# Vibrational-$\gamma$ bands in even $^{104-118}$Pd isotopes

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The structure of the vibrational-$\gamma$ bands in even $^{104-118}$Pd isotopes has been investigated in the framework of the IBA-2 model. A detailed comparison of experimental and predicted values of the available spectroscopic data has been performed. The role played by states of mixed symmetry character in the proton and neutron degrees of freedom in reproducing the properties of the vibrational-$\gamma$ bands has been highlighted.

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## I. INTRODUCTION

In recent years measurements concerning even-even nuclei of the $A \simeq 100$ mass region have led to the identification of several well-developed vibrational-$\gamma$ bands. These bands, built on the $2_2^+$ state, have even- and odd-spin components, which may show the peculiar energy staggering characterizing the $\gamma$ bands in rotational nuclei.

In even palladium ($Z = 46$) isotopes, which belong to a vibrational to $\gamma$-soft transitional region [1], such bands have been clearly identified in several isotopes [2]. Their knowledge is noticeably improved in recent years, mainly because of the possibility of populating some of these isotopes via spontaneous and induced fission and of using large multidetector arrays (see, e.g., Refs. [3–10]). This has lead to the identification of new members of the vibrational-$\gamma$ bands in several isotopes, and, in some cases, to highlight the existence of whole bands not previously observed. Indeed, for example, in $^{112-116}$Pd isotopes the bands have been extended from the $4_2^+$ level up to the $13_1^+$, $15_1^+$, and $13_1^+$ state, respectively. At present, the information on even $^{104-118}$Pd is sufficiently detailed to make it possible to perform an in-depth experiment-theory comparison. However, in several cases the vibrational-$\gamma$ bands in these isotopes have been investigated only to extract information on the nuclear deformation from the odd-even-spin energy staggering, whereas comparisons between the observed decay schemes and the ones predicted by current models have been extremely limited.

Aim of the present work is to investigate to what extent the IBA-2 model (the version of the IBA model [11] which distinguishes between proton- and neutron-bosons) can describe the structure of the vibrational-$\gamma$ bands in even $^{104-118}$Pd isotopes, through an analysis where all the available spectroscopic data are taken into account. Particular attention is paid to the role played by states of mixed symmetry (MS) character (partly antisymmetric under the interchange of proton and neutron boson degrees of freedom [12–14]) in reproducing the properties of the vibrational-$\gamma$ band along the isotopic chain.

Over the years the importance of MS states in determining the structure of even-even nuclei has become increasingly evident even though, in many cases, the identification concerns only one or a few low-lying MS states in single nuclei belonging to different mass regions (see, e.g., the review by Pietralla *et al.* [15]).

The present work is part of phenomenological analyses intended to achieve a systematic identification of states of mixed symmetry character through the study of their properties along whole isotopic chains, in transitional and vibrational nuclei of the $A \simeq 80, 100$ mass region (see Refs. [16–19]). The aim of such a kind of study is to avoid possible wrong identifications by imposing the constraint that the properties of a given MS state smoothly change as a function of the neutron number, as required for states of collective nature. Nuclei having just one valence neutron boson are not considered in those analyses, as the presence of single-particle degrees of states can be particularly important, making it difficult to disentangle the collective properties. Another specific feature of the aforementioned analyses is the identification of states of predominant MS character not limited to the lowest-lying ones.

## II. IBA-2 ANALYSIS AND RESULTS

In the 1990s two IBA-2 analyses [17,20], concerning the spectroscopic properties known at that time of the even Pd chain, were carried out. Afterwards, exploiting the new available experimental data, the study of Ref. [17] was extended to investigate the evolution of the ground state (g.s.) band in the heavier isotopes of the chain [19]. In the analyses of Refs. [17,19] a satisfactory description of the properties of the states of collective nature was achieved.

In the present study of the vibrational-$\gamma$ bands in the even $^{104-118}$Pd isotopes, excitation energies, quadrupole moments, $B(M1)$- and $B(E2)$-reduced transition probabilities, branchings, and mixing ratios are considered. The analysis has been performed by using the Hamiltonian of Refs. [17,19]:

$$H = \varepsilon \left(\hat{n}_{d_\pi} + \hat{n}_{d_\nu}\right) + \kappa\, \hat{Q}_\pi[\chi_\pi] \cdot \hat{Q}_\nu[\chi_\nu] + w_{\pi\nu}\, \hat{L}_\pi \cdot \hat{L}_\nu + \hat{M}_{\pi\nu}[\xi_1, \xi_2, \xi_3]. \quad (1)$$

Here, the indexes $\pi$ and $\nu$ refer to proton- and neutron-bosons, respectively. In the first term, $\hat{n}_d$ ($\hat{n}_d = \hat{n}_{d_\pi} + \hat{n}_{d_\nu}$) is the $d$-boson number operator. The second and third terms represent the quadrupole and dipole interactions. The Majorana

TABLE I. Hamiltonian parameters used in the IBA-2 calculations. The values are taken from Ref. [17] for $^{104-112}$Pd and from Ref. [19] for $^{114-118}$Pd. All parameters are in MeV, except $\chi_\nu$ (dimensionless). The value of the parameter $\chi_\pi$ (dimensionless), kept fixed along the isotopic chain, is −0.90.

| $A$ | $\varepsilon$ | $\kappa$ | $\chi_\nu$ | $w_{\pi,\nu}$ | $\xi_2$ | $\xi_3$ |
|---|---|---|---|---|---|---|
| 104 | 0.800 | −0.08 | −0.65 | 0.030 | 0.24 | −0.28 |
| 106 | 0.741 | −0.08 | −0.55 | 0.030 | 0.20 | −0.25 |
| 108 | 0.678 | −0.08 | −0.50 | 0.040 | 0.12 | −0.25 |
| 110 | 0.624 | −0.08 | −0.40 | 0.050 | 0.11 | −0.20 |
| 112 | 0.604 | −0.10 | 0.10 | 0.060 | 0.00 | −0.19 |
| 114 | 0.547 | −0.10 | 0.20 | 0.060 | 0.03 | −0.20 |
| 116 | 0.550 | −0.10 | 0.20 | 0.060 | 0.07 | −0.21 |
| 118 | 0.580 | −0.095 | 0.20 | 0.060 | 0.12 | −0.25 |

operator, $\hat{M}_{\pi\nu}[\xi_1, \xi_2, \xi_3]$, properly accounts for MS states and is responsible for their location with respect to the FS states. The parameters $\xi_1, \xi_2, \xi_3$ affect only the excitation energy of states having MS components.

The $\hat{T}(E2)$ and $\hat{T}(M1)$ operators utilized to study the electromagnetic properties of Pd isotopes have the standard expressions [11]

$$\hat{T}(E2) \equiv e_\nu\, \hat{T}_\nu(E2) + e_\pi\, \hat{T}_\pi(E2), \tag{2}$$

$$\hat{T}(M1) \equiv g_\nu\, \hat{T}_\nu(M1) + g_\pi\, \hat{T}_\pi(M1), \tag{3}$$

where $e_\nu, e_\pi$ and $g_\nu, g_\pi$ are the effective quadrupole charges and gyromagnetic ratios, respectively

Because of the form of the transition operators, $E2$ and $M1$ transitions obey the selection rules $\Delta n_d = 0, \pm 1$ and $\Delta n_d = 0$, respectively.

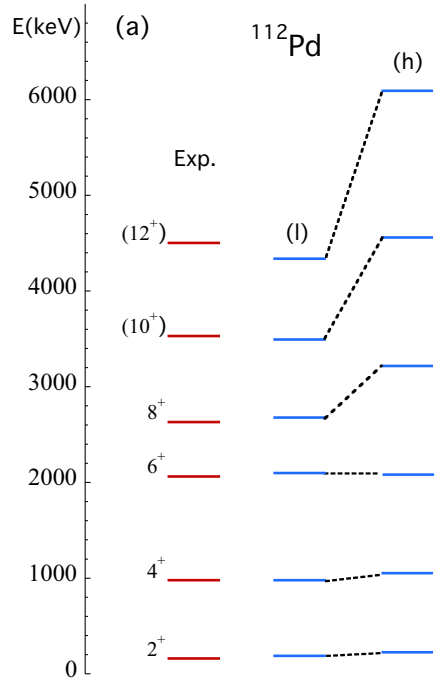
Excitation energies and electromagnetic (e.m.) properties of even $^{100-116}$Pd isotopes have been investigated in Ref. [17], keeping fixed six ($\chi_\pi, \xi_1, \epsilon_\pi, \epsilon_\nu, g_\pi, g_\nu$) out of the 12 model parameters. As to the other ones, the criteria adopted for their choice, with particular attention to $\xi_2$ and $\xi_3$ Majorana parameters, are described in details in Sec. III of that paper. In the subsequent study of the g.s. bands in $^{110-118}$Pd [19] it is explained how the parameters have been fixed in $^{118}$Pd and the reasons that have led to a slight change of the $\xi_2$ and $\xi_3$ values for $^{114,116}$Pd isotopes, with respect to those adopted in Ref. [17]. In both works the $\xi_1$ Majorana parameter has been arbitrarily kept fixed to 1 MeV. This value is sufficiently high to move the lowest $1^+$ state at an energy higher than 2 MeV, which is justified by the absence of any observed $1^+$ state below this excitation energy in even Pd isotopes. It was also checked that the calculated excitation energies and e.m. properties of the levels studied in the two papers mentioned above were quite insensitive to the value of $\xi_1$. Such a test has been repeated in the present work, finding that also the properties of the vibrational-$\gamma$ bands under study do not change when the value of this parameter is varied over a large range of positive values centered around 1 MeV.

The values of the parameters varying as a function of the mass number, $A$, are reported in Table I. It is seen that they change smoothly along the isotopic chain, which is just what is to be expected when the states under study have a collective structure.

