Effect of shell corrections on the α-decay properties of $^{280-305}$Fl isotopes

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The α-decay half-lives of $^{285-289}$Fl isotopes and their decay chains are investigated by employing the generalized liquid-drop model (GLDM), the unified fission model, the Royer's analytical formula, and the universal decay law. For the GLDM, we take into account the shell correction. The agreement between the experimental data and the calculations indicates that all the methods we used are successful to reproduce α-decay half-lives of $^{285-289}$Fl. For the unknown nuclei, the α-decay half-lives have been predicted by inputting α-decay energies (Q_α) extracted from the finite-range droplet model and the updated Weizsäcker-Skyrme-4 (WS4) model. It is found that the shell correction would enlarge the calculated α-decay half-lives in the region from ^{292}Fl to ^{298}Fl, where the shell effects are evident. We confirm that $N = 184$ is the neutron magic number and $N = 178$ is the submagic number by analyzing the α-decay half-lives and the shell correction energies. The competition between α-decay and spontaneous fission is discussed in detail and the decay modes of $^{280-283}$Fl and $^{290-305}$Fl have been predicted. Our calculations are in good agreement with the experiments for the decay properties of $^{284-289}$Fl. We also predict ^{284,286}Fl with both α-decay and spontaneous fission. The $^{280-283,290-295,297}$Fl isotopes are α decay, $^{300-305}$Fl undergo spontaneous fission, and ^{296,298,299}Fl would have both α-decay and spontaneous fission. We also predict the decay chains of $^{280-283,290,291}$Fl.

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I. INTRODUCTION

With the development of low-energy high intensity ion beam facilities and the advanced detectors, more and more superheavy nuclei (SHN) have been synthesized in the laboratory. It is a great challenge to synthesize the superheavy elements and to reach the "island of stability", where filled proton and neutron shells will give rise to extraordinarily stable and hence long-lived nuclei [1,2]. Many theoretical works predict the center of the island of stability, macroscopic-microscopic models regarded nucleus $Z = 114$, $N = 184$ as the center of the island of the stability [3]. The relativistic and nonrelativistic mean field models suggested $Z = 120$, $N = 172$ and $Z = 114, 126$, $N = 184$ are magic numbers [4,5]. However the nonrelativistic Skyrme-Hartree-Fock models predicted that $Z = 124, 126$ and $N = 184$ are magic numbers [6,7]. The α-decay properties, such as decay energies and half-lives, reflect the location of shell structures.

Fl is one of the most remarkable elements, since the isotopes are much close to the predicted spherical nuclei ^{298}Fl and, consequently, being relatively stable. From 1999, six Fl isotopes, $^{284-289}$Fl have been synthesized by using the hot-fusion mechanism [8–10]. The known most heavy isotope ^{289}Fl has only nine fewer neutrons than the predicted spherical shell closure ($Z = 114$, $N = 184$). Hence, both the experimental and theoretical investigations are endeavoring to produce heavier Fl isotopes. On the other hand, the known most neutron deficient isotope ^{284}Fl was identified recently, which was observed with spontaneous fission (SF) and might have a potential observation of the α decay [11,12]. Measurements of the α-decay energy of ^{285}Fl were carried out for the first time, and the decay properties of the nuclei on the decay chain were determined with high precision. Since the α-decay properties indicate the stability of the nucleus, predict the decay modes and the shell positions, it is important to systematically calculate the α-decay properties accurately for Fl isotopes with the updated experimental data.

The α decay is one of the most important decay modes of the SHN, which provides an efficient approach to identify the experimental synthesized SHN by detecting the α-decay chains in the experiments and to extract detailed nuclear structure properties of these nuclei. The basic process of α decay is explained as quantum-tunneling effect [13,14]. The tunneling of the α particle across the Coulomb barrier for heavy and superheavy nuclei was well described by semiclassical models such as the generalized liquid drop model (GLDM) [15–17], unified fission model (UFM) [18], and density-dependent

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cluster model (DDCM) [19], etc. Many empirical relationships, e.g., the Brown formula [20], Viola-Seaborg formula [21,22], Royer's formula [23], etc., were used to calculate the α-decay half-life in the framework of Geiger-Nuttall law [24]. To describe α decay in a fully microscopic way was difficult although still many works considering microscopic modifications in the calculations, such as multistep shell model (MSM) [25], multichannel cluster model (MCCM) [26] which is based on the coupled-channel Schrodinger equation, and using quartetting wave function approach to describe preformation factor in a microscopic way [27].

In this work, we employed the GLDM, the UFM, the Royer's formula, and the UDL [28,29] to calculate the α-decay half-lives. The shell correction has been considered in the GLDM. For the experimentally synthesized nuclei, we use experimental Q_α values as input. For the unknown nuclei, the FRDM [30] and the WS4 [31] model were used to calculate Q_α. It is shown that $N = 184$ is a neutron magic number and $N = 178$ is a submagic number. To predict the decay modes of unknown Fl isotopes, we use a modified shell-induced Swiatecki's formula to calculate theoretical SF half-lives [32,33]. For the known nuclei, our calculations show that ^{284,286}Fl may have both α-decay and spontaneous fission. For the unknown isotopes, $^{280\text{–}283,290\text{–}295,297}$Fl undergo α decay, $^{300\text{-}305}$Fl are spontaneous fission, and ^{296,298,299}Fl have both α-decay and spontaneous fission. We also present the theoretical decay chains of $^{280\text{–}283,290,291}$Fl.

