Tests of the standard model in neutron β decay with a polarized neutron and electron and an unpolarized proton

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We analyze the electron-energy and angular distribution of neutron β^- decay with a polarized neutron and electron and an unpolarized proton, calculated by Ivanov *et al.* [Phys. Rev. C **95**, 055502 (2017)] within the standard model (SM), by taking into account the contributions of interactions beyond the SM. After the absorption of vector and axial-vector contributions by the axial coupling constant and Cabibbo-Kobayashi-Maskawa (CKM) matrix element [Bhattacharya *et al.*, Phys. Rev. D **85**, 054512 (2012) and so on] these are the contributions of scalar and tensor interactions only. The neutron lifetime, correlation coefficients and their averaged values, and asymmetries of neutron β^- decay with a polarized neutron and electron are adapted to the analysis of experimental data in search of contributions of interactions beyond the SM. Using the obtained results we propose some estimates of the values of the scalar and tensor coupling constants of interactions beyond the SM. We use the estimate of the Fierz interference term $b = -0.0028 \pm 0.0026$ by Hardy and Towner [Phys. Rev. C **91**, 025501 (2015)], the neutron lifetime $\tau_n = 880.2(1.0)$ s [Particle Data Group, Chin. Phys. C **40**, 100001 (2016)] and the experimental data $N_{\rm exp} = 0.067 \pm 0.011_{\rm stat.} \pm 0.004_{\rm syst.}$ for the averaged value of the correlation coefficient of the neutron-electron spin-spin correlations, measured by Kozela *et al.* [Phys. Rev. C **85**, 045501 (2012)]. The contributions of G-odd correlations are calculated and found at the level of 10^{-5} in agreement with the results obtained by Gardner and Plaster [Phys. Rev. C **87**, 065504 (2013)].

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I. INTRODUCTION

Recently [1] we calculated in the standard model (SM) the electron-energy and angular distribution of neutron β^- decay with a polarized neutron and electron and an unpolarized proton by taking into account the contributions of the weak magnetism and proton recoil of order $O(E_e/M)$, where M is an averaged nucleon mass and E_e is the electron energy, and the radiative corrections of order $O(\alpha/\pi)$, where α is the fine-structure constant [2]. These contributions define a complete set of corrections of order 10^{-3} to the correlation coefficients of neutron β^- decay with a polarized neutron and electron and an unpolarized proton. The obtained results together with Wilkinson's corrections of order 10^{-5} [3], which we have also adapted to the correlation coefficients of the neutron β^- decay under consideration [1], may provide a robust SM theoretical background for the analysis of experimental data in search of contributions of interactions beyond the SM at the level of 10^{-4} [4] or even better [5] (see also [1,6]) if they are supplemented by a complete set of corrections of order 10^{-5}. This set of corrections is caused by the weak magnetism and proton recoil of order $O(E_e^2/M^2)$, calculated to next-to-next-to-leading order in the large nucleon mass expansion, the radiative corrections of order $O(\alpha E_e/M)$, calculated to next-to-leading order in the large nucleon mass expansion, and the radiative corrections of order $O(\alpha^2/\pi^2)$, calculated to leading order in the large nucleon mass expansion [7,8]. The first steps towards the experimental search for contributions of interactions beyond the SM in neutron β^- decay with a polarized neutron and electron and an unpolarized proton have been done by Kozela *et al.* [9,10].

The paper is organized as follows. In Sec. II we give the electron-energy spectrum and angular distribution of neutron β^- decay with a polarized neutron and electron and an unpolarized proton, which has been calculated within the SM in [1]. The correlation coefficients $A_W(E_e)$, $G(E_e)$, $N(E_e)$, $Q_e(E_e)$ and $R(E_e)$ [see Eq. (1)] are calculated at the level of 10^{-3} by taking into account the contributions of the weak magnetism and proton recoil to next-to-leading order in the large proton mass expansion and radiative corrections of order $O(\alpha/\pi)$ calculated to leading order in the large proton mass expansion [1,6]. In Sec. III we calculate

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TABLE I. Scalar and tensor coupling constants of interactions beyond the SM and their contributions to the neutron lifetime and the measurable correlation coefficients of neutron β^- decay with a polarized neutron and electron for the correlation coefficient $b_E = -0.0107$. With an accuracy of about 7×10^{-5} the values of the correlation coefficient b_F define the values of the Fierz interference term b.

b_F	b_E	C_S	C_T	$\Delta\tau_n^{(\mathrm{BSM})}/\tau_n$	$\langle \bar{N}^{(\mathrm{BSM})}(E_e)\rangle_{\mathrm{SM}}$	$\langle N^{(\mathrm{BSM})}(E_e)\rangle_{\mathrm{SM}}$	$\langle A_W^{(\mathrm{BSM})}(E_e)\rangle_{\mathrm{SM}}$
−0.0002	−0.0107	0.0186	0.0050	6.0×10^{-5}	−0.00944	−0.01067	4.0×10^{-5}
−0.0054	−0.0107	0.0149	0.0080	3.5×10^{-3}	−0.00942	−0.01066	6.3×10^{-5}
−0.0017	−0.0107	0.0175	0.0059	10^{-3}	−0.00944	−0.01067	4.7×10^{-5}
−0.0004	−0.0107	0.0184	0.0051	1.9×10^{-4}	−0.00944	−0.01067	4.1×10^{-5}

the contributions of interactions beyond the SM to the correlation coefficients $A_W(E_e)$, $G(E_e)$, $N(E_e)$, $Q_e(E_e)$, and $R(E_e)$, calculated to leading order in the large nucleon mass expansion, and arrive at the correlation coefficients $A_{W,\mathrm{eff}}(E_e)\,G_{\mathrm{eff}}(E_e)$, $N_{\mathrm{eff}}(E_e)$, $Q_{e,\mathrm{eff}}(E_e)$, and $R_{\mathrm{eff}}(E_e)$ [11–23] (see also [6]). In the linear approximation for vector and axial-vector interactions beyond the SM, the obtained contributions are defined by scalar and tensor nucleon-lepton four-fermion couplings beyond the SM only in agreement with [18–23] (see also [6]). A possible dominant role of scalar and tensor interactions beyond the SM has been also discussed by Jackson *et al.* [12]. In Sec. IV we give the neutron lifetime, correlation coefficients $A_{W,\mathrm{eff}}(E_e)$, $G_{\mathrm{eff}}(E_e)$, $N_{\mathrm{eff}}(E_e)$, $Q_{e,\mathrm{eff}}(E_e)$ and $R_{\mathrm{eff}}(E_e)$ and asymmetries of neutron β^- decay with a polarized neutron and electron in a form suitable for the analysis of experimental data in search of interactions beyond the SM, including the complete set of the SM corrections of order 10^{-3}, Wilkinson's corrections of order 10^{-5} [1], and contributions of interactions beyond the SM. The contributions of interactions beyond the SM agree well with the results obtained by Jackson *et al.* [12,13] and Severijns *et al.* [17] up to redefinition of the metric and normalization. In Sec. V we calculation the G-odd corrections to the neutron lifetime and correlation coefficients of neutron β^- decay with a polarized neutron and electron and an unpolarized proton. We estimate these corrections to be at the level of 10^{-5} and even smaller, in agreement with the results obtained by Gardner and Plaster [23]. In Sec. VI we discuss the obtained results and propose some estimates of the values of scalar and tensor coupling constants of interactions beyond the SM. We follow Severijns *et al.* [17] and use for simplicity a real coupling constant approximation and nucleon-lepton four-fermion couplings with left-handed neutrinos only. The obtained results are given in Tables I and II. For the analysis of experimental data in search of contributions of interactions beyond the SM at the level of 10^{-4} and even better [5], we argue that there is an important role of the theoretical background with SM corrections of order 10^{-5}, including Wilkinson's corrections [1] and corrections caused by the weak magnetism and proton recoil of order $O(E_e^2/M^2)$, radiative corrections of order $O(\alpha E_e/M)$, and radiative corrections of order $O(\alpha^2/\pi^2)$ [7,8].

II. ELECTRON ENERGY AND ANGULAR DISTRIBUTION IN THE SM

The electron-energy and angular distribution of neutron β^- decay with a polarized neutron and electron, introduced for the first time by Jackson *et al.* [12,13] but using in notations as in [1], is given by

$$\frac{d^3\lambda_n(E_e,\vec{k}_e,\vec{\xi}_n,\vec{\xi}_e)}{dE_e d\Omega_e} = (1+3\lambda^2)\frac{G_F^2|V_{ud}|^2}{8\pi^4}(E_0-E_e)^2\sqrt{E_e^2-m_e^2}\,E_e\,F(E_e,Z=1)\,\zeta(E_e)\bigg\{1 + A_W(E_e)\frac{\vec{\xi}_n\cdot\vec{k}_e}{E_e} + G(E_e)\frac{\vec{\xi}_e\cdot\vec{k}_e}{E_e} + N(E_e)\,\vec{\xi}_n\cdot\vec{\xi}_e + Q_e(E_e)\frac{(\vec{\xi}_n\cdot\vec{k}_e)(\vec{k}_e\cdot\vec{\xi}_e)}{E_e(E_e+m_e)} + R(E_e)\frac{\vec{\xi}_n\cdot(\vec{k}_e\times\vec{\xi}_e)}{E_e}\bigg\}, \tag{1}$$

where $G_F = 1.1664\times10^{-11}\ \mathrm{MeV}^{-2}$ is the Fermi weak constant; $V_{ud} = 0.97417(21)$ is the Cabibbo-Kobayashi-Maskawa (CKM) matrix element [2], extracted from the $0^+\to0^+$ transitions; $\lambda = -1.2750(9)$ is the axial coupling constant, which is real [6]; $E_0 = (m_n^2 - m_p^2 + m_e^2)/2m_n = 1.2927$ MeV is the endpoint energy of the electron-energy spectrum, calculated for $m_n = 939.5654$ MeV, and $m_p = 938.2721$ MeV, and $m_e = 0.5110$ MeV [2]; $\vec{\xi}_n$ and $\vec{\xi}_e$ are unit polarization vectors of the neutron and electron, respectively; and $F(E_e, Z=1)$ is the relativistic Fermi function [13,24–26]

$$F(E_e,Z=1) = \left(1+\frac{1}{2}\gamma\right)\frac{4(2r_p m_e\beta)^{2\gamma}}{\Gamma^2(3+2\gamma)}\frac{e^{\pi\alpha/\beta}}{(1-\beta^2)^\gamma}\left|\Gamma\left(1+\gamma+i\frac{\alpha}{\beta}\right)\right|^2, \tag{2}$$

where $\beta = k_e/E_e = \sqrt{E_e^2-m_e^2}/E_e$ is the electron velocity, $\gamma = \sqrt{1-\alpha^2}-1$, and r_p is the electric radius of the proton.