The Hamiltonian has been diagonalized in the $U_{\pi,\nu}(5)$ basis, using the NPBOS code [21]. In the $U_{\pi,\nu}(5)$ limit of the IBA-2 model $F$-spin and $d$-boson number are good quantum numbers. The $F$-spin characterizes the symmetry of a state in the proton and neutron degrees of freedom. Fully symmetric states are completely symmetric with respect to the exchange of any two bosons and have $F = F_{\max} = N/2$ (where $N$ is the total boson number). They correspond to the states predicted by the IBA-1 model. Mixed symmetry states contain also antisymmetric boson pairs. Their $F$-spin can assume the values $F = F_{\max} - 1$, $F_{\max} - 2$..., down to the minimum value $F_{\min} = 1/2\,|N_\pi - N_\nu|$. As to the $F$-spin selection rule, both $E2$ and $M1$ transitions have to satisfy the $\Delta F = 0, \pm 1$ constraint and transitions between FS states are forbidden [22,23]. This last property provides one of the most important signature in the search of states of MS character. Another important signature is related to the possibility of reproducing the excitation energies of possible MS candidates only by a proper choice of the Majorana parameters. An example is shown in Fig. 1, where the experimental excitation energies of the even- and odd-spin members of the vibrational-$\gamma$ band in $^{112}$Pd are compared with those calculated using for $\xi_2$ and $\xi_3$ values sufficiently high to force an almost pure FS character of the states [column (h)] and then reducing their values to those reported in Table I [column (l)]. It is seen that in case (h) no satisfactory match to the experimental excitation energies for most of the states can be obtained. On lowering the values of the Majorana parameters, a subset of the levels decrease their energies so as to match the experimental values for the $\xi_2$ and $\xi_3$ values given in Table I. This gives a clue for the presence of important MS components in the structure of the corresponding experimental states.

In the even $^{104-118}$Pd isotopes, which have $N_\pi = 2$ and $N_\nu$ in the 2−8 range, the $F$-spin can assume the values $\mathrm{F}_{\max}$, $\mathrm{F}_{\max} - 1$, and $\mathrm{F}_{\max} - 2$. In a realistic Hamiltonian, as that of Eq. (1), the eigenstates can have more $F$-spin components. The NPBOS code [21] gives, as outputs, the $F$-spin and $n_d$ components of each state.

Normally, the experimental even-spin states of the vibrational-$\gamma$ band are yrare states, i.e., their spin $J_i$ has an ordinal index equal to 2, while the odd-spin states are yrast states ($i = 1$); a stretched separate cascade connects the even-spin as well as the odd-spin states. In most cases, the corresponding predicted states have the same properties, so that the even-spin states have index $i = 2$ and those associated to the odd-spin states are yrast states. However, in some cases, a different association has been found necessary, as in the following example, which concerns the odd-spin levels of $^{114}$Pd (see Fig. 2). The calculated states corresponding to the $11_1^+$, $13_1^+$, and $15_1^+$ experimental levels have been identified as the $11_2^+$, $13_2^+$, and $15_2^+$ states. Indeed, their excitation energies match quite well the experimental values. In addition, they are part of a strongly connected sequence of states, as revealed by the calculated $B(E2)$ values, reported for each transition in the figure. This is at variance with the properties of the calculated $11_1^+$, $13_1^+$, and $15_1^+$ yrast states.



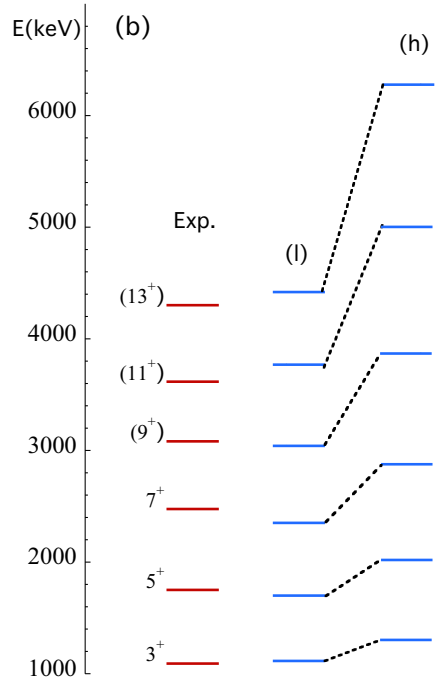




FIG. 1. The experimental excitation energies of the even-spin [panel (a)] and odd-spin [panel (b)] members of the vibrational-$\gamma$ band in $^{112}$Pd are compared to those calculated with the parameters of Table I [column (l)] and keeping the values of the Majorana parameters at 1 MeV [column (h)].

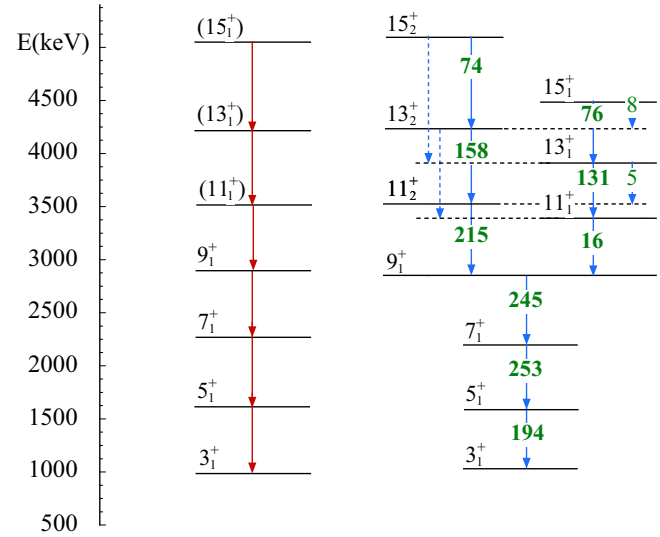



FIG. 2. Comparison of the experimental (left side) decay of the odd-spin band in $^{114}$Pd with the IBA-2 model predictions (right side). The calculated $B(E2)$ values, reported on each transition, are given in $10^{-3}e^2$ b$^2$ units. Vertical dashed lines indicate transitions of negligible intensity.

To be able to perform the theory-experiment comparison it is necessary to clarify particular issues pertinent to some experimental data of specific Pd isotopes.

The studies of Refs. [24,25] on $^{104}$Pd have provided new information compared to that given in NDS [26]. As to the vibrational-$\gamma$ band, Bellizzi *et al.* [24] have assigned spin 5 to the positive-parity 2445 keV level. Sohler *et al.* [25] have identified the $6^+$, $8^+$, and $10^+$ states of the even-spin band at 2678, 3593, and 4619 keV, respectively, and proposed to extend the odd-spin band up to the $9^+$ state. Their findings are based on the results of DCO (directional correlation of oriented nuclei) and polarization measurements as well as on the observed decay pattern and on its similarity to that of $^{102}$Ru. However, NDS [26] had previously reported $J^\pi = 4^+$ for the 2678 keV state. Such an assignment was based on of the observation of the transition de-exciting the 2678 keV state to the $4_1^+$ state in both $\beta^+$/EC decays of the ground ($J^\pi = 5^+$) and metastable ($J^\pi = 2^+$) state of $^{104}$Ag. As a consequence of the uncertainty on the spin of this state, the spin assignments to the states having $J \geqslant 6$, made in Ref. [25], are also doubtful. Additional doubts on the identification of the $J \geqslant 6$ members of the band arise from the preferential decay of the $7_1^+$ state to the $6_1^+$ state, compared to that to the $5_1^+$ state, (intensity ratio $7_1^+ \rightarrow 6_1^+/7_1^+ \rightarrow 5_1^+ = 2$), which is at variance with what observed in the well developed odd-spin bands in $^{112-116}$Pd.

$^{106}$Pd is the only isotope in the even Pd chain where the gyromagnetic factor ($g$) of a state ($2_2^+$) belonging to the vibrational-$\gamma$ band has been measured. The $g(2_2^+)$ value has been calculated in an IBA-2 study of the $g$-factors in even Pd isotopes [28], performed using the parameters given in Table I. The predicted value ($0.35\,\mu_N$) matches well the experimental one ($0.34(4)\ \mu_N$ [29–31]). This is of particular interest for the identification of MS states. Indeed, as mentioned before, MS states are characterized by the possibility of decaying through $M1$ transitions, the strength of which depend on the same parameters $g_\pi$ and $g_\nu$ that determine the $g$-factors. In this isotope, He *et al.* [32] propose the 2757 keV level as the $6^+$ member of the vibrational-$\gamma$ band. However, the definite

TABLE II. The values of $B(M1)$, $B(E2)$ reduced transition strengths, $\delta(E2/M1)$ mixing ratios, and $Q(J)$ quadrupole moments (calculated using $e_\pi = 0.15$, $e_\nu = 0.095$ $e$ b effective charges and $g_\pi = 0.58$, $g_\nu = 0.21$ $\mu_N$ gyromagnetic ratios of Ref. [17]) are compared to the experimental ones. Energies of levels and $\gamma$ transitions are given in keV, $B(M1)$ in $\mu_N^2$, $B(E2)$ in $e^2$ $b^2$, and quadrupole moment in $eb$. The experimental $B(E2)$ values of the $3_1^+ \to 4_1^+$ and $4_2^+ \to 3_1^+$ transitions in $^{106}$Pd have been evaluated assuming pure $E2$ multipolarity [33].

