450.4	4300.6	4179.7		−135.0	4757.3	4532.3	4499.8
	−575.8	4137.8	3988.0	3867.0		−375.7	4516.5	4291.5	4259.0

baryon)-(bottom meson) and (bottom baryon)-(charmed meson). The results and relevant reference systems are presented in Tables X and XI. The masses obtained with the two types of thresholds are slightly different. We use results estimated with the (charmed baryon)-(bottom meson)-type threshold for further discussions. In Fig. 3, the relative positions for these pentaquark states and relevant baryon-meson thresholds are plotted. For the $bcss\bar{n}$ and $bcss\bar{s}$ states, only one value of isospin is possible and we do not label the subscripts of the baryon-meson states into which the pentaquarks may decay.

From Figs. 3(a) and 3(d), we see that the $bcnn\bar{n}$ system has more than 12 possible rearrangement decay channels and the $bcss\bar{s}$ system has more than 6. However, one cannot simply distinguish a pentaquark from a conventional baryon or from a $3q$ and $5q$ mixed state just from these decay channels if the isospin is not 3/2. The discussions are similar to the previous systems. On the other hand, in the $bcnn\bar{s}$ and $bcss\bar{n}$ cases, good pentaquark candidates may be searched for in their relevant decay patterns shown in Figs. 3(b) and 3(c). If we use $m_{\Xi_{bc}} = 6922$ MeV, $m_{\Xi_{bc}'} = 6948$ MeV, and

TABLE XIII. The estimated masses for the $bbns\bar{q}$ systems in units of MeV. The values in the third column are obtained with the effective quark masses and are theoretical upper limits. The masses after this column are determined with relevant thresholds.

J^P	Eigenvalue	Mass	$(\Xi_b' \bar{B})$	J^P	Eigenvalue	Mass	$(\Xi_b' \bar{B}_s)$
		$bbns\bar{n}(I=0,1)$				$bbns\bar{s}$ $(I=\frac{1}{2})$	
$\frac{5}{2}^-$	175.5	11545.3	11400.7	$\frac{5}{2}^-$	116.4	11664.8	11432.3
	98.2	11468.0	11323.4		47.9	11596.3	11363.8
$\frac{3}{2}^-$	231.0	11600.8	11456.2	$\frac{3}{2}^-$	144.8	11693.2	11460.7
	166.8	11536.6	11392.0		105.8	11654.2	11421.7
	85.7	11455.5	11310.9		41.2	11589.6	11357.1
	55.2	11425.0	11280.4		28.2	11576.6	11344.1
	12.6	11382.4	11237.8		4.9	11553.3	11320.8
	−233.8	11136.0	10991.4		−83.3	11465.1	11232.7
	−502.8	10867.0	10722.4		−301.6	11246.8	11014.3
$\frac{1}{2}^-$	248.5	11618.3	11473.7	$\frac{1}{2}^-$	163.5	11711.9	11479.4
	162.7	11532.5	11387.9		109.2	11657.6	11425.1
	118.3	11488.1	11343.5		74.1	11622.5	11390.0
	65.0	11434.8	11290.2		21.6	11570.0	11337.5
	51.5	11421.3	11276.7		3.4	11551.8	11319.3
	−89.5	11280.3	11135.7		−80.8	11467.6	11235.1
	−245.4	11124.4	10979.8		−104.1	11444.3	11211.8
	−529.9	10839.9	10695.3		−328.0	11220.4	10987.9