TABLE II. Scalar and tensor coupling constants of interactions beyond the SM and their contributions to the neutron lifetime and the measurable correlation coefficients of neutron β^- decay with a polarized neutron and electron for the correlation coefficient b_E taken at the level of 10^{-4}.

b_F	b_E	C_S	C_T	$\Delta\tau_n^{(\rm BSM)}/\tau_n$	$\langle \bar{N}^{(\rm BSM)}(E_e)\rangle_{\rm SM}$	$\langle N^{(\rm BSM)}(E_e)\rangle_{\rm SM}$	$\langle A_W^{(\rm BSM)}(E_e)\rangle_{\rm SM}$
-0.0002	-0.0002	$+2.1\times 10^{-4}$	$+2.1\times 10^{-4}$	1.3×10^{-4}	-1.77×10^{-4}	-2.00×10^{-4}	$+2.95\times 10^{-8}$
-0.0002	$+0.0002$	-4.9×10^{-4}	$+2.5\times 10^{-5}$	1.3×10^{-4}	$+1.77\times 10^{-4}$	$+2.00\times 10^{-4}$	-3.99×10^{-9}
-0.0002	-0.0001	$+3.3\times 10^{-5}$	$+1.6\times 10^{-4}$	1.3×10^{-4}	-8.85×10^{-5}	-1.00×10^{-4}	$+1.08\times 10^{-8}$
-0.0002	$+0.0001$	-3.2×10^{-4}	$+7.1\times 10^{-5}$	1.3×10^{-4}	$+8.85\times 10^{-5}$	$+1.00\times 10^{-4}$	-5.93×10^{-9}

In the numerical calculations we will use $r_p = 0.841$ fm [27] used in [7], which is smaller than $r_p = 0.875$ fm reported in [28] and used in [1]. The Fermi function (2) describes the contribution of the electron-proton final-state Coulomb interaction. The analysis of different approximations of the Fermi function (2) has been carried out by Wilkinson [3] (see also [1]). In the SM the correlation coefficients of the electron-energy and angular distribution, Eq. (1), we calculate with the Hamiltonian of V-A weak interactions and the weak magnetism [6]:

$$\mathcal{H}_W(x) = \frac{G_F}{\sqrt{2}}\,V_{ud}\left\{[\bar{\psi}_p(x)\gamma_\mu(1+\lambda\gamma^5)\psi_n(x)] + \frac{\kappa}{2M}\partial^\nu[\bar{\psi}_p(x)\sigma_{\mu\nu}\psi_n(x)]\right\}[\bar{\psi}_e(x)\gamma^\mu(1-\gamma^5)\psi_{\nu_e}(x)], \tag{3}$$

where $\psi_p(x)$, $\psi_n(x)$, $\psi_e(x)$ and $\psi_{\nu_e}(x)$ are the field operators of the proton, neutron, electron, and antineutrino, respectively; γ^μ, $\sigma^{\mu\nu} = \frac{i}{2}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)$, and γ^5 are the Dirac matrices; $\kappa = \kappa_p - \kappa_n = 3.7058$ is the isovector anomalous magnetic moment of the nucleon, defined by the anomalous magnetic moments of the proton $\kappa_p = 1.7928$ and neutron $\kappa_n = -1.9130$ and measured in nuclear magnetons [2]; and $M = (m_n + m_p)/2$ is the average nucleon mass. The correlation coefficients $\zeta(E_e)$ and $A_W(E_e)$ have been calculated in [6]. They read

$$\begin{aligned}\zeta(E_e) &= \Big(1 + \frac{\alpha}{\pi}\,g_n(E_e)\Big) + \frac{1}{M}\,\frac{1}{1+3\lambda^2}\Big[-2\,(\lambda^2 - (\kappa+1)\,\lambda)\,E_0 + (10\lambda^2 - 4(\kappa+1)\,\lambda + 2)\,E_e\\ &\quad - 2\,(\lambda^2 - (\kappa+1)\,\lambda)\,\frac{m_e^2}{E_e}\Big],\\ \zeta(E_e)\,A_W(E_e) &= \zeta(E_e)\Big(A(E_e) + \frac{1}{3}\,Q_n(E_e)\Big) = A_0\Big(1 + \frac{\alpha}{\pi}\,g_n(E_e) + \frac{\alpha}{\pi}\,f_n(E_e)\Big) + \frac{1}{M}\,\frac{1}{1+3\lambda^2}\\ &\quad\times\Big[\Big\{\frac{4}{3}\,\lambda^2 - \Big(\frac{4}{3}\kappa + \frac{2}{3}\Big)\,\lambda - \frac{2}{3}(\kappa+1)\Big\}\,E_0 - \Big\{\frac{22}{3}\lambda^2 - \Big(\frac{10}{3}\kappa - \frac{4}{3}\Big)\,\lambda - \frac{2}{3}(\kappa+1)\Big\}\,E_e\Big],\end{aligned} \tag{4}$$

where the correlation coefficients $A(E_e)$ and $Q_n(E_e)$ are given in [6] (see also [29]). The correlation coefficient $A_W(E_e)$ without the contribution of the radiative corrections, defined by the function $f_n(E_e)$, has been calculated by Wilkinson [3]. We would like to remind the reader that for the first time the calculation of corrections to order $O(E_e/M)$ to the correlation coefficients of neutron β^- decay with a polarized neutron and an unpolarized proton and electron, caused by the weak magnetism and proton recoil, has been carried out by Bilen'kii *et al.* [30,31]. The radiative corrections $g_n(E_e)$ and $f_n(E_e)$ have been calculated by Sirlin [32] and Shann [33], respectively (for details of these calculations one may consult [29] and [6]). The calculation of the contributions of the W-boson and Z-boson exchanges and the QCD corrections to the function $g_n(E_e)$ have been performed by Czarnecki *et al.* [34]. The other correlation coefficients in the electron-energy and angular distribution Eq. (1) have been calculated in [1]. They are equal to

$$\begin{aligned}G(E_e) &= -\Big(1 + \frac{\alpha}{\pi}\,f_n(E_e)\Big)\Big(1 + \frac{1}{M}\,\frac{1}{1+3\lambda^2}\,(2\lambda^2 - 2(\kappa+1)\,\lambda)\,\frac{m_e^2}{E_e}\Big),\\ N(E_e) &= +\Big(1 + \frac{\alpha}{\pi}\,h_n^{(1)}(E_e)\Big)\,\frac{m_e}{E_e}\Big\{-A_0 + \frac{1}{M}\,\frac{1}{1+3\lambda^2}\Big[\Big(\frac{16}{3}\,\lambda^2 - \Big(\frac{4}{3}\kappa - \frac{16}{3}\Big)\,\lambda - \frac{2}{3}(\kappa+1)\Big)E_e\\ &\quad - \Big(\frac{4}{3}\,\lambda^2 - \Big(\frac{4}{3}\kappa - \frac{1}{3}\Big)\,\lambda - \frac{2}{3}(\kappa+1)\Big)E_0\Big] - \frac{1}{M}\,\frac{A_0}{1+3\lambda^2}\Big[-(10\lambda^2 - 4(\kappa+1)\,\lambda + 2)\,E_e\\ &\quad + (2\lambda^2 - 2(\kappa+1)\,\lambda)\Big(E_0 + \frac{m_e^2}{E_e}\Big)\Big]\Big\},\end{aligned}$$

$$
\begin{aligned}
Q_e(E_e) &= \Big(1+\frac{\alpha}{\pi}\,h^{(2)}_n(E_e)\Big)\Big\{-A_0+\frac{1}{M}\,\frac{1}{1+3\lambda^2}\Big[\Big(\frac{22}{3}\,\lambda^2-\Big(\frac{10}{3}\kappa-\frac{10}{3}\Big)\,\lambda-\frac{2}{3}(\kappa+1)\Big)E_e\\
&\quad -\Big(\frac{4}{3}\,\lambda^2-\Big(\frac{4}{3}\kappa-\frac{1}{3}\Big)\,\lambda-\frac{2}{3}(\kappa+1)\Big)E_0+(2\lambda^2-2(\kappa+1)\,\lambda)m_e\Big]-\frac{1}{M}\,\frac{A_0}{1+3\lambda^2}\\
&\quad\times\Big[-(10\lambda^2-4(\kappa+1)\,\lambda+2)\,E_e+(2\lambda^2-2(\kappa+1)\,\lambda)\,\Big(E_0+\frac{m^2_e}{E_e}\Big)\Big]\Big\},\\
R(E_e) &= -\alpha\,\frac{m_e}{k_e}\,A_0,\quad A_0=-2\,\frac{\lambda(1+\lambda)}{1+3\lambda^2},
\end{aligned}
\tag{5}
$$

where the terms of order $(\alpha/\pi)(E_e/M) < 3\times 10^{-6}$ are neglected. The correlation coefficients in Eq. (5) are defined at the level of 10^{-3} of a complete set of contributions, caused by the weak magnetism and proton recoil of order $O(E_e/M)$ and radiative corrections of order $O(\alpha/\pi)$ [1]. The functions $h^{(1)}_n(E_e)$ and $h^{(2)}_n(E_e)$, defining the radiative corrections to the correlation coefficients of $N(E_e)$ and $Q_e(E_e)$, were calculated for the first time in [1].

III. ELECTRON-ENERGY AND ANGULAR DISTRIBUTION BEYOND THE SM

For the calculation of contributions of interactions beyond the SM we use the effective low-energy Hamiltonian of weak nucleon-lepton four-fermion local interactions, taking into account all phenomenological couplings beyond the SM [11–17]. Using notation as in [6], such a Hamiltonian takes the form

$$
\begin{aligned}
\mathcal{H}_W(x) &= \frac{G_F}{\sqrt{2}}\,V_{ud}\Big\{[\bar{\psi}_p(x)\gamma_\mu\psi_n(x)][\bar{\psi}_e(x)\gamma^\mu(C_V+\bar{C}_V\gamma^5)\psi_{\nu_e}(x)]+[\bar{\psi}_p(x)\gamma_\mu\gamma^5\psi_n(x)][\bar{\psi}_e(x)\gamma^\mu(\bar{C}_A+C_A\gamma^5)\psi_{\nu_e}(x)]\\
&\quad+[\bar{\psi}_p(x)\psi_n(x)][\bar{\psi}_e(x)(C_S+\bar{C}_S\gamma^5)\psi_{\nu_e}(x)]+[\bar{\psi}_p(x)\gamma^5\psi_n(x)][\bar{\psi}_e(x)(C_P+\bar{C}_P\gamma^5)\psi_{\nu_e}(x)]\\
&\quad+\frac{1}{2}[\bar{\psi}_p(x)\sigma^{\mu\nu}\gamma^5\psi_n(x)][\bar{\psi}_e(x)\sigma_{\mu\nu}(\bar{C}_T+C_T\gamma^5)\psi_{\nu_e}(x)\Big\}.
\end{aligned}
\tag{6}
$$

