Structure of the low-lying positive-parity states in ^{154}Sm

H. G. Ganev
Joint Institute for Nuclear Research, Dubna, Russia
and Institute of Nuclear Research and Nuclear Energy, Bulgarian Academy of Sciences, Sofia, Bulgaria

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The proton-neutron symplectic model with Sp(12,R) dynamical algebra is applied to the simultaneous description of the microscopic structure of the low-lying states of the lowest ground, β, and γ bands in ^{154}Sm. For this purpose, the model Hamiltonian is diagonalized in a U(6)-coupled basis, restricted to the state space spanned by the fully symmetric U(6) irreps. A good description of the energy levels of the three bands under consideration as well as the intraband $B(E2)$ transition strengths between the states of the ground band is obtained without the use of an effective charge. The microscopic structure of low-lying collective states in ^{154}Sm shows that there are no admixtures from the higher shells and hence shows the presence of a very good U(6) dynamical symmetry. It is also shown that, in contrast to the Sp(6,R) case, the lowest excited bands, e.g., the β and γ bands, naturally appear together with the ground band within a single Sp(12,R) irreducible representation. The obtained results is given a simple geometrical multiphonon interpretation, based on the algebraic realization of the coupled two-rotor picture, which in turn suggests an interpretation of the low-lying excited bands as relative proton-neutron excitations of the two-component nuclear system, governed by the $Q_p \cdot Q_n$ interaction.

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I. INTRODUCTION

Experimental spectra in heavy nuclei show the emergence of simple collective patterns represented primarily by the nuclear collective rotation. The microscopic shell-model structure of these low-lying rotational states is still a challenge for the microscopic many-particle nuclear theory. This is particularly so because the model space dimensionalities rule out the use of standard shell-model theory. As a consequence, different algebraic models which capitalize on symmetries, exact or approximate, have been developed to reduce the model space to manageable size.

The structure of observed collective patterns in the heavy nuclei is, however, well understood in simple geometrical terms and is described very successfully within the macroscopic nuclear structure physics theories, like the Bohr-Mottelson (BM) [1] and the interacting boson model (IBM) [2] ones. For heavy nuclei, the BM collective model has provided the basic concepts and language in terms of which the observed low-energy nuclear rotational states are described. In this regard, many efforts has been made in attempts to give the BM model a microscopic foundation. Among them the algebraic approach plays an important role. The microscopic evolution of the collective models and their underlying foundations from the algebraic perspective are given in many review articles [3–5]. It was also shown that the Bohr-Mottelson models have expressions as macroscopic limits of microscopic models that have precisely defined expressions in many-nucleon quantum mechanics [4]. Along these lines, it has been shown that a microscopic version of the Bohr-Mottelson collective model augmented by the vortex spin degrees of freedom and compatible with the microscopic shell-model nucleon structure of nucleus is represented by the one-component Sp(6,R)[1] symplectic model [6,7].

The shell-model approach to the many-particle description of low-energy states of a nucleus starts with a decomposition of the infinite-dimensional Hilbert space into an energy-ordered sequence of subspaces of the three-dimensional harmonic oscillator. This makes it possible to diagonalize the model Hamiltonian in a truncated space spanned by a selection of some number of leading subspaces in the decomposition with decreasing contributions to the structure of observed nuclear collective states.

The first microscopic, algebraic model of nuclear collective motion in light nuclei is the Elliott SU(3) model [8], which showed how states with rotational properties could emerge within the framework of the nuclear shell model. It defined a relevant coupling scheme for identifying the collective dynamics and performing large shell-model calculations. However, the calculations within the framework of the one-component Sp(6,R) symplectic model [6,7], which is a natural multi-major-shell extension of the Elliott SU(3) model, showed that the standard spherical shell model is not appropriate for the description of the rotational states of strongly deformed heavy nuclei [9,10]. This is so, in particular, because the conventional shell-model configurations available are not enough deformed to describe observed quadrupole collectivity. The calculations showed that the highly deformed rotational states which lie low in energy have their dominant

[1]The notation Sp($2n$, R) is used for the group of linear canonical transformations in $2n$-dimensional phase space. Some authors denote the Sp($2n$, R) group by Sp(n, R).

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components in much higher spherical harmonic oscillator shells and have essentially zero overlaps with the standard shell-model states [9,10].

Another symmetry-based, microscopic shell-model approach to the structure of strongly deformed heavy nuclei is provided by the pseudo-SU(3) scheme [11–13], based on the observation that, because of the spin-orbit interaction, the single-particle energy levels of the shell model regroup into pseudo-oscillator shells. As a result, another good SU(3) symmetry appears, called pseudo-SU(3) symmetry. Based on the this symmetry, the Sp(6,R) pseudo-symplectic model and its contracted version for heavy nuclei have been developed [14] and applied to the description of the microscopic structure of the low-lying rotational states of the ground or ground and γ bands for some heavy well deformed even-even nuclei from the rare-earth and actinide regions [14,15]. A simple microscopic picture for these collective states was obtained, in which several SU(3) multiplets from the first few harmonic oscillator shells exhaust the structure. The low-lying rotational states observed in strongly deformed nuclei can therefore be considered as renormalized SU(3) states due to the coupling to giant resonance degrees of freedom by involving a vertical mixing of different SU(3) irreducible representations. The one-component Sp(6,R) symplectic model irreducible representation is determined by a single SU(3) irreducible representation, which can be associated with the ground band (and possibly with a few other K bands, depending on the value of μ) and hence no other low-lying β bands appear. In order to include them in the theory, one needs to consider other symplectic Sp(6,R) irreducible representations and to perform much more complicated configuration mixed symplectic model calculations by taking into account the symplectic symmetry-braking part of the nuclear interaction [3,16], which involves a horizontal mixing of different SU(3) irreps within the $0\hbar\omega$ valence shell nucleon space.

Recently, the fully microscopic proton-neutron symplectic model (PNSM) of nuclear collective motion with Sp(12,R) dynamical algebra was introduced by considering the symplectic geometry and possible collective flows in the two-component many-particle nuclear system [17]. The more general motion group GL(6,R) $\subset$ Sp(12,R) of the PNSM, which allows for the separate treatment of the collective dynamics of proton and neutron subsystems, as well as the combined proton-neutron collective excitations, is parametrized by 36 real parameters which are related to the 21 irrotational-flow collective and 15 intrinsic vortex degrees of freedom. In this way, one obtains a richer algebraic structure than in the case of two copies of Sp(6,R) by considering the algebraic structure Sp(6,R) $\otimes$ Sp(6,R) [or equivalently, Sp(6,R) $\otimes$SU$_T$(2)] with totals of 15 collective and 6 intrinsic vortex spin degrees of freedom. In this way, from the hydrodynamic perspective, the PNSM appears as an irrotational-flow collective model of the two-component nuclear system of Bohr-Mottelson type, coupled to the intrinsic U(6) vortex degrees of freedom which are related to the valence shell protons and neutrons. The need to consider intrinsic degrees of freedom and their coupling to the irrotational-flow collective dynamics was realized long time ago (cf. Ref. [3] and references therein). The U(6) intrinsic degrees of freedom play an important role in the construction of the microscopic wave functions because they allow ensuring the full antisymmetry of the total wave function and are responsible for the appearance of the low-lying collective states. In this way the extra degrees of freedom contained in this larger U(6) algebraic structure therefore embrace the basic SU(3) rotor as well as the low-lying vibrational degrees of freedom.