The paper is assembled as follows. In Sec. II the theoretical framework is introduced. The results and corresponding discussions are presented in Sec. III. In the last section, the conclusions are given.

II. THEORETICAL FRAMEWORK

A. α decay

1. GLDM

In the framework of the GLDM, the decay width is defined as $\lambda = P_\alpha \nu_0 P$. The preformation factor P_α is considered as a constant. According to the experiments as well as model calculations, we adopt $P_\alpha = 0.43$ for even-even nuclei, $P_\alpha = 0.35$ for odd-A nuclei, and $P_\alpha = 0.18$ for doubly odd nuclei [34]. The assault frequency ν_0 is phenomenologically calculated by [35]

$$\nu_0 = \frac{1}{2R}\sqrt{\frac{2E_\alpha}{M_\alpha}}, \tag{1}$$

where R is the radius of the parent nucleus, E_α is the kinetic energy of the α particle, corrected for recoil, and M_α is its mass.

The barrier penetrability P is calculated via Wenzel-Kramers-Brillouin (WKB) approximation

$$P = \exp\left[-\frac{2}{h}\int_{R_{\rm in}}^{R_{\rm out}}\sqrt{2B(r)(E(r)-E(\text{sphere}))}dr\right], \tag{2}$$

where $E(R_{\rm in}) = E(R_{\rm out}) = Q_\alpha^{\rm exp}$, $B(r) = \mu$, μ is the reduced mass of the α particle, and the daughter nucleus, $E_{\rm sphere}$ is the ground state energy of the parent nucleus.

2. UFM

In the UFM, the decay constant is defined as $\lambda = \nu_0 P$. The assault frequency ν_0 is calculated via quantum mechanism [18]

$$\nu_0 = \frac{\omega}{2\pi} = \frac{\left(G+\frac{3}{2}\right)\hbar}{1.2\pi\mu R_0^2}. \tag{3}$$

Here, ω is the oscillation frequency, R_0 is the radii of the parent nucleus. The barrier penetrability P is determined within the action integral,

$$P = \exp\left[-\frac{2}{h}\int_{R_{\rm in}}^{R_{\rm out}}\sqrt{2\mu(V(r)-Q_\alpha)}dr\right]. \tag{4}$$

The potential $V(r)$ is constructed by Coulomb potential, nuclear proximity potential (V_p), and the centrifugal potential (V_ℓ) for $r \geqslant R_1 + R_2$; for $r < R_1 + R_2$, $V(r)$ is written as a polynomial,

$$V(r) = \begin{cases} V_p + V_l + \frac{Z_1 Z_2 e^2}{r}, & r \geqslant R_1 + R_2 \\ a_0 + a_1 r + a_2 r^2, & R_0 \leqslant r < R_1 + R_2 \end{cases}, \tag{5}$$

where R_1 and R_2 are the radii of the daughter nucleus and the emitted particle, respectively. The coefficients a_0, a_1, and a_2 were obtained by the following boundary conditions:

(i) At $r = R_0$, $V(r) = Q_\alpha$;
(ii) At $r = R_1 + R_2$, $V(r)$ is a continuous function;
(iii) At $r = R_1 + R_2$, $dV(r)/dr$ is a continuous function.

The centrifugal potential V_l(r) takes the form as

$$V_l(r) = \frac{l(l+1)\hbar^2}{2\mu r^2}. \tag{6}$$

3. Royer's formula

The analytical formula by Royer was adopted to calculate α-decay half-lives [23]. By fitting 131 even-even nuclei with the root mean square (rms) deviation of 0.285, half-life could be calculated as

$$\log_{10}[T_{1/2}(s)] = -25.31 - 1.1629A^{1/6}Z^{1/2} + 1.5864Z/\sqrt{Q_\alpha}. \tag{7}$$

For the even-odd nuclei, 106 nuclei were used to obtain the following formula with rms deviation of 0.39

$$\log_{10}[T_{1/2}(s)] = -26.65 - 1.0859A^{1/6}Z^{1/2} + 1.5848Z/\sqrt{Q_\alpha}. \tag{8}$$

For the subset of 86 odd-even nuclei, the following equation was obtained with rms deviation of 0.36

$$\log_{10}[T_{1/2}(s)] = -25.68 - 1.1423A^{1/6}Z^{1/2} + 1.592Z/\sqrt{Q_\alpha}. \tag{9}$$

For the odd-odd nuclei, 50 nuclei were adopted to get $\log_{10} T_{1/2}$ with an rms deviation of 0.35

$$\log_{10}[T_{1/2}(s)] = -29.48 - 1.113A^{1/6}Z^{1/2} + 1.6971Z/\sqrt{Q_\alpha}. \tag{10}$$

4. UDL

The UDL gives the relationship between α-decay half-lives and the properties of daughter nucleus and the α particle [28,29],

$$\log_{10}[T_{1/2}(s)] = aZ_\alpha Z_d\sqrt{\frac{A}{Q_\alpha}} + b\sqrt{AZ_\alpha Z_d\left(A_d^{1/3}+A_\alpha^{1/3}\right)} + c, \quad (11)$$

where $A = \frac{A_d A_\alpha}{A_d + A_\alpha}$, the constant $a = 0.4314$, $b = -0.4087$, and $c = -25.7725$, which are determined by fitting to experimental data.