This is the most general form of the effective low-energy weak interactions, where the phenomenological coupling constants C_i and $\bar{C}_i$ for $i = V, A, S, P$, and T can be induced by the left-handed and right-handed hadronic and leptonic currents [12–17]. They are related to the phenomenological coupling constants, analogous to those which were introduced by Herczeg [16], as follows:

$$
\begin{aligned}
&C_V = 1+a^h_{LL}+a^h_{LR}+a^h_{RR}+a^h_{RL},\quad \bar{C}_V=-1-a^h_{LL}-a^h_{LR}+a^h_{RR}+a^h_{RL},\\
&C_A = -\lambda+a^h_{LL}-a^h_{LR}+a^h_{RR}-a^h_{RL},\quad \bar{C}_A=\lambda-a^h_{LL}+a^h_{LR}+a^h_{RR}-a^h_{RL},\\
&C_S = A^h_{LL}+A^h_{LR}+A^h_{RR}+A^h_{RL},\quad \bar{C}_S=-A^h_{LL}-A^h_{LR}+A^h_{RR}+A^h_{RL},\\
&C_P = -A^h_{LL}+A^h_{LR}+A^h_{RR}-A^h_{RL},\quad \bar{C}_P=A^h_{LL}-A^h_{LR}+A^h_{RR}-A^h_{RL},\\
&C_T = 2\big(\alpha^h_{LL}+\alpha^h_{RR}\big),\quad \bar{C}_T=2\big(-\alpha^h_{LL}+\alpha^h_{RR}\big),
\end{aligned}
\tag{7}
$$

where the index h means that the phenomenological coupling constants are introduced at the *hadronic* level but not at the *quark* level, as was done by Herczeg [16]. In the SM the phenomenological coupling constants C_i and $\bar{C}_i$ for $i = V, A, S, P$, and T are equal to $C_S = \bar{C}_S = C_P = \bar{C}_P = C_T = \bar{C}_T = 0$, $C_V = -\bar{C}_V = 1$, and $C_A = -\bar{C}_A = -\lambda$ [6]. The phenomenological coupling constants a^h_{ij}, A^h_{ij}, and α^h_{jj} for $i\ (j) = L$ or R are induced by interactions beyond the SM. The coupling constants in Eq. (6) are related to the coupling constants by Gudkov *et al.* [29]) as follows:

$$
\begin{aligned}
&C_V = C_V,\quad \bar{C}_V = C'_V,\quad \bar{C}_A = -C_A,\quad C_A = -C'_A,\quad C_S = C_S,\quad \bar{C}_S = C'_S,\\
&C_P = C'_P,\quad \bar{C}_P = C_P,\quad C_T = C_T,\quad \bar{C}_T = C'_T.
\end{aligned}
\tag{8}
$$

Thus, our definition of the coupling constants of interactions beyond the SM, used in [6,35], differs from Gudkov's definition only for the axial-vector coupling constants. However, the contributions to the correlation coefficients, obtained in the linear approximation with respect to deviations of the vector and axial-vector coupling constants from their values in the SM and expressed in terms of the scalar and tensor coupling constants, are related by the redefinition $(C_j, \bar{C}_j) \to (C_j, C'_j)$ for $j = S, T$ [see Eq. (15) and Sec. IV].

The structure of the phenomenological coupling constants in Eq. (7) agrees well with the coupling constants of interactions beyond the SM used by Cirigliano *et al.* [20] for consideration of the role of precision measurements of β decays and light meson semileptonic decays in probing physics beyond the SM in the Large Hadron Collider era. For this aim, using an effective field theory framework, all low-energy charged-current processes within and beyond the SM were described, and theoretical hadronic input, which in these precision tests plays a crucial role in setting the baseline for new physics searches, was discussed.

The contribution of interactions beyond the SM, given by the Hamiltonian of weak interactions Eq.(6), to the amplitude of neutron β^- decay, calculated to leading order in the large nucleon mass expansion, takes the form

$$M(n\to pe^-\bar{\nu}_e) = -2m_n\,\frac{G_F}{\sqrt{2}}\,V_{ud}\,\{[\varphi^\dagger_p\varphi_n][\bar{u}_e\gamma^0(C_V+\bar{C}_V\gamma^5)v_{\bar{\nu}}] - [\varphi^\dagger_p\vec{\sigma}\,\varphi_n]\cdot[\bar{u}_e\vec{\gamma}\,(\bar{C}_A+C_A\gamma^5)v_{\bar{\nu}}]$$
$$+[\varphi^\dagger_p\varphi_n][\bar{u}_e(C_S+\bar{C}_S\gamma^5)v_{\bar{\nu}}] + [\varphi^\dagger_p\vec{\sigma}\,\varphi_n]\cdot[\bar{u}_e\gamma^0\vec{\gamma}\,(\bar{C}_T+C_T\gamma^5)v_{\bar{\nu}}]\}. \tag{9}$$

The Hermitian conjugate amplitude is

$$M^\dagger(n\to pe^-\bar{\nu}_e) = -2m_n\,\frac{G_F}{\sqrt{2}}\,V^*_{ud}\{[\varphi^\dagger_n\varphi_p][\bar{v}_{\bar{\nu}}\gamma^0(C^*_V+\bar{C}^*_V\gamma^5)u_e] - [\varphi^\dagger_n\vec{\sigma}\,\varphi_p]\cdot[\bar{v}_{\bar{\nu}}\vec{\gamma}\,(\bar{C}^*_A+C^*_A\gamma^5)u_e]$$
$$+[\varphi^\dagger_n\varphi_p][\bar{v}_{\bar{\nu}}(C^*_S-\bar{C}^*_S\gamma^5)u_e] - [\varphi^\dagger_n\vec{\sigma}\,\varphi_p]\cdot[\bar{v}_{\bar{\nu}}\gamma^0\vec{\gamma}\,(\bar{C}^*_T-C^*_T\gamma^5)u_e]\}. \tag{10}$$

The contributions of interactions with the strength defined by the phenomenological coupling constants C_P and $\bar{C}_P$ may appear only of order $O(C_PE_e/M)$ and $O(\bar{C}_PE_e/M)$ and can be neglected to leading order in the large nucleon mass expansion. We have also neglected the contributions of the neutron-proton mass difference. The squared absolute value of the amplitude (9), summed over polarizations of massive fermions, is equal to

$$\sum_{\rm pol.}|M(n\to pe^-\bar{\nu}_e)|^2$$
$$= 8m^2_nG^2_F|V_{ud}|^2E_\nu E_e\Big\{\frac{1}{2}(|C_V|^2+|\bar{C}_V|^2+3|C_A|^2+3|\bar{C}_A|^2+|C_S|^2+|\bar{C}_S|^2+3|C_T|^2+3|\bar{C}_T|^2)$$
$$+\frac{m_e}{E_e}\,{\rm Re}(C_VC^*_S+\bar{C}_V\bar{C}^*_S-3C_AC^*_T-3\bar{C}_A\bar{C}^*_T)+\frac{\vec{\xi}_n\cdot\vec{k}_e}{E_e}\,{\rm Re}(2C_A\bar{C}^*_A-2C_T\bar{C}^*_T-C_V\bar{C}^*_A-\bar{C}_VC^*_A-C_S\bar{C}^*_T-\bar{C}_SC^*_T)$$
$$+\frac{\vec{k}_e\cdot\vec{\xi}_e}{E_e}\,{\rm Re}(C_V\bar{C}^*_V+3C_A\bar{C}^*_A-C_S\bar{C}^*_S-3C_T\bar{C}^*_T)+\vec{\xi}_n\cdot\vec{\xi}_e\,{\rm Re}\Big[\frac{m_e}{E_e}\,(|C_A|^2+|\bar{C}_A|^2-C_VC^*_A-\bar{C}_V\bar{C}^*_A+|C_T|^2+|\bar{C}_T|^2$$
$$+C_SC^*_T+\bar{C}_S\bar{C}^*_T)+C_VC^*_T+\bar{C}_V\bar{C}^*_T-C_AC^*_S-\bar{C}_A\bar{C}^*_S-2C_AC^*_T-2\bar{C}_A\bar{C}^*_T\Big]+\frac{(\vec{\xi}_n\cdot\vec{k}_e)(\vec{k}_e\cdot\vec{\xi}_e)}{E_e(E_e+m_e)}\,{\rm Re}(|C_A|^2+|\bar{C}_A|^2$$
$$-C_VC^*_A-\bar{C}_V\bar{C}^*_A-C_VC^*_T-\bar{C}_V\bar{C}^*_T+C_AC^*_S+\bar{C}_A\bar{C}^*_S+2C_AC^*_T+2\bar{C}_A\bar{C}^*_T+|C_T|^2+|\bar{C}_T|^2+C_SC^*_T+\bar{C}_S\bar{C}^*_T)$$
$$+\frac{\vec{\xi}_n\cdot(\vec{k}_e\times\vec{\xi}_e)}{E_e}\,{\rm Im}(C_V\bar{C}^*_T+\bar{C}_VC^*_T-C_A\bar{C}^*_S-\bar{C}_AC^*_S-2C_A\bar{C}^*_T-2\bar{C}_AC^*_T)\Big\}. \tag{11}$$

In Eq. (11) the structure of the contributions of interactions beyond the SM agrees well with the structure of the corresponding expressions obtained by Jackson *et al.* [12,13]. The first term on the second line of Eq. (11) is the Fierz interference term. It appears as a result of the calculation of the traces over the Dirac matrices on the same footing as that in the paper by Lee and Yang [36] (see the Appendix of Refs. [36] and [38]). In the linear approximation for coupling constants of vector and axial-vector interactions beyond the SM [6], we get