From the shell-model perspective, from another side, the PNSM appears as a natural multi-major-shell extension of the generalized proton-neutron SU(3) scheme, which takes into account the core collective excitations of monopole and quadrupole, as well as dipole type associated with the giant resonance vibrational degrees of freedom. This becomes evident by considering the reduction chain U(6) $\supset$ SU$_p$(3) $\otimes$ SU$_n$(3) $\supset$ SU(3) $\supset$ SO(3), which defines a shell-model coupling scheme of the PNSM and through the subgroup chain SU$_p$(3) $\otimes$SU$_n$(3) $\supset$ SU(3) $\supset$ SO(3) relates the PNSM to the pseudo-SU(3) model [11–13] of nuclear rotations. The appearance of a U(6) intrinsic structure in the PNSM, which in turn contains many SU(3) irreducible representations appropriate for the description of different rotational bands, turns out to be of significant importance for the microscopic theory of nuclear collective excitations. Recall in this regard that the popular IBM [2] has clearly demonstrated that simple algebraic ways exist to get collective spectra within a U(6)-based scheme. Then, within the framework of the PNSM, the low-lying states could be described by a microscopically based U(6) structure along the lines of the IBM, albeit, in contrast to the latter, renormalized by their coupling to the giant resonance vibrations. This result could not be overestimated, recalling also that, as mentioned earlier, in order to obtain the low-lying excited collective bands (e.g., β bands) within the framework of the one-component symplectic model [6] one needs to involve a representation mixing caused by, e.g., pairing, spin-orbit, and other symplectic-breaking components of the nuclear interaction (cf. Ref. [18]). In this way, the intrinsic U(6) structure provides a single shell-model framework for the simultaneous description of the low-lying collective bands in strongly deformed nuclei, which exhibit a simple rotational patterns and shape vibrational excitations (horizontal mixing) over different SU(3) irreps from the $0\hbar\omega$ valence proton-neutron shell-model space. Moreover, as will be shown further, these low-lying vibrational excitations in the proton-neutron nuclear system, which are related to the relative proton-neutron configurations, can be given a simple geometrical multiphonon interpretation.

Finally, ending the short consideration of the evolution of algebraic microscopic models of nuclear collective motion, another characteristic of the PNSM should be pointed out. Sp(12,R) appears as a dynamical group in two other approaches to nuclear structure. In the first one, the microscopic Sp(12,R) model, introduced in Refs. [19,20], the components of the mass quadrupole tensor are used as collective variables. Expressing these variables and their derivatives through the boson creation and annihilation operators, among the reduction chains considered in [19,20], the three algebraic structures [U(5), O(6), and SU(3)] of the IBM-1 [2] were obtained, which are embedded in Sp(12,R) through the group U(6) $\subset$ Sp(12,R) associated with the six quadrupole collective degrees of freedom. In the second one, the phenomenological interacting vector boson model (IVBM) [21] of nuclear collective motion, Sp(12,R) appears as the full dynamical symmetry group of the system of two interacting vector bosons, by means of which the collective excitations of the nuclear systems are built up. But, although it is mathematically isomorphic to that of PNSM, the realization of the Sp(12,R) algebra of IVBM is very different. The latter, in contrast to the PNSM, admits only two irreducible representations: the even (scalar) and odd (one-particle) ones with even and odd numbers of excitation quanta. In Ref. [22] it is shown that the representation space of the IVBM is a very particular and physically unimportant case of the representation space of the PNSM. The IVBM then corresponds to the two-component irrotational-flow collective model of Bohr-Mottelson type. In this way, the microscopic proton-neutron symplectic model of nuclear collective motions can be considered as a generalization of both the IVBM and the Sp(6,R) $\subset$ Sp(12,R) model of Rowe and Rosensteel for the case of two-component many-particle nuclear systems. The PNSM contains also the extended Sp(6,R) $\otimes$ Sp(6,R) model as a submodel, since Sp(6,R) $\otimes$ Sp(6,R) $\subset$ Sp(12,R).

In the present paper, the proton-neutron symplectic model with Sp(12,R) dynamical algebra is applied to the simultaneous description of the microscopic structure of the low-lying states of the ground, β, and γ bands in ^{154}Sm. For this purpose, the model Hamiltonian is diagonalized in a U(6)-coupled basis, restricted to state space spanned by the fully symmetric U(6) irreps up to $40\hbar\omega$. The results for the energy levels of the three bands under consideration, as well as the intraband $B(E2)$ transition strengths between the states of the ground band obtained without the usage of an effective charge, are presented. As will become clear later, the results obtained for the microscopic structure of low-lying collective states in ^{154}Sm reveal the presence of a very good U(6) dynamical symmetry, which shows that the observed collective dynamics is already covered by the symplectic Sp(12,R) bandhead structure. The results obtained for the lowest three collective bands in ^{154}Sm extend the previously obtained ones in the framework of the one-component Sp(6,R) symplectic model in the description of the rotational states of the ground band up to $L = 6$ only within a single axially symmetric Sp(6,R) irreducible representation, determined by its single lowest-grade SU(3) irrep (82,0) in the stretched SU(3) [23] and in the full Sp(6,R) model spaces [9], respectively. In this regard, the present paper represents a further step towards the more comprehensive treatment of collective motion in this nucleus within the microscopic symplectic-based framework.

II. THE PROTON-NEUTRON SYMPLECTIC MODEL

Collective observables of the proton-neutron symplectic model, which span the Sp(12,R) algebra, are given by the following one-body operators [17]:

$$Q_{ij}(\alpha,\beta) = \sum_{s=1}^{m} x_{is}(\alpha)x_{js}(\beta), \tag{1}$$

$$S_{ij}(\alpha,\beta) = \sum_{s=1}^{m}(x_{is}(\alpha)p_{js}(\beta) + p_{is}(\alpha)x_{js}(\beta)), \tag{2}$$

$$L_{ij}(\alpha,\beta) = \sum_{s=1}^{m}(x_{is}(\alpha)p_{js}(\beta) - x_{js}(\beta)p_{is}(\alpha)), \tag{3}$$

$$T_{ij}(\alpha,\beta) = \sum_{s=1}^{m} p_{is}(\alpha)p_{js}(\beta), \tag{4}$$

where $i, j = 1, 2, 3, \alpha, \beta = p, n$, and $s = 1, \ldots, m = A - 1$. In Eqs. (1)–(4), $x_{is}(\alpha)$ and $p_{is}(\alpha)$ denote the coordinates and corresponding momenta of the translationally invariant Jacobi vectors of the m-quasiparticle two-component nuclear system, and A is the number of protons and neutrons.

In terms of the harmonic oscillator creation and annihilation operators

$$b^{\dagger}_{i\alpha,s} = \sqrt{\frac{m_\alpha\omega}{2\hbar}}\left(x_{is}(\alpha) - \frac{i}{m_\alpha\omega}p_{is}(\alpha)\right),$$
$$b_{i\alpha,s} = \sqrt{\frac{m_\alpha\omega}{2\hbar}}\left(x_{is}(\alpha) + \frac{i}{m_\alpha\omega}p_{is}(\alpha)\right), \tag{5}$$

the many-particle realization of the Sp(12,R) Lie algebra is given by [22]

$$F_{ij}(\alpha,\beta) = \sum_{s=1}^{m} b^{\dagger}_{i\alpha,s}b^{\dagger}_{j\beta,s}, \tag{6}$$

$$G_{ij}(\alpha,\beta) = \sum_{s=1}^{m} b_{i\alpha,s}b_{j\beta,s}, \tag{7}$$

$$A_{ij}(\alpha,\beta) = \frac{1}{2}\sum_{s=1}^{m}(b^{\dagger}_{i\alpha,s}b_{j\beta,s} + b_{j\beta,s}b^{\dagger}_{i\alpha,s}). \tag{8}$$