B. Spontaneous fission

The spontaneous fission was calculated based on the Swiateckis formula [32]. The generalized Swiateckis formula by Xu, Ren, and Guo [36] was constructed to calculate half-lives of nuclei without experimental mass values. The fissionability parameter and the isospin effect play important roles in determining the spontaneous fission half-lives of heavy and superheavy nuclei. The shell effect is also important in the calculation of spontaneous fission half-lives [32]. In this paper, we use a recently modified Swiateckis formula, which takes into account the isospin effect [$I = (N-Z)/A$], as well as the shell effect, to calculate the SF half-lives [33]

$$\log_{10}[T_{1/2}(yr)] = c_1 + c_2\left(\frac{Z^2}{(1-kI^2)A}\right) + c_3\left(\frac{Z^2}{(1-kI^2)A}\right)^2 + c_4E_{sh} + h_i. \quad (12)$$

Here, $Z^2/(1-kI^2)A$ denotes the fissionability parameter when the isospin effect has been included. The shell correction energy E_{sh} was obtained from $E_{\rm mic}$ in FRDM [30]. The fixed value of k is 2.6 from [23]. The coefficients c_1, c_2, c_3, c_4 were obtained by fitting 45 even-even nuclei experimental spontaneous fission data: $c_1 = 1174.353441$, $c_2 = -47.666855$, $c_3 = 0.471307$, and $c_4 = 3.378848$. The h_i represents the blocking effect of unpaired nucleon. For the odd-N nuclei $h_{eo} = 2.609374$, for the odd-Z nuclei $h_{oe} = 2.619768$, which were obtained by fitting the experimental spontaneous fission half-lives of 12 odd-N and12 odd-Z nuclei, respectively. For odd-odd nuclei $h_{oo} = h_{eo} + h_{oe}$ and for even-even nuclei $h_{ee} = 0$.

C. The shell correction in the GLDM

The shape-dependent shell corrections have been introduced in the GLDM [37]

$$E_{\rm shell} = E_{\rm shell}^{\rm sphere}(1-2.6\alpha^2)e^{-\alpha^2}, \quad (13)$$

where $\alpha^2 = (\delta R)^2/a^2$ represent the root mean square of the deviation of the particle surface from the sphere, which include all types of deformation indiscriminately. The whole shell correction energy decreases to zero with the increasing distortion of the nucleus due to the attenuating factor $e^{-\alpha^2}$.

The $E_{\rm shell}^{\rm sphere}$ is the shell correction for a spherical nucleus

$$E_{\rm shell}^{\rm sphere} = cE_{sh}, \quad (14)$$

where E_{sh} denote the shell energy of a nucleus and is obtained by the Strutinsky procedure [38] by setting the smoothing parameter $\gamma = 1.2\hbar\omega_0$ and the order $p = 6$ of the Gauss-Hermite polynomials and $\hbar\omega_0 = 41A^{-1/3}$ is the mean distance between the gross shells. The parameter c is a scaled parameter adopted to adjust the division of the binding energy between the microscopic correction and the macroscopic part [39].

III. RESULTS AND DISCUSSION

A. The decay chains of $^{285-289}$Fl isotopes

The α-decay half-lives of $^{285-289}$Fl isotopes and the nuclei on the decay chains have been calculated by inputting the experimental Q_α in analytical formulas, the UFM, the GLDM with and without the shell correction. The results are listed in Table I. The first two columns are elements and nuclear mass A. The third column is the experimental Q_α used to calculate α-decay half-lives [10–12]. The forth column is the experimental $\log_{10} T_{1/2}^\alpha$ [10–12]. The last five columns are α-decay half-lives calculated by Royer's formula, the UDL, the UFM, the GLDM, and the GLDM with shell correction.

For Fl isotopes, with increasing nuclear mass numbers, the nuclei half-lives are getting large, which means they are relatively more stable and close to the shell structures. The difference of half-lives by the models with and without shell correction are also getting large. The ^{289}Fl is the heaviest and the most long-lived known Fl isotope, only nine fewer than the predicted doubly magic spherical nucleus ^{298}Fl. Thus the shell correction effect of ^{289}Fl is the most obvious comparing to the other known Fl isotopes. The $\log_{10} T_{1/2}^\alpha$ of ^{289}Fl calculated by the GLDM with shell correction is about 32% larger than that without shell correction. This is because the addition of the shell correction energies would change the shape of the fission barrier and change the penetration probability. The P value of ^{289}Fl is decreased from 0.62×10^{-20} to 0.39×10^{-20} after considering the shell correction.