| $A$ | $E_{\rm lev}$ | $J_i^\pi$ | $J_f^\pi$ | $E_\gamma$ | $B(M1)^{\rm exp}$ | $B(M1)^{\rm calc}$ | $B(E2)^{\rm exp}$ | $B(E2)^{\rm calc}$ | $\delta^{\rm exp}$ | $\delta^{\rm calc}$ | $Q^{\rm exp}$ | $Q^{\rm calc}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 104 | 556 | $2_1^+$ | $2_1^+$ | | | | | | | | $-0.47(10)$ | $-0.37$ |
| | | $2_1^+$ | $0_1^+$ | 556 | | | 0.109(6) | 0.096 | | | | |
| | 1324 | $4_1^+$ | $2_1^+$ | 768 | | | 0.145(21) | 0.153 | | | | |
| | 1342 | $2_2^+$ | $2_1^+$ | 786 | 0.0011(19) | 0.0002 | 0.065(5) | 0.112 | $-4.8(42)$ | $-16$ | | |
| | | $2_2^+$ | $0_1^+$ | 1342 | | | 0.0038(3) | 0.0010 | | | | |
| | 1821 | $3_1^+$ | $2_2^+$ | 479 | | | | | $<$ **1.2**[a] | $-0.67$ | | |
| | | $3_1^+$ | $4_1^+$ | 498 | | | | | $M1$, $E2$ | $-0.4$ | | |
| | | $3_1^+$ | $2_1^+$ | 1265 | | | | | 0.23(7) | $-1.44$ | | |
| | 2082 | $4_2^+$ | $2_2^+$ | 741 | | | 0.077(38)[b] | 0.081 | | | | |
| | | $4_2^+$ | $4_1^+$ | 759 | **0.016(16)** | 0.0003 | 0.030(30) | 0.057 | $-0.84(24)$ | $-8$ | | |
| | | $4_2^+$ | $2_1^+$ | 1527 | | | 0.002(2) | 0.0002 | | | | |
| | 2444 | **5$_1^+$** [a] | $4_2^+$ | 362 | | | | | **M1** [a] | $-0.19$ | | |
| | | **5$_1^+$** | $4_1^+$ | 1120 | | | | | **$-0.35(15)$ or $-3.9^{+1.7a}_{-4.7}$** | $-0.34$ | | |
| 106 | 512 | $2_1^+$ | $2_1^+$ | | | | | | | | $-0.55(5)$[d] | $-0.42$ |
| | | $2_1^+$ | $0_1^+$ | 512 | | | 0.134(5) | 0.121 | | | | |
| | 1128 | $2_2^+$ | $2_2^+$ | | | | | | | | $+0.39^{+0.05}_{-0.04}$ [d] | $+0.28$ |
| | | $2_2^+$ | $2_1^+$ | 616 | 0.0004(2) | 0.0002 | 0.134(12) | 0.141 | $-9.4(20)$ | $-13$ | | |
| | | $2_2^+$ | $0_1^+$ | 1128 | | | 0.0036(3) | 0.0016 | | | | |
| | 1229 | $4_1^+$ | $4_1^+$ | | | | | | | | $-0.77^{+0.05}_{-0.08}$ [d] | $-0.60$ |
| | | $4_1^+$ | $2_2^+$ | 101 | | | $0.002^{+0.021}_{-0.001}$ | 0.003 | | | | |
| | | $4_1^+$ | $2_1^+$ | 717 | | | 0.231(33) | 0.196 | | | | |
| | 1557 | $3_1^+$ | $4_1^+$ | 328 | | | **$0.018^{+0.003}_{-0.018}$**[c] | 0.028 | E2(+M1) | $-0.69$ | | |
| | | $3_1^+$ | $2_2^+$ | 430 | **0.0002(1)**[e] | 0.0058 | 0.049(9)[d] | 0.091 | $-7.9(8)$ | $-1.4$ | | |
| | | $3_1^+$ | $2_1^+$ | 1046 | **0.00011(4)**[c] | 0.00042 | **0.0013(2)**[d] | 0.0027 | $-3.8(4)$ | $-2.2$ | | |
| | 1932 | $4_2^+$ | $4_2^+$ | | | | | | | | $-0.23^{+0.14}_{-0.40}$ [d] | $-0.03$ |
| | | $4_2^+$ | $3_1^+$ | 374 | | | **$0.100^{+0.003}_{-0.010}$** [c] | 0.014 | **M1(+E2)** | 6.3 | | |
| | | $4_2^+$ | $4_1^+$ | 703 | 0.0039(7) | 0.0003 | 0.064(9) | 0.075 | $-2.30(2)$ | $-9.9$ | | |
| | | $4_2^+$ | $2_2^+$ | 804 | | | 0.106(18) | 0.108 | | | | |
| | | $4_2^+$ | $2_1^+$ | 1419 | | | $(2^{+2}_{-1}) \times 10^{-5}$ | $0.3 \times 10^{-5}$ | | | | |
| | 2077 | $6_1^+$ | $6_1^+$ | | | | | | | | $-1.02^{+0.16}_{-0.09}$ [d] | $-0.70$ |
| | | $6_1^+$ | $4_1^+$ | 848 | | | 0.267(27) | 0.229 | | | | |
| 108 | 434 | $2_1^+$ | $2_1^+$ | | | | | | | | $-0.58(4)$ | $-0.52$ |
| | | $2_1^+$ | $0_1^+$ | 434 | | | 0.157(5) | 0.153 | | | | |
| | 931 | $2_2^+$ | $2_2^+$ | | | | | | | | $+0.55(6)$[c] | $+0.37$ |
| | | $2_2^+$ | $2_1^+$ | 497 | 0.0039(11) | 0.0004 | 0.224(19) | 0.161 | $-3.1(4)$ | $-8.3$ | | |
| | | $2_2^+$ | $0_1^+$ | 931 | | | 0.0026(3) | 0.0029 | | | | |
| | 1048 | $4_1^+$ | $4_1^+$ | | | | | | | | $-0.59(8)$[d] | $-0.70$ |
| | | $4_1^+$ | $2_2^+$ | 117 | | | 0.0038(4) | 0.0017 | | | | |
| | | $4_1^+$ | $2_1^+$ | 614 | | | 0.236(28) | 0.247 | | | | |
| | 1335 | $3_1^+$ | $2_2^+$ | 901 | | | | | **$\leqslant -5$ or $\leqslant 0.2$** | $-1.7$ | | |
| | 1624 | $4_2^+$ [d] | $4_2^+$ | | | | | | | | $-0.015^{+0.06}_{-0.10}$ [d] | $+0.007$ |
| | | $4_2^+$ | $4_1^+$ | 577 | | | 0.093(22) | 0.088 | | | | |
| | | $4_2^+$ | $2_2^+$ | 694 | | | 0.168(34) | 0.135 | | | | |
| | | $4_2^+$ | $2_1^+$ | 1191 | | | 0.0004(4) | 0.0002 | | | | |
| | 1771 | $6_1^+$ | $6_1^+$ | | | | | | | | $-0.53(13)$[c] | $-0.78$ |
| | | $6_1^+$ | $4_1^+$ | 723 | | | 0.333(40) | 0.291 | | | | |
| | 2548 | $8_1^+$ | $8_1^+$ | | | | | | | | $-0.71^{+0.13}_{-0.26}$ [d] | $-0.85$ |
| | | | $6_1^+$ | 777 | | | 0.460(53) | 0.292 | | | | |
| 110 | 374 | $2_1^+$ | $2_1^+$ | | | | | | | | $-0.61(12)$[e] | $-0.61$ |
| | | $2_1^+$ | $0_1^+$ | 374 | | | 0.177( 3) | 0.191 | | | | |
| | 814 | $2_2^+$ | $2_2^+$ | | | | | | | | $+0.48(10)$[e] | $+0.48$ |
| | | $2_2^+$ | $2_1^+$ | 440 | | | 0.140(10) | 0.183 | $-4.6^{+19}_{-12}$ | $-8.6$ | | |
| | | $2_2^+$ | $0_1^+$ | 814 | | | 0.0024(3) | 0.0047 | | | | |