An Sp(12,R) unitary irreducible representation is characterized by the U(6) quantum numbers $\sigma = [\sigma_1, \ldots, \sigma_6]$ of its lowest-weight state $|\sigma\rangle$; i.e., $|\sigma\rangle$ satisfies

$$G_{ab}|\sigma\rangle = 0$$
$$A_{ab}|\sigma\rangle = 0, \quad a < b,$$
$$A_{aa}|\sigma\rangle = \left(\sigma_a + \frac{m}{2}\right)|\sigma\rangle \tag{9}$$

for the indices $a \equiv i\alpha$ and $b \equiv j\beta$ taking the values $1, \ldots, 6$. If the following notation for the U(6) tensor product operators $P^{(n)}(F) = [F \times \cdots \times F]^{(n)}$ is introduced, where $n = [n_1, \ldots, n_6]$ is a partition with even integer parts, then by a U(6) coupling of these tensor products to the lowest-weight state $|\sigma\rangle$, one constructs the whole basis of states for an Sp(12,R) irrep:

$$|\Psi(\sigma n\rho E\eta)\rangle = [P^{(n)}(F) \times |\sigma\rangle]^{\rho E}_{\eta}, \tag{10}$$

where $E = [E_1, \ldots, E_6]$ indicates the U(6) quantum numbers of the coupled state, η labels a basis of states for the coupled U(6) irrep E, and ρ is a multiplicity index. In this way one obtains a basis of Sp(12,R) states that reduces the subgroup chain Sp(12,R) $\supset$ U(6). To fix the basis η one has to consider further the reduction of the U(6) to the three-dimensional rotational group SO(3). Thus, in order to completely classify the basis states, the following reduction chain is further used [22]:

$$\begin{array}{l} \mathrm{Sp}(12,R) \supset \\ \quad \sigma \quad n\rho \\ \quad\quad \supset \mathrm{U}(6) \supset \mathrm{SU}_p(3)\otimes \mathrm{SU}_n(3) \\ \quad\quad\quad E \;\; \gamma \quad (\lambda_p,\mu_p)\;(\lambda_n,\mu_n) \\ \quad\quad \supset \mathrm{SU}(3) \supset \mathrm{SO}(3) \supset \mathrm{SO}(2), \\ \quad\quad \varrho\;(\lambda,\mu)\;K \quad L \qquad M \end{array} \qquad (11)$$

which defines a shell-model coupling scheme. The chain (11) corresponds to the following choice of the index $\eta = \gamma(\lambda_p,\mu_p)(\lambda_n,\mu_n)\varrho(\lambda,\mu)KLM$, labeling the basis states (10) of an Sp(12,R) irrep. Each Sp(12,R) irreducible representation is determined by a symplectic bandhead or an intrinsic U(6) space, which in turn is fixed by the underlying proton-neutron shell-model structure. So, the theory becomes completely compatible with the Pauli principle.

III. APPLICATION

The early applications of the one-component Sp(6,R) symplectic model and its contracted version showed that the dominant contributions to the wave functions are presented by the so-called stretched SU(3) states. The latter are defined as the set of SU(3) states $(\lambda_0 + 2n, \mu_0)$ [3], where (λ_0, μ_0) is the leading irreducible representation for the combined proton-neutron nuclear system and $n = 0, 1, 2, 3, \ldots$. The calculations within the framework of the stretched approximation is often called the Sp(2,R) submodel [24–26] of Sp(6,R) because the set of these basis states can be generated only by the raising symplectic generators adding oscillator quanta only along the z-axis, which are the generators of the subgroup Sp(2,R) $\subset$ Sp(6,R). The stretched SU(3) states within the framework of the one-component Sp(6,R) symplectic model and its contracted version usually contribute between 80% and 90% to the ground state band wave functions. For example, 90% of the ^{20}Ne ground state comes from the (8,0), (10,0), and (12,0) stretched states [7]. Similarly, the stretched states contribute up to 93.7% to the ground state in ^{238}U using the contracted symplectic model [14]. The same picture was obtained in the recent applications of the symplectic Sp(6,R) scheme with algebraic and schematic many-particle interactions to light and intermediate-mass nuclei [27–29]. Hence, the restriction of the full symplectic basis to the subset of stretched SU(3) states seems to be a valuable initial approximation for the symplectic model calculations in the heavy nuclei. So, the stretched SU(3) approximation is used as a first step in the application of the present theory and as a calibration for its further usage.

The first point in the practical application of the theory to the description of low-lying collective states in strongly deformed nuclei is the determination of the relevant irreducible representation of Sp(12,R). Different approaches might be used to determine the symplectic irrep by fixing the shell-model structure of the ground state using the isotropic or anisotropic harmonic oscillator with or without spin-orbit interaction. It is also well known that, for heavy mass nuclei from the rare-earth and actinide regions, spin-orbit interaction is strong and destroys the oscillator structure. Due to this, the pseudo-SU(3) scheme [11–13], which effectively takes into account the spin-orbit part of the nuclear interaction, is used to determine the relevant irreducible representation of Sp(12,R). The symplectic bandhead for ^{154}Sm is determined by fixing the corresponding underlying proton-neutron shell-model structure $\mathrm{SU}_p(3) \otimes \mathrm{SU}_n(3) \supset \mathrm{SU}(3)$ embedded in the U(6) irrep $[\sigma_1, \ldots, \sigma_6]$. Thus, for ^{154}Sm, by compactly filling pairwise the three-dimensional pseudo-oscillator potential with six normal parity protons and six normal parity neutrons, one obtains the SU(3) irreps (12,0) and (18,0) for the parent proton and neutron subsystem, respectively. Then, the direct product irrep $(12,0)\otimes(18,0)$ of $\mathrm{SU}_p(3)\otimes\mathrm{SU}_n(3)$ allows one to fix the lowest-grade U(6) irreducible representation $\sigma = [72, 42, 42, 42, 42, 42]_6 \equiv [30]_6$, which in turn determines the ^{154}Sm irreducible representation $\langle\sigma\rangle = \langle 72 + \frac{153}{2}, 42 + \frac{153}{2}, 42 + \frac{153}{2}, 42 + \frac{153}{2}, 42 + \frac{153}{2}, 42 + \frac{153}{2}\rangle$ of Sp(12,R). The latter is given in Table I. As can be seen from it, the majority of relevant U(6) irreps composing the symplectic irreducible representation under consideration are not fully symmetric. Unfortunately, the isoscalar factors which reduce the $\mathrm{U}(6) \supset \mathrm{SU}_p(3) \otimes \mathrm{SU}_n(3)$ subgroup chain of (11) for generic U(6) irreducible representations, required for the calculation of the matrix elements of relevant physical operators, are not available at present. However, one expects the most symmetric U(6) irreps E to be dominant in the low-energy spectra of the heavy deformed even-even nuclei. Hence, in the present application, only the state space spanned by the fully symmetric U(6) irreps is considered in what follows.

TABLE I. The U(6) irreps contained in the Sp(12,R) irreducible representation $\langle\sigma\rangle = \langle 72 + \frac{153}{2}, 42 + \frac{153}{2}, 42 + \frac{153}{2}, 42 + \frac{153}{2}, 42 + \frac{153}{2}, 42 + \frac{153}{2}\rangle$ for ^{154}Sm.

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[34], [33,1], 2[32,2], [31,3], [31,2,1], [30,4], [30,2,2]
[32], [31,1], [30,2]
[30]

Equation (11) implies a strong coupling of the proton and neutron distributions to form a composite distribution of the combined proton-neutron system with different possible deformations. The maximum deformation is obtained by restricting the direct product irrep $(\lambda_p,\mu_p)\otimes(\lambda_n,\mu_n)$ of $\mathrm{SU}_p(3)\otimes\mathrm{SU}_n(3)$ to the leading irreducible representation $(\lambda_p+\lambda_n, \mu_p+\mu_n)$ of SU(3), which for the Hamiltonian with a quadrupole-quadrupole interaction will lie lowest in energy. Thus, the leading SU(3) irrep (30,0) with maximal value of the second-order Casimir operator of SU(3) will correspond to the ground state. The direct product of proton and neutron SU(3) subrepresentations allows one further to write down the remaining SU(3) irreps (with decreasing deformation and, respectively, increasing energy) of the combined proton-neutron system, contained in symplectic Sp(12,R) bandhead, which

TABLE II. Symplectic classification of the SU(3) basis states for ^{154}Sm.