Comparing the half-lives calculated by the GLDM with and without shell correction, we find that the effect of shell correction seems much obvious for the medium-mass nuclei, such as ^{269,271}Sg and ^{273,275}Hs. Take ^{269}Sg for example, the $\log_{10} T_{1/2}^\alpha$ is changed from 1.25 s to 2.06 s, which has been increased by about 60% after considering the shell correction. For ^{277,279,281}Ds, ^{281,283,285}Cn, the shell correction effects are not very evident; the $\log_{10} T_{1/2}^\alpha$ values are increased by 0.1–0.01 s. For Fl isotopes, we find the trend that the shell corrections become more apparent with the increasing mass numbers. It is noticed that when nuclei are close to the magic or submagic numbers, such as $Z = 106$, 108, 114, $N = 162$ and 184, the shell correction effects become much more important.

To test the agreement between the experimental half-lives and the theoretical ones, we calculate the average deviation by

TABLE I. The experimental and theoretical α-decay half-lives of nuclei on the decay chains of $^{285-289}$Fl isotopes. The theoretical results are calculated by Royer's formula, UDL, UFM, and GLDM by inputting the experimental Q_α [10–12]. The last column is result calculated by the GLDM considering shell correction. The $\bar{\delta}$ is the average deviation between the experiments and the theoretical calculations by Eq. (15).

Element	A	$Q_\alpha^{\rm exp}$ (MeV)	$\log_{10} T_{1/2}^{\rm exp}$ (s)	$\log_{10} T_{1/2}$ (s) Royer	$\log_{10} T_{1/2}$ (s) UDL	$\log_{10} T_{1/2}$ (s) UFM	$\log_{10} T_{1/2}$ (s) GLDM	$\log_{10} T_{1/2}$ (s) $\rm GLDM_{shell}$
Fl	289	9.98 ± 0.02	0.279	0.727	0.259	0.166	−0.642	−0.438
	288	10.07 ± 0.03	−0.180	−0.227	0.003	−0.081	−0.946	−0.817
	287	10.17 ± 0.02	−0.319	0.225	−0.280	−0.354	−1.091	−1.021
	286	10.35 ± 0.04	−0.921	−0.966	−0.789	−0.846	−1.615	−1.587
	285	10.56 ± 0.05	−1.000	−0.796	−1.370	−1.407	−2.025	−2.028
Cn	285	9.32 ± 0.02	1.447	2.010	1.625	1.515	0.642	0.676
	283	9.66 ± 0.02	0.623	1.013	0.560	0.483	−0.307	−0.369
	281	10.45 ± 0.04	−0.745	−1.154	−1.745	−1.751	−2.319	−2.447
Ds	281	8.85 ± 0.03	1.114	2.802	2.467	2.359	1.462	1.373
	279	9.85 ± 0.02	−0.538	−0.217	−0.743	−0.760	−1.419	−1.381
	277	10.7 ± 0.04	−2.456	−2.434	−3.101	−3.047	−3.464	−3.356
Hs	275	9.45 ± 0.02	−0.699	0.250	−0.244	−0.253	−0.929	−0.506
	273	9.65 ± 0.04	−0.292	−0.295	−0.828	−0.819	−1.436	−1.002
Sg	271	8.67 ± 0.08	1.982	1.961	1.573	1.540	0.784	1.464
	269	8.54 ± 0.04	2.924	2.429	2.063	2.020	1.250	2.055
$\bar{\delta}$				0.410	0.430	0.421	0.921	0.756

the following expression:

$$\bar{\delta} = \frac{1}{N}\sum_{i=1}^{N}\left|\log_{10} T_{1/2}^{\rm theo} - \log_{10} T_{1/2}^{\rm exp}\right|. \tag{15}$$

The $\bar{\delta}$ values of results by Royer's formula, the UDL, the UFM, the GLDM, and the GLDM with shell correction are 0.41, 0.43, 0.42, 0.92, and 0.76, respectively. This means the calculated half-lives are in good agreement with the experimental ones. The GLDM $\log_{10} T_{1/2}^{\alpha}$ values are systematically lower than the experiments. This maybe because the preformation factor we adopted is constant which is not fully accurate. However the GLDM results show similar trend with the others, which meas the GLDM can also reproduce the α-decay half-lives accurately. After the consideration of shell correction, the precision of the GLDM would be improved, since the average deviation is reduced by 0.16. All the discussions show that it is appropriate to include the shell correction to calculate α-decay half-lives, especially for nuclei near the shell closures or subshell closures.

B. Predictions of α-decay properties of $^{280-283}$Fl and $^{290-305}$Fl isotopes

To test the sensitivity of the α-decay half-lives on the changing of Q_α, we extract Q_α values as input of GLDM from two models: the FRDM [30] and the newest WS4 model [31]. To make a self-consistent result, the shell correction energies in the model are adopted directly from $E_{\rm mic}$ in Ref. [30] and E_{sh} in Ref. [31]. We present the predicted α-decay half-lives of the even-even Fl isotopes in Fig. 1. Panel (a) is calculated by the FRDM Q_α, panel (b) is from the WS4 Q_α. Both results show that the α-decay half-lives quickly decrease at $N = 184$. It is confirmed that $N = 184$ is the neutron magic number. The FRDM results have another peak at $N = 178$, which might be a submagic number.