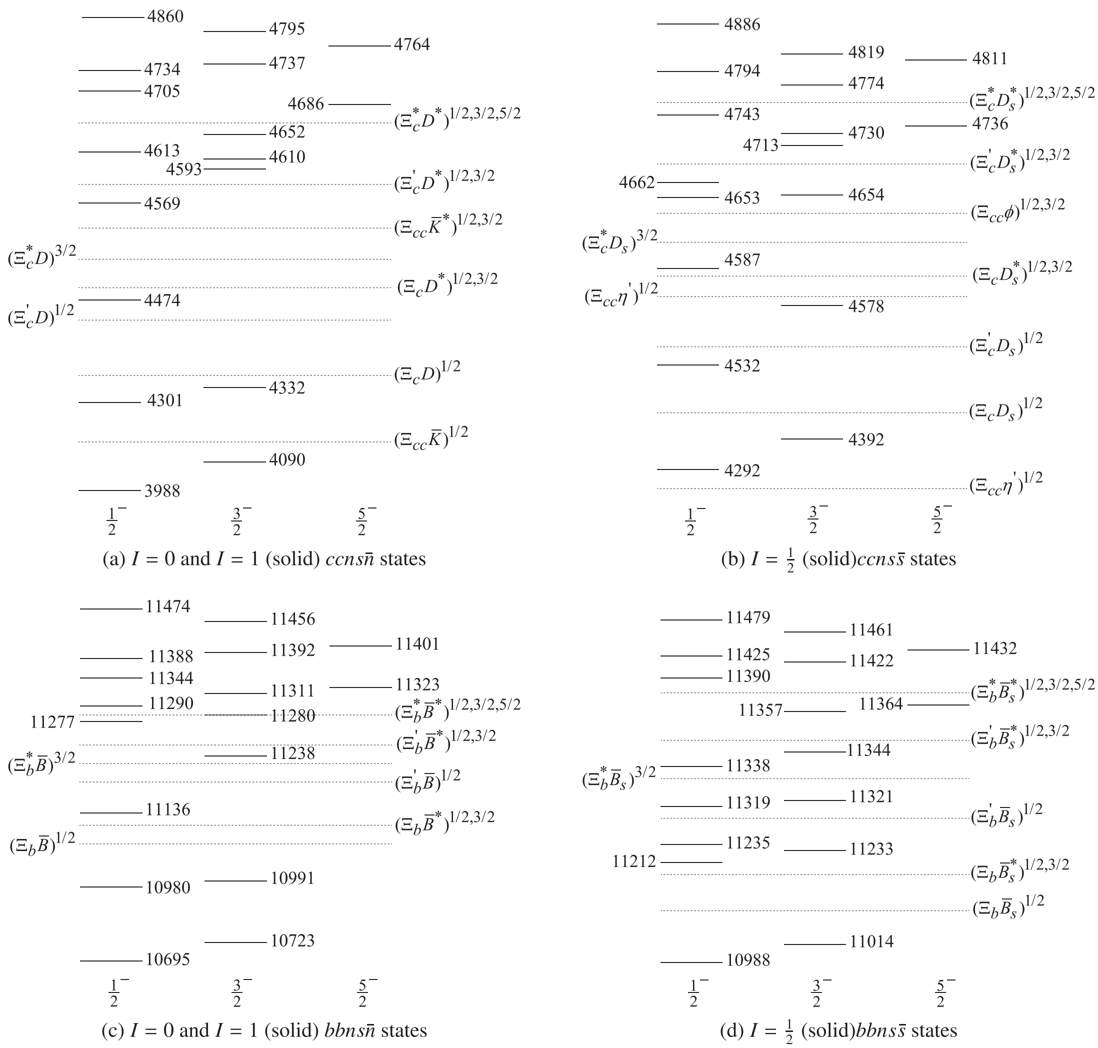


FIG. 4. Relative positions (units: MeV) for the $ccns\bar{q}$ and $bbns\bar{q}$ pentaquark states labeled with solid lines. The dotted lines indicate various baryon-meson thresholds. The $I = 0$ and $I = 1$ $ccns\bar{n}$ states have the same mass spectrum and are shown in the diagram (a). The doubly bottom analog is shown in the diagram (c). When the spin of an initial pentaquark state is equal to a number in the superscript of a baryon-meson state, its decay into that baryon-meson channel through S or D wave is allowed by the angular momentum conservation. We have adopted the masses estimated with the reference thresholds of (a) $\Xi_c' D$, (b) $\Xi_c' D_s$, (c) $\Xi_b' \bar{B}$, and (d) $\Xi_b' \bar{B}_s$.