$2n$	$[E_1,\ldots,E_6]$	(λ_p,μ_p)	(λ_n,μ_n)	(λ,μ)
0	[30]	… (12, 0) …	… (18, 0) …	(30, 0), (28, 1), (26, 2), …, (10, 10), (8, 11), (6, 12)
2	[32]	… (14, 0) …	… (18, 0) …	(32, 0), (30, 1), (28, 2), …, (8, 12), (6, 13), (4, 14)
4	[34]	… (16, 0) …	… (18, 0) …	(34, 0), (32, 1), (30, 2), …, (6, 14), (4, 15), (2, 16)
⋮	⋮	⋮	⋮	⋮

together with the leading one are

$$(12,0)\otimes(18,0)\rightarrow(30,0),(28,1),(26,2),(24,3),\ldots,(10,10),(8,11),(6,12). \tag{12}$$

Obviously, this $0\hbar\omega$ valence shell state space of the Sp(12,R) bandhead, spanned by the SU(3) irreducible representations appearing in (12), corresponds to the different relative configurations of the proton system with respect to the neutron one. Further, it is well known that each SU(3) irreducible representation can be associated with one or several rotational bands (depending on the value of μ). In this regard, for example, the IBM [2] has demonstrated that different rotational bands can be easily described by considering several SU(3) irreps, contained in a single U(6) irreducible representation. Thus, the symplectic bandhead structure of PNSM provides us with a singe microscopic shell-model framework for the simultaneous description of the lowest collective bands, observed in the spectrum of ^{154}Sm. This is in contrast to the one-component Sp(6,R) case, whose bandhead is determined by a single SU(3) irrep, and hence there is no other low-lying bands, e.g., β bands. Thus, in order to obtain them in that case, one needs to involve a mixing of different Sp(6,R) symplectic irreps cased by various symplectic symmetry-breaking interactions, e.g., the spin-orbit and pairing. In this way, the relative proton-neutron collective dynamics beyond the Sp(6,R) is implicitly presented in the extended state space of the Sp(12,R) bandhead structure.

The symplectic classification of the SU(3) basis states for ^{154}Sm according to the decompositions given by the chain (11) for the Sp(12,R) irrep $\langle\sigma\rangle = \langle 72+\frac{153}{2}, 42+\frac{153}{2}, 42+\frac{153}{2}, 42+\frac{153}{2}, 42+\frac{153}{2}, 42+\frac{153}{2}\rangle$, restricted to the space of fully symmetric U(6) partitions, is given in Table II. Note that the SU(3) basis states so obtained, comprising different U(6) irreps, are precisely those which can be obtained by acting on the intrinsic base space states (λ,μ) by the SU(3) (2,0) and/or (0,1) symplectic raising operators (6).

In the present application, the following model Hamiltonian is used:

$$H = N\hbar\omega - \frac{1}{2}\chi[Q_p\cdot Q_n - (Q_p\cdot Q_n)_{TE}] - \xi C_2[SU(3)] + aL^2, \tag{13}$$

where $N = N_p + N_n$ and the operators $Q_\alpha \equiv Q(\alpha,\alpha)$ with $\alpha = p, n$ are given by Eq. (1). A similar Hamiltonian has been used in the pseudo-SU(3) scheme calculations within the framework of the contracted symplectic model [14,15]. The trace-equivalent part $(Q_p\cdot Q_n)_{TE}$ [30–32] is subtracted from the collective potential in order to preserve the mean-field shell structure [14,15,33] under the action of the proton-neutron quadrupole-quadrupole interaction. The SU(3) second-order Casimir operator C_2[SU(3)] splits energetically different SU(3) multiplets and in this way determines the bandhead energies of excited bands with respect to the ground state band. Finally, the last term in (13), which represents a residual rotor part, allows the experimentally observed moment of inertia to be reproduced without altering the wave functions. The Hamiltonian (13) preserves the symplectic symmetry, thus having Sp(12,R) as its dynamical algebra in the sense that the physical operators are obtained in terms of its generators, and the whole spectrum is provided by a single irreducible representation of it. The full dynamics for it therefore occurs within a single irreducible representation of Sp(12,R). More precisely, the Hamiltonian (13) is in the enveloping algebra of Sp(6,R) ⊗ Sp(6,R) ⊂ Sp(12,R) subgroup, so it does not go beyond the considered Sp(12,R) irreducible space and mixes different SU(3) irreps within and between major shells (horizontal and vertical mixing).

After obtaining the appropriate symplectic irrep, the model Hamiltonian (13) is further used to determine the microscopic structure of the low-lying collective states in ^{154}Sm. For this purpose, as was mentioned, its diagonalization in the space of stretched SU(3) states is first performed. In order to do this, the matrix elements of an U(6) tensor $T^{[p,-q]_6}$ in a U(6)-coupled basis, for the case of the fully symmetric U(6) irreps, are obtained using a generalized Wigner-Eckart theorem:

$$\begin{aligned}
&\langle\sigma n'\rho' E';(E_1',0),(E_2',0);(\lambda',\mu');K'L'||T^{[p,-q]_6}_{[\chi_p]_3[\chi_n]_3}{}^{lm}_{[\chi]_3}||\sigma n\rho E;(E_1,0),(E_2,0);(\lambda,\mu);KL\rangle\\
&=\sum\langle\sigma n'\rho' E'||||T^{[p,-q]_6}_{[\chi_p]_3[\chi_n]_3}{}^{lm}_{[\chi]_3}||||\sigma n\rho E\rangle\left\langle\begin{matrix}[E]\\ [E_1][E_2]\end{matrix}\quad\begin{matrix}[p,-q]\\ [\chi_p][\chi_n]\end{matrix}\middle|\begin{matrix}[E']\\ [E_1'][E_2']\end{matrix}\right\rangle\\
&\times\left\{\begin{matrix}(E_1,0) & (\lambda_p^T,\mu_p^T) & (E_1',0) & 1\\ (E_2,0) & (\lambda_n^T,\mu_n^T) & (E_2',0) & 1\\ (\lambda,\mu) & (\lambda^T,\mu^T) & (\lambda',\mu') & \rho_f\\ 1 & 1 & 1 & \end{matrix}\right\}\langle(\lambda,\mu)KL,(\lambda^T,\mu^T)kl||(\lambda',\mu')K'L'\rangle_{\rho_f},
\end{aligned} \tag{14}$$

where the notations $[\chi_p]_3 \equiv [\chi_{p1}, \chi_{p2}, \chi_{p3}]_3$, $[\chi_n]_3 \equiv [\chi_{n1}, \chi_{n2}, \chi_{n3}]_3$, and $[\chi]_3 \equiv [\chi_1, \chi_2, \chi_3]_3$ for the $U_\alpha(3)$ ($\alpha = p, n$) and U(3) irreps, respectively, are used. The standard Elliott notations [8] for the SU(3) quantum numbers are also used, for example $\lambda_p^T = \chi_{p1} - \chi_{p2}$, $\mu_p^T = \chi_{p2} - \chi_{p3}$, and similarly for the others. In (14), $\langle \sigma n'\rho' E' |||| T^{[p,-q]_6 \;\; lm}_{[\chi_p]_3[\chi_n]_3 \; [\chi]_3} |||| \sigma n \rho E\rangle$ is the U(6) reduced matrix element, $\langle {}^{[E]}_{[E_1][E_2]} \;\; {}^{[p,-q]}_{[\chi_p][\chi_n]} | {}^{[E']}_{[E'_1][E'_2]} \rangle$ is the isoscalar factor reducing the $U(6) \supset SU_p(3) \otimes SU_n(3)$ chain, and $\{\ldots\}$ and $\langle (\lambda,\mu)KL, (\lambda^T,\mu^T)kl || (\lambda',\mu')K'L'\rangle_{\rho_f}$ are the SU(3) recoupling and coupling coefficients, respectively. The latter are evaluated numerically using the available code [34], while the relevant $U(6) \supset SU_p(3) \otimes SU_n(3)$ isoscalar factors are given in Ref. [35].

