Lifetime measurements in ^{156}Gd

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Background: The nature of low-lying oscillations or excitations around the equilibrium deformed nuclear shape remains an open question in a nuclear structure. The question revolves around the possible degrees of freedom in deformed nuclei. Rotational motion is an expected feature of deformed nuclei; the open challenge is whether the "granularity" of the nuclei allows single or multiple quanta of vibrational oscillations or excitations superimposed on the equilibrium deformed shape of the nucleus. Special emphasis is placed on the $K^\pi = 0^+$, β vibration whose existence is open to debate some 40 years after Bohr-Mottelson-Rainwater's Nobel prize for connecting nucleon motion to the emergence of collectivity.

Purpose: The ^{156}Gd nucleus is an excellent test case for the search of the predicted oscillations since it has one of the most developed level schemes up to 2.35 MeV and it lies in the well-deformed rare-earth region of the chart of nuclides. This nucleus has previously been studied by (n,γ), (n,e^-), (e,e'), (p,p'), (d,p), and (d,t) reactions with six known excited $K^\pi = 0^+$ bands. We measured level lifetimes of ^{156}Gd in order to determine the nature of the low-lying excited bands.

Method: The lifetimes of the excited states in the ^{156}Gd nucleus were measured following neutron capture using the GRID technique at the Institut Laue-Langevin in Grenoble, France.

Results: Twelve level lifetimes were measured from four excitation bands in the ^{156}Gd nucleus including the lifetimes of three of the $K^\pi = 0^+$ bands.

Conclusions: There are two $K^\pi = 0^+$ bands in this nucleus connected to the ground-state band. Transitions from the $K^\pi = 0^+_2$ band at 1049.5 keV to the ground-state band are more collective than the ones from the $K^\pi = 0^+_3$ band at 1168.2 keV. The moments of inertia of the various $K^\pi = 0^+$, the $K^\pi = 2^+$, and the $K^\pi = 4^+$ bands show that all the bands except the $K^\pi = 0^+_3$ band at 1168.2 keV have nearly identical moments of inertia with the ground-state band pointing to the fact that all of the bands discussed here with the exception of this indicating $K^\pi = 0^+_3$ band seem to be collective excitations built on the ground state. This result is consistent with various theoretical predictions. $B(E2)$ calculations for transitions from the $K^\pi = 2^+$ band to the ground-state band supports the assignment of this band as the γ band. Also, the $K^\pi = 0^+_4$ and the $K^\pi = 4^+_1$ bands at 1715.2 and 1510.6 keV, respectively, are shown to be strongly connected to the $K^\pi = 2^+$ γ band presenting evidence for the observation of a second set of two-phonon $\gamma\gamma$ vibrational excitations albeit with greatly varying degrees of anharmonicity in comparison to the case of ^{166}Er.

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I. INTRODUCTION

The 1975 Nobel prize in Physics was awarded to Bohr, Mottelson, and Rainwater for the discovery of the connection between nucleon motion and the emergent collective behavior. Bohr-Mottelson-Rainwater described nuclei geometrically as a shape and the oscillations of the nucleus around that shape. The lowest-lying shape effecting oscillations or vibrations would be quadrupole ($\lambda = 2$) in nature, resulting in two types of vibrations in deformed nuclei: β with oscillations along the symmetry axis ($K^\pi = 0^+$) and γ-breaking axial symmetry with a projection of $K^\pi = 2^+$ on the symmetry axis. The γ vibration seems to be well characterized as the first $K^\pi = 2^+_1$ (or 2^+_γ) band and exhibits a systematic behavior across the region of deformed nuclei with typical $B(E2{:}\,2^+_\gamma \rightarrow 0^+_{\rm g.s.})$ values of a few Weisskopf units (W.u.) and where "g.s." represents the "ground state" [1].

Today, over 40 years later, the existence and characterization of the low-lying β vibration remains an open question in nuclear structures [1–35]. This is due to the lack of sufficient experimental data on the identification and characterization of 0^+ excitations in deformed nuclei and to the interpretation of what is expected of a β vibration. The absence of a β-vibrational excitation in deformed nuclei will call into question why nuclei, unlike all other quantum systems, do not exhibit this mode of oscillatory motion.

