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Signatures of higher-multipole deformations and non-coplanarity as essential, additional degrees-of-freedom in heavy-ion reactions

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Eloquent display of signatures of including higher-multipole (octupole and hexadecupole) deformations $\beta_{\lambda i}$, $i = 1, 2$, $\lambda = 2$–4, and non-coplanar degrees-of-freedom Φ are shown here for the first time as essential, additional variables in low-energy heavy-ion reactions (HIR), within the dynamical cluster-decay model (DCM) of the collective clusterization process. In other words, a description in terms of quadrupole-deformed, "optimally" oriented coplanar nuclei is not sufficient, which therefore calls for both higher-multipole deformations (β_{3i} and β_{4i}; $i = 1, 2$) and non-coplanarity ($\Phi \neq 0^\circ$) as the additional degrees-of-freedom. Such a convincing, real result appears in terms of the non-compound-nucleus (nCN) effect, the nCN cross section $\sigma_{\rm nCN}$-content in $\sigma_{\rm fusion}$, which is different for different compound nuclei (CN), i.e., CN specific, varying from an almost *complete impure* to *pure* CN decay while going successively from (β_{2i}, $\theta_{2i}^{\rm opt}$, $\Phi = 0^\circ$) to (β_{2i}-β_{4i}, "compact" oriented θ_{ci}, $\Phi \neq 0^\circ$), which is further independent of the radioactive or nonradioactive nature of CN formed in different entrance channels, i.e., also independent of entrance channel target-projectile nuclei. This study plus our earlier study of spontaneous ^{14}C-cluster radioactivity provide a new set of variables, in terms of higher-multipole deformations and non-coplanarity degrees-of-freedom, to explore the real outcome of any low-energy HIR (plus spontaneous decay), which could in fact be already close to Nature.

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The true nature of experimental data can be best understood in terms of a model if the variables of the problem are chosen judiciously. For a quantum mechanical, dynamical description, the variables are referred to as good "quantum numbers" or "degrees-of-freedom" of the system. Here, in this work, we investigate the heavy-ion reaction (HIR) problem in terms of the quantum mechanical fragmentation theory (QMFT)-based dynamical cluster-decay model (DCM) [1–3], where the formation and/or decay of a compound nucleus (CN) formed in HIRs is formulated in terms of the mass (and charge) asymmetries η (and η_Z) [$\eta = (A_1 - A_2)/(A_1 + A_2)$ [4], $\eta_Z = (Z_1 - Z_2)/(Z_1 + Z_2)$ [5]] and the coordinate of relative separation R, for quadrupole (β_{2i})-deformed, (optimum $\theta_i^{\rm opt}$) oriented nuclei lying in the same plane (coplanar nuclei, $\Phi = 0^\circ$) [6–11]. The nuclei are, however, observed to have higher-multipole (β_{3i}, β_{4i}) deformations and to lie in two different planes (non-coplanar nuclei, $\Phi \neq 0$; refer to Fig. 1, which is based on Fig. 1 in Ref. [12]). Only a very few studies have been made for coplanar ($\Phi = 0^\circ$) nuclei with higher-multipole deformations (mostly β_{4i}) [13–20] included, the exceptions being a couple of very recent studies of our own group where both deformations including higher multipoles [$\beta_{\lambda i}$, $\lambda = 2, 3, 4$; taken from Möller *et al.* [21], supplemented by Audi *et al.* [22] and relativistic mean field (RMF) calculations for $A < 16$ nuclei] and non-coplanar ($\Phi \neq 0$) degrees-of-freedom are considered, together with hot-compact orientations (θ_{ci}) [23–26] (hot-compact orientations are ones with the highest barriers and smallest interaction radii). Of particular interest to our present study, where the role of (β_{3i}, β_{4i}) is found to be explicitly important is the spontaneous exotic ^{14}C-cluster radioactivity [17], predicted by Săndulescu, Poenaru, and Greiner in 1980 [27] on the basis of the same theory as described here, prior to the very first signatures of ^{14}C decay of ^{223}Ra by Rose and Jones in 1984 [28]. For such decays, the corresponding preformed cluster model (PCM), a zero temperature ($T = 0$) version of DCM is used for coplanar ($\Phi = 0^\circ$) configurations.