TABLE II. *(Continued.)*

| $A$ | $E_{\rm lev}$ | $J_i^\pi$ | $J_f^\pi$ | $E_\gamma$ | $B(M1)^{\rm exp}$ | $B(M1)^{\rm calc}$ | $B(E2)^{\rm exp}$ | $B(E2)^{\rm calc}$ | $\delta^{\rm exp}$ | $\delta^{\rm calc}$ | $Q^{\rm exp}$ | $Q^{\rm calc}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 921 | $4_1^+$ | $4_1^+$ | | | | | | | | $-1.21(48)$[e] | $-0.78$ |
| | | $4_1^+$ | $2_2^+$ | 107 | | | $0.029^{+0.013}_{-0.036}$ [e] | 0.001 | | | | |
| | | $4_1^+$ | $2_1^+$ | 547 | | | 0.287(22) | 0.305 | | | | |
| | 1212 | $(3_1^+)$ | $4_1^+$ | 292 | | | 0.039(16)[e] | 0.066 | | | | |
| | | $(3_1^+)$ | $2_2^+$ | 399 | | | 0.078(31)[e] | 0.220 | | | | |
| | | $(3_1^+)$ | $2_1^+$ | 838 | | | 0.0012(5)[e] | 0.008 | | | | |
| | 1398 | $4_2^+$ | $4_2^+$ | | | | | | | | $-0.007(3)$[e] | $+0.082$ |
| | | $4_2^+$ | $3_1^+$ | 186 | | | 0.028(11)[e] | 0.036 | | | | |
| | | $4_2^+$ | $4_1^+$ | 478 | | | 0.064(26)[e] | 0.105 | **[E2 + M1]** | $-6.6$ | | |
| | | $4_2^+$ | $2_2^+$ | 585 | | | 0.108(19) | 0.167 | | | | |
| | 1574 | $6_1^+$ | $6_1^+$ | | | | | | | | $-0.98(39)$[e] | $-0.85$ |
| | | $6_1^+$ | $4_2^+$ | 176[e] | | | 0.0003(13)[e] | 0.0003 | | | | |
| | | $6_1^+$ | $4_1^+$ | 653 | | | 0.344(35) | 0.361 | | | | |
| | 2092[e] | $6_2^+$ | $6_2^+$ | | | | | | | | $-0.77(31)$[e] | $+0.01$ |
| | | $6_2^+$ | $6_1^+$ | 519[e] | | | 0.012(23)[e] | 0.066 | | | | |
| | | $6_2^+$ | $4_2^+$ | 694[e] | | | 0.097(39)[e] | 0.224 | | | | |
| | | $6_2^+$ | $4_1^+$ | 1171[e] | | | 0.0001(19)[e] | 0.0001 | | | | |
| | 2296 | $8_1^+$ | $8_1^+$ | | | | | | | | $-1.03(41)$[e] | $-0.90$ |
| | | $8_1^+$ | $6_2^+$ | 204[e] | | | $0.018^{+0.019}_{-0.035}$ [e] | 0.001 | | | | |
| | | $8_1^+$ | $6_1^+$ | 722 | | | 0.451(182)[e] | 0.375 | | | | |
| | 3062 | $(8_2^+)$[e] | $6_2^+$ | | | | 0.285(114)[e] | 0.174 | | | | |
| 112 | 349 | $2_1^+$ | $0_1^+$ | 349 | | | **0.131(23)** | 0.233 | | | | |
| | 736 | $2_2^+$ | $2_1^+$ | 388 | | | | | $\mathbf{-4.7^{+1.7}_{-3.5}}$ | $-3.2$ | | |
| | 1097 | $3_1^+$ | $2_2^+$ | 360 | | | | | **M1 + E2** | $-2.5$ | | |
| | | $3_1^+$ | $2_1^+$ | 748 | | | | | **−1.7(1)** | $-2.0$ | | |
| 114 | 333 | $2_1^+$ | $0_1^+$ | 333 | | | **0.167(25)**[f] | 0.209 | | | | |
| | 695 | $2_2^+$ | $2_1^+$ | 362 | | | | | **−20**[g] | $-4$ | | |
| 116 | 340 | $2_1^+$ | $0_1^+$ | 340 | | | 0.116(34) | 0.177 | | | | |
| | 738 | $2_2^+$ | $2_1^+$ | 398 | | | | | **−20**[g] | $-4$ | | |
| | 1066 | $3_1^+$ | $2_2^+$ | 328 | | | | | **(M1 + E2)** | $-1.9$ | | |

[a]Reference [24].
[b]Reference [27].
[c]Reference [33].
[d]Deduced from Ref. [34].
[e]Reference [38].
[f]Reference [40].
[g]Reference [8].

$J^\pi = 5^+$ assignment to this state (based also on its observed decay to the $3^+$ state) [29] excludes such a possibility. However, Prados *et al.* [33] propose, for the same role, the new 2812 keV, $(6^+)$ level they observe in $(n, n'\gamma)$ reaction.

In $^{108}$Pd, Alcántara-Núñez *et al.* [35] add two new levels ($7^+$ and $10^+$) to the vibrational-$\gamma$ band reported by Lalkovski *et al.* [5] and find that the $10^+$ state has comparable branchings to the 2548 and 2954 keV, $8^+$ states. NDS [36] report in addition an $(8^+)$ state at 2397 keV. Such a level has been observed by Pohl *et al.* [37] in heavy-ion fusion-evaporation reactions. Through an accurate analysis of its decay these authors reach the conclusion that the $J^\pi = 8^+$ assignment to this state would be strongly questionable. In addition, such a level has not been observed in the experimental works reported in Refs. [5,35]. The 2548 keV and 2954 keV levels are therefore identified as the $8_1^+$ and $8_2^+$ states, respectively.

The possible existence of a 1579 keV level in $^{110}$Pd, suggested by Lalkovski *et al.* [5] as the $5^+$ member of the vibrational-$\gamma$ band, has been questioned by Banerjee *et al.* [9], who populated this nucleus in heavy-ion induced fusion-fission reaction without observing this level. Both authors of Refs. [5,9] propose the 1987 and 2651 keV levels as the $6^+$ and $8^+$ states of the vibrational-$\gamma$ band. However, in measurements of polarized deuteron inelastic scattering, Hertenberger *et al.* [38] assigned $J^\pi = 4^+$ to the 1987 keV level. This result also excludes $J = 8$ for the 2651 keV level. Lee *et al.* [39] were the first to hypothesize that the 2092 keV level, populated in multiple-phonon Coulomb excitation, is the $6_2^+$ state. Such a possibility was supported by the results obtained by Hertenberger *et al.* [38], who also proposed the 3062 keV level as the $8_2^+$ state. The properties of the calculated $6_2^+$ and $8_2^+$ states in $^{110}$Pd have been compared to those of these states for a possible association.

The information on the vibrational-$\gamma$ bands in even $^{112-118}$Pd isotopes has been widely extended by Luo *et al.* [8], who populated these isotopes through the $^{252}$Cf fission. By exploiting the high statistics obtained in multiple-$\gamma$ coincidences and $\gamma$-$\gamma$ angular correlations measurements they

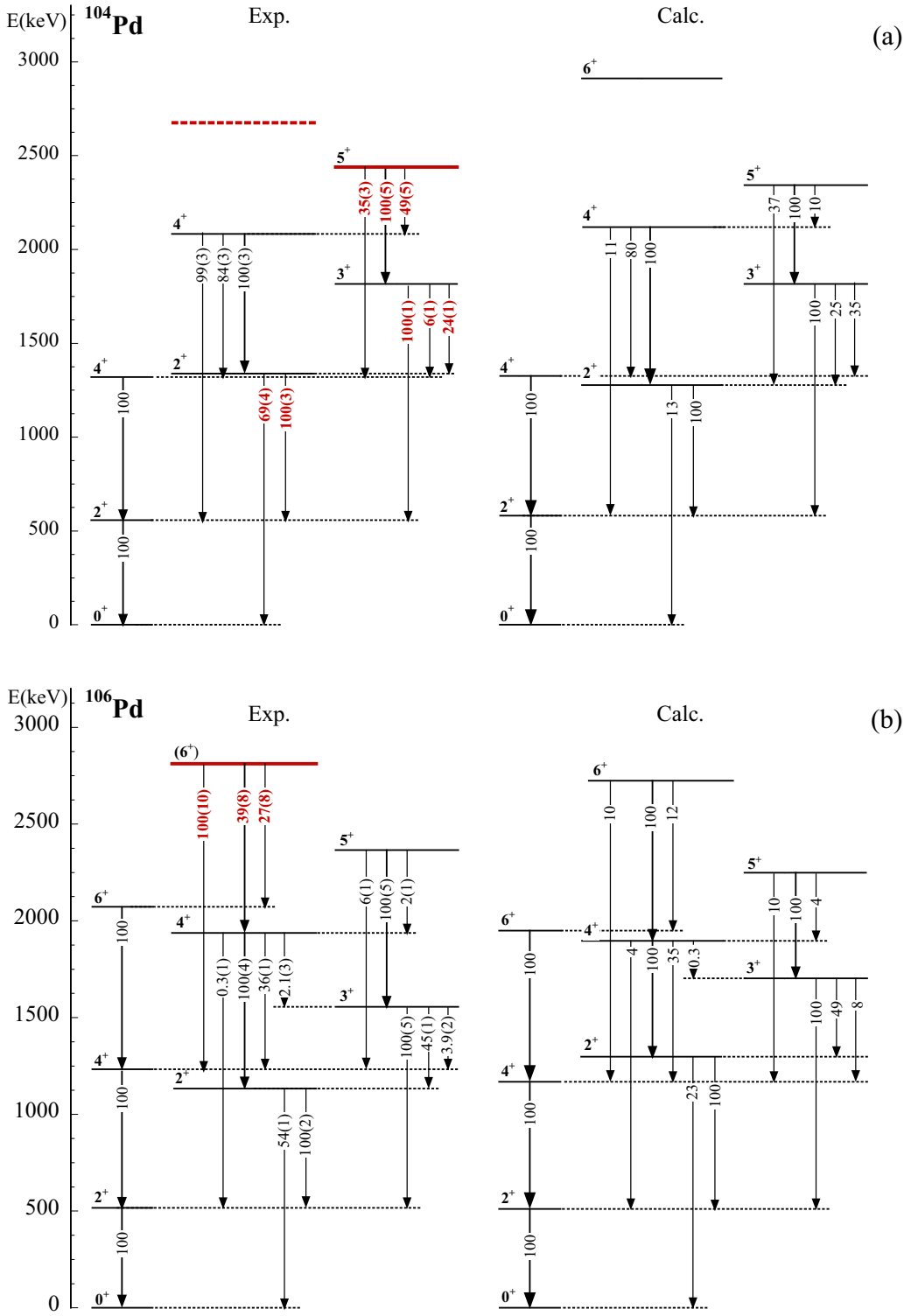



FIG. 3. Comparison of experimental and calculated decay pattern of the vibrational-$\gamma$ bands in $^{104}$Pd [panel (a)] and $^{106}$Pd [panel (b)]. New experimental levels are represented by thick lines; new data on the branching ratios are given in bold face. In the upper panel a dashed line represents the experimental 2678 keV, positive parity state, to which it has been assigned spin $J = 6$ in Ref. [25] but $J = 4$ in Ref. [26] (see text for details).

have been able to observe very weak linking among the lower members of the $\gamma$-bands and to assign spin/parities to several levels. However, one has to note that in $^{114}$Pd, these authors report a 5011.9 keV, $(15^+)$ level (at exactly the same energy as the $(16^+)$ level), which de-excites to the $(13^+)$ level via a 844 keV transition. By adding the energy of this transition to that of the $(13^+)$ level it turns out that the $(15^+)$ level has an excitation energy of 5050 keV. In $^{116}$Pd they