Figure 1(b) shows that the results using the GLDM without shell correction seems closer to the experimental data than those by the GLDM with shell correction. As shown in Table I, the $\log_{10} T_{1/2}^{\alpha}$ values by the GLDM with experimental Q_α are systematically lower than the experimental half-lives. However, the theoretical Q_α values are lower than the experimental Q_α and then enlarge the calculated half-lives. Thus the $\log_{10} T_{1/2}^{\alpha}$ values by the GLDM with theoretical Q_α values seem fit the experiments well. With similar input Q_α values, the results by the GLDM with shell correction are slightly larger than those without shell correction. Thus the results with shell correction are slightly higher than the experimental data. Since it is difficult to reproduce Q_α exactly, we need to use the experimental Q_α to test the precision of the model. As shown in Table I, the half-lives calculated by the GLDM with shell correction are more precise than those without shell correction, according to the average deviation $\bar{\delta}$.

Comparing the results calculated by models with and without shell correction, the half-life differences become much obvious in the region from $N = 178$ to 184, where the isotopes are relatively long-lived. It shows that the shell correction leads to a larger α-decay half-life in this region. To describe the shell correction, we show the shell correction energies of Fl even-even isotopes in Fig. 2. Lower shell correction energy means higher stability [40]. There exist valleys in the range from $N = 178$ to 184, indicating that the isotopes in this region are relatively stable. The maximum shell correction occurs at $N = 178$ for both the FRDM and the WS4 mass formula. However the deformations of nuclei indicate that nuclei with $N = 184$ are nearly spherical in shape [39]. The reason for this deviation has been discussed that the difference comes from the adopted values for the spin-orbit and diffuseness parameters of the single-particle potential [41]. It is difficult to

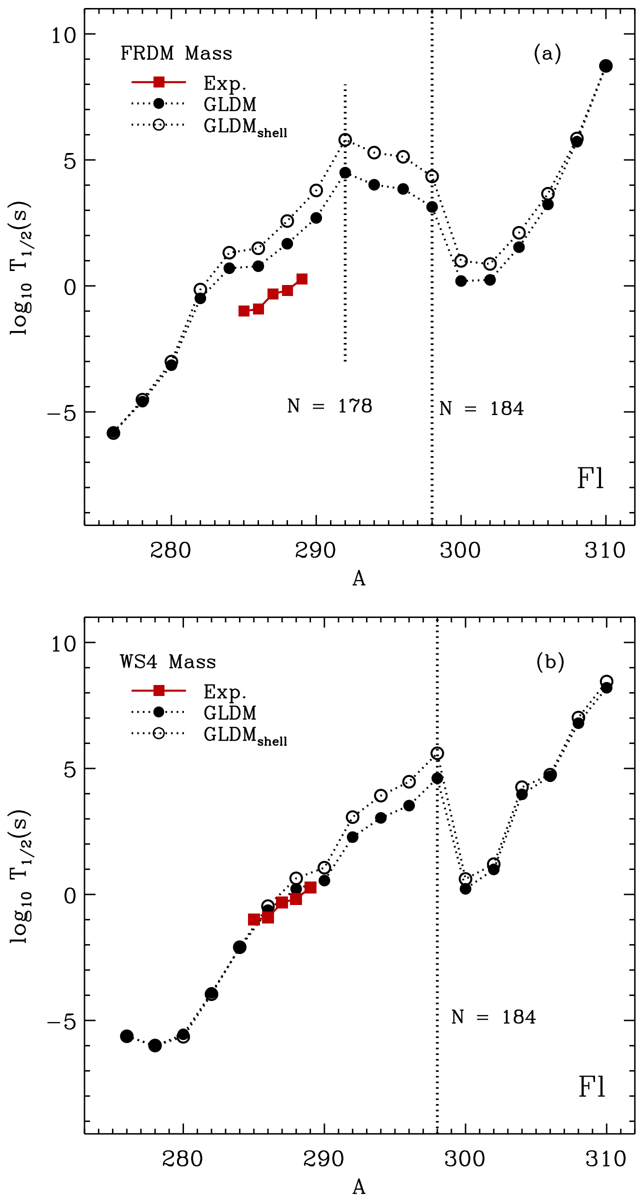

FIG. 1. The α-decay half-lives of even-even Fl isotopes calculated with FRDM [30] (a) and with WS4 model [31] (b). The solid circles are results calculated by GLDM. The open circles are calculated by GLDM including shell correction. The red squares are experimental data [10–12].

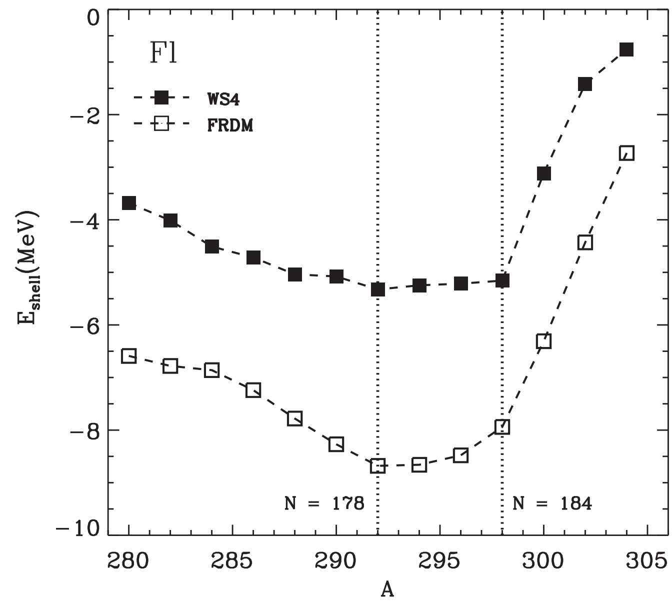

FIG. 2. The shell correction energies of the even-even nuclei with $Z = 114$. The WS4 data are from E_{sh} in Ref. [31]. The FRDM data is adopted from E_{mic} in Ref. [30].