$$\sum_{\rm pol.}|M(n\to pe^-\bar{\nu}_e)|^2$$
$$= 8m^2_nG^2_F|V_{ud}|^2E_\nu E_e\,(1+3\lambda^2)\Big\{\Big[1+\frac{1}{2}\,\frac{1}{1+3\lambda^2}\,(|C_S|^2+|\bar{C}_S|^2+3|C_T|^2+3|\bar{C}_T|^2)\Big]$$
$$+\frac{m_e}{E_e}\,\frac{1}{1+3\lambda^2}\,{\rm Re}((C_S-\bar{C}_S)+3\lambda\,(C_T-\bar{C}_T))+\frac{\vec{\xi}_n\cdot\vec{k}_e}{E_e}\Big(A_0-\frac{1}{1+3\lambda^2}\,{\rm Re}(C_S\bar{C}^*_T+\bar{C}_SC^*_T+2C_T\bar{C}^*_T)\Big)$$
$$+\frac{\vec{k}_e\cdot\vec{\xi}_e}{E_e}\Big(-1-\frac{1}{1+3\lambda^2}\,{\rm Re}(C_S\bar{C}^*_S+3C_T\bar{C}^*_T)\Big)+\vec{\xi}_n\cdot\vec{\xi}_e\Big[\frac{m_e}{E_e}\Big(-A_0+\frac{1}{1+3\lambda^2}\,{\rm Re}(C_SC^*_T+\bar{C}_S\bar{C}^*_T$$
$$+|C_T|^2+|\bar{C}_T|^2)\Big)+\frac{1}{1+3\lambda^2}\,{\rm Re}(\lambda(C_S-\bar{C}_S)+(1+2\lambda)(C_T-\bar{C}_T))\Big]+\frac{(\vec{\xi}_n\cdot\vec{k}_e)(\vec{k}_e\cdot\vec{\xi}_e)}{E_e(E_e+m_e)}\Big[-A_0+\frac{1}{1+3\lambda^2}$$
$$\times{\rm Re}(-\lambda(C_S-\bar{C}_S)-(1+2\lambda)(C_T-\bar{C}_T)+C_SC^*_T+\bar{C}_S\bar{C}^*_T+|C_T|^2+|\bar{C}_T|^2)\Big]$$
$$+\frac{\vec{\xi}_n\cdot(\vec{k}_e\times\vec{\xi}_e)}{E_e}\,\frac{1}{1+3\lambda^2}\,{\rm Im}(\lambda(C_S-\bar{C}_S)+(1+2\lambda)(C_T-\bar{C}_T))\Big\}, \tag{12}$$

where we have replaced C_j and $\bar{C}_j$ with $j = V, A$ by $C_V = 1 + \delta C_V$, $\bar{C}_V = -1 + \delta\bar{C}_V$, $C_A = -\lambda + \delta C_A$, and $\bar{C}_A = \lambda + \delta\bar{C}_A$ [29] (see also [6]) and neglected the contributions of the products $(\delta C_j)^2$, $\delta C_j \delta\bar{C}_j$, $\delta C_j C_k$, and $\delta\bar{C}_j C_k$ for $j = V, A$ and $k = S, T$. Following [19] (see also [6]) we have absorbed the contributions the vector and axial vector interactions beyond the SM by the axial coupling constant λ and the CKM matrix element V_{ud}.

Thus, the electron-energy and angular distribution, Eq. (1), taking into account the contributions of interactions beyond the SM, can be transcribed into the following standard form [12] (see also [29] and [19–23]):

$$\begin{aligned}\frac{d^3\lambda_n(E_e, \vec{k}_e, \vec{\xi}_n, \vec{\xi}_e)}{dE_e d\Omega_e} &= (1 + 3\lambda^2)\,\frac{G_F^2|V_{ud}|^2}{8\pi^4}(E_0 - E_e)^2\sqrt{E_e^2 - m_e^2}\,E_e F(E_e, Z = 1)\zeta^{(\mathrm{SM})}(E_e)\\ &\quad\times (1 + \zeta^{(\mathrm{BSM})}(E_e))g\bigg\{1 + b\,\frac{m_e}{E_e} + A_{W,\mathrm{eff}}(E_e)\,\frac{\vec{\xi}_n\cdot\vec{k}_e}{E_e} + G_{\mathrm{eff}}(E_e)\frac{\vec{\xi}_e\cdot\vec{k}_e}{E_e} + N_{\mathrm{eff}}(E_e)\vec{\xi}_n\cdot\vec{\xi}_e\\ &\quad + Q_{e,\mathrm{eff}}(E_e)\frac{(\vec{\xi}_n\cdot\vec{k}_e)(\vec{k}_e\cdot\vec{\xi}_e)}{E_e(E_e + m_e)} + R_{\mathrm{eff}}(E_e)\frac{\vec{\xi}_n\cdot(\vec{k}_e\times\vec{\xi}_e)}{E_e}g\bigg\},\end{aligned} \tag{13}$$

where the indices "SM" and "BSM" mean "standard model" and "beyond standard model," respectively. The correlation coefficient $\zeta^{(\mathrm{SM})}(E_e)$ is given by Eq. (4), whereas the analytical expressions for the correlation coefficient $\zeta^{(\mathrm{BSM})}(E_e)$ are given in Eq. (15). Other correlation coefficients are defined by

$$\begin{aligned} b &= \frac{b_F}{1 + \zeta^{(\mathrm{BSM})}(E_e)}, \quad A_{W,\mathrm{eff}}(E_e) = \frac{A_W^{(\mathrm{SM})}(E_e) + A_W^{(\mathrm{BSM})}(E_e)}{1 + \zeta^{(\mathrm{BSM})}(E_e)},\\ G_{\mathrm{eff}}(E_e) &= \frac{G^{(\mathrm{SM})}(E_e) + G^{(\mathrm{BSM})}(E_e)}{1 + \zeta^{(\mathrm{BSM})}(E_e)}, \quad N_{\mathrm{eff}}(E_e) = \frac{N^{(\mathrm{SM})}(E_e) + N^{(\mathrm{BSM})}(E_e)}{1 + \zeta^{(\mathrm{BSM})}(E_e)},\\ Q_{e,\mathrm{eff}}(E_e) &= \frac{Q_e^{(\mathrm{SM})}(E_e) + Q_e^{(\mathrm{BSM})}(E_e)}{1 + \zeta^{(\mathrm{BSM})}(E_e)}, \quad R_{\mathrm{eff}}(E_e) = \frac{R^{(\mathrm{SM})}(E_e) + R^{(\mathrm{BSM})}(E_e)}{1 + \zeta^{(\mathrm{BSM})}(E_e)},\end{aligned} \tag{14}$$

where b is the Fierz interference term. The correlation coefficients with index "SM" are given by Eqs. (4) and (5). These expressions should be also supplemented by Wilkinson's corrections of order 10^{-5} [3], calculated for the neutron β^- decay under consideration in [1]. The correlation coefficients b_F, b_E and others with index "BSM" are given by

$$\begin{aligned} b_F &= \frac{1}{1 + 3\lambda^2}\,\mathrm{Re}[(C_S - \bar{C}_S) + 3\lambda\,(C_T - \bar{C}_T)],\\ b_E &= \frac{1}{1 + 3\lambda^2}\,\mathrm{Re}[\lambda\,(C_S - \bar{C}_S) + (1 + 2\lambda)\,(C_T - \bar{C}_T)],\\ \zeta^{(\mathrm{BSM})}(E_e) &= \frac{1}{2}\,\frac{1}{1 + 3\lambda^2}\,(|C_S|^2 + |\bar{C}_S|^2 + 3|C_T|^2 + 3|\bar{C}_T|^2),\\ A_W^{(\mathrm{BSM})}(E_e) &= -\frac{1}{1 + 3\lambda^2}\,\mathrm{Re}(C_S\bar{C}_T^* + \bar{C}_S C_T^* + 2C_T\bar{C}_T^*),\\ G^{(\mathrm{BSM})}(E_e) &= -\frac{1}{1 + 3\lambda^2}\,\mathrm{Re}(C_S\bar{C}_S^* + 3C_T\bar{C}_T^*),\\ N^{(\mathrm{BSM})}(E_e) &= \frac{m_e}{E_e}\,\frac{1}{1 + 3\lambda^2}\,\mathrm{Re}(C_S C_T^* + \bar{C}_S\bar{C}_T^* + |C_T|^2 + |\bar{C}_T|^2) + b_E,\\ Q_e^{(\mathrm{BSM})}(E_e) &= \frac{1}{1 + 3\lambda^2}\,\mathrm{Re}(C_S C_T^* + \bar{C}_S\bar{C}_T^* + |C_T|^2 + |\bar{C}_T|^2) - b_E,\\ R^{(\mathrm{BSM})}(E_e) &= \frac{1}{1 + 3\lambda^2}\,\mathrm{Im}[\lambda(C_S - \bar{C}_S) + (1 + 2\lambda)(C_T - \bar{C}_T)].\end{aligned} \tag{15}$$

The correlation coefficients in Eq. (15) can be redefined in Gudkov's notation [29]) by the replacement $(C_, \bar{C}_j) \to (C_j, C'_j)$ for $j = S, T$ [see Eq. (8)]. In Eq. (15) the structure of the contributions of interactions beyond the SM agrees well with the structure of corresponding expressions taken in the linear approximation with respect to vector and axial-vector interactions beyond the SM obtained by Jackson *et al.* [12,13]. For the calculation of Eq. (13) we have carried out the integration over the directions of the antineutrino momentum. This gives the correlation coefficient $A_W^{(\mathrm{SM})}(E_e)$ equal to $A_W^{(\mathrm{SM})}(E_e) = A^{(\mathrm{SM})}(E_e) + \frac{1}{3}\,Q_n^{(\mathrm{SM})}(E_e)$ [3,6] [see also Eq. (26)].

IV. NEUTRON LIFETIME, AVERAGED VALUES OF CORRELATION COEFFICIENTS, AND ASYMMETRIES OF NEUTRON β^- DECAY WITH A POLARIZED NEUTRON AND ELECTRON

For the analysis of experimental data in search of interactions beyond the SM in neutron β^- decay with a polarized neutron and electron, we propose to use a complete set of contributions of scalar and tensor interactions beyond the SM including linear, crossing, and quadratic terms, which are given in Eq. (15).

A. Neutron lifetime

Having integrated the electron-energy and angular distribution, Eq. (13), with contributions of interactions beyond the SM we get

$$\tau_n^{-1}(\vec{\xi}_n,\vec{\xi}_e) = \tau_n^{-1}\left(1 + \frac{1}{2}\,\frac{1}{1+3\lambda^2}\,(|C_S|^2 + |\bar{C}_S|^2 + 3|C_T|^2 + 3|\bar{C}_T|^2) + \left\langle\frac{m_e}{E_e}\right\rangle_{\rm SM} b_F + \langle \bar{N}_{\rm eff}(E_e)\rangle\,\vec{\xi}_e\cdot\vec{\xi}_e\right). \tag{16}$$

Here we have denoted

$$\langle \bar{N}_{\rm eff}(E_e)\rangle = \langle N^{(\rm SM)}(E_e) + \frac{1}{3}\left(1 - \frac{m_e}{E_e}\right) Q_e^{(\rm SM)}(E_e)\rangle_{\rm SM} + \langle \bar{N}^{(\rm BSM)}(E_e)\rangle_{\rm SM}. \tag{17}$$

For the calculation of the averaged value $\langle \bar{N}_{\rm eff}(E_e)\rangle$ we use the electron-energy density

$$\rho_e(E_e) = \rho_e^{(\rm SM)}(E_e)\left(1 + \frac{1}{2}\,\frac{1}{1+3\lambda^2}\,(|C_S|^2 + |\bar{C}_S|^2 + 3|C_T|^2 + 3|\bar{C}_T|^2)\right), \tag{18}$$

where the electron-energy density $\rho_e^{(\rm SM)}(E_e)$ is defined by Eq. (D-59) of Ref. [6]. The notation $\langle\cdots\rangle_{\rm SM}$ means that the integration over the electron-energy spectrum is carried out with the electron-energy density $\rho_e^{(\rm SM)}(E_e)$. Then, $\langle \bar{N}^{(\rm BSM)}(E_e)\rangle_{\rm SM}$ is equal to

$$\langle \bar{N}^{(\rm BSM)}(E_e)\rangle_{\rm SM} = \left(\frac{2}{3} + \frac{1}{3}\left\langle\frac{m_e}{E_e}\right\rangle_{\rm SM}\right) b_E + \left(\frac{1}{3} + \frac{2}{3}\left\langle\frac{m_e}{E_e}\right\rangle_{\rm SM}\right)\frac{1}{1+3\lambda^2}\,{\rm Re}(C_S C_T^{*} + \bar{C}_S\bar{C}_T^{*} + |C_T|^2 + |\bar{C}_T|^2), \tag{19}$$

where $\langle m_e/E_e\rangle_{\rm SM} = 0.6556$ and $\tau_n = 879.6(1.1)$ s [6]. Recent analysis of experimental data on the neutron lifetime, carried out by Czarnecki *et al.* [39], has led to the *favored* neutron lifetime $\tau^{(\rm favored)} = 879.4(6)$ s and the *favored* axial coupling constant $\lambda^{(\rm favored)} = -1.2755(11)$, which agree very well with $\tau_n = 879.6(1.1)$ s and $\lambda = -1.2750(9)$, respectively [6].