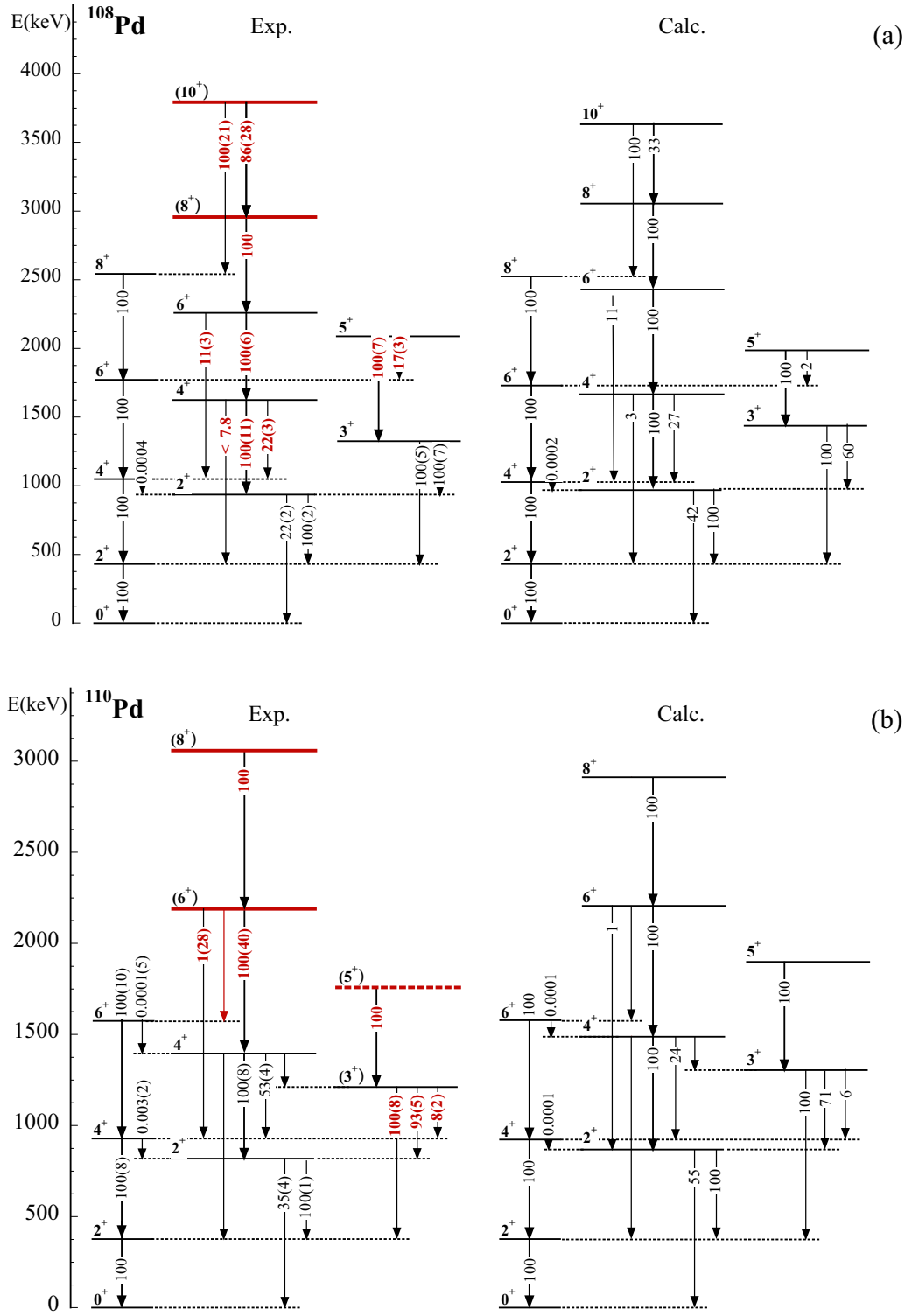



FIG. 4. As Fig. 3, for $^{108}$Pd and $^{110}$Pd. In $^{110}$Pd the experimental $5^+$ state, whose existence is doubtful (see text), is represented by a dashed line.

mention a 352.7 keV transition, de-exciting the $6_2^+$ state, but no final level differing by such an energy from the $6_2^+$ state exists.

The whole set of the experimental data concerning the excitation energies and the e.m. properties of the states of the vibrational-gamma bands in even $^{104-118}$Pd isotopes is compared to the calculated ones in in Figs. 3–7 and in Table II. The data already considered in Ref. [17] are also included so that, to point out the information become available after the completion of that work, the new experimental data in Table II

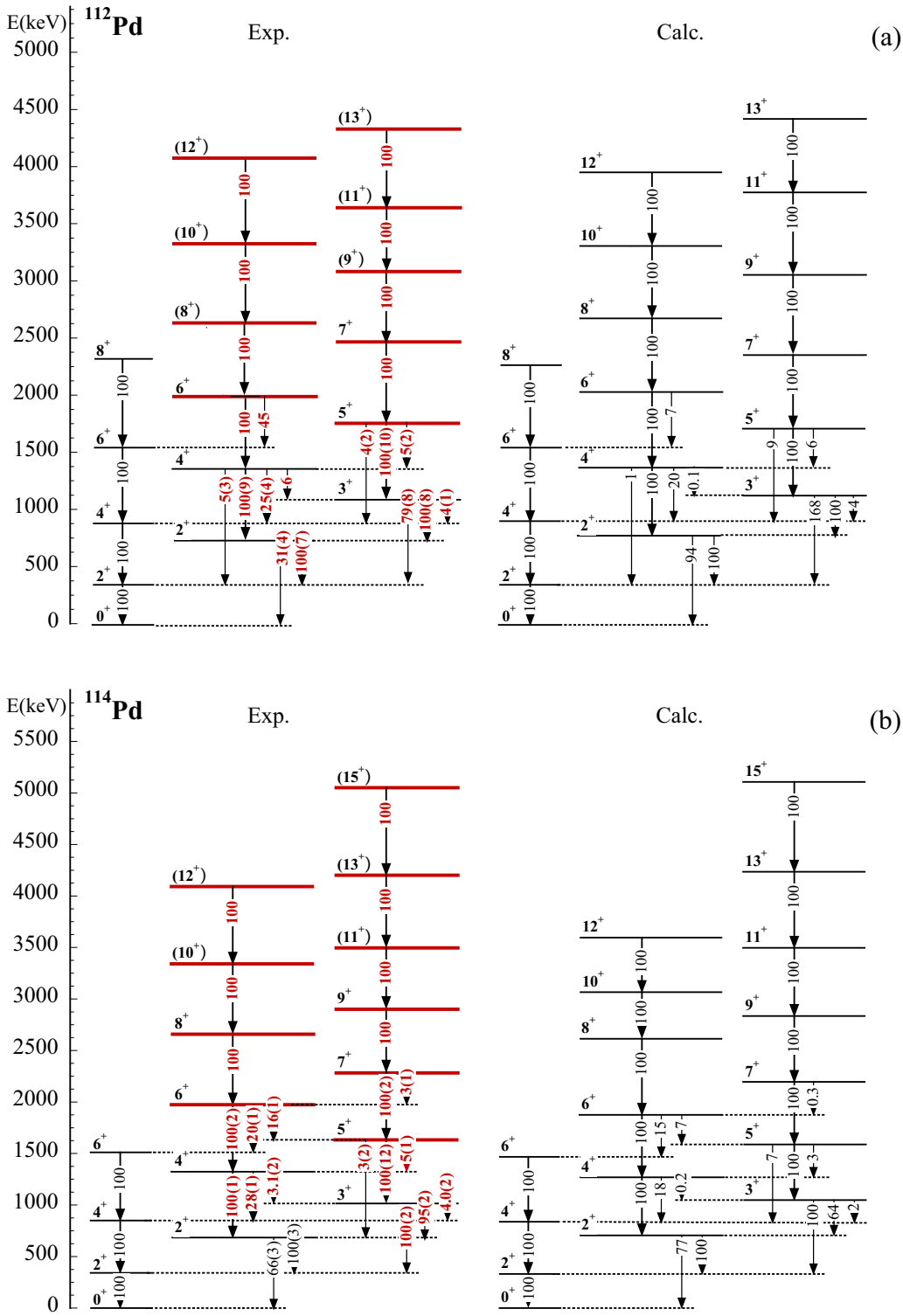



FIG. 5. As Fig. 3, for $^{112}$Pd and $^{114}$Pd. The relative intensities reported without errors in $^{112}$Pd and the excitation energies of the $12^+$ and $15^+$ states in $^{114}$Pd are taken from Ref. [8].

are reported in bold face character. In the figures newly found excited levels and branching ratios are reported as thick lines and in bold face character, respectively. All the experimental values are from Nuclear Data Sheets (NDS), unless noted otherwise. It is to remark that to associate a calculated state to an experimental one an agreement on the excitation energies within 10% has been always required.