In well-deformed regions of nuclei, excitations built on a deformed ground state have traditionally been described in

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terms of quadrupole excitations leading to the classifications of the first excited 0^+ bands as single-phonon β-vibrational excitations. However, discussions in recent years have focused on a debate about the absence, or lack of, a ($K^\pi = 0^+$) β vibration with a multitude of possible interpretations including the possibility for phase changes at the onset of deformation (for example, at $N = 90$ and $Z = 64$) and the application of new symmetries to describe these nuclei [4,36–41] or the change in the expectation of a β vibration. The discussions on the existence or absence of the $K^\pi = 0^+$ β-vibrational excitations in nuclei have spanned a wide spectrum of possibilities from shape coexistence where a competing shape is not the lowest favored shape but occurs low in the excitation spectrum of a given nucleus [2] to a redefinition of what can, in fact, be interpreted as a β vibration [42]. In the interacting boson model [43–45], the first excited 0^+ and 2^+ bands are members of the same representation and in a pure SU(3) limit would not decay to the g.s. Most deformed nuclei however are not pure SU(3) and the breaking of that symmetry gives rise to interband transitions from both of the low-lying $K^\pi = 2^+$ and $K^\pi = 0^+$ (γ and β) bands of significant strengths. Another recent development describes nuclei at the point of phase change from spherical to deformed in terms of β- and γ-shape parameters, the SU(3) symmetry [17,18,20] or the pseudo-SU(3) [35]. There is also the possibility for partial dynamical symmetries where the SU(3) symmetry is obeyed by some of the states and broken in others [46]. A systematic theoretical study of even-even deformed nuclei in the Hartree-Fock-Bogoliubov approach extended by the generator coordinate method and mapped into a five-dimensional collective Hamiltonian for even-even nuclei from $Z = 10$ to 110 provides guidelines for distinguishing between coexistence and β-vibrational oscillations. These studies and others [30,32] on the nature of the $K^\pi = 0^+$ bands in deformed nuclei predict widely varying levels of collectivity depopulating the first excited 0^+ states [1].

The Gd isotopes lie in a well-deformed region of the chart of nuclides with the ratio of the first two excited states $4^+/2^+$ ($R_{4/2}$) varying from 3.0 to 3.3 for ^{154}Gd to ^{160}Gd allowing a fertile testing ground for previous theories. The focus of this paper is ^{156}Gd with a $R_{4^+/2^+}$ ratio of 3.24, a well-known level scheme up to an excitation energy of 2.35 MeV and six excited $K^\pi = 0^+$ bands, four of them below the pairing gap at approximately 2 MeV. The ^{156}Gd nucleus has been studied extensively and with high precision using bent crystals GAMS 2/3 for (n, γ), the BILL electron spectrometer for (n, e^-) measurements [47], the early tests of the GRID (Gamma-ray-induced Doppler broadening) technique [48–50] using the GAMS 4 spectrometer at the Institut Laue Langevin (ILL) in Grenoble, and by (d, t) and (d, p) reactions at the Munich Tandem Accelerator [47]. Numerous other studies included electron, proton, and photon scatterings [51,52]. The ^{156}Gd nucleus was used for comparisons with early tests of the interacting boson model numerical studies for the SU(3) limit [53] as well as the later tests of partial dynamical symmetry tests [46]. The focus of this paper is the excited $K^\pi = 0^+$ bands. We have measured 12 level lifetimes in these bands in an attempt to characterize the nature of these states in search of the β and other low-lying vibrational excitations.

II. EXPERIMENT

The experiment was performed at the ILL neutron High Flux Reactor in Grenoble, France. The ^{156}Gd nucleus was populated by neutron capture on a 5.349-g Gd_2O_3 (gadolinium oxide) target. The GRID technique [49,54] of lifetime measurements is based on measuring the broadening of decay γ-ray lines using perfect crystals to measure the associated γ-ray wavelength. The broadening is due to the initial recoil velocity of the nucleus where the width of a given γ-ray transition emitted in flight results from the competition between the slowing-down process and the level lifetime. Knowing the slowing-down process from simulations, the nuclear level lifetime is extracted. The recoil velocities are typically 10^{-4} to 10^{-6} c, resulting in a broadening of only a few eV. The γ-ray wavelengths rely on crystal diffraction from nearly perfect flat crystals made of silicon or germanium. GAMS4 is a double-flat-crystal spectrometer with remarkable energy resolution and high precision of a few eV [50,55–57]. The broadened γ-ray peaks were fitted using the code GRIDDLE [58]. Figure 1 shows the broadening of the widths of γ-ray transitions depopulating from two different levels at 1129.44 and 1248.01 keV in the ^{156}Gd nucleus.