B. Averaged values of correlations coefficients

In terms of the correlation coefficients b_F and b_E the phenomenological scalar and tensor coupling constants ${\rm Re}(C_S - \bar{C}_S)$ and ${\rm Re}(C_T - \bar{C}_T)$ are defined by

$$\begin{aligned}{\rm Re}(C_S - \bar{C}_S) &= \frac{3\lambda^2+1}{3\lambda^2 - 2\lambda - 1}\,[-(1+2\lambda)\,b_F + 3\lambda\, b_E],\\ {\rm Re}(C_T - \bar{C}_T) &= \frac{3\lambda^2+1}{3\lambda^2 - 2\lambda - 1}\,(\lambda\, b_F - b_E).\end{aligned} \tag{20}$$

The averaged values of the correlation coefficients $A_{W,\rm eff}(E_e)$, $G_{\rm eff}(E_e)$, $N_{\rm eff}(E_e)$, $Q_{e,\rm eff}(E_e)$, and $R_{\rm eff}(E_e)$, taking into account the contributions of the SM and interactions beyond the SM, are given by

$$\begin{aligned}\langle A_{W,\rm eff}(E_e)\rangle &= \langle A_W^{(\rm SM)}(E_e)\rangle_{\rm SM} - \frac{1}{1+3\lambda^2}\,{\rm Re}(C_S\bar{C}_T^{*} + \bar{C}_S C_T^{*} + 2C_T\bar{C}_T^{*})\\ &= -0.12121 - \frac{1}{1+3\lambda^2}\,{\rm Re}(C_S\bar{C}_T^{*} + \bar{C}_S C_T^{*} + 2C_T\bar{C}_T^{*}),\\ \langle G_{\rm eff}(E_e)\rangle &= \langle G^{(\rm SM)}(E_e)\rangle_{\rm SM} - \frac{1}{1+3\lambda^2}\,{\rm Re}(C_S\bar{C}_S^{*} + 3C_T\bar{C}_T^{*})\\ &= -1.00242 - \frac{1}{1+3\lambda^2}\,{\rm Re}(C_S\bar{C}_S^{*} + 3C_T\bar{C}_T^{*}),\\ \langle N_{\rm eff}(E_e)\rangle &= \langle N^{(\rm SM)}(E_e)\rangle_{\rm SM} + \left\langle\frac{m_e}{E_e}\right\rangle_{\rm SM}\frac{1}{1+3\lambda^2}\,{\rm Re}(C_S C_T^{*} + \bar{C}_S\bar{C}_T^{*} + |C_T|^2 + |\bar{C}_T|^2) + b_E\\ &= 0.07767 + 0.6556\,\frac{1}{1+3\lambda^2}\,{\rm Re}(C_S C_T^{*} + \bar{C}_S\bar{C}_T^{*} + |C_T|^2 + |\bar{C}_T|^2) + b_E,\end{aligned}$$

$$\begin{aligned}\langle Q_{\mathrm{e,eff}}(E_e)\rangle &= \langle Q^{(\mathrm{SM})}_e(E_e)\rangle_{\mathrm{SM}} + \frac{1}{1+3\lambda^2}\,\mathrm{Re}(C_S C^*_T + \bar{C}_S \bar{C}^*_T + |C_T|^2 + |\bar{C}_T|^2) - b_E\\ &= 0.12279 + \frac{1}{1+3\lambda^2}\,\mathrm{Re}(C_S C^*_T + \bar{C}_S \bar{C}^*_T + |C_T|^2 + |\bar{C}_T|^2) - b_E,\\ \langle R_{\mathrm{eff}}(E_e)\rangle &= \langle R^{(\mathrm{SM})}(E_e)\rangle_{\mathrm{SM}} + \frac{1}{1+3\lambda^2}\,\mathrm{Im}[\lambda(C_S - \bar{C}_S) + (1+2\lambda)(C_T - \bar{C}_T)]\\ &= 0.00089 + \frac{1}{1+3\lambda^2}\,\mathrm{Im}[\lambda(C_S - \bar{C}_S) + (1+2\lambda)(C_T - \bar{C}_T)]. \end{aligned} \tag{21}$$

For the calculation of $\langle X_{\mathrm{eff}}(E_e)\rangle$, where $X = A_W$, G, N, Q_e, and R, we have used the electron-energy density from Eq. (18).

C. Asymmetries of neutron β^- decay with a polarized neutron and electron

Asymmetry of neutron-electron spin-momentum correlations: Electron asymmetry

For the electron asymmetry $A_{\exp}(E_e)$ [4,40–47] we obtain the following expression (see [6]):

$$A_{\exp}(E_e) = \frac{\mathcal{N}^+_A(E_e) - \mathcal{N}^-_A(E_e)}{\mathcal{N}^+_A(E_e) + \mathcal{N}^-_A(E_e)} = \frac{1}{2}\,\beta\,\mathcal{A}_{W,\mathrm{eff}}(E_e) P_n(\cos\theta_1 + \cos\theta_2), \tag{22}$$

where $P_n = |\vec{\xi}_n| \leqslant 1$ is the neutron spin polarization, β is the electron velocity, and $\mathcal{N}^{\pm}_A(E_e)$ are the numbers of events of the emission of the electron forward $(+)$ and backward $(-)$ with respect to the neutron spin into the solid angle $\Delta\Omega_{12} = 2\pi(\cos\theta_1 - \cos\theta_2)$ with $0 \leqslant \varphi \leqslant 2\pi$ and $\theta_1 \leqslant \theta_e \leqslant \theta_2$. They are determined by [4] (see also [6])

$$\begin{aligned}\mathcal{N}^+_A(E_e) &= 2\pi\mathcal{N}(E_e)\int_{\theta_1}^{\theta_2}(1 + A_{W,\mathrm{eff}}(E_e)\,P_n\,\beta\,\cos\theta_e)\sin\theta_e\,d\theta_e\\ &= 2\pi\mathcal{N}(E_e)\left(1 + \frac{1}{2}\,A_{W,\mathrm{eff}}(E_e)\,P_n\,\beta\,(\cos\theta_1 + \cos\theta_2)\right)(\cos\theta_1 - \cos\theta_2),\\ \mathcal{N}^-_A(E_e) &= 2\pi\mathcal{N}(E_e)\int_{\pi-\theta_1}^{\pi-\theta_2}(1 + \bar{A}_{W,\mathrm{eff}}(E_e)\,P_n\,\beta\,\cos\theta_e)\sin\theta_e\,d\theta_e\\ &= 2\pi\mathcal{N}(E_e)\left(1 - \frac{1}{2}\,\bar{A}_{W,\mathrm{eff}}(E_e)\,P_n\,\beta\,(\cos\theta_1 + \cos\theta_2)\right)(\cos\theta_1 - \cos\theta_2), \end{aligned} \tag{23}$$

where $\mathcal{N}(E_e)$ is the normalization factor equal to

$$\begin{aligned}\mathcal{N}(E_e) &= (1+3\lambda^2)\,\frac{G^2_F|V_{ud}|^2}{8\pi^4}(E_0 - E_e)^2\sqrt{E^2_e - m^2_e}\,E_e\,F(E_e, Z=1)\,\zeta^{(\mathrm{SM})}(E_e)\\ &\quad\times\left(1 + \frac{1}{2}\,\frac{1}{1+3\lambda^2}\,(|C_S|^2 + |\bar{C}_S|^2 + 3|C_T|^2 + 3|\bar{C}_T|^2) + b_F\,\frac{m_e}{E_e}\right). \end{aligned} \tag{24}$$

The correlation coefficient $\mathcal{A}_{W,\mathrm{eff}}(E_e)$ in Eq. (22) is given by

$$\mathcal{A}_{W,\mathrm{eff}}(E_e) = \frac{A^{(\mathrm{SM})}_W(E_e) - \frac{1}{1+3\lambda^2}\,\mathrm{Re}(C_S\bar{C}^*_T + \bar{C}_S C^*_T + 2C_T\bar{C}^*_T)}{1 + \frac{1}{2}\,\frac{1}{1+3\lambda^2}\,(|C_S|^2 + |\bar{C}_S|^2 + 3|C_T|^2 + 3|\bar{C}_T|^2) + b_F\,\frac{m_e}{E_e}} = \frac{A_{W,\mathrm{eff}}(E_e)}{1 + b\,\frac{m_e}{E_e}}, \tag{25}$$

with the correlation coefficient $A^{(\mathrm{SM})}_W(E_e)$ equal to [6]

$$\begin{aligned}A^{(\mathrm{SM})}_W(E_e) &= \left(1 + \frac{\alpha}{\pi}\,f_n(E_e)\right)A_0\left\{1 - \frac{1}{M}\,\frac{1}{2\lambda(1+\lambda)(1+3\lambda^2)}\left(A^{(W)}_1 E_0 + A^{(W)}_2 E_e + A^{(W)}_3\frac{m^2_e}{E_e}\right)\right\},\\ A^{(W)}_1 &= \frac{2}{3}\,(-3\lambda^3 + (3\kappa+5)\,\lambda^2 - (2\kappa+1)\,\lambda - (\kappa+1)) = -2(\lambda - (\kappa+1))\left(\lambda^2 - \frac{2}{3}\,\lambda - \frac{1}{3}\right),\\ A^{(W)}_2 &= \frac{2}{3}\,(-3\lambda^4 + (3\kappa+12)\,\lambda^3 - (9\kappa+14)\,\lambda^2 + (5\kappa+4)\,\lambda + (\kappa+1)) = -2(\lambda - (\kappa+1))\left(\lambda^3 - 3\lambda^2 + \frac{5}{3}\,\lambda + \frac{1}{3}\right),\\ A^{(W)}_3 &= -4\,\lambda^2(\lambda+1)\,(\lambda - (\kappa+1)). \end{aligned} \tag{26}$$

The electron asymmetry, Eq. (25), can be used for the extraction of contributions of interactions beyond the SM from new experimental data, which can be obtained in new runs of experiments in search of interactions beyond the SM with cold and ultracold neutrons [5].