identify the shell positions with shell correction energies only. We additionally need α-decay half-lives to give more reliable information about the shell structures.

As generally known that a slight variation of Q_α values by 1 MeV leads to a change of $\log_{10} T_{1/2}^{\alpha}$ values by about 3 orders of magnitude. Since the FRDM half-lives are slightly larger than the experimental data, we adopt WS4 Q_α to predict the α-decay half-lives. To describe the decay-modes of Fl isotopes, we put the theoretical spontaneous fission together with the α-decay half-lives in Table II. The first column is the nuclear mass number A. The second column presents the Q_α values extracted from WS4 model. The α-decay half-lives are calculated by the GLDM, the GLDM with shell correction and the UFM. As shown in Table I, Royer's formula shows powerful predictive ability. We calculate α-decay half-lives by Royer's formula and present the results in the sixth column. The calculated half-lives are close to the results by the UFM. The seventh column presents the theoretical SF half-lives. In the last column, we present the predicted decay-modes of these isotopes. The α-decay and SF half-lives show that the competition between α-decay and spontaneous fission is quite obvious. The $\log_{10} T_{1/2}^{\mathrm{SF}}$ values of ^{284}Fl and ^{286}Fl are quite similar with their $\log_{10} T_{1/2}^{\alpha}$ values. It seems that ^{284}Fl and ^{286}Fl probably undergo both spontaneous fission and α decay. Similar suspects had also been presented that ^{286}Fl has a probability of 40% undergoing spontaneous fission [10]; ^{284}Fl might has an α-decay branch of about 20% [12].

The calculations of decay modes of the known $^{284-289}$Fl isotopes are in good agreement with the experiments. We then confidently present the decay properties of the unknown isotopes $^{280-283,290-305}$Fl. The α-decay half-lives are getting large with increasing nuclear mass until $A = 198$. Then the $T_{1/2}^{\alpha}$ values decrease quickly. Thus the most stable Fl isotope is ^{298}Fl, which is the predicted shell closure. The α-decay half-lives are obviously smaller than SF half-lives for $^{280-283,290-295,297}$Fl, which undergo α decay. The $^{300-305}$Fl nuclei probably have spontaneous fission since their SF half-lives are relatively smaller. The ^{296,298,299}Fl isotopes would have both α-decay and spontaneous fission, because their $T_{1/2}^{\alpha}$ values are much similar with $T_{1/2}^{\mathrm{SF}}$ values.

C. Decay chains of $^{280-283,290,291}$Fl

At last, we present the decay chains of $^{280-283,290,291}$Fl in Fig. 3. The decay chain of ^{280}Fl is terminated to the known

TABLE II. The theoretical α-decay half-lives and SF half-lives of $^{280-305}$Fl isotopes. The Q_α^{theo} values are extracted from WS4 model [31]. Columns 3–6 are α-decay half-lives calculated by the GLDM, the GLDM with shell correction, the UFM and the Royer's formula. Column 7 presents SF half-lives calculated by Eq. (12). The last column lists predicted decay modes. For isotopes $^{284-289}$Fl (shown in boldface font) the entries are for nuclei that have been experimentally synthesized.