The theory-experiment comparisons shown in Figs. 3–6 concern the decay patterns of the single isotopes, that in Fig. 7 the excitation energies of the vibrational-$\gamma$ bands

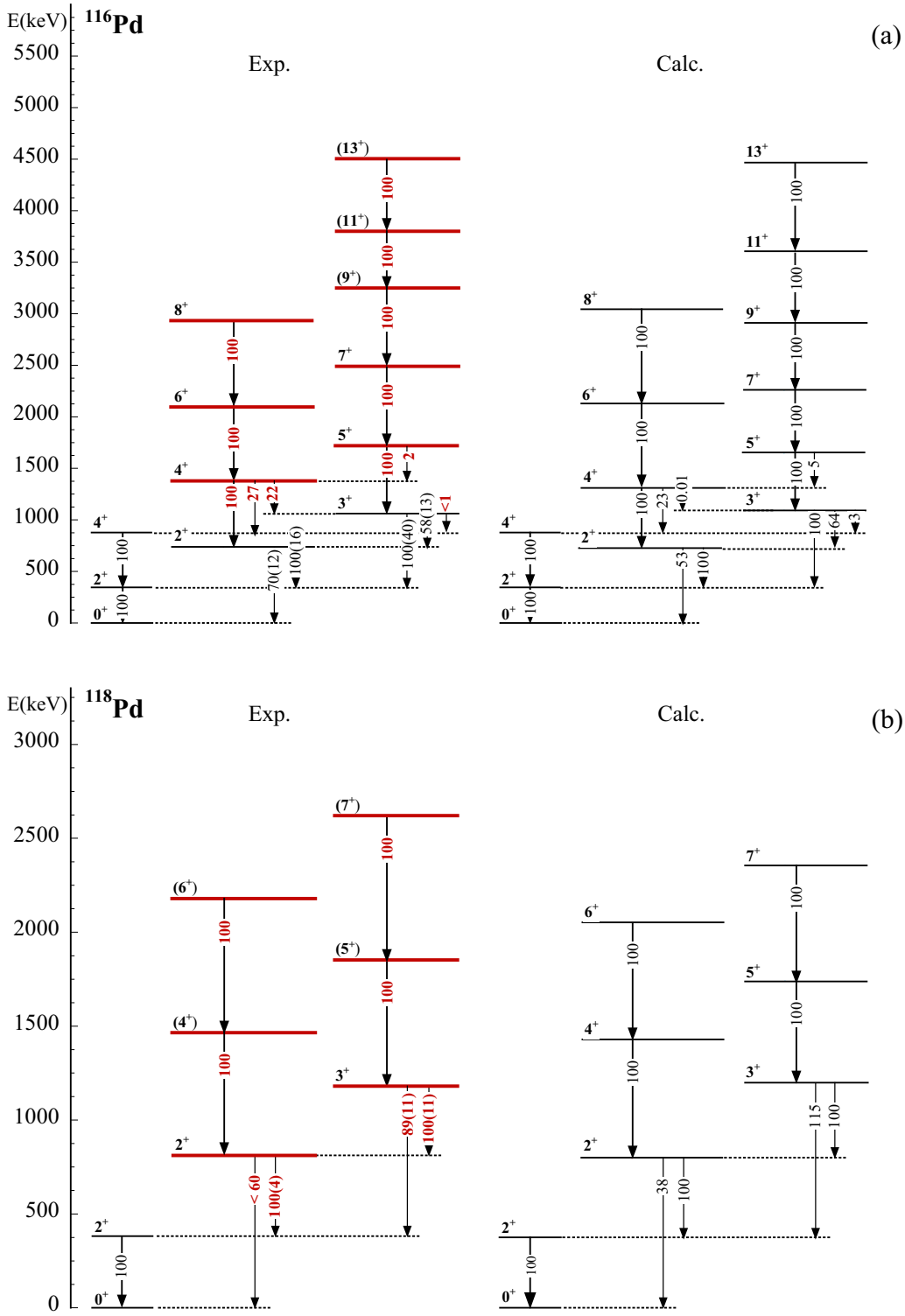



FIG. 6. As Fig. 3, for $^{116}$Pd and $^{118}$Pd. The experimental relative intensities of the $4_2^+ \rightarrow 3_1^+$ and $5_1^+ \rightarrow 4_2^+$ transitions in $^{116}$Pd are taken from Ref. [8].

as a function of the mass number. Experimental and predicted quadrupole moments, $E2$- and $M1$-reduced transition strengths and $\delta(E2/M1)$ mixing ratios are reported in Table II. For completeness, the data concerning the e.m. properties of the states of the g.s. band, fed by members of the vibrational-$\gamma$ band, are also reported.

Some remarks, useful to clarify the comparisons, are listed hereafter.

Because of the spin uncertainties mentioned above for the $J \geqslant 6$ states in $^{104}$Pd, the comparison reported in the upper panel of Fig. 3 is only up to the $5^+$ state. In the same figure, the experimental branching ratios of the $2_2^+$, $3_1^+$,

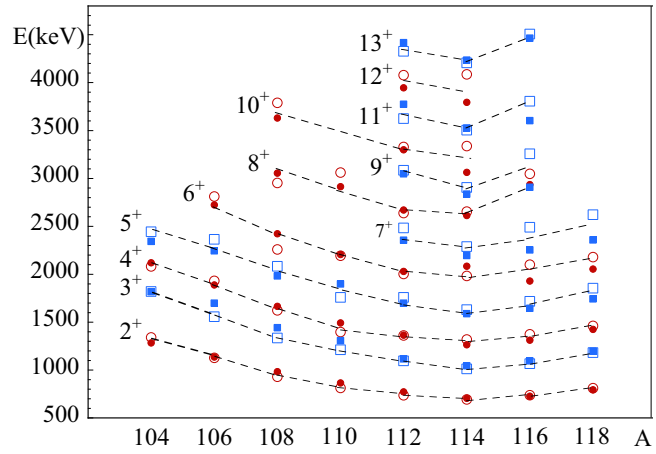



FIG. 7. Comparison of experimental (open symbols) and calculated (closed symbols) excitation energies of the even-spin yrare states (circles) and odd-spin yrast states (squares) of the vibrational-$\gamma$ bands in even $^{104-118}$Pd isotopes. The lines connecting the levels are drawn to guide the eye.

$4_2^+$, and $5_1^+$ levels are weighted averages of the values given in Refs. [24–26], when compatible, and just the averages for the branchings concerning the $2_2^+ \rightarrow 0_1^+$ and $4_2^+ \rightarrow 4_1^+$ transitions. The relative intensity of the $3_1^+ \rightarrow 4_1^+$ transition is taken from Ref. [24].

In the lower panel of Fig. 3, concerning $^{106}$Pd, the strongly preferential decay of the 2812 keV level (proposed as the $6^+$ member of the vibrational-$\gamma$ band in Ref. [33]) to the $4_1^+$ state is not reproduced by the model, which instead predicts a predominant decay of the $6_2^+$ state to the $4_2^+$ state.

The calculated $7^+$ level in $^{108}$Pd has an excitation energy 12% lower than that of the $7^+$ state, proposed by Alcántara-Núñez *et al.* [35] as the state of highest odd-spin of the vibrational-$\gamma$ band. On the basis of the adopted constraint (maximum 10% theory-experiment difference in the excitation energies) its association to the experimental state has been discarded, so that the $7^+$ state is not reported in the upper panel of Fig. 4.

The experimental quadrupole moments and $B(E2)$ transition strengths of $^{110}$Pd reported in Table II have been deduced from the $E2$ matrix elements given in Ref. [38]. The intensity ratios of the transitions de-exciting the $4_1^+$ state to the $2_1^+$ and $2_2^+$ states and the $6_1^+$ and the $6_2^+$ states to the $4_1^+$ and $4_2^+$ states have been deduced from the $B(E2)$ values shown in Table II. The possible presence of $M1$ components prevents the evaluation of the branchings of the $4_2^+ \rightarrow 3_1^+$ and $6_2^+ \rightarrow 6_1^+$ transitions, so that their relative intensities is not given in Fig. 4(b). In addition to the data reported by NDS [41], two new sets of data concerning the branching ratios of the states in $^{114}$Pd are available [8,10]. Those of Ref. [8] are given without errors. The values reported in the figure are the weighted average of the data of Refs. [10,41]. The branching ratio of the 2291 keV, $7^+$ state is from Ref. [10].

The calculated excitation energy of the $15^+$ state (6184 keV) in $^{116}$Pd turns out to be 16% higher than that of the experimental 5340 keV state, proposed as the $15^+$ state in Ref. [8], so that, also in this case, the possible association of the two states has been discarded.

## III. DISCUSSION

Several theoretical analyses have bee performed over the last years on the vibrational-$\gamma$ band in even Pd isotopes.

In $^{104}$Pd and $^{106}$Pd such a band has been interpreted by Sholer *et al.* [25] and He *et al.* [32], respectively, as due to excitations associated to a $\gamma$-soft deformation, on the basis of the observed odd-even spin energy staggering [42]. For $^{106}$Pd a different interpretation has been given by Prados-Estévez *et al.* [33], who suggest that this isotope has an axially asymmetric rotor character, with possibly some $\gamma$ softness. They attain such a conclusion by comparing the summed $E2$ strengths of the transitions populating the $0_2^+$, $2_2^+$, and $4_1^+$ states with different models and by arranging the experimental positive-parity states in bands, using $K$ as a convenient label. As to the IBA-2 model, these authors claim that "mixed-symmetry collectivity does not play a significant role at low energy in $^{106}$Pd, contrary to the implications of the calculations of Refs. [17,20]."

The vibrational-$\gamma$ bands in even $^{106-110}$Pd have been studied by Zajac *et al.* [43] within a microscopic approach, based on the general collective Bohr model, which includes the effect of coupling with the pairing vibrations. The calculated excitation energies of the states up to $6^+$ in $^{106,108}$Pd, $8^+$ in $^{110}$Pd and the $B(E2)$ transition strengths of the de-exciting transitions are compared to the experimental data, achieving a satisfactory agreement.

Lalkovski *et al.* [5] find that the observed odd-even staggering in $^{108,110}$Pd supports the theoretical predictions for $\gamma$ instability of their shapes. Alcántara-Núñez *et al.* [35] identify the vibrational-$\gamma$ band in $^{108}$Pd through its similarity with the corresponding bands identified in $^{104-108}$Ru isotopes [44]. Banerjee *et al.* [9] remark that in $^{110}$Pd only the $3^+$ state has definite spin-parity assignment among the odd-spin states, so that more information is essential for a clear understanding of the underlying nuclear shape.