$m_{\Xi_{bc}^*} = 6973$ MeV [66], one finds that the lowest two $bcnn\bar{s}$ pentaquarks should be stable and the lowest $J^P = 3/2^-$ state is probably narrow. Since the three $I = 3/2$ $bcnn\bar{n}$ states are more than 350 MeV lower than the $\Lambda_b D$ threshold and just above the $\Xi_{bc}\pi$ threshold, they probably have narrow widths and we may use the $\Xi_{bc}\pi$ channels to identify such pentaquarks. Similarly, the $\Omega_{bc}\bar{K}$ channels may be used to identify the $bcss\bar{n}$ pentaquarks if $m_{\Omega_{bc}}$ is around 7011 MeV [66].

C. The $ccns\bar{q}$ and $bbns\bar{q}$ pentaquark states

In the mass estimation for the $ccns\bar{q}$ $(q = n, s)$ system, we use two types of thresholds: (charmed baryon)-(charmed meson) and (doubly charmed baryon)-(light meson). For the $bbns\bar{q}$ system, we only adopt the (bottom baryon)-(bottom meson)-type threshold. The pentaquark masses estimated with the help of the doubly charmed baryon are smaller than those with the (charmed baryon)-(charmed meson)-type threshold. We present the numerical results for the $ccns\bar{q}$ and $bbns\bar{q}$ systems in Tables XII and XIII, respectively. The relative positions for these pentaquark states and the relevant rearrangement decay states are shown in Fig. 4. From the figure, we can see that both the heaviest state and the lightest state are the $J^P = \frac{1}{2}^-$ pentaquarks in each system. Because all these systems contain a quark-antiquark pair, it is not easy to distinguish a pentaquark state from a $3q$ baryon state if the isospin of the decay product is less than 1. Also, the widths of the lowest pentaquark states are probably not narrow if we take $m_{\Omega_{cc}} = 3715$ MeV and $m_{\Omega_{bb}} = 10230$ MeV [65]. In Ref. [69], a bound state with $I = 0$ below the $\Xi_{cc}\bar{K}$ threshold is predicted. If experiments observed one state with the quark content $ccns\bar{n}$, irrespective of its nature, its partner states could also be searched for in the $\Omega_{cc}\pi$, $\Omega_{cc}K$, $\Omega_{bb}\pi$, and $\Omega_{bb}K$ channels and whether they exist or not can test the simple model we use.

TABLE XIV. The estimated masses for the $bcns\bar{q}$ systems in units of MeV. The values in the third column are obtained with the effective quark masses and are theoretical upper limits. The masses after this column are determined with relevant thresholds.