To calculate the matrix elements of the collective potential $\upsilon(Q) = Q_p \cdot Q_n$, a normal-ordered expansion in U(6) unitary irreducible terms is required. The result is

$$
\begin{aligned}
Q_p \cdot Q_n = {} & \frac{22}{25}(C_2[SU(3)] - C_2[SU_p(3)] - C_2[SU_n(3)]) - \frac{34}{25}\left[A^2(p,p) \times A^2(n,n)\right]^{[2,-2]_6 \;\; l=0m=0}_{[1,-1]_3[1,-1]_3 \; [2,-2]_3} \\
& + \Bigg\{ \frac{1}{4}\Bigg(\sqrt{\frac{5}{6}}\left[F^2(p,p) \times G^2(n,n)\right]^{[2,-2]_6 \;\; 00}_{[2]_3[2]^*_3 \; [2,-2]_3} + \sqrt{\frac{5}{6}}\left[F^2(n,n) \times G^2(p,p)\right]^{[2,-2]_6 \;\; 00}_{[2]^*_3[2]_3 \; [2,-2]_3} \Bigg) \\
& + \frac{1}{2}\Bigg(\sqrt{5}\left[A^2(p,p) \times G^2(n,n)\right]^{[2]^*_6 \;\; 00}_{[1,-1]_3[2]^*_3 \; [2]^*_3} + \sqrt{5}\left[A^2(n,n) \times G^2(p,p)\right]^{[2]^*_6 \;\; 00}_{[2]^*_3[1,-1]_3 \; [2]^*_3} \Bigg) \\
& + \frac{1}{2}\Bigg(\sqrt{\frac{5}{6}}\frac{2}{3}\left[G^2(p,p) \times G^2(n,n)\right]^{[4]^*_6 \;\; 00}_{[2]^*_3[2]^*_3 \; [4]^*_3} + \sqrt{\frac{5}{6}}\frac{\sqrt{5}}{3}\left[G^2(p,p) \times G^2(n,n)\right]^{[4]^*_6 \;\; 00}_{[2]^*_3[2]^*_3 \; [2]_3} \\
& + \sqrt{\frac{5}{3}}\frac{2}{3}\left[G^2(p,p) \times G^2(n,n)\right]^{[2,2]^*_6 \;\; 00}_{[2]^*_3[2]^*_3 \; [4]^*_3} + \sqrt{\frac{5}{3}}\frac{\sqrt{5}}{3}\left[G^2(p,p) \times G^2(n,n)\right]^{[2,2]^*_6 \;\; 00}_{[2]^*_3[2]^*_3 \; [2]_3} \Bigg) + \text{H.c.} \Bigg\},
\end{aligned}
\tag{15}
$$

where

$$C_2[\mathrm{SU}(3)] = \widetilde{Q} \cdot \widetilde{Q} + \frac{1}{2}L^2 \tag{16}$$

is the SU(3) second-order Casimir operator with eigenvalue $\langle C_2[\mathrm{SU}(3)]\rangle = \frac{2}{3}(\lambda^2 + \mu^2 + \lambda\mu + 3\lambda + 3\mu)$. The $SU_p(3)$ and $SU_n(3)$ second-order Casimir operators are similarly defined. The quantities $\widetilde{Q}_{m,\alpha} \equiv \widetilde{Q}_m(\alpha,\alpha) = A^{2m}(\alpha,\alpha)$ ($\alpha = p, n$) are the components of SU(3) truncated Elliott's quadrupole operators of the proton and neutron subsystem, respectively, and $\widetilde{Q}_m = \widetilde{Q}_m(p,p) + \widetilde{Q}_m(n,n)$. In Eq. (15), the following notations for the U(6) tensors are also used: $[2]_6 \equiv [2,0,0,0,0,0]_6$, $[2]^*_6 \equiv [0,0,0,0,0,-2]_6$, $[2,-2]_6 \equiv [2,0,0,0,0,-2]_6$, $[1,-1]_6 \equiv [1,0,0,0,0,-1]_6$, $[2,2]^*_6 \equiv [0,0,0,0,-2,-2]_6$, etc. The same notations are valid for the U(3) tensor operators. For convenience, instead of SU(3) labels in Eq. (15), the corresponding U(3) ones are also used to characterize the tensor properties of different interaction terms with respect to the chain (11). With such a normal-ordering expansion, the inclusion of intermediate states external to the truncated space is avoided. The required matrix elements for the basic irreducible terms which appear in (15) are given in Ref. [36].

The results for the low-lying energy levels of the ground, β, and γ bands, obtained in the space of stretched SU(3) states, are compared with experiment [37] in Fig. 1. Recall that the calculations within this space introduce only a vertical mixing, caused by the coupling to the states from the higher shells. The major shell separation energy $\hbar\omega$ is determined by the standard formula $41A^{-1/3}$ MeV. The adopted values for the model parameters (in MeV), obtained by fitting to the energies and $B(E2)$ value for the transition from 2^+ to 0^+ states of the ground band, are as follows: $\chi = 0.0032$, $\xi = 0.0051$, and $a = 0.013$. From Fig. 1 one sees a good description of the energy levels.

The reduced intraband $E2$ electromagnetic transition strengths between the states of the ground band are also computed:

$$
\begin{aligned}
B(E2; L_i \to L_f) \\
= \frac{2L_f + 1}{2L_i + 1}\left(\frac{5}{16\pi}\right)\left(\frac{eZ}{A-1}\right)^2 |\langle f || Q(p,p) || i\rangle|^2.
\end{aligned}
\tag{17}
$$

Note that in the definition of the operator $Q(p,p)$ [cf. Eq. (1)], the summation is over the $(A-1)$ Jacobi quasiparticles. Thus, in order to obtain the proton charge quadrupole operator, the $Q(p,p)$ operator is multiplied by the factor

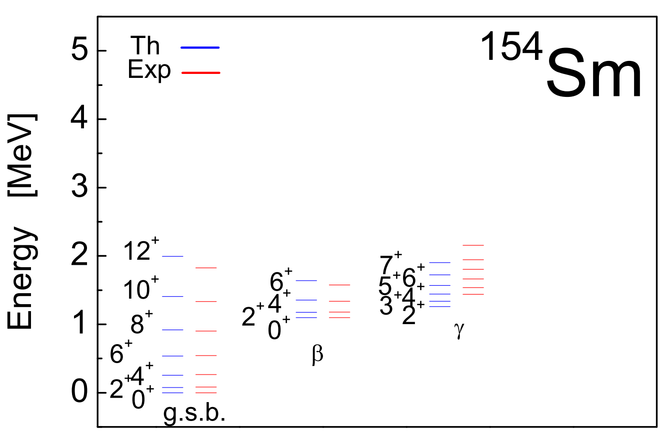

FIG. 1. Comparison of the theoretical [within the space of stretched SU(3) states] and experimental energy levels for the ground, β, and γ bands in ^{154}Sm.

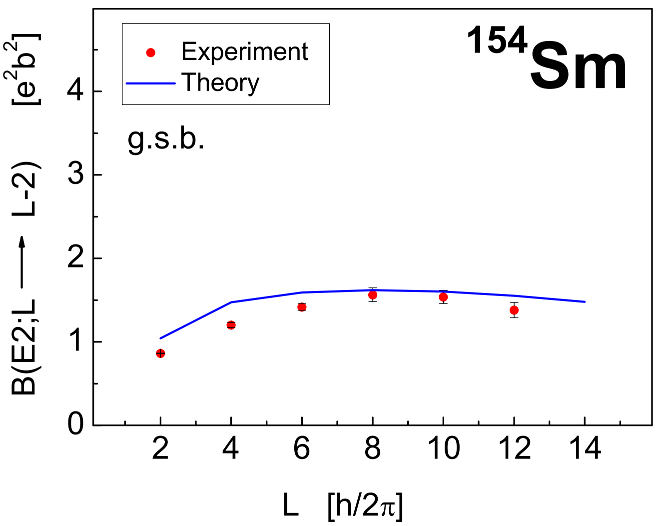

FIG. 2. Calculated [within the space of stretched SU(3) states] and experimental intraband $B(E2)$ values between the states of the ground band in ^{154}Sm. No effective charge is used.

$Z/(A-1)$. The calculated reduced intraband $E2$ electromagnetic transition strengths are compared with experiment [37] in Fig. 2. One sees that the theory slightly overestimates the experimental data. Notice that no effective charge is used in the calculation, i.e., $e=1$.