Gore *et al.* [6] analyze the energy level splitting of the even-odd spin states in even $^{112-116}$Pd. According to these authors, it shows rapid variations with spin and neutron number, so that, e.g., for explaining the staggering in $^{112}$Pd one would need the Wilets and Jean model [45] and for that in $^{114}$Pd the Davydov and Filippov model [46]. Luo *et al.* [8] identify the onset of wobbling motion in $^{114}$Pd and, probably, in $^{116}$Pd isotopes from the sign of the signature splitting in the observed quasi-$\gamma$ band. The triaxial projected shell model [47,48] calculations performed by Huang *et al.* [10] satisfactorily reproduce the excitation energies of the vibrational-$\gamma$ band in $^{114}$Pd.

From the theoretical analyses just summarized, it appears that no systematic and thorough study of the properties of the vibrational-$\gamma$-bands in even $^{104-118}$Pd isotopes has been yet carried out. Moreover, no mention is ever made of the presence of $M1$ components (as revealed by the measured mixing ratios) in transitions de-exciting states of the band. Indeed, the interpretation of such a presence provides a great challenge to the current nuclear models.

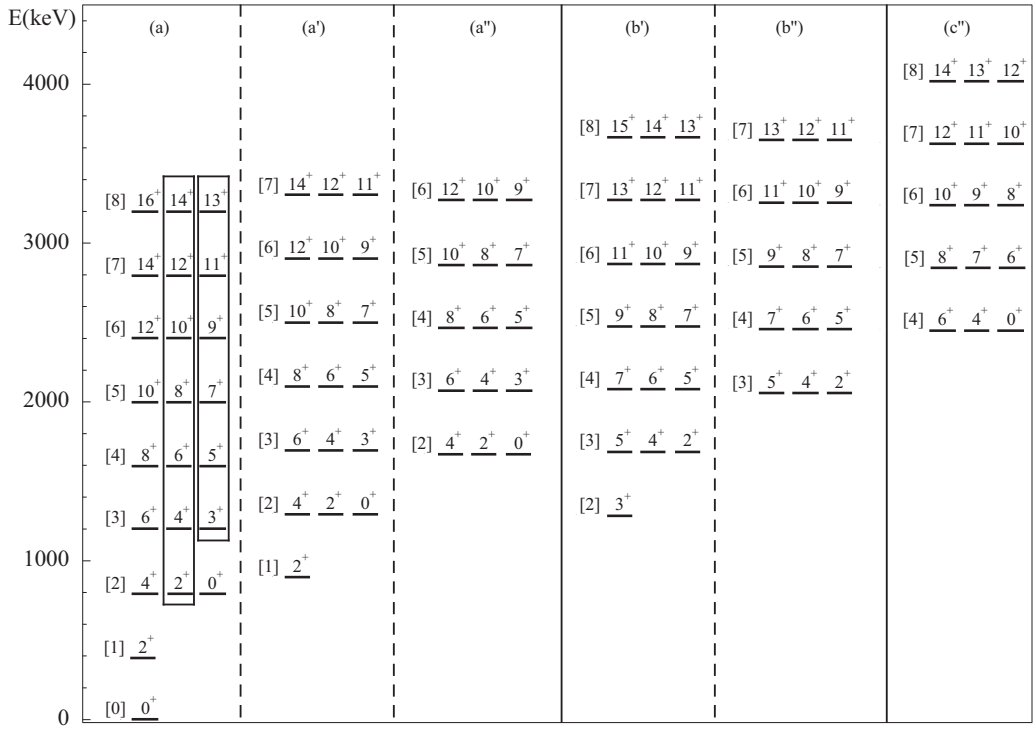



FIG. 8. Excitation energies of FS and MS states in the $U_{\pi\nu}(5)$ limit, for the particular case $N_\pi = 2$ and $N_\nu = 6$. States are separated in six groups, according to the different dependence of their excitation energies on the Majorana parameters (see, e.g., Ref. [51]). The $d$-boson number is reported in square brackets on the left of each $n_d$ multiplet.

In the present IBA-2 analysis, an attempt has been made of providing an interpretation, as extended as possible, of the large amount of available spectroscopic data concerning the vibrational-$\gamma$ bands in the even $^{104-118}$Pd isotopes. As a general comment on the results (Figs. 3–7 and Table II) one could say that, on the whole and without resorting to any parameter adjustment, the calculations are able to satisfactorily match the experimental data. As to some specific points, one can observe the following:

(i) The regular trend of the experimental states, as a function of $A$ (see Fig. 7), is well reproduced by the calculations (average difference of experimental and calculated excitation energies less than 1%).

(ii) The model correctly predicts the positive sign of $Q(2_2^+)$ [the only positive one among the measured $Q(J_i^+)$] and the small absolute value of $Q(4_2^+)$ in even $^{106-110}$Pd isotopes. Note also that there is a reasonable general agreement with the measured $B(E2)$ values, which are spread over several orders of magnitude.

(iii) Overall, the calculations match order of magnitude and sign of the experimental mixing ratios or predict the presence of M1 components in transitions where only the presence of a mixing of $M1$ and $E2$ multipolarities is reported in the literature. Since small changes in the percentage of MS basis states in the wave functions can strongly affect the $\delta$ values (see, e.g., Ref. [16]) such a result may be considered satisfactory.

(iv) The possibility of reproducing the branching ratios in the decay patterns reported in Figs. 3–6 is related, to a large extent, to the presence of important or predominant $M1$ component predicted for many $\Delta J = 1$ transitions of the band.

To achieve a deeper insight on the structure of the vibrational-$\gamma$ bands in even $^{104-118}$Pd, the wave functions of their members have been considered in detail. This in turn implies a preliminary description of the $U_{\pi,\nu}(5)$ basis states, on which the realistic wave functions [eigenstates of Hamiltonian Eq. (1)] are expanded. According to their $F$-spin and $n_d$ quantum numbers, the basis states can be arranged in groups, whose excitation energies have different dependence on the Majorana parameters [49,50]. The excitation energy pattern of a nucleus having $N_\pi = 2$ and $N_\nu = 6$ (as $^{108}$Pd and $^{116}$Pd, which have six $\nu$-boson and six $\nu$-boson holes, with respect to the $N = 50$ and $N = 82$ neutron shell closure, respectively), obtained using the $U_{\pi,\nu}(5)$ Hamiltonian

$$H = \varepsilon\left(\hat{n}_{d_\pi} + \hat{n}_{d_\nu}\right) + \hat{M}_{\pi\nu}[\xi_1, \xi_2, \xi_3], \tag{4}$$

is shown in Fig. 8. Here, the groups having the $J^\pi = 1^+$ state as the lowest one (whose excitation energy depends on all the Majorana parameters) are not reported, since, as mentioned above, they do not practically affect the structure of the states under study. In the figure, only the three states of highest spin for each $n_d$ multiplet are shown. An example of a complete $U_{\pi,\nu}(5)$ excitation energy pattern, which includes the group of states having the $1^+$ state as the lowest one, is given in Ref. [51].

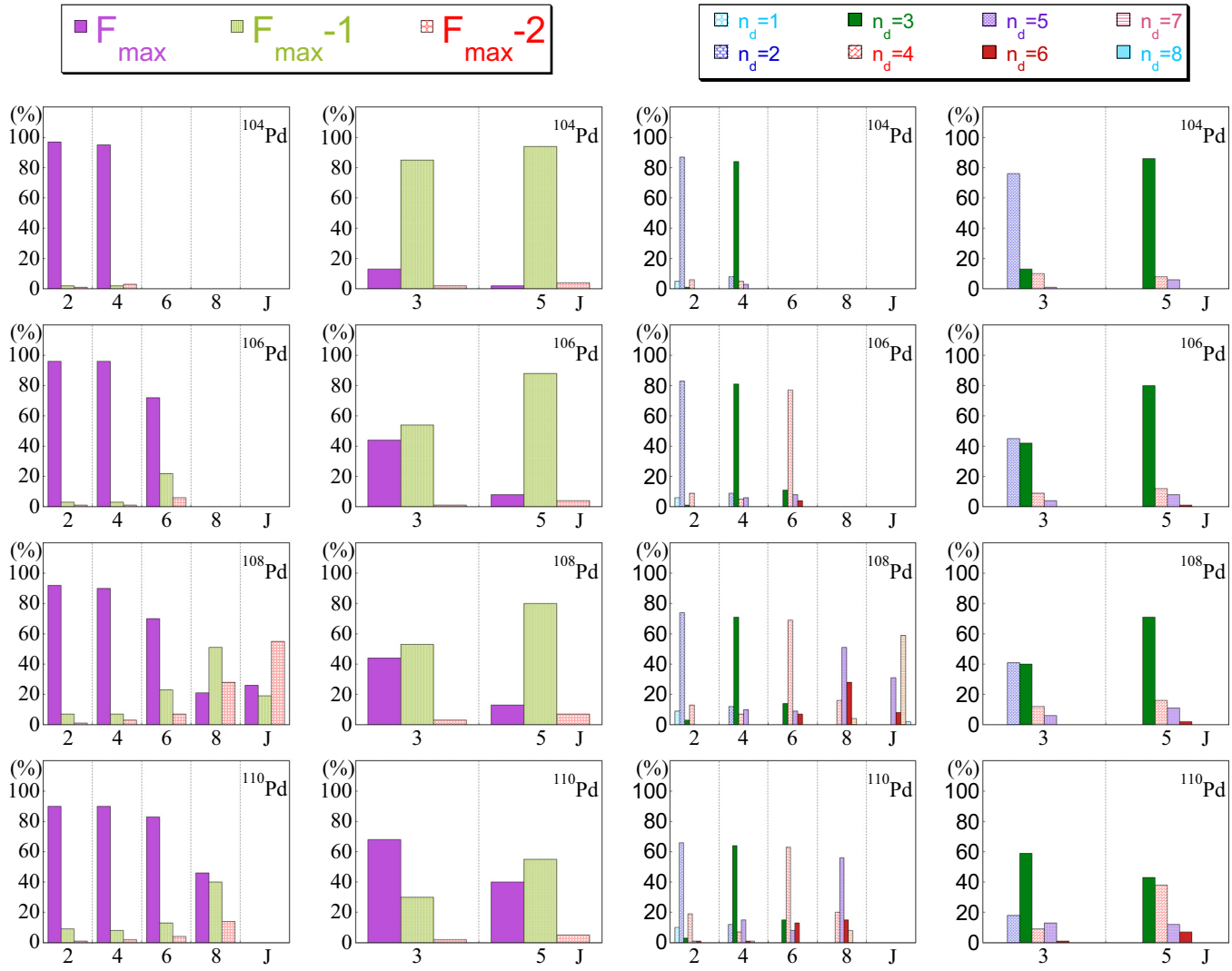



FIG. 9. $F$-spin and $n_d$ components (amplitude square) of even- and odd-spin states of the vibrational-$\gamma$ bands in $^{104-110}$Pd isotopes.