A	Q_α^{theo} (MeV)	T_α^{GLDM} (s)	$T_\alpha^{\text{GLDM}_{\text{shell}}}$ (s)	T_α^{UFM} (s)	T_α^{Royer} (s)	T_{SF} (s)	Decay-mode$^{\text{theo}}$
280	12.23	2.82×10^{-6}	2.25×10^{-6}	4.35×10^{-6}	4.51×10^{-6}	9.40×10^{-2}	α
281	11.82	2.03×10^{-5}	1.66×10^{-5}	3.44×10^{-5}	1.72×10^{-4}	2.07×10^{2}	α
282	11.38	1.25×10^{-4}	1.10×10^{-4}	3.55×10^{-4}	3.24×10^{-4}	3.97×10^{-1}	α
283	10.88	1.81×10^{-3}	1.73×10^{-3}	6.06×10^{-3}	2.62×10^{-2}	2.35×10^{2}	α
284	**10.57**	$\mathbf{7.45 \times 10^{-3}}$	$\mathbf{8.07 \times 10^{-3}}$	$\mathbf{3.76 \times 10^{-2}}$	$\mathbf{2.99 \times 10^{-2}}$	$\mathbf{7.20 \times 10^{-1}}$	$\boldsymbol{\alpha}$**/SF**
285	**10.28**	$\mathbf{4.74 \times 10^{-2}}$	$\mathbf{5.63 \times 10^{-2}}$	$\mathbf{2.32 \times 10^{-1}}$	$\mathbf{9.02 \times 10^{-1}}$	$\mathbf{6.51 \times 10^{3}}$	α
286	**9.97**	$\mathbf{2.39 \times 10^{-1}}$	$\mathbf{3.44 \times 10^{-1}}$	**1.76**	**1.25**	$\mathbf{1.36 \times 10^{1}}$	$\boldsymbol{\alpha}$**/SF**
287	**9.77**	$\mathbf{9.80 \times 10^{-1}}$	**1.88**	**6.73**	$\mathbf{2.36 \times 10^{1}}$	$\mathbf{1.23 \times 10^{5}}$	α
288	**9.65**	**1.66**	**4.36**	$\mathbf{1.57 \times 10^{1}}$	$\mathbf{1.03 \times 10^{1}}$	$\mathbf{8.98 \times 10^{2}}$	α
289	**9.61**	**2.57**	**7.11**	$\mathbf{2.01 \times 10^{1}}$	$\mathbf{6.78 \times 10^{1}}$	$\mathbf{8.84 \times 10^{6}}$	α
290	9.52	3.56	1.16×10^{1}	3.60×10^{1}	2.28×10^{1}	4.05×10^{4}	α
291	9.27	2.38×10^{1}	1.04×10^{2}	2.25×10^{2}	7.02×10^{2}	1.83×10^{8}	α
292	8.95	1.89×10^{2}	1.17×10^{3}	2.58×10^{3}	1.42×10^{3}	9.83×10^{5}	α
293	8.78	8.18×10^{2}	5.53×10^{3}	9.94×10^{3}	2.73×10^{4}	2.80×10^{9}	α
294	8.71	1.09×10^{3}	8.34×10^{3}	1.73×10^{4}	8.87×10^{3}	8.47×10^{5}	α
295	8.60	2.99×10^{3}	2.43×10^{4}	4.07×10^{4}	1.06×10^{5}	1.78×10^{9}	α
296	8.56	3.35×10^{3}	2.97×10^{4}	5.88×10^{4}	2.87×10^{4}	2.11×10^{5}	α/SF
297	8.35	2.19×10^{4}	2.19×10^{5}	3.38×10^{5}	8.15×10^{5}	1.10×10^{8}	α
298	8.27	4.08×10^{4}	3.99×10^{5}	7.09×10^{5}	3.16×10^{5}	3.23×10^{3}	α/SF
299	8.91	2.28×10^{2}	1.27×10^{3}	2.87×10^{3}	8.02×10^{3}	9.59×10^{2}	α/SF
300	9.56	1.67	4.12	1.94×10^{1}	1.17×10^{1}	1.03×10^{-2}	SF
301	9.58	1.69	2.48	1.59×10^{1}	5.15×10^{1}	3.77×10^{-4}	SF
302	9.28	9.82	1.59×10^{1}	1.34×10^{2}	7.58×10^{1}	4.76×10^{-9}	SF
303	8.95	1.38×10^{2}	2.25×10^{2}	1.78×10^{3}	4.96×10^{3}	6.56×10^{-10}	SF
304	8.43	9.38×10^{3}	1.81×10^{4}	1.38×10^{5}	6.25×10^{4}	9.03×10^{-15}	SF
305	8.28	4.53×10^{4}	6.42×10^{4}	4.73×10^{5}	1.09×10^{6}	4.02×10^{-15}	SF

nuclei ^{264}Sg and ^{260}Rf. The experiments have observed that ^{264}Sg has both α-decay and spontaneous fission, and ^{260}Rf undergoes spontaneous fission. Our calculations of the decay modes are in good agreement with the experiments. The first three nuclei on the decay chain of ^{280}Fl probably undergo α decay, and then terminate to ^{260}Rf.

The decay chain of ^{281}Fl is contaminated to the known nuclei ^{277}Cn, ^{273}Ds, ^{269}Hs, ^{265}Sg, and ^{261}Rf. The experiments show that these nuclei all undergo α decay, and ^{265}Sg have both α-decay and spontaneous fission. Our calculations reproduce the experiments very well.

The decay chain of ^{282}Fl is contaminated to the synthesized nuclei ^{270}Hs, ^{266}Sg, and ^{262}Rf. The experiments observed ^{262}Rf has spontaneous fission, which is consistent with our results. The nuclei ^{270}Hs was observed with α decay, and ^{266}Sg had both α-decay and spontaneous fission. Our calculations predict that ^{270}Hs would also have spontaneous fission.

The nuclei ^{273}Cn and ^{263}Rf on the decay chain of ^{283}Fl have been synthesized. The experiments have shown that ^{273}Cn has both α-decay and spontaneous fission, which is coincide with our calculations. The ^{263}Rf was observed undergo α decay. Our results predict it also has spontaneous fission.

Overall, since our calculations reproduce the experimental decay modes very well, we can confidently trust the predictions. The decay chains of ^{290,291}Fl are not contaminated to the known nuclei. We could predict that the sequence nuclei ^{286}Cn and ^{287}Cn may have both α-decay and spontaneous fission. Then the decay chains would end up to nuclei ^{282}Ds and ^{283}Ds with spontaneous fission.