In the following, we carry out a systematic analysis of some randomly chosen, DCM-studied HIRs [7–11,18,23–26] where both $\beta_{\lambda i}$, $\lambda = 2, 3, 4$, and Φ degrees-of-freedom play their roles, compared with ones using β_{2i} alone or $\beta_{\lambda i}$, $\lambda = 2, 3, 4$ included for coplanar, $\Phi = 0^\circ$ nuclei. The DCM used here to study the hot and rotating CN is based on the nuclear proximity potential of Blocki *et al.* [29], and the chosen CN, formed in various entrance-channel reactions, are the three weakly fissioning CN ^{105}Ag* [18,25], ^{164}Yb* [8,24], and ^{196}Pt* [9], and three strongly fissioning radioactive CN ^{202}Po* [11], ^{220}Th* [10,26], and ^{246}Bk* [7,23]. It is interesting to find that both multipole deformations ($\beta_{\lambda i}$, $i = 1, 2$, $\lambda > 2$) and non-coplanarity ($\Phi > 0^\circ$) play their roles, which are CN specific, i.e., different for different compound systems, also entrance-channel independent, and hence act as the very essential, additional good quantum numbers, characterizing the CN formed in any low-energy HIR.

The DCM, based on the above-noted collective coordinates, defines for ℓ partial waves the CN decay and/or formation cross section for each pair of fragments as

$$\sigma_{(A_1 {\rm or} A_2)} = \frac{\pi}{k^2}\sum_{\ell=0}^{\ell_{\rm max}}(2\ell+1)P_0 P, \quad k = \sqrt{\frac{2\mu E_{\rm c.m.}}{\hbar^2}}, \qquad (1)$$

where P_0, the relative preformation probability referring to η motion, is given as the (normalized) solution of the stationary Schrödinger equation in η, namely, $P_0(A_i) = |\psi(\eta(A_i))|^2 \sqrt{B_{\eta\eta}}\frac{2}{A}$, where $B_{\eta\eta}$ are the smooth classical hydrodynamical masses [30]; and P, the penetration probability referring to R motion, is given by the WKB integral between the first turning point R_a and second turning point R_b, where

$$R_a(T) = R_1(\alpha_1, T) + R_2(\alpha_2, T) + \Delta R(\eta, T), \qquad (2)$$

being dependent on both angular momentum ℓ and temperature T. The above relations are used for each decay channel, whose sum for light-particle decay $A_2 = 1$–4 or 5 gives the evaporation residue (ER) cross section $\sigma_{\rm ER}$ ($= \sum_{A_2=1}^{4{\rm or}5} \sigma_{A_2}$), the intermediate mass fragmentation (IMF) $A_2 = 5$ or $6 \leqslant A_2 \leqslant 20$, and the fusion-fission cross section $\sigma_{\rm ff}$ ($= 2\sum_{5{\rm or}6}^{A/2} \sigma_{A_2}$). The definition of $\sigma_{\rm ff}$ is also applicable to IMFs with the sum taken up to the maximum measured value of IMFs and without the multiplying factor 2 (i.e., only for light fragments); in fact, the IMFs are a part of fusion-fission. Thus, the pure CN cross section $\sigma_{\rm CN}^{\rm pure} = \sigma_{\rm ER} + \sigma_{\rm ff}$.

Equation (1) is also applicable to nCN decay, treated as the quasifission (qf)-like decay channel where $P_0 = 1$ since the target and projectile nuclei are considered to have not yet lost their identities. Then, $\sigma_{\rm fusion} = \sigma_{\rm CN}^{\rm pure} + \sigma_{\rm nCN}$.

In Eq. (2), the neck-length parameter ΔR, which varies smoothly with $E^*_{\rm CN}$, is directly related to "barrier lowering" and hence to the fusion hindrance phenomenon in HIRs. The choice of ΔR, for a best fit to experimental data on excitation function $\sigma_{\rm fusion}^{\rm Expt}(E^*_{\rm CN})$ for each and every reaction, allows us to relate in a simple way the $V(R_a, \ell)$ at $R = R_a$ to the top of the barrier $V_B(\ell)$ for each ℓ, by defining the difference $\Delta V_B(\ell) = V(R_a, \ell) - V_B(\ell)$ as the effective "lowering of the barrier."