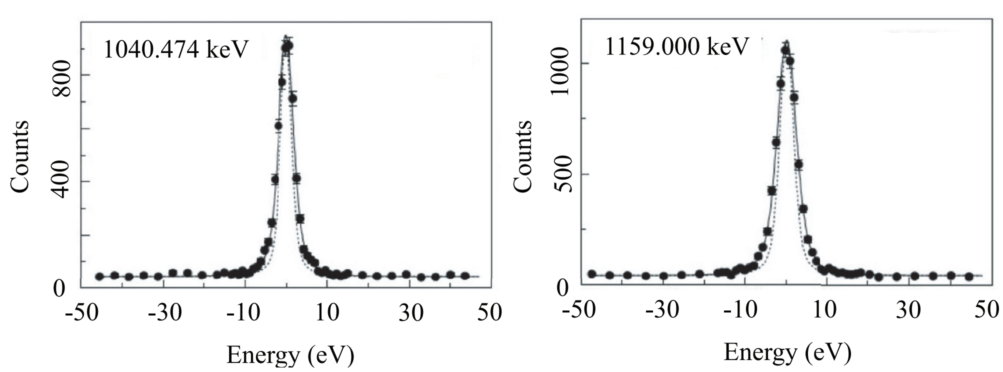

FIG. 1. Broadening curves for two γ-ray transitions 1040.474- and 1159.000-keV depopulating levels at 1129.440 and 1248.008 keV, respectively. The dotted line is the instrumental response expected without broadening, and the solid black line is the fit of the data points. The lifetime for a given transition is extracted from the observed broadening.

TABLE I. The energy levels and depopulating transitions measured in ^{156}Gd, the missing feeding of each level, and the upper and lower lifetime limits, resulting in the conservative overall range for the lifetimes.

E_x (keV)	J^π	E_γ (keV)	Feeding[a]	τ_{upper} (ps)	τ_{lower} (ps)	τ_{range} (ps)
$K^\pi = 0_2^+$ band at 1049.479 keV						
1049.479	0^+	960.510	44.3%	$2.86^{+1.88}_{-0.82}$	$2.52^{+1.59}_{-0.71}$	$1.81 < \tau < 4.75$
1129.440	2^+	1040.474	20.4%	$2.08^{+0.50}_{-0.34}$	$1.48^{+0.30}_{-0.22}$	$1.26 < \tau < 2.58$
1297.825	4^+	1009.622	40.5%	$2.52^{+0.84}_{-0.51}$	$1.84^{+0.54}_{-0.34}$	$1.49 < \tau < 3.36$
$K^\pi = 2^+$ band at 1154.151 keV						
1154.151	2^+	1065.182	43.7%	$1.20^{+0.14}_{-0.12}$	$0.96^{+0.11}_{-0.09}$	$0.87 < \tau < 1.35$
1248.008	3^+	1159.000	52.3%	$0.92^{+0.09}_{-0.08}$	$0.74^{+0.08}_{-0.06}$	$0.68 < \tau < 1.01$
1355.425	4^+	1067.236	34.8%	$0.91^{+0.10}_{-0.09}$	$0.64^{+0.07}_{-0.06}$	$0.58 < \tau < 1.02$
1506.868	5^+	1218.710	21.8%	$1.21^{+0.61}_{-0.31}$	$0.34^{+0.18}_{-0.10}$	$0.25 < \tau < 1.82$
$K^\pi = 0_3^+$ band at 1168.190 keV						
1168.190	0^+	1079.230	19.1%	$6.34^{+8.35}_{-2.31}$	$4.61^{+5.04}_{-1.60}$	$3.00 < \tau < 14.7$
1258.075	2^+	1169.092	14.4%	$3.46^{+1.60}_{-0.84}$	$3.17^{+1.84}_{-0.86}$	$2.31 < \tau < 5.06$
$K^\pi = 0_4^+$ band at 1715.181 keV						
1715.181	0^+	472.700	11.4%	$3.69^{+3.39}_{-1.21}$	1.88 ± 1.86	$0.02 < \tau < 7.08$
1771.089	2^+	1682.184	25.7%	$0.60^{+0.13}_{-0.10}$	$0.17^{+0.06}_{-0.04}$	$0.13 < \tau < 0.73$
1893.395	4^+	1605.217	1.3%	$0.37^{+0.08}_{-0.06}$	$0.0002^{+0.0001}_{-0.00005}$	$0.00015 < \tau < 0.45$