In Fig. 3, the squares of the amplitudes (probabilities) of the SU(3) components $(\lambda+2n,\mu)$ from each $n\hbar\omega$ shell are plotted for the 0^+ states of the ground and β bands, and for the 2^+ state of the γ band, as a function of n. The starting SU(3) configurations for the ground and β, γ bands are (30,0) and (26,0), respectively, on which the quadrupole giant resonance excitations are built up. From the figure, the structure of the eigenstates obtained in the diagonalization of the model Hamiltonian becomes evident. One sees almost a pure SU(3) structure, exhausted by the SU(3) irreps (30,0) and (26,2) for the ground and β, γ bands, respectively.

It is interesting to see what happens when the model space is extended beyond the stretched SU(3) approximation. Thus, as a next step, the other SU(3) irreps from the Sp(12,R) bandhead and the SU(3) states built on them—which introduce also a horizontal mixing among different SU(3) multiplets within the different U(6) irreps in addition to the vertical (between different major shells) one—are included in the diagonalization of the model Hamiltonian. [Note that the $Q_p \cdot Q_n$ interaction mixes different SU(3) multiplets within and between different major shells.]

In Fig. 4, the $B(E2)$ transition strength between the first excited and ground states of the ground band in ^{154}Sm is shown as a function of the model parameter χ in the range of physically relevant values. No effective charge is used. From the figure one sees a reduction of the $B(E2)$ transition strength with the increase of χ. As can be seen, the experimentally observed value is obtained for $\chi \simeq 0.0032$. The values of the rest of the model parameters in the Hamiltonian are kept the same, except the value of the parameter ξ, which is slightly changed from 0.0051 to 0.0053.

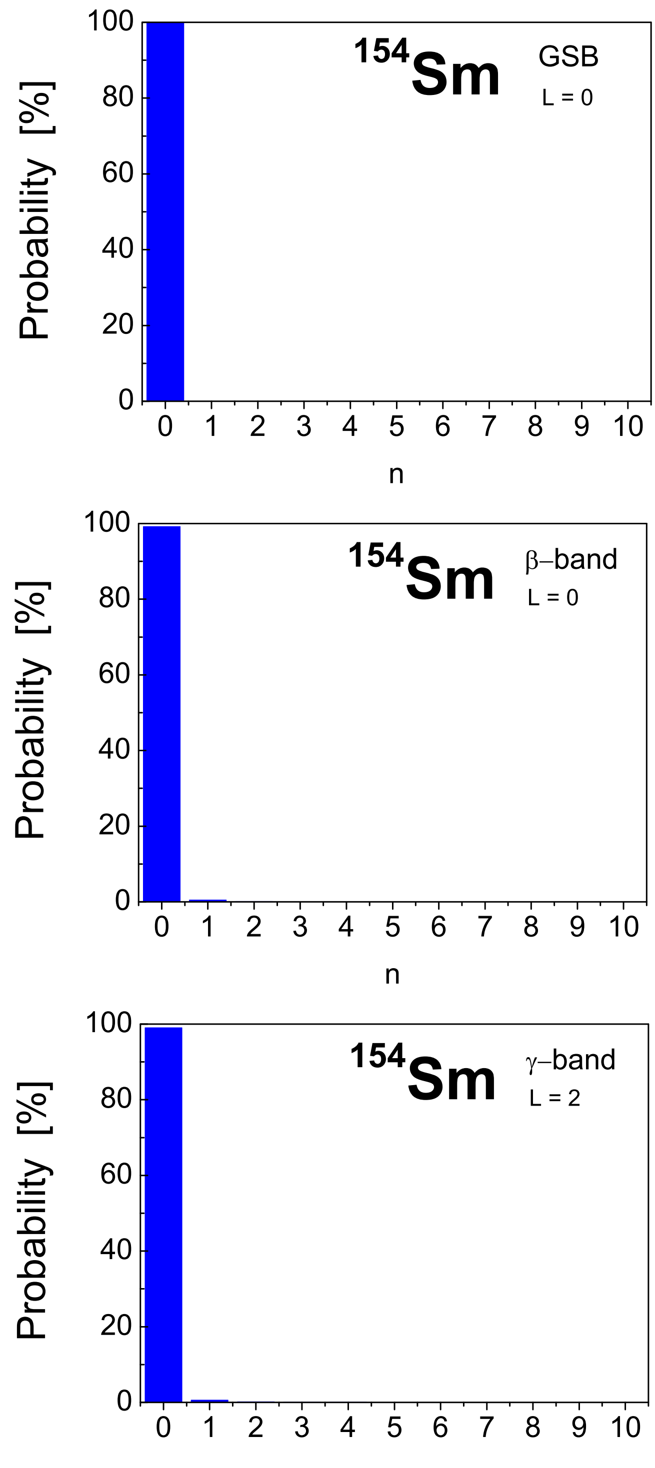

FIG. 3. Calculated [within the space of stretched SU(3) states] probability distributions for the SU(3) wave function components $(\lambda+2n,\mu)$ from each $n\hbar\omega$ shell for the 0^+ states of the ground and β bands and for the 2^+ state of the γ band, as a function of n. The starting SU(3) configurations for the ground and β, γ bands are (30,0) and (26,0), respectively.

The results for the energies of the ground, β, and γ bands, as well as the intraband $B(E2)$ transition strengths between the states of the ground band for $\chi=0.0032$, are shown,

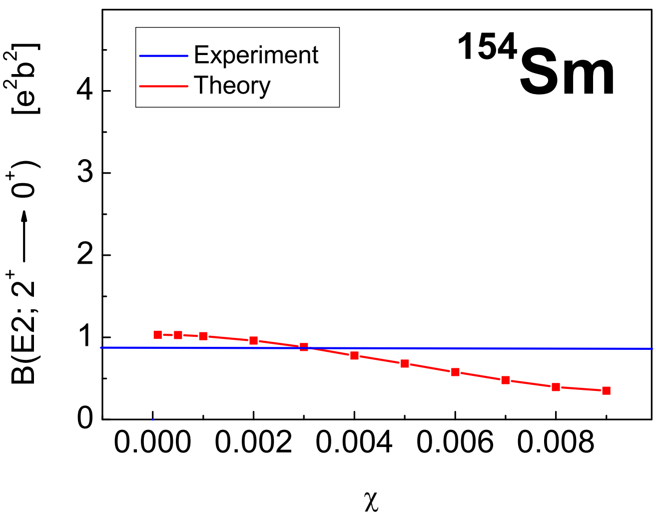

FIG. 4. Calculated $B(E2;2_1^+ \rightarrow 0_1^+)$ transition strength in ^{154}Sm as a function of the model parameter χ. No effective charge is used.

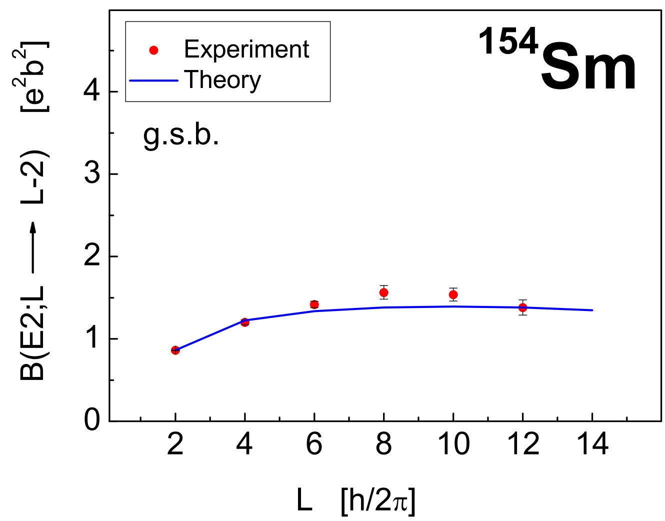

FIG. 6. Calculated and experimental intraband $B(E2)$ values between the states of the ground band in ^{154}Sm. No effective charge is used. The values for the model parameters are as follows (in MeV): $\chi = 0.0032$, $\xi = 0.0053$, and $a = 0.013$.

respectively, in Figs. 5 and 6. The energies are practically not affected by the extension of the model space for the used values of the model parameters. From Fig. 6 one sees a reduction of quadrupole collectivity, caused by extending horizontally the model space, which allows the proper reproduction of the observed $B(E2)$ values.