To calculate the cross sections for non-coplanar, $\Phi \neq 0^\circ$ nuclei (refer to Fig. 1), we use the same formalism as for $\Phi = 0^\circ$ [31], but by replacing for the out-of-plane nucleus ($i = 1$ or 2) the corresponding radius parameter $R_i(\alpha_i)$ ($= R_{0i}[1 + \sum_\lambda \beta_{\lambda i} Y_\lambda^{(0)}(\alpha_i)]$, $i = 1, 2$, R_{0i} being the equivalent spherical nucleus radius), with its projected radius parameter $R_i^P(\alpha_i)$ in both Coulomb and nuclear proximity potentials [12]. For the Coulomb potential, it enters via $R_i(\alpha_i)$, and for the proximity potential via the definitions of both the mean curvature radius $\bar{R}$ and the shortest distance s_0, i.e., the compact configurations with orientations θ_{ci} and Φ_c. For further details, see Refs. [6,32].

Using the DCM, we present in Fig. 2, our calculated *pure* CN or fusion cross sections ($\sigma_{\rm CN}^{\rm Cal}$), compared with experimental data ($\sigma_{\rm fusion}^{\rm Expt}$), of the six chosen compound systems, mentioned above, three being strongly fissioning radioactive CN (^{202}Po*, ^{220}Th*, and ^{246}Bk*) and the other three weakly fissioning nonradioactive CN (^{105}Ag*, ^{164}Yb*, and ^{196}Pt*). The main aim here is to see the role of including higher-multipole deformations ($\beta_{\lambda i}$, $\lambda = 2$–4) and non-coplanarity ($\Phi \neq 0^\circ$) degrees-of-freedom toward estimating the nCN content $\sigma_{\rm nCN}$ in $\sigma_{\rm fusion}$, and hence establishing the nature of the two variables ($\beta_{\lambda i}$, $\lambda = 2$–4 and Φ) as independent degrees-of-freedom which must be considered in all heavy-ion-reaction studies, otherwise the net result could be erroneous.

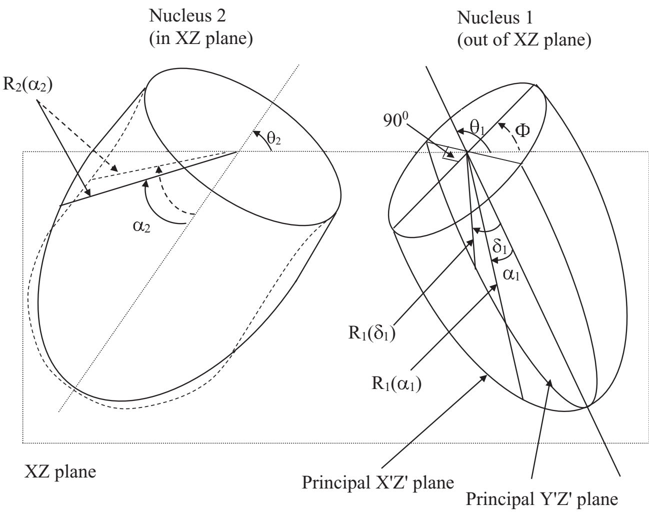

FIG. 1. Two unequal nuclei (one β_2 deformed and the other up to β_4), oriented at angles θ_1 and θ_2, with their principal planes $X'Z'$ and XZ making an azimuthal angle Φ. The angle Φ is shown by a dashed line, since it is meant to be an angle coming out of plane XZ. Nucleus 2 is in the XZ plane and for the out-of-plane nucleus 1, another principal plane $Y'Z'$, perpendicular to $X'Z'$, is also shown. Only the lower halves of the two nuclei are shown. This figure is based on Fig. 1 of Ref. [12].

The interesting result of Fig. 2 is that the effect of non-coplanarity ($\Phi \neq 0^\circ$) is different in different reactions. In the studied cases of $\Phi = 0^{\rm o}$ configurations with β_2 deformations and $\theta^{\rm opt}$ orientations, the $\sigma_{\rm nCN}$ content in $\sigma_{\rm fusion}$ is different for different reactions, i.e., it is zero or of a considerable amount which gets reduced or remains constant in going from β_2 alone to $\beta_{\lambda i}$, $\lambda = 2$–4.