Asymmetry of electron spin-momentum correlations

For a polarized electron and an unpolarized neutron and proton the correlation coefficient $G_{\rm eff}(E_e)$ defines the following electron-energy and angular distribution:

$$\frac{d^3\lambda_n(E_e,\vec{k}_e,\vec{\xi}_e)}{dE_e d\Omega_e} = (1+3\lambda^2)\,\frac{G_F^2|V_{ud}|^2}{8\pi^4}\,(E_0-E_e)^2\sqrt{E_e^2-m_e^2}\,E_e\,F(E_e,Z=1)\,\zeta^{(\rm SM)}(E_e)$$
$$\times\left(1+\frac{1}{2}\,\frac{1}{1+3\lambda^2}\,(|C_S|^2+|\bar{C}_S|^2+3|C_T|^2+3|\bar{C}_T|^2)\right)\left(1+b\,\frac{m_e}{E_e}+G_{\rm eff}(E_e)\,\frac{\vec{\xi}_e\cdot\vec{k}_e}{E_e}\right). \qquad (27)$$

Following Kozela *et al.* [10] (see also [48]) the corresponding asymmetry can be defined as follows:

$$G_{\rm exp}(E_e) = \frac{\frac{d^3\lambda_n(E_e,\vec{k}_e,\vec{\xi}_e)}{dE_e d\Omega_e}\Big|_+ - \frac{d^3\lambda_n(E_e,\vec{k}_e,\vec{\xi}_e)}{dE_e d\Omega_e}\Big|_-}{\frac{d^3\lambda_n(E_e,\vec{k}_e,\vec{\xi}_e)}{dE_e d\Omega_e}\Big|_+ + \frac{d^3\lambda_n(E_e,\vec{k}_e,\vec{\xi}_e)}{dE_e d\Omega_e}\Big|_-} = \beta\,\mathcal{G}_{\rm eff}(E_e)\,P_{e\parallel}, \qquad (28)$$

where $P_{e\parallel}$ is the longitudinal polarization of the electron. The signs $(|_\pm)$ mean parallel and antiparallel polarizations of the electron with respect to its momentum. For a comparison with Ref. [10] we have to set $P_{e\parallel} = \sigma_L$. The correlation coefficient $\mathcal{G}_{\rm eff}(E_e)$ is equal to

$$\mathcal{G}_{\rm eff}(E_e) = \frac{G^{(\rm SM)}(E_e) - \frac{1}{1+3\lambda^2}\,{\rm Re}(C_S\bar{C}_S^* + 3C_T\bar{C}_T^*)}{1+\frac{1}{2}\,\frac{1}{1+3\lambda^2}\,(|C_S|^2+|\bar{C}_S|^2+3|C_T|^2+3|\bar{C}_T|^2)+b_F\,\frac{m_e}{E_e}} = \frac{G_{\rm eff}(E_e)}{1+b\,\frac{m_e}{E_e}}, \qquad (29)$$

where $G^{(\rm SM)}(E_e)$ is given in Eq. (5).

Asymmetry of neutron-electron spin-spin correlations

For the decay electrons in polarization states with polarization $P_{e\perp}$ or $\sigma_{\rm T_1}$ in the notation of Ref. [10], lying in the decay plane spanned by the neutron spin polarization vector $\vec{\xi}_n$ and electron momentum $\vec{k}_e$ (see Fig. 1 of Ref. [10]), we may define the asymmetry [48], caused by the neutron-electron spin-spin correlations

$$N_{\rm exp}(E_e) = \mathcal{N}_{\rm eff}(E_e)\,P_n\,P_{e\perp}\cos\gamma, \qquad (30)$$

where we have denoted $\vec{\xi}_n\cdot\vec{\xi}_e = P_n\,P_{e\perp}\cos\gamma$. The correlation coefficient $\mathcal{N}_{\rm eff}(E_e)$ in Eq. (30) is given by

$$\mathcal{N}_{\rm eff}(E_e) = \frac{N^{(\rm SM)}(E_e) + \frac{m_e}{E_e}\,\frac{1}{1+3\lambda^2}\,{\rm Re}(C_SC_T^* + \bar{C}_S\bar{C}_T^* + |C_T|^2 + |\bar{C}_T|^2) + b_E}{1+\frac{1}{2}\,\frac{1}{1+3\lambda^2}\,(|C_S|^2+|\bar{C}_S|^2+3|C_T|^2+3|\bar{C}_T|^2)+b_F\,\frac{m_e}{E_e}} = \frac{N_{\rm eff}(E_e)}{1+b\,\frac{m_e}{E_e}}, \qquad (31)$$

where $N^{(\rm SM)}(E_e)$ is defined in Eq.(5). The results, obtained in this section, can be used for the analysis of experimental data in search of interactions beyond the SM in neutron β^- decay with a polarized neutron and electron and an unpolarized proton. The expressions for the correlations coefficients and asymmetries obtained in this section can be trivially defined in Gudkov's notation [29] (see also [19]) by a replacement $(C_j,\bar{C}_j)\to(C_j,C'_j)$ for $j = S, T$ [see Eq. (8)].

V. *G*-ODD CORRELATIONS

The G-parity transformation, i.e., $G = C\,e^{i\pi I_2}$, where C and I_2 are the charge conjugation and isospin operators, was introduced by Lee and Yang [49] as a symmetry of strong interactions. According to the G-transformation properties of hadronic currents, Weinberg divided hadronic currents into two classes: G-even first-class and G-odd second-class currents [50]. Thus, in agreement with Weinberg's classification of hadronic currents, the effective phenomenological interactions beyond the SM, Eq. (6), are induced by the first-class hadronic currents.

Following Weinberg [50] and Gardner and Plaster [23], the G-odd contributions or the contributions of the second-class hadronic currents to the matrix element of the hadronic $n\to p$ transition in the V-A theory of weak interactions can be taken in the following form:

$$\langle p(\vec{k}_p,\sigma_p)|J^{(+)}_\mu(0)|n(\vec{k}_n,\sigma_n)\rangle_{G\text{-odd}} = \bar{u}_p(\vec{k}_p,\sigma_p)\left(\frac{q_\mu}{M}\,f_3(0) + i\,\frac{1}{M}\,\sigma_{\mu\nu}\gamma^5 q^\nu g_2(0)\right)u_n(\vec{k}_n,\sigma_n), \qquad (32)$$

where $J^{(+)}_\mu(0) = V^{(+)}_\mu(0) - A^{(+)}_\mu(0)$, $\bar{u}_p(\vec{k}_p,\sigma_p)$ and $u_n(\vec{k}_n,\sigma_n)$ are the Dirac wave functions of the proton and neutron [51], and $f_3(0)$ and $g_2(0)$ are the phenomenological coupling constants defining the strength of the second-class currents in the weak decays. The contribution of the second-class currents, Eq. (32), to the amplitude of neutron β^- decay in the nonrelativistic baryon approximation is defined by

$$M(n\to pe^-\bar{\nu}_e)_{G\text{-odd}} = -2m_n\,\frac{G_F}{\sqrt{2}}\,V_{ud}\,\Big\{f_3(0)\,\frac{m_e}{M}\,[\varphi^\dagger_p\varphi_n][\bar{u}_e(1-\gamma^5)v_{\bar{\nu}}] + g_2(0)\,\frac{1}{M}\,[\varphi^\dagger_p(\vec{\sigma}\cdot\vec{k}_p)\varphi_n][\bar{u}_e\gamma^0(1-\gamma^5)v_{\bar{\nu}}] - g_2(0)\,\frac{E_0}{M}\,[\varphi^\dagger_p\vec{\sigma}\varphi_n]\cdot[\bar{u}_e\vec{\gamma}\,(1-\gamma^5)v_{\bar{\nu}}]\Big\}, \tag{33}$$

where we have kept only the leading $1/M$ terms in the large baryon mass expansion. The Hermitian conjugate contribution is

$$M^\dagger(n\to pe^-\bar{\nu}_e)_{G\text{-odd}} = -2m_n\,\frac{G_F}{\sqrt{2}}\,V_{ud}\,\Big\{f^*_3(0)\,\frac{m_e}{M}\,[\varphi^\dagger_n\varphi_p][\bar{v}_\nu(1+\gamma^5)u_e] + g^*_2(0)\,\frac{1}{M}\,[\varphi^\dagger_n(\vec{\sigma}\cdot\vec{k}_p)\varphi_p][\bar{v}_\nu\gamma^0(1-\gamma^5)u_e] - g^*_2(0)\,\frac{E_0}{M}\,[\varphi^\dagger_n\vec{\sigma}\varphi_p]\cdot[\bar{v}_\nu\vec{\gamma}\,(1-\gamma^5)u_e]\Big\}. \tag{34}$$

The contributions of the G-odd correlations to the squared absolute value of the amplitude of neutron β^- decay of a polarized neutron and electron and an unpolarized proton, summed over polarizations of massive fermions, are equal to

$$\begin{aligned}
&\sum_{\text{pol.}}(M^\dagger(n\to pe^-\bar{\nu}_e)M(n\to pe^-\bar{\nu}_e)_{G-\text{odd}} + M^\dagger(n\to pe^-\bar{\nu}_e)_{G\text{-odd}}M(n\to pe^-\bar{\nu}_e))\\
&= 8m_n^2G_F^2|V_{ud}|^2\,\frac{1}{M}\Big\{2\,\mathrm{Re}\,f_3(0)\,\frac{m_e^2}{E_e} + 2\,\lambda\,\mathrm{Re}\,f_3(0)\,m_e\Big(\vec{\xi}_n\cdot\vec{\xi}_e - \frac{(\vec{\xi}_n\cdot\vec{k}_e)(\vec{k}_e\cdot\vec{\xi}_e)}{E_e(E_e+m_e)}\Big) + 2\,\lambda\,\mathrm{Im}\,f_3(0)\,m_e\,\frac{\vec{\xi}_n\cdot(\vec{k}_e\times\vec{\xi}_e)}{E_e}\\
&\quad + 2\,\mathrm{Re}g_2(0)\Big[-\Big(\frac{4}{3}\,E_0+\frac{2}{3}\,E_e\Big)\frac{\vec{\xi}_n\cdot\vec{k}_e}{E_e} + \Big(\frac{4}{3}\,E_0-\frac{1}{3}\,E_e\Big)\frac{m_e}{E_e}\,(\vec{\xi}_n\cdot\vec{\xi}_e) + \Big(\frac{4}{3}\,E_0+\frac{2}{3}\,E_e+m_e\Big)\frac{(\vec{\xi}_n\cdot\vec{k}_e)(\vec{k}_e\cdot\vec{\xi}_e)}{E_e(E_e+m_e)}\Big]\\
&\quad + 2\,\lambda\,\mathrm{Re}g_2(0)\Big[\Big(4E_0-\frac{m_e^2}{E_e}\Big) - 4\,E_0\,\frac{\vec{\xi}_e\cdot\vec{k}_e}{E_e} + \Big(-\frac{8}{3}\,E_0+\frac{2}{3}\,E_e\Big)\frac{\vec{\xi}_n\cdot\vec{k}_e}{E_e} + \Big(\frac{8}{3}\,E_0-\frac{2}{3}\,E_e\Big)\frac{m_e}{E_e}\,(\vec{\xi}_n\cdot\vec{\xi}_e)\\
&\quad + \Big(\frac{8}{3}\,E_0-\frac{2}{3}\,E_e\Big)\frac{(\vec{\xi}_n\cdot\vec{k}_e)(\vec{k}_e\cdot\vec{\xi}_e)}{E_e(E_e+m_e)}\Big] - 2\,\lambda\,\mathrm{Im}g_2(0)\,m_e\,\frac{\vec{\xi}_n\cdot(\vec{k}_e\times\vec{\xi}_e)}{E_e}\Big\}.
\end{aligned} \tag{35}$$