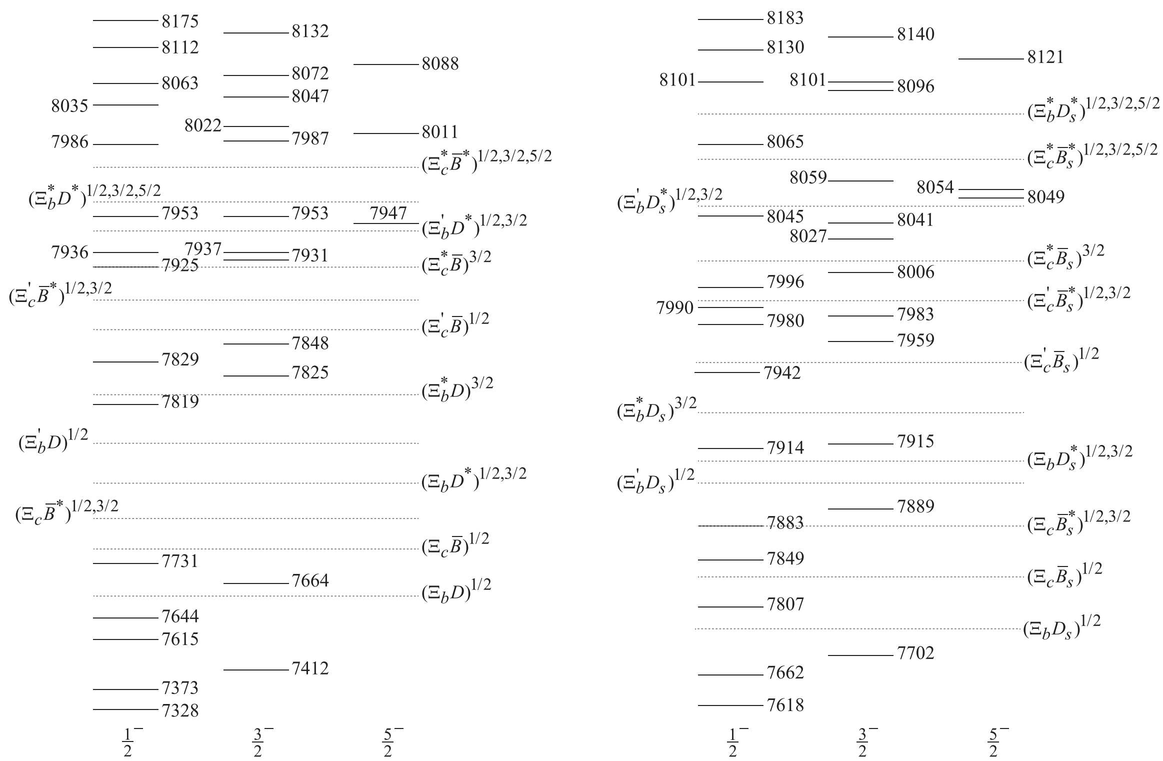
J^P	Eigenvalue	Mass	$(\Xi_c^{'}\bar{B})$	$(\Xi_b^{'}D)$	J^P	Eigenvalue	Mass	$(\Xi_c^{'}\bar{B}_s)$	$(\Xi_b^{'}D_s)$
		$bcns\bar{n}$ $(I=0,1)$					$bcns\bar{s}$ $(I=\frac{1}{2})$		
$\frac{5}{2}^-$	185.8	8227.5	8088.3	8073.1	$\frac{5}{2}^-$	127.8	8348.1	8121.0	8118.6
	108.0	8149.7	8010.5	7995.4		61.2	8281.5	8054.4	8051.9
	44.2	8085.9	7946.6	7931.5		55.8	8276.1	8049.0	8046.6
$\frac{3}{2}^-$	229.0	8270.7	8131.5	8116.3	$\frac{3}{2}^-$	147.0	8367.3	8140.2	8137.8
	169.4	8211.1	8071.8	8056.7		107.6	8327.9	8100.8	8098.4
	144.8	8186.5	8047.3	8032.1		102.9	8323.2	8096.1	8093.7
	119.1	8160.8	8021.5	8006.4		66.2	8286.5	8059.4	8057.0
	84.3	8126.0	7986.8	7971.6		48.0	8268.3	8041.1	8038.7
	50.9	8092.6	7953.4	7938.2		33.6	8253.9	8026.8	8024.4
	34.7	8076.4	7937.2	7922.0		13.1	8233.4	8006.3	8003.9
	28.6	8070.3	7931.1	7915.9		−10.0	8210.3	7983.2	7980.8
	−54.2	7987.5	7848.3	7833.1		−34.5	8185.8	7958.7	7956.3
	−77.1	7964.6	7825.4	7810.2		−78.2	8142.1	7915.0	7912.6
	−238.7	7803.0	7663.8	7648.6		−104.6	8115.8	7888.6	7886.2
	−490.7	7551.0	7411.8	7396.6		−291.3	7929.0	7701.9	7699.4
$\frac{1}{2}^-$	272.1	8313.8	8174.5	8159.4	$\frac{1}{2}^-$	189.9	8410.2	8183.1	8180.7
	209.2	8251.0	8111.7	8096.6		137.1	8357.4	8130.3	8127.9
	160.8	8202.5	8063.3	8048.1		107.7	8328.0	8100.9	8098.5
	132.5	8174.2	8034.9	8019.8		71.7	8292.0	8064.9	8062.5
	83.9	8125.6	7986.4	7971.2		52.1	8272.4	8045.3	8042.8
	50.4	8092.1	7952.8	7937.7		3.0	8223.3	7996.2	7993.8
	33.4	8075.1	7935.8	7920.7		−3.7	8216.6	7989.5	7987.1
	22.1	8063.8	7924.6	7909.4		−13.3	8207.0	7979.9	7977.4
	−73.8	7967.9	7828.7	7813.5		−51.6	8168.7	7941.6	7939.1
	−83.1	7958.6	7819.4	7804.3		−79.7	8140.6	7913.5	7911.1
	−171.6	7870.1	7730.9	7715.7		−110.5	8109.8	7882.7	7880.3
	−258.1	7783.6	7644.4	7629.3		−144.4	8075.9	7848.8	7846.4
	−287.5	7754.2	7615.0	7599.8		−186.4	8033.9	7806.8	7804.3
	−530.0	7511.7	7372.5	7357.4		−330.8	7889.5	7662.4	7660.0
	−574.3	7467.4	7328.2	7313.0		−375.5	7844.8	7617.7	7615.2