Let us first take up in detail the three (strongly fissioning) radioactive nuclei in Figs. 2(d)–2(f). In Fig. 2(d) for ^{246}Bk* formed via the ^{11}B + ^{235}U reaction, we notice that for $\Phi = 0^{\rm o}$, the nCN contribution is nonzero both for β_{2i} alone, $\theta^{\rm opt}$ as well as for higher-multipole deformations included, i.e., β_{2i}-β_{4i}, θ_{ci} (compare empty squares with empty stars), and that it goes successively to zero for $\Phi \neq 0^\circ$ together with β_{2i}-β_{4i} (compare empty triangles with experimental data shown as filled circles). Thus, a description in terms of quadrupole deformed, optimally oriented, coplanar nuclei is not sufficient, i.e., the appearance of the nCN effect here is apparently an artefact of our calculations, seen only at the highest two CN excitation energies $E^*_{\rm CN}$, which gets reduced to zero for β_{2i}-β_{4i} and $\Phi \neq 0^\circ$, thereby calling for both higher-multipole deformations (β_{3i} and β_{4i}; $i = 1, 2$) and non-coplanarity ($\Phi \neq 0^\circ$) as the possible essential degrees-of-freedom in such HIR studies.

On the other hand, in Fig. 2(e) for ^{220}Th* formed via the ^{48}Ca + ^{172}Yb reaction, the nCN content (due to the 4n-decay channel) is very large ($\sim$95%) of channel cross section $\sigma_{xn}^{\rm Cal}$, i.e., the magnitude of $\sigma_{\rm nCN}/\sigma_{xn}^{\rm Cal} = \sigma_{\rm nCN}/(\sigma_{xn}^{\rm CN} + \sigma_{\rm nCN}^{\rm Cal})$ as a function of $E^*_{\rm CN}$ remains nearly a constant ($\simeq$0.95) for $x = 4$ in the present case, out of the measured $x = 3$–5 decay channels, and is further same for adding or not adding higher-multipole deformations β_{3i}, β_{4i} in coplanar ($\Phi = 0^{\rm o}$)