The SU(3) probability distributions for the 0^+ states of the ground and β bands, and for the 2^+ state of the γ band, obtained in the calculations in the new space, are shown in Fig. 7. From these distributions one sees that the SU(3) dynamical symmetry is slightly broken due to the mixing. In particular, for the states of ground and β bands one sees a comparatively simple structure in which a few SU(3) multiplets contribute. For the 2^+ states of the γ band one sees almost a pure SU(3) structure, determined by the SU(3) irrep (26,2) which exhausts 99.92%. But what is remarkable from these results is the fact that all SU(3) states, contributing to the structure of the collective states under consideration,

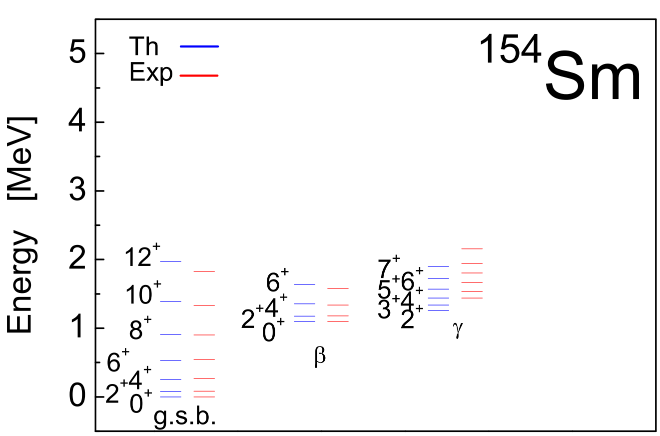

FIG. 5. Comparison of the theoretical and experimental energy levels for the ground, β, and γ bands in ^{154}Sm. The values for the model parameters are as follows (in MeV): $\chi = 0.0032$, $\xi = 0.0053$, and $a = 0.013$.

belong to a single U(6) irrep, namely that of the symplectic bandhead. More precisely, the U(6) irrep of the lowest-weight state exhausts up to 99.925% of the structure for the ground state, and 99.926% and 99.991% for the 0^+ and 2^+ states of the β and γ bands, respectively. The same picture is obtained for the other collective states.

At this point it should be pointed out that the eigenvectors of the model Hamiltonian (13), obtained for different values of the parameter χ (shown in Fig. 4), also belong to a single U(6) irreducible representation. In other words, the microscopic structure of the low-lying collective states in ^{154}Sm shows the presence of a very good U(6) dynamical symmetry. This explains the success of different U(6)-based models, e.g., the IBM [2].

As can be seen from Figs. 3 and 7, the Hamiltonian (13) leads to an almost exact SU(3) symmetry for the wave functions of the β and γ bands, which have almost similar SU(3) probability distributions, determined by the predominant SU(3) irrep (26,2), plus small admixtures from the other $0\hbar\omega$ SU(3) irreps for the β band of the order of $\sim 5.4\%$. From the similar probability distributions of the β and γ bands it follows that the two bands will be almost degenerate in energy, as can be seen from the theoretical values in Figs. 1 and 5. In experiment, however, these two bands are not degenerate. This experimental observation can easily be reproduced in the theory, at the price of one more parameter, by adding for example the K^2 term to the model Hamiltonian which will split the degeneracy between the β and γ bands.

Concerning the interband $E2$ transitions, which are not the subject of the present work, it will be noticed that within the stretched SU(3) approximation they will be zero for $\beta \rightarrow g$ and $\gamma \rightarrow g$, and nonzero for $\gamma \rightarrow \beta$ since the latter two bands lie in the same set of stretched SU(3) states $(26+2n,2)$. The extension of the model space produces a small mixing of the two sets of SU(3) irreps, corresponding to the ground and β (γ) bands, which will result in nonzero interband

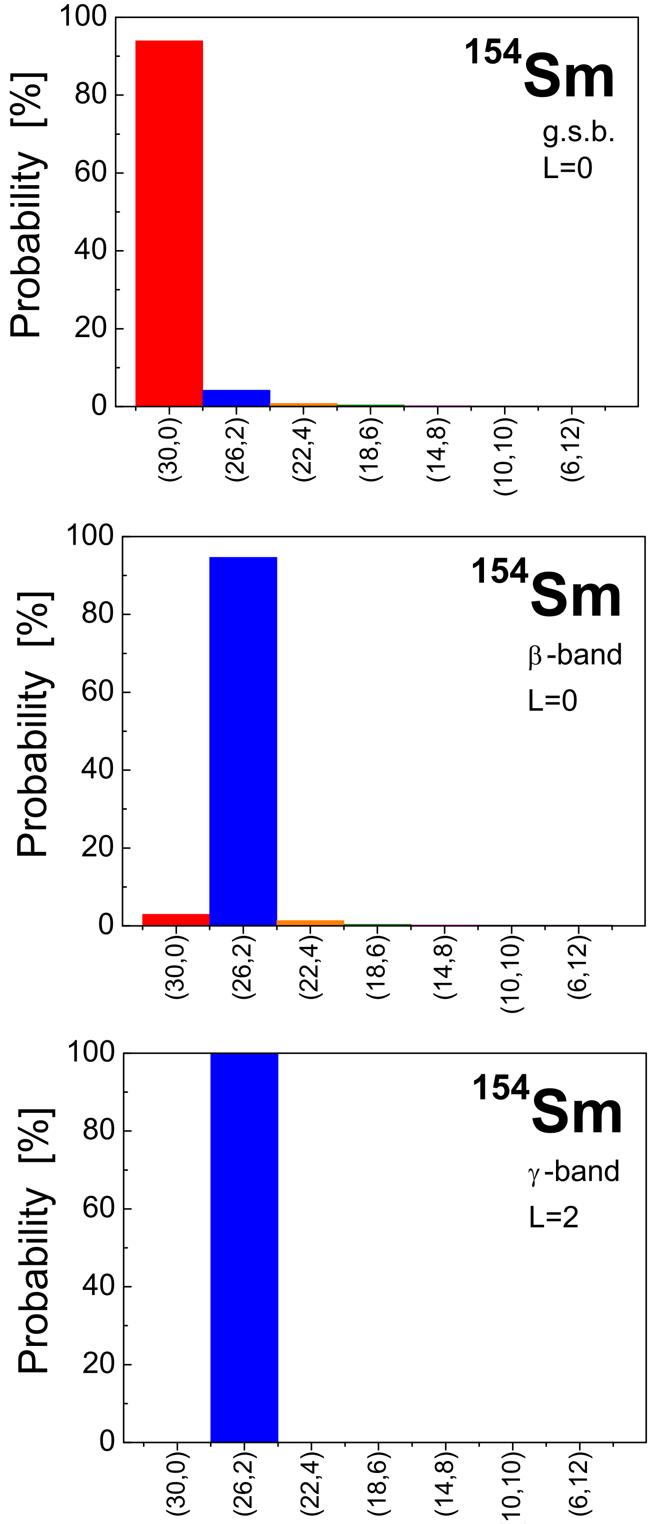

FIG. 7. Calculated SU(3) probability distributions for the wave functions for the 0^+ states of the ground and β bands, and for the 2^+ state of the γ band. The values for the model parameters are as follows (in MeV): $\chi = 0.0032$, $\xi = 0.0053$, and $a = 0.013$.

transitions between them. For instance, for $0^+_\beta \to 2^+_g$ and $2^+_\gamma \to 0^+_g$ transitions one obtains (with no effective charge) 5.22 and 4.08 w.u., which are in reasonable agreement with the experimental values [37] 12 and 2.13 w.u., respectively.