D. The $bcns\bar{q}$ pentaquark states

For the $bcns\bar{q}$ states, the wave functions do not get constraints from the Pauli principle and the number of wave function bases for a given quantum number is bigger than that for other systems. After diagonalizing the Hamiltonian, one gets numbers of possible pentaquark states. Here we use two types of thresholds to estimate their masses: (charmed baryon)-(bottom meson) and (bottom baryon)-(charmed meson). The results are presented in Table XIV. One finds that these two types of thresholds lead to comparable values. With the masses from the (charmed baryon)-(bottom meson)-type thresholds, we plot the relative positions for these pentaquarks and their relevant decay patterns in Fig. 5. The quantum numbers of the heaviest state and the lightest state are both $J^P=\frac{1}{2}^-$. The mass of the lightest state for the $bcns\bar{n}$ system is around 7313 MeV, which is above the thresholds of $\Omega_{bc}\pi$, $\Omega_{bc}'\pi$, and $\Omega_{bc}^*\pi$ and is much lower than other two-body baryon-meson thresholds if we adopt the masses obtained in Ref. [66]. This feature is helpful for us to identify compact $I=1$ pentaquarks once the bcs-type baryons can be used to spectrum reconstruction. On the other hand, the identification of a $bcns\bar{s}$ pentaquark is not easy since it may share the same decay products with an excited Ξ_{bc} baryon.

V. DISCUSSIONS AND SUMMARY

Up to now, some candidates of the tetraquark states have been confirmed by different experiments. The observation of the $P_c(4380)$ and $P_c(4450)$ at LHCb gave us significant evidence for the existence of pentaquak states and opened a new door for studying hidden-charm exotic states. More possible pentaquarks have been predicted in various theoretical calculations and await further confirmation. In this paper, motivated by the $P_c(4380)$ and $P_c(4450)$ and the observation of the Ξ_{cc} at LHCb, we have discussed the doubly heavy $QQqq\bar{q}$ pentaquark states in a CMI model and shown their possible rearrangement decay patterns. Although the model we adopt is simple and is not a dynamical model, it may give us some qualitative properties with which the experimentalists may be used to search for such exotic baryons. In the early-stage studies of the multiquark properties, chromomagnetic effects were also intensively considered as the primary contribution in an