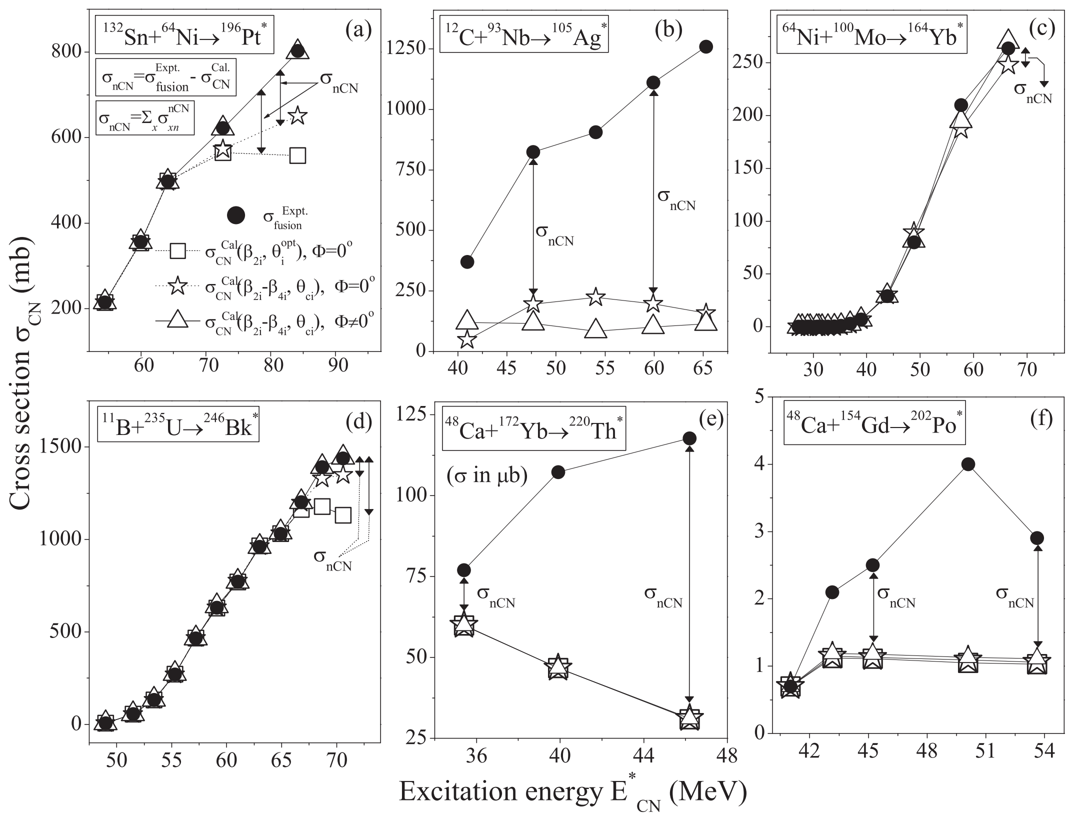

FIG. 2. Calculated CN cross sections $\sigma_{\rm CN}^{\rm Cal}$ for three cases of quadrupole (β_{2i}) deformed and optimally oriented ($\theta_i^{\rm opt}$), $\Phi = 0^{\rm o}$ configuration, and higher-multipole deformations (β_{3i}, β_{4i}) included, "compact" oriented (θ_{ci}) nuclei for both $\Phi_c = 0^{\rm o}$ and $\Phi_c \neq 0^{\rm o}$ configurations compared with experimental data for various CN formed in different entrance channels. The $\sigma_{\rm nCN}$ content in each case is also shown. For ^{105}Ag* and ^{164}Yb*, calculations are available for only two of the three cases, and the nCN content is more than 90%.

or non-coplanar ($\Phi \neq 0^\circ$) configuration [see Fig. 3(c)]. Thus, the role of including or not including multipole deformations and using coplanar or non-coplanar degrees-of-freedom in CN ^{220}Th* is different from, say, the CN ^{246}Bk*, and $\sigma_{\rm nCN}$ is of a large constant value as a function of $E^*_{\rm CN}$ and hence a real, measurable effect. Similarly, in Fig. 2(f) for another radioactive CN ^{202}Po*, formed via the ^{48}Ca + ^{154}Gd reaction,

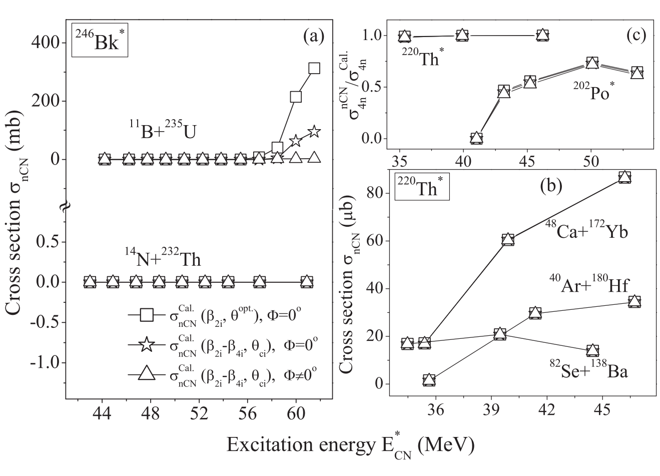

FIG. 3. Same as Fig. 2, but for nCN cross sections $\sigma_{\rm nCN}^{\rm Cal.}$ and for different entrance channels forming (a) ^{246}Bk* and (b) ^{220}Th*. (c) Relative $\sigma_{\rm nCN}/\sigma_{xn}^{\rm Cal}$ for ^{220}Th* vs ^{202}Po* CN.

we observe that its behavior is nearly the same as for the above studied ^{220}Th* CN, i.e., the nCN content is the same for adding or not adding higher-multipole deformations in coplanar ($\Phi = 0^{\rm o}$) or non-coplanar ($\Phi \neq 0^\circ$) configuration, but instead its magnitude with $E^*_{\rm CN}$ varies from zero to a maximum of ∼70% of the channel cross section $\sigma_{xn}^{\rm Cal}$; $x = 4$ [see Fig. 3(c)]. The similarity of the above result is possibly due to the same projectile (^{48}Ca) and target nucleus belonging to the same strongly deformed rare-earth mass region, though the variation of $\sigma_{\rm nCN}$ with $E^*_{\rm CN}$ in Fig. 3(c) is still CN specific.