For the relative G-odd contributions to the correlation coefficients we obtain the following expressions:

$$\begin{aligned}
\frac{\delta\zeta(E_e)_{G\text{-odd}}}{\zeta^{(\mathrm{SM})}(E_e)} &= \frac{2}{1+3\lambda^2}\,\frac{1}{M}\,\Big\{\mathrm{Re}\,f_3(0)\,\frac{m_e^2}{E_e} + \lambda\,\mathrm{Re}g_2(0)\,\Big(4E_0-\frac{m_e^2}{E_e}\Big)\Big\},\\
\frac{\delta A_W(E_e)_{G\text{-odd}}}{A_W^{(\mathrm{SM})}(E_e)} &= \frac{2}{1+3\lambda^2}\,\frac{1}{M}\,\frac{\mathrm{Re}g_2(0)}{A_0}\,\Big\{\Big(-\frac{8}{3}\,\lambda-\frac{4}{3}\Big)E_0 + \Big(\frac{2}{3}\,\lambda-\frac{2}{3}\Big)E_e\Big\} - \delta\zeta(E_e)_{G\text{-odd}},\\
\frac{\delta G(E_e)_{G\text{-odd}}}{G^{(\mathrm{SM})}(E_e)} &= \frac{2\lambda}{1+3\lambda^2}\,\frac{4E_0}{M}\,\mathrm{Re}g_2(0) - \delta\zeta(E_e)_{G\text{-odd}},\\
\frac{\delta N(E_e)_{G\text{-odd}}}{N^{(\mathrm{SM})}(E_e)} &= -\frac{2}{1+3\lambda^2}\,\frac{1}{M}\,\frac{1}{A_0}\,\Big\{\lambda\,\mathrm{Re}\,f_3(0)\,E_e + \mathrm{Re}g_2(0)\Big[\Big(\frac{8}{3}\,\lambda+\frac{4}{3}\Big)E_0 - \Big(\frac{2}{3}\,\lambda+\frac{1}{3}\Big)E_e\Big]\Big\} - \delta\zeta(E_e)_{G\text{-odd}},\\
\frac{\delta Q_e(E_e)_{G\text{-odd}}}{Q_e^{(\mathrm{SM})}(E_e)} &= -\frac{2}{1+3\lambda^2}\,\frac{1}{M}\,\frac{1}{A_0}\,\Big\{-\lambda\,\mathrm{Re}\,f_3(0)\,m_e + \mathrm{Re}g_2(0)\Big[\Big(\frac{8}{3}\,\lambda+\frac{2}{3}\Big)E_0 - \Big(\frac{2}{3}\,\lambda-\frac{2}{3}\Big)E_e + m_e\Big]\Big\}\\
&\quad -\delta\zeta(E_e)_{G\text{-odd}},\\
\frac{\delta R(E_e)_{G\text{-odd}}}{R_e^{(\mathrm{SM})}(E_e)} &= -\frac{2\lambda}{1+3\lambda^2}\,\frac{k_e}{M}\,\frac{1}{\alpha A_0}\,\mathrm{Im}[f_3(0)-g_2(0)].
\end{aligned} \tag{36}$$

For $\lambda = -1.2750$ [40] we get

$$\begin{aligned}
\frac{\delta\zeta(E_e)_{G\text{-odd}}}{\zeta^{(\mathrm{SM})}(E_e)} &= 1.85\times10^{-4}\,\mathrm{Re}\,f_3(0)\,\frac{m_e}{E_e} + \Big(-2.39\times10^{-3} + 2.36\times10^{-4}\,\frac{m_e}{E_e}\Big)\mathrm{Re}g_2(0),\\
\frac{\delta A_W(E_e)_{G\text{-odd}}}{A_W^{(\mathrm{SM})}(E_e)} &= -1.85\times10^{-4}\,\mathrm{Re}\,f_3(0)\,\frac{m_e}{E_e} + \Big(-5.73\times10^{-3} + 5.96\times10^{-3}\,\frac{E_e}{E_0} - 2.36\times10^{-4}\,\frac{m_e}{E_e}\Big)\mathrm{Re}g_2(0),\\
\frac{\delta G(E_e)_{G\text{-odd}}}{G^{(\mathrm{SM})}(E_e)} &= -1.85\times10^{-4}\,\mathrm{Re}\,f_3(0)\,\frac{m_e}{E_e} - 2.36\times10^{-4}\,\mathrm{Re}g_2(0)\,\frac{m_e}{E_e},
\end{aligned}$$

$$
\begin{aligned}
\frac{\delta N(E_e)_{G\text{-odd}}}{N^{(\mathrm{SM})}(E_e)} &= \left(-5.00\times 10^{-3}\,\frac{E_e}{E_0} - 1.85\times 10^{-4}\,\frac{m_e}{E_e}\right)\mathrm{Re}\,f_3(0)\\
&\quad + \left(-5.73\times 10^{-3} + 2.03\times 10^{-3}\,\frac{E_e}{E_0} - 2.36\times 10^{-4}\,\frac{m_e}{E_e}\right)\mathrm{Re}g_2(0),\\
\frac{\delta Q_e(E_e)_{G\text{-odd}}}{Q_e^{(\mathrm{SM})}(E_e)} &= \left(1.98\times 10^{-3} - 1.85\times 10^{-4}\,\frac{m_e}{E_e}\right)\mathrm{Re}\,f_3(0)\\
&\quad + \left(-6.79\times 10^{-3} + 5.96\times 10^{-3}\,\frac{E_e}{E_0} - 2.36\times 10^{-4}\,\frac{m_e}{E_e}\right)\mathrm{Re}g_2(0),\\
\frac{\delta R(E_e)_{G\text{-odd}}}{R_e^{(\mathrm{SM})}(E_e)} &= -0.69\,\frac{k_e}{E_0}\,\mathrm{Im}[f_3(0) - g_2(0)]. \qquad (37)
\end{aligned}
$$

The G-odd correction to the neutron lifetime is

$$
\begin{aligned}
\frac{1}{\tau_n^{(\mathrm{eff})}} &= \frac{1}{\tau_n}\left\{1 + \frac{2}{1+3\lambda^2}\,\frac{1}{M}\left[\mathrm{Re}\,f_3(0)\left\langle\frac{m_e^2}{E_e}\right\rangle_{\mathrm{SM}} + \lambda\,\mathrm{Re}g_2(0)\left(4E_0 - \left\langle\frac{m_e^2}{E_e}\right\rangle_{\mathrm{SM}}\right)\right]\right\}\\
&= \frac{1}{\tau_n}\,[1 + 1.21\times 10^{-4}\,\mathrm{Re}\,f_3(0) - 22.35\times 10^{-4}\,\mathrm{Re}g_2(0)], \qquad (38)
\end{aligned}
$$

where $\langle m_e/E_e\rangle_{\mathrm{SM}} = 0.6556$. For $|\mathrm{Re}\,f_3(0)| < 0.1$ and $|\mathrm{Re}g_2(0)| < 0.01$ the contributions of the G-odd correlations to the neutron lifetime and correlation coefficients can appear at the level of 10^{-5} or even smaller. This agrees well with results obtained by Gardner and Plaster [23].

VI. CONCLUSION

In this paper we have continued our work on the precision analysis of the neutron lifetime and the correlation coefficients of the electron-energy and angular distribution of neutron β^- decay with a polarized neutron and electron and an unpolarized proton. The correlation coefficients, calculated within the SM with Wilkinson's corrections [1], we have supplemented by the contributions of interactions beyond the SM. Since the contributions of vector and axial-vector interactions beyond the SM, calculated to linear approximation, can be absorbed by the axial coupling constant λ and the CKM matrix element V_{ud} [6], the observable contributions of interactions beyond the SM are defined by scalar and tensor interactions only. We have taken into account a complete set of contributions of scalar and tensor interactions, which have been calculated in the linear approximation for the vector and axial-vector interactions beyond the SM [6] and to leading order in the large nucleon mass expansion. The neutron lifetime and asymmetries of neutron β^- decay with a polarized neutron and electron, calculated in Sec. IV, can be used for the analysis of experimental data in search of contributions of interactions beyond the SM.

Numerical estimates of contributions of interactions beyond the SM

Using the results obtained in this paper, we attempt to make some estimates of the values of scalar and tensor coupling constants and contributions of scalar and tensor interactions to observables. Following [6] and [5] we assume that the contributions of scalar and tensor interactions beyond the SM to neutron lifetime are at the level 10^{-4}. As we have shown [see Eq. (20)], the real parts of the scalar and tensor coupling constants are defined in terms of the correlation coefficients b_F and b_E. According to recent analysis by Hardy and Towner [52], the value of the Fierz interference term is equal to $b = -0.0028 \pm 0.0026$. This allows us to analyze the values of the Fierz interference term from the interval $-0.0054 \leqslant b \leqslant -0.0002$. Of course, we understand that the estimate of the Fierz interference term $b = -0.0028 \pm 0.0026$ is obtained from the pure Fermi $0^+ \to 0^+$ transitions, caused by the vector part of the effective V-A weak interactions. Thus, according to the definition of the Fierz interference term [see Eqs. (14) and(15)], it should be induced only by scalar interactions beyond the SM. Nevertheless, in spite of this fact we propose to use such an estimate $b = -0.0028 \pm 0.0026$ in a more extended interpretation, allowing us to understand the order of contributions of interactions beyond the SM to the neutron lifetime and correlation coefficients of neutron β^- decays at the level of 10^{-4}.