As seen from Fig. 8, the number of $U_{\pi,\nu}(5)$ basis states noticeably increases with respect to that of the $U(5)$ limit of the IBA-1 model. Indeed, this limit, which is the counterpart of the vibrational model in a geometric description, predicts only the $F = F_{\max}$ states of group (a). The MS states of group (a′) and (a″) are the counterparts of the FS states of column (a), i.e., they have the same wave function structure but a lower symmetry (characterized by the quantum number $F$) decreasing from (a′) to (a″). The states of group (b″) are the counterpart of the states of group (b′). For each group reported in Fig. 8 the eigenvalues of Hamiltonian Eq. (4) can be given in a close analytic form, as a function of $\epsilon$ [group (a)], $\epsilon, \xi_2$ [groups (a′), (a″)], and $\epsilon, \xi_2, \xi_3$ [groups (b′), (b″), (c″)] [49,50]. Depending on the values of these parameters, it can happens that MS states of even-spin (odd-spin) become yrare (yrast) states. Moreover, it can happens that some basis states of a given spin, belonging to different groups, fall in a narrow energy range. It follows that the resulting structure of a realistic vibrational-$\gamma$ band is expected to be more complicated than that of the $U(5)$ limit, which consists of the states included in the rectangles shown in Fig. 8. Indeed, the present calculations, which use the realistic Hamiltonian Eq. (1), point to a rather complicated structure of the vibrational-$\gamma$ band.

This is apparent from Figs. 9 and 10, where the $F$-spin and $n_d$ components of the states associated to the members of the experimental vibrational-$\gamma$ bands in even $^{104-118}$Pd are displayed. Starting from the structure of the even-spin states, it is possible to observe that, all along the isotopic chain, the $2_2^+$ and $4_2^+$ predicted states have strongly predominant FS character and main $n_d = 2, 3$ components, respectively. In the lighter isotopes, the similarity of their structure to that of the $2_2^+$ and $4_2^+$ basis states of group (a), respectively, is particularly evident. As to the $6_2^+$ state, the $F_{\max}$ and $n_d = 4$ components are very strong in the lighter isotopes, making its structure quite similar to that of the $6_2^+$ state in group (a). In going toward the heavier isotopes the percentage of the $F_{\max} - 1$ and $F_{\max} - 2$ components increases and in $^{112-118}$Pd the MS character of this state becomes predominant. In $^{108-116}$Pd, all states of spin $J \geqslant 8$ have predominant MS character, while the $F_{\max} - 2$ component becomes the strongest one in the $8^+$ state of $^{114}$Pd and $10^+$ states of $^{108,114}$Pd.

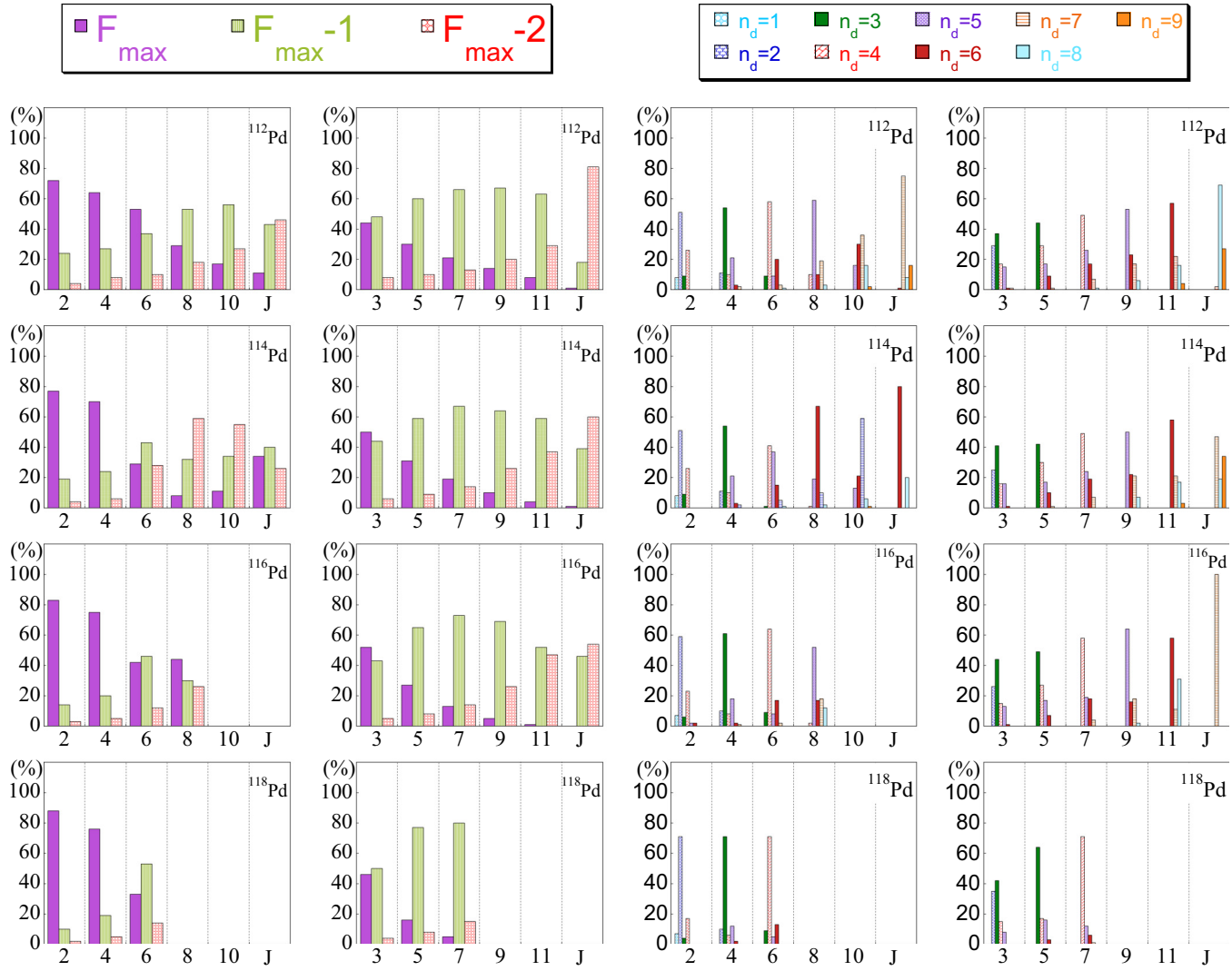



FIG. 10. $F$-spin and $n_d$ components (amplitude square) of even- and odd-spin states of the vibrational-$\gamma$ bands in $^{112-118}$Pd isotopes.

As to the odd-spin states, it turns out that all the states have mixed symmetry character, apart from the $3^+$ state in $^{106-116}$Pd. Indeed, in these isotopes this state has almost comparable $F_{\max}, n_d = 3$ and $F_{\max} - 1, n_d = 2$ components, so that its wave function results from a mixing of the lowest $3^+$ states of groups (a) and (b′) of Fig. 8. By looking at the wave functions of the $5^+$, $7^+$, $9^+$, and $11^+$ states, it appears that their main components are the lowest $5^+$, $7^+$, $9^+$, and $11^+$ basic states of group (b′), respectively, all along the isotopic chain. Finally, in $^{112-116}$Pd the $F_{\max} - 2$ component noticeably increases in going from $J = 3$ to $J = 13$ state and becomes predominant in the last one.

As a more general comment, the capability of the IBA-2 model of predicting sets of even-spin and odd-spin states (which have remarkably different structure) whose properties match those experimentally observed, joined to the negligible intensity of intraband transitions among states of spin $J \geqslant 5$, disfavor an interpretation of the "so-called" vibrational-$\gamma$ band in terms of a unique band.

## IV. CONCLUSIONS

The information on the "vibrational-$\gamma$ bands" in even $^{104-118}$Pd isotopes has been amply extended by the large body of newly available experimental data. In the present work, their properties have been studied in the framework of the IBA-2 model. The analysis has been carried out using the parameters of Ref. [17] for $^{104-112}$Pd and of Ref. [19] for $^{114-118}$Pd. The agreement between experimental and calculated excitation energies is better than 1%, on average. On the whole, the e.m. properties of the "vibrational-$\gamma$ bands" are reasonably reproduced by the calculations. The possibility of fitting a large amount of new experimental data without performing any tuning of the parameters supports the correctness of their choice and underlines the predictive capability of the IBA-2 model.

The analysis of the relevant wave functions (in terms of $F$-spin and $n_d$ components) reveals that the percentage of MS components increases as the spin increases and that most of the states have predominant MS character.

Even though additional degrees of freedom might contribute to the structure of the "vibrational-$\gamma$ bands," it turns out that their main properties are correctly reproduced once the presence of states of MS character are taken into account.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