Next, including the weakly fissioning nonradioactive nuclei (^{105}Ag*, ^{164}Yb*, and ^{196}Pt*) in the discussion, we first notice that the characterization of CN into radioactive and nonradioactive CN is irrelevant, since the nonradioactive CN ^{196}Pt* formed via ^{132}Sn + ^{64}Ni in Fig. 2(a) behaves exactly the same as the radioactive ^{246}Bk* formed via ^{11}B + ^{235}U in Fig. 2(d), both behaving as pure CN reactions for the β_{2i}-β_{4i}, $\Phi \neq 0^\circ$ configuration, and ^{12}C + ^{93}Nb → ^{105}Ag* in Fig. 2(b) the same (95 ± 5% impure) as ^{48}Ca + ^{172}Yb → ^{220}Th* in Fig. 2(e). Furthermore, ^{164}Yb* formed via the ^{64}Ni + ^{100}Mo reaction in Fig. 2(c) fits the measured data nearly exactly, with a strongly reduced nCN contribution in going from $\Phi = 0^{\rm o}$ to β_{2i}-β_{4i}, $\Phi \neq 0^\circ$. Thus, for ^{164}Yb*, the nCN content is reduced almost to zero, and hence the reaction ^{64}Ni + ^{100}Mo for $\Phi \neq 0^\circ$ could be taken as a pure CN reaction, similar to ^{246}Bk* in Fig. 2(d). Thus, the result of Fig. 2 is clearly CN specific, irrespective of being radioactive or nonradioactive, or adding or not adding the higher-multipole deformations and non-coplanar degrees-of-freedom.

Finally, considering the entrance channel effect, we notice in Figs. 3(a) and 3(b) that in the case of ^{220}Th* [Fig. 3(b)], for each entrance channel, the $\sigma_{\rm nCN}$ value (refer to open square, open star, and open triangle symbols) remains exactly the same, i.e., independent of adding or not adding higher-multipole deformations and non-coplanarity Φ degrees-of-freedom, whereas for ^{246}Bk* [Fig. 3(a)] formed via entrance channel ^{14}N + ^{232}Th, the $\sigma_{\rm nCN} = 0$ for all three cases of β_{2i}-alone, $\theta^{\rm opt}$, higher-multipole deformations included, i.e., β_{2i}-β_{4i}, θ_{ci}, $\Phi_c = 0^\circ$, or $\Phi_c \neq 0^\circ$, and that for ^{246}Bk* formed via another entrance channel ^{11}B + ^{235}U, the $\sigma_{\rm nCN}$ reduces successively to zero. In other words, with higher-multipole deformations and non-coplanarity Φ_c included, ^{246}Bk* formed via both entrance channels is a *pure* CN decay, which revokes the experimental result [33] that one of the entrance channels (^{14}N + ^{232}Th) contain nCN content, though no data were recorded. Thus, once again, our result is CN specific.

Concluding, the higher-multipole deformations (β_{3i}, β_{4i}) together with non-coplanar ($\Phi_c \neq 0$) configurations are very important additional degrees-of-freedom for analyzing a true and real outcome of a compound nucleus fusion reaction, i.e., its CN-specific character. Thus, putting this result together with our earlier result for spontaneous exotic cluster radioactivity, the above-stated two additional variables of higher-multipole deformations and non-coplanarity must be included in each and every HIR and spontaneous ($T = 0$) decay study.

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[1] R. K. Gupta, S. K. Arun, R. Kumar, and Niyti, Int. Rev. Phys. (IREPHY) **2**, 369 (2008).

[2] R. K. Gupta, Review in *Clusters in Nuclei*, Lecture Notes in Physics 18, edited by C. Beck, Vol. 1 (Springer, Berlin, 2010) p. 223.

[3] R. K. Gupta, Review in *Nuclear Particle Correlations and Cluster Physics*, edited by W.-U. Schröder (World Scientific, Singapore, 2017), Chap. 18.