For an estimate of the strengths of the scalar and tensor interactions beyond the SM, we accept for simplicity the approximation by real scalar and tensor coupling constants and nucleon-lepton four-fermion interactions with the left-handed neutrinos only [17], i.e., $C_S = -\bar{C}_S$ and $C_T = -\bar{C}_T$. In order to fit the mean value of the experimental data on the averaged value of the correlation coefficient of the neutron-electron spin-spin correlations, $\langle N_{\mathrm{eff}}(E_e)\rangle = 0.0670$, we have to set $b_E = -0.0107$. In Table I we give the values of the scalar and tensor coupling constants and their contributions to the neutron lifetime and measurable correlation coefficients.

Below, without loss of generality, instead of the Fierz interferance term b we use the correlation coefficients b_F. From Table I one may see that the correlation coefficient b_F coincides with the Fierz interference term with an accuracy of about 7×10^{-5}. Setting $b_F = -0.0054$ we find that the contribution of interactions beyond the SM to the neutron lifetime is at the level of 3σ with respect to the world

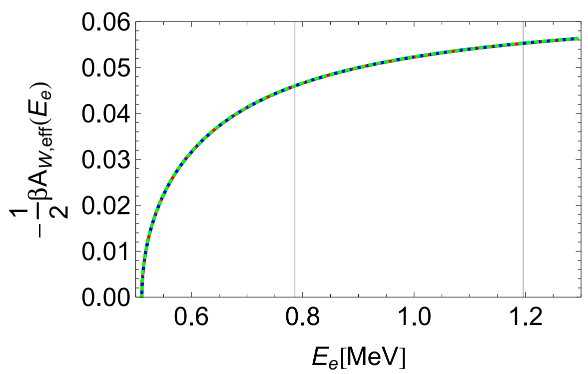

FIG. 1. The theoretical electron asymmetry, calculated in the SM (blue dotted curve) and with interactions beyond the SM (red dotted curve) for $b_F = -0.0002$ and (green dotted curve) for $b_F = -0.0004$.

averaged value $\tau_n = 880.2(1.0)$ s [2] and the experimental one $\tau_n = 880.2(1.2)$ s [53]. This provides sufficiently strong deviation of the theoretical value of the neutron lifetime of about 3 s from the experimental one $\tau_n = 880.2(1.2)$ s [53], and a deviation of about 5 s from the recent experimental value $\tau_n = 877.7 \pm 0.7_{\rm stat.}{}^{+0.3}_{-0.1}{}_{\rm syst.}$ s, which was reported by the UCNA Collaboration [54]. If we want to keep the contribution of interactions beyond the SM to the neutron lifetime at the level of 1σ or 10^{-3} we have to restrict the values of the correlation coefficients b_F to $-0.0017 \leqslant b_F \leqslant -0.0002$. However, in order to keep the contribution of interactions beyond the SM to the neutron lifetime at the level of 10^{-4} or even smaller we have to analyze the contributions of the correlation coefficient b_F with values taken from the interval $-0.0004 \leqslant b_F \leqslant -0.0002$.

In Fig. 1 we plot the theoretical electron asymmetry, calculated within the SM (blue dotted line) with $A_{W,\mathrm{eff}}(E_e) = A^{(\mathrm{SM})}_W(E_e)$ [see Eq. (26)] only and within the SM with the contributions of interactions beyond the SM, given by Eq. (25) and calculated for the correlation coefficients $b_F = -0.0002$ (red dotted line) and $b_F = -0.0004$ (green dotted line). The vertical lines in Fig. 1 define the experimental electron-energy region of the electron asymmetry observation. We would like to emphasize that the obtained estimates of contributions of scalar and tensor interactions beyond the SM to the electron asymmetry do not contradict the experimental data on the correlation coefficient A_0 and the axial coupling constant λ extracted from recent measurements of the electron asymmetry, namely $A_0 = -0.11933(34)$ [$\lambda = -0.12750(9)$] [40], $A_0 = -0.11966(89)(^{+123}_{-140})$ [$\lambda = -1.27590(239)(^{+331}_{-377})$] [44], $A_0 = -0.11972(45)_{\rm stat.}(^{+32}_{-44})_{\rm syst.}$ [$\lambda = -1.2761(12)_{\rm stat.}(^{+9}_{-12})_{\rm syst.}$] [43], $A_0 = -0.11954(55)_{\rm stat.}(98)_{\rm syst.}$ [$\lambda = -1.2756(30)$] [45], and $A_0 = -0.12015(34)_{\rm stat.}(63)_{\rm syst.}$ [$\lambda = -1.2772(20)$] and $A_0 = -0.12054(44)_{\rm stat.}(68)_{\rm syst.}$ [$\lambda = -1.2783(22)$] [47]. Varying the axial coupling constant from $\lambda = -1.2750$ [40] to $\lambda = -1.2783$ [47] and keeping $b_F = -0.0002$ and $b_E = -0.0107$, one may show that the scalar and tensor coupling constants change their values by $\Delta C_S = 3.56 \times 10^{-5}$ and $\Delta C_T = -2.99 \times 10^{-6}$, respectively.

It is important to emphasize that the contributions of scalar and tensor interactions beyond the SM to the correlation coefficients $\bar{N}_{\rm eff}(E_e)$ and $N_{\rm eff}(E_e)$, caused by neutron-electron spin-spin correlations, are of order 10^{-2} and do not practically depend on the values of the correlation coefficient b_F (or the Fierz interference term b at the level of accuracy of about 7×10^{-5}) taken from the interval $-0.0054 \leqslant b_F \leqslant -0.0002$. The contributions of interactions beyond the SM to these correlation coefficients are practically defined by the correlation coefficient b_E, which we have set equal to $b_E = -0.0107$. Of course, our estimate depends strongly on the experimental mean value $N_{\rm exp} = 0.067 \pm 0.011_{\rm stat.} \pm 0.004_{\rm syst.}$ of the correlation coefficient of the neutron-electron spin-spin correlations, which we have accepted as a signal for a trace of contributions of interactions beyond the SM. Of course, such an assumption seems to be sufficiently strong if we take into account that the experimental value $N_{\rm exp} = 0.067 \pm 0.011_{\rm stat.} \pm 0.004_{\rm syst.}$ agrees with the theoretical one $\langle N^{(\mathrm{SM})}(E_e)\rangle_{\rm SM} = 0.07767$, calculated in the SM [1], within one standard deviation.

Of course, the contributions of interactions beyond the SM of order 10^{-2} to the correlation coefficients $N_{\rm eff}(E_e)$, $Q_{e\rm eff}(E_e)$, and $\bar{N}_{\rm eff}(E_e)$ seem to be unreal, and we have to keep them at the level of 10^{-4}. In Table II we give some estimates of the scalar and tensor coupling constants obtained for the correlation coefficients $b_F = -0.0002$ and $|b_E| \sim 10^{-4}$.

It is interesting that for $b_F = -0.0002$, and keeping the value of the correlation coefficient b_E at the level of 10^{-4}, i.e., $|b_F| \sim |b_E| \sim 10^{-4}$, we get the results in Table II. One may see that for the correlation coefficients b_F and b_E kept at the level of 10^{-4} the values of scalar and tensor coupling constants are at the level of 10^{-5}–10^{-4}. In this case the contributions of scalar and tensor interactions beyond the SM can be taken in the linear approximation and fully defined by b_F and b_E. As a result, expected experimental mean values of the correlation coefficient $\langle N_{\rm eff}(E_e)\rangle$ of the neutron-electron spin-spin correlations, averaged over the electron-energy spectrum, may appear, for example, from the interval $0.07747 \leqslant \langle N_{\rm eff}(E_e)\rangle \leqslant 0.07787$.

Towards a robust SM theoretical background with corrections to order 10^{-5} for analysis of experimental data in search of interactions beyond the SM at the level of 10^{-4}

It is obvious that the analysis of experimental data in search of contributions of interactions beyond the SM at the level of 10^{-4} or even better [5] demands a robust SM theoretical background with corrections at the level of 10^{-5}. These are (i) Wilkinson's corrections [1] and (ii) corrections of order $O(E_e^2/M^2)$ defined by the weak magnetism and proton recoil, calculated to next-to-next-to-leading order in the large nucleon mass expansion, the radiative corrections of order $O(\alpha E_e/M)$, calculated to next-to-leading order in the large nucleon mass expansion, and the radiative corrections of order $O(\alpha^2/\pi^2)$, calculated to leading order in the large nucleon mass expansion [8]. These theoretical corrections should provide, for the analysis of experimental data of "discovery" experiments, the required 5σ level of experimental uncertainties of a few parts in 10^{-5} [1]. An important role of strong low-energy interactions for a correct gauge invariant calculation of radiative corrections of order $O(\alpha E_e/M)$ and $O(\alpha^2/\pi^2)$ as functions of the electron energy E_e has been pointed out in [8]. This agrees with Weinberg's assertion about the important role of strong low-energy interactions in decay processes [55]. A procedure for the calculation of these radiative corrections to neutron β^- decays with a consistent account for contributions of strong low-energy interactions, leading to gauge invariant observable expressions dependent on the electron energy E_e determined at the confidence level of Sirlin's radiative corrections [32], has been proposed in [8].

The contributions of the G-odd correlations or the contributions of the second-class hadronic currents [50] we have found to be at the level of 10^{-5} or even smaller. Such an estimate does not contradict the estimates performed by Gardner and Plaster [23]. It is just the level of the SM corrections by Wilkinson [3] and corrections of order $O(\alpha E_e/M)$, $O(\alpha^2/\pi^2)$, and $O(E_e^2/M^2)$ pointed out in [8]. These SM corrections should be taken into account for experimental searches for interactions beyond the SM of order 10^{-4}, caused by the contributions of the first-class hadronic currents [see Eq. (6)] [50], whereas a "discovery" experiment with the required 5σ sensitivity will require experimental uncertainties of a few parts in 10^{-5} [1]. An estimate of the G-odd correlations or contributions of the second-class hadronic currents at the level of 10^{-5} implies an urgent necessity of the robust theoretical background, caused by the Wilkinson corrections and corrections of order $O(\alpha E_e/M)$, $O(\alpha^2/\pi^2)$, and $O(E_e^2/M^2)$. A specific dependence of the G-odd corrections on the electron energy should allow us to distinguish them from the SM background corrections of order 10^{-5} and the contributions of interactions beyond the SM of order 10^{-4}, caused by the first-class hadronic currents. Thus, one may argue that just after the calculation of the theoretical background of order 10^{-5}, caused by the Wilkinson corrections and corrections $O(\alpha E_e/M)$, $O(\alpha^2/\pi^2)$, and $O(E_e^2/M^2)$, a perspective of an experimental discovery of the contributions of the second-class hadronic currents (or the G-odd corrections), as well as the contributions of the first-class hadronic currents beyond the SM, should not be illusive and unfeasible.

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