We also find that our calculations for some elements, such as Rf and Sg, would not completely match the experiments. This is because these elements have complex decay modes. Some isotopes of Rf and Sg would have α decay, spontaneous fission, as well as electron capture (EC) [42]. We need to deal with more kinds of decay processes for these elements in the future works.

IV. CONCLUSIONS

The shell correction has been included in the GLDM. We employ the GLDM with and without shell correction, the UFM, the Royer's formula, and the UDL to calculate the α-decay half-lives of $^{285-289}$Fl decay chains by inputting the experimental Q_α. The calculations are in good agreement with the experiments, which means the methods we used could reproduce α-decay half-lives well. It is shown that the effect of shell correction would be obvious for

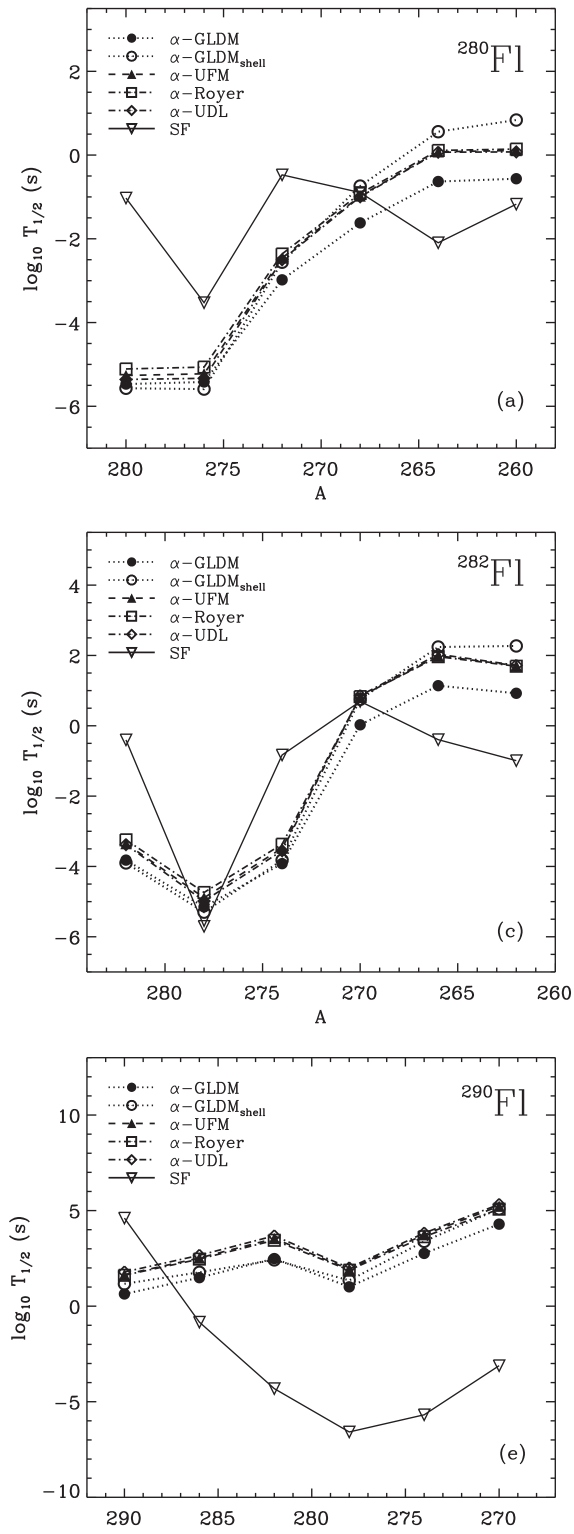

FIG. 3. The predicted decay chains of $^{280-283,290,291}$Fl isotopes.

the nuclei near the magic or sub-magic numbers. The shell correction would improve the calculation precision of the GLDM.

To calculate the α-decay properties of $^{280-305}$Fl isotopes, we adopt the FRDM and the newest WS4 model to extract Q_α values. The α-decay half-lives of $^{292-298}$Fl isotopes would be increased after considering the shell correction, where the corresponding shell correction energies show obvious valleys. The α-decay half-lives and the shell correction energies show that ^{298}Fl is a shell closure, and ^{292}Fl is a subshell closure.

To present the decay modes of $^{280-305}$Fl isotopes, we calculate the SF half-lives by a generalized Swiateckis formula including shell correction energies. The calculations of the known $^{284-289}$Fl isotopes match the experiments well. Besides, we predict that ^{284}Fl might have a branch of α decay, ^{286}Fl may also undergo spontaneous fission. Our calculations show that the decay modes of the unknown isotopes $^{280-283,290-295,297}$Fl nuclei are α decay. The $^{300-305}$Fl isotopes undergo spontaneous fission. The nuclei ^{296,298,299}Fl have both α-decay and spontaneous fission.

We also predict the decay-chains of $^{280-283,290,291}$Fl. The $^{280-283}$Fl decay-chains are contaminated to the known nuclei. Our calculations for these nuclei are in good agreement with the experiments. Thus we would predict the decay modes for the unknown nuclei and the ^{290,291}Fl decay chains.

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