(a) $I = 0$ and $I = 1$ (solid) $bcns\bar{n}$ states

(b) $I = \frac{1}{2}$ (solid) $bcns\bar{s}$ states

FIG. 5. Relative positions (units: MeV) for the $bcns\bar{q}$ pentaquark states labeled with solid lines. The dotted lines indicate various baryon-meson thresholds. The $I = 0$ and $I = 1$ $bcns\bar{n}$ states have the same mass spectrum and are shown in the diagram (a). When the spin of an initial pentaquark state is equal to a number in the superscript of a baryon-meson state, its decay into that baryon-meson channel through S or D waves is allowed by the angular momentum conservation. We have adopted the masses estimated with the reference thresholds of (a) $\Xi_b^{'}D$ and (b) $\Xi_b^{'}D_s$.

attempt to explain the narrow hadronic resonances [70]. In recent years, this model as a widely used method was adopted to study the multiquark states, such as the investigations in Refs. [71–78].

In the estimation of the rough masses, we have used two approaches for comparison: one with the quark masses and the other with a reference threshold. The results obtained with the former approach are larger and can be treated as theoretical upper limits. In the estimation with the latter approach, we mainly adopt the (heavy baryon)-(heavy meson)-type thresholds. Although no enough experimental data for the doubly heavy $3q$ baryons are available, we may employ the masses calculated in the quark model [65,66]. For the investigated systems, we find that stable pentaquarks with $I = 0$ are possible in the $bcnn\bar{s}$ case. The lowest threshold of the rearrangement decay product is for the $\Xi_{bc}K$ state while the lowest pentaquarks can be below such thresholds. The typical examples are the two lowest $I = 0$ $bcnn\bar{s}$ states in Fig. 3(b) and the $I = 0$ $ccnn\bar{s}$ state in Fig. 2(b). In the $Q_1Q_2nn\bar{n}$ and $Q_1Q_2ns\bar{n}$ cases, the lowest threshold of the rearrangement decay product is for the $\Xi_{Q_1Q_2}\pi$ or $\Omega_{Q_1Q_2}\pi$, but the lowest pentaquarks we obtain are hard to be below such thresholds. The good news is that the lowest pentaquark may be below the (heavy baryon)-(heavy meson) threshold and one may search for such pentaquarks with the strong decay modes containing a pion. In the $Q_1Q_2ss\bar{n}$ and $Q_1Q_2ns\bar{s}$ cases, the lowest pentaquarks may be above the $\Omega_{Q_1Q_2}\bar{K}$ or $\Omega_{Q_1Q_2}K$ threshold and can be discovered with the decay modes containing a kaon. Contrary to the above systems, the strong decay channel with lowest threshold in the $Q_1Q_2ss\bar{s}$ case may be the (heavy baryon)-(heavy meson)-type. Since the doubly heavy baryons are very difficult to use to reconstruct pentaquark spectra, maybe one should notice the $Q_1Q_2ss\bar{s}$ pentaquarks experimentally. Alternatively, the $J = 5/2$ pentaquarks may be searched for first since they have many D-wave decay modes but one or two S-wave decay modes and probably are not broad. This feature is similar to that of the hidden-charm pentaquarks [58].

In the study of multiquark states, the number of color-spin structures may be more than ten. The mixing or channel-coupling effects could be important. The lowest pentaquarks we obtain get significant contributions from such effects. More studies are needed to determine whether there are substructures in multiquark states and whether the configuration mixing effects are that important. In the near future, further experimental and theoretical studies on pentaquarks are still important, especially with running LHC at 13 TeV and the forthcoming BelleII.

In summary, we have preliminarily studied the mass spectra of doubly heavy pentaquark states in a color-magnetic model. We find candidates of possible narrow states. If they do exist, the identification may be not difficult from their exotic quantum numbers. We hope that the present study may inspire experimental exploration to exotic states.

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