It should also be pointed out that, in the present approach, the structure of the low-lying collective states of the ground band has an SU(3) probability distribution similar to that obtained in the contracted symplectic model for ^{160}Dy, ^{168}Er, and ^{234,236,238}U nuclei [15], but in which the role of "vertical" and "horizontal" mixing is interchanged. This means that, in the PNSM, the mixing among the SU(3) states within the symplectic bandhead $0\hbar\omega$ subspace is preferable over the major intershell mixing for the present choice of the collective potential.

The results obtained with the $Q_p \cdot Q_n$ interaction show that it can be replaced by its U(6)-restricted part, $\widetilde{Q}_p \cdot \widetilde{Q}_n$, acting only within a single shell. Therefore, using this replacement and the algebraic realization [38,39] of a coupled two-rotor picture, in which one rotor represents the proton and another represents the neutron distribution, for the axial-axial case considered in this paper, one can associate the U(6)-restricted part of Hamiltonian (13) with a one-dimensional harmonic oscillator restoring potential with a frequency ω_θ. The correspondence is based on the well-known relation of the SU(3) symmetry group to the symmetry group of the rigid-rotor, Rot(3) [40]. The angle θ determines the relative orientation of the two rotors and each orientation, in turn, implies a specific SU(3) configuration. According to Littlewood rules, the geometrical image of two axial coupled rotors is in one-to-one correspondence to the $SU_p(3) \otimes SU_n(3) \supset SU(3)$ reduction $(\lambda_p, \mu_p = 0) \otimes (\lambda_n, \mu_n = 0) \to (\lambda, \mu)$, which can be expressed in terms of a single quantum number r:

$$(\lambda_p, 0) \otimes (\lambda_n, 0) = \bigoplus_r (\lambda_p + \lambda_n - 2r, r). \quad (18)$$

Then, a discrete orientation angle

$$\theta_r = \sin^{-1}\sqrt{\frac{(\lambda_p + \lambda_n - r)r}{\lambda_p \lambda_n}} \quad (19)$$

can be associated with the joined proton-neutron SU(3) irrep $(\lambda_p + \lambda_n - 2r, r)$, where $r = 0, \ldots, \min(\lambda_p, \lambda_n)$ [38]. For $\theta = 0$ the two axially symmetric ellipsoids overlap maximally, whereas when $\theta = \pi/2$ the principal axes are perpendicular to one another and the resulting overlap of the two distributions is minimal.

Using the correspondence of the invariant operators of SU(3) and Rot(3) groups, the second-order SU(3) Casimir operator can be expressed as a function of the angle θ in the form [39]

$$C_2[SU(3)](\theta) = [(\lambda_p + \lambda_n + 2)^2 + 2(\lambda_p + \lambda_n + 2) + 4] - 12(\lambda_p + 1)(\lambda_n + 1)\theta^2 - 3. \quad (20)$$

Then the eigenvalues of the U(6)-restricted part of the model Hamiltonian (13) can be expressed as

$$E - E_{g.s.} = r\hbar\omega_\theta - \frac{\chi}{4} L_p \cdot L_n + aL(L+1) + E'_0, \quad (21)$$

where $\hbar\omega_\theta = \hbar\omega_\infty[1 + \frac{1-r}{\lambda_p+\lambda_n}]$ and $\hbar\omega_\infty = (\frac{\chi}{4} + \xi)2(\lambda_p + \lambda_n)$. For fixed initial proton and neutron distributions, the constant E_0' is related to the $SU_\alpha(3)$ quantum numbers λ_α ($\alpha = p, n$) and is not important for the calculation of nuclear spectra since only one $SU_\alpha(3)$ irrep for protons and neutrons is considered. From Eq. (21) it follows that the low-lying rotational bands in nuclear spectra can be considered as being built on different multiphonon excitations (which are slightly mixed by the nondiagonal $L_p \cdot L_n$ term) with a phonon energy $\hbar\omega_\theta$ and phonon spin $1\hbar$. The quantum number r therefore can be associated with the number of oscillator phonons, i.e., $r = n_\theta$.

Recall that in the general case of two triaxial coupled rotors, one needs three angles (θ, ϕ_p, ϕ_n), corresponding to the three quantum numbers (r, s, ϱ) in the decomposition $SU_p(3) \otimes SU_n(3) \supset SU(3)$, which specify the relative orientation of the proton and neutron ellipsoids [39].

IV. CONCLUSIONS

In the present paper, the proton-neutron symplectic model with Sp(12,R) dynamical algebra is applied to the simultaneous description of the microscopic structure of the low-lying states of the ground, β, and γ bands in ^{154}Sm. For this purpose, the model Hamiltonian, consisting of a spherical harmonic oscillator shell-model part and the full major shell-mixing proton-neutron quadrupole-quadrupole interaction, plus an SU(3) scalar term which take into account the bandhead energies and a residual rotational part, is diagonalized in a U(6)-coupled basis, restricted to the state space spanned by the fully symmetric U(6) irreps. Although the model Hamiltonian used is very schematic, it nevertheless reveals the kind of possible proton-neutron collective dynamics that can be investigated within the framework of the PNSM.

A good description of the energy levels of the three bands under consideration, as well as the intraband $B(E2)$ transition strengths between the states of the ground band, is obtained without the use of an effective charge. The results obtained reveal a simple structure of the collective states in which only a few SU(3) multiplets contribute. A remarkable observation, which follows from the present calculations, is that all the SU(3) irreducible representations that contribute to the structure of collective states belong to a single U(6) irreducible representation, namely that of the symplectic bandhead. This reveals the presence of a very good U(6) dynamical symmetry in the low-energy spectrum of ^{154}Sm, at least for the present choice of the model Hamiltonian. The obtained results are governed by the full multi-major-shell mixing quadrupole-quadrupole interaction, $Q_p \cdot Q_n$, which favors the horizontal mixing of different SU(3) multiplets within the $0\hbar\omega$ space of the Sp(12,R) bandhead.

It is also shown that, in contrast to the Sp(6,R) case, the lowest excited bands, e.g., the β and γ bands, naturally appear together with the ground band within a single Sp(12,R) irreducible representation. In this regard, the extension of the one-component Sp(6,R) model to the two-component Sp(12,R) one already includes at the level of model state space, besides the basic rotational SU(3), also the low-lying vibrational degrees of freedom, represented by the presence of other excited collective bands in the experimentally observed spectrum. It is clear that both the rotational and low-lying vibrational degrees of freedom are contained in this larger intrinsic U(6) structure of PNSM. Moreover, the PNSM calculations in a space up to $40\hbar\omega$ with the full major-shell mixing $Q_p \cdot Q_n$ interaction for different values of χ give eigenvectors which belong to the Hilbert space of the Sp(12,R) bandhead state space only. This fact shows that the required quadrupole collective dynamics is already covered by the Sp(12,R) bandhead intrinsic structure. This is a remarkable observation, which suggests an even simpler interpretation of the observed experimental data and possible usage of a U(6)-truncated Hamiltonian in practical applications. The latter explains the success of the U(6)-based macroscopic theories of nuclear collective motion, e.g., the IBM, in describing the observed low-lying collective states in strongly deformed nuclei.

The results, obtained by extending the model space beyond that of the stretched SU(3) states, showed a reduction of the collective nuclear dynamics. In this regard, the $Q_p \cdot Q_n$ interaction through its horizontal mixing causes effects similar to those of the symplectic symmetry-breaking interactions, e.g., the spin-orbit and pairing, within the framework of the one-component Sp(6,R) symplectic model or its contracted version. This effect becomes understandable if one considers a decomposition of a given Sp(12,R) collective space into different Sp(6,R) irreducible representations. Then it becomes clear that the $Q_p \cdot Q_n$ interaction, which is in the enveloping algebra of Sp(12,R), will naturally incorporate the effects of horizontal mixing of different Sp(6,R) multiplets.

Finally, the obtained results is given a simple geometrical multi-phonon interpretation, based on the algebraic realization of the coupled two-rotor picture. This suggests an interpretation of the low-lying excited bands as relative proton-neutron excitations of the two-component nuclear system, governed by the $Q_p \cdot Q_n$ interaction.

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