[4] J. Maruhn and W. Greiner, Phys. Rev. Lett. **32**, 548 (1974).

[5] R. K. Gupta, W. Scheid, and W. Greiner, Phys. Rev. Lett. **35**, 353 (1975).

[6] R. K. Gupta *et al.*, J. Phys. G: Nucl. Part. Phys. **31**, 631 (2005).

[7] B. B. Singh, M. K. Sharma, and R. K. Gupta, Phys. Rev. C **77**, 054613 (2008).

[8] S. K. Arun, R. Kumar, and R. K. Gupta, J. Phys. G: Nucl. Part. Phys. **36**, 085105 (2009).

[9] M. K. Sharma, S. Kanwar, G. Sawhney, R. K. Gupta, and W. Greiner, J. Phys. G: Nucl. Part. Phys. **38**, 055104 (2011).

[10] Hemdeep, S. Chopra, A. Kaur, and R. K. Gupta, Phys. Rev. C **95**, 014609 (2017).

[11] P. Kaushal, A. Kaur, Hemdeep, S. Chopra, and R. K. Gupta, Phys. Rev. C **98**, 014602 (2018).

[12] M. Manhas and R. K. Gupta, Phys. Rev. C **72**, 024606 (2005).

[13] M. J. Rhoades-Brown and V. E. Oberacker, Phys. Rev. Lett. **50**, 1435 (1983).

[14] J. O. Fernández Niello, M. di Tada, A. O. Macchiavelli, A. J. Pacheco, D. Abriola, M. Elgue, A. Etchegoyen, M. C. Etchegoyen, S. Gil, and J. E. Testoni, Phys. Rev. C **43**, 2303 (1997).

[15] Ş. Mişicu and W. Greiner, Phys. Rev. C **69**, 054601 (2004).

[16] V. Yu. Denisov and N. A. Pilipenko, Phys. At. Nucl. **73**, 1152 (2010).

[17] G. Sawhney, M. K. Sharma, and R. K. Gupta, Phys. Rev. C **83**, 064610 (2011).

[18] S. Chopra, M. Bansal, M. K. Sharma, and R. K. Gupta, Phys. Rev. C **88**, 014615 (2013).

[19] X. J. Bao, S. Q. Guo, H. F. Zhang, and J. Q. Li, J. Phys. G: Nucl. Part. Phys. **43**, 125105 (2016).

[20] G. Kaur, K. Hagino, and N. Rowley, Phys. Rev. C **97**, 064606 (2018).

[21] P. Möller, J. R. Nix, W. D. Myers, and W. J. Swiatecki, At. Nucl. Data Tables **59**, 185 (1995).

[22] G. Audi, A. H. Wapstra, and C. Thiboult, Nucl. Phys. A **729**, 337 (2003).

[23] R. K. Gupta and M. Bansal, Int. Rev. Phys. (IREPHY) **5**, 74 (2011).

[24] M. Bansal, S. Chopra, R. K. Gupta, R. Kumar, and M. K. Sharma, Phys. Rev. C **86**, 034604 (2012).

[25] S. Chopra, Hemdeep, A. Kaur, and R. K. Gupta, Phys. Rev. C **93**, 024603 (2016).

[26] Hemdeep, S. Chopra, A. Kaur, P. Kaushal, and R. K. Gupta, Phys. Rev. C **97**, 044623 (2018).

[27] A. Săndulescu, D. N. Poenaru, and W. Greiner, Sovt. J. Part. Nucl. **11**, 528 (1980).

[28] H. J. Rose and G. A. Jones, Nature (London) **307**, 245 (1984).

[29] J. Blocki, J. Randrup, W. J. Swiatecki, and C. F. Tsang, Ann. Phys. (NY) **105**, 427 (1977).

[30] H. Kröger and W. Scheid, J. Phys. G **6**, L85 (1980).

[31] R. K. Gupta, N. Singh, and M. Manhas, Phys. Rev. C **70**, 034608 (2004).

[32] R. K. Gupta, M. Manhas, and W. Greiner, Phys. Rev. C **73**, 054307 (2006).

[33] B. R. Behera *et al.*, Phys. Rev. C **64**, 041602(R) (2001); **69**, 064603 (2004).