[a]Percentage of known level feeding [47,59].

The largest uncertainties in these measurements arise from the unknown feeding of the level of interest. Therefore, in cases where the feeding of a particular nuclear level is not completely understood, rather extreme and conservative assumptions have been made in order to extract conservative *upper* and *lower* limits. The upper limit of the extracted lifetime is determined assuming that the level is totally fed by cascades of γ-ray transitions from the compound capture state at 8.536 MeV. The cascades are typically several MeV unobserved transitions via two-step cascades connecting the level of interest to the compound state. This would yield the maximum broadening for the γ rays depopulating a given level or the longest slowing-down times resulting in the longest possible lifetime for the level of interest. Lifetimes shorter than the upper limit would yield more collective $B(E2)$ values. The lower limit of the lifetime is extracted by assuming that the missing feeding comes from the unplaced low-energy transitions measured in this nucleus. These are extreme and conservative limits for the measured lifetimes. The more realistic scenario would probably lie somewhere in the middle of the lifetimes resulting from these intentionally extreme feeding assumptions. Table I lists the percentage of known feeding for the levels of interest as well the results of the calculations for the extraction of upper and lower limits on the lifetimes. The extracted lifetime ranges are listed in Table II along with a listing for comparison of six previously measured lifetimes [47,59,60] for the states of interest, four were measured using the GAMS4 spectrometer in its early incarnation [47] and two were measured by Coulomb excitation [59,60]. In all the cases where previous measurements existed, there is agreement. A $0.6 \times \tau_{max}$ was used as a calibration factor for figures which correspond to the approximate middle of the range of the GRID measurement to match the lifetime of the 1154.2-keV level. A more detailed explanation on the calibration is found in Ref. [61].

Figure 2 presents a partial level scheme of ^{156}Gd showing in red the levels whose lifetimes were measured in this paper. A value of $0.6 \times \tau_{max}$ was used to calculate the $B(E2)$ values of the transitions for all the levels of interest that were measured with GRID. The $B(E2)$ values from the $K^\pi = 4^+$, $J^\pi = 4^+$ level was previously measured by Coulomb excitation.

III. RESULTS

The results of the lifetime measurements are shown in Table II with the uncertainties that have led to the broad

TABLE II. Level energies and lifetime ranges for all 12 states measured in this paper in comparison with previous measurements. Four of the previous values were measured by the GRID technique using the GAMS4 spectrometer [47]; another three were measured by Coulomb excitation [59,60].

E_x (keV)	τ_{grid} (ps)	Previous measurement (ps)
1049.479	$1.81 < \tau < 4.75$	$1.39 < \tau < 12.97$ [47]
1129.440	$1.26 < \tau < 2.58$	2.27 ± 0.17 [60]
1154.151	$0.87 < \tau < 1.35$	$0.87 < \tau < 1.44$ [47]
		0.82 ± 0.03 [59]
1168.190	$3.00 < \tau < 14.69$	
1248.008	$0.68 < \tau < 1.01$	
1258.075	$2.31 < \tau < 5.06$	2.22 ± 0.22 [59]
1297.825	$1.49 < \tau < 3.36$	
1355.425	$0.58 < \tau < 1.02$	$1.15 < \tau < 7.71$ [47]
1506.868	$0.24 < \tau < 1.82$	
1715.181	$0.02 < \tau < 7.08$	
1771.089	$0.13 < \tau < 0.73$	$0.14 < \tau < 1.44$ [47]
1893.395	$0.00015 < \tau < 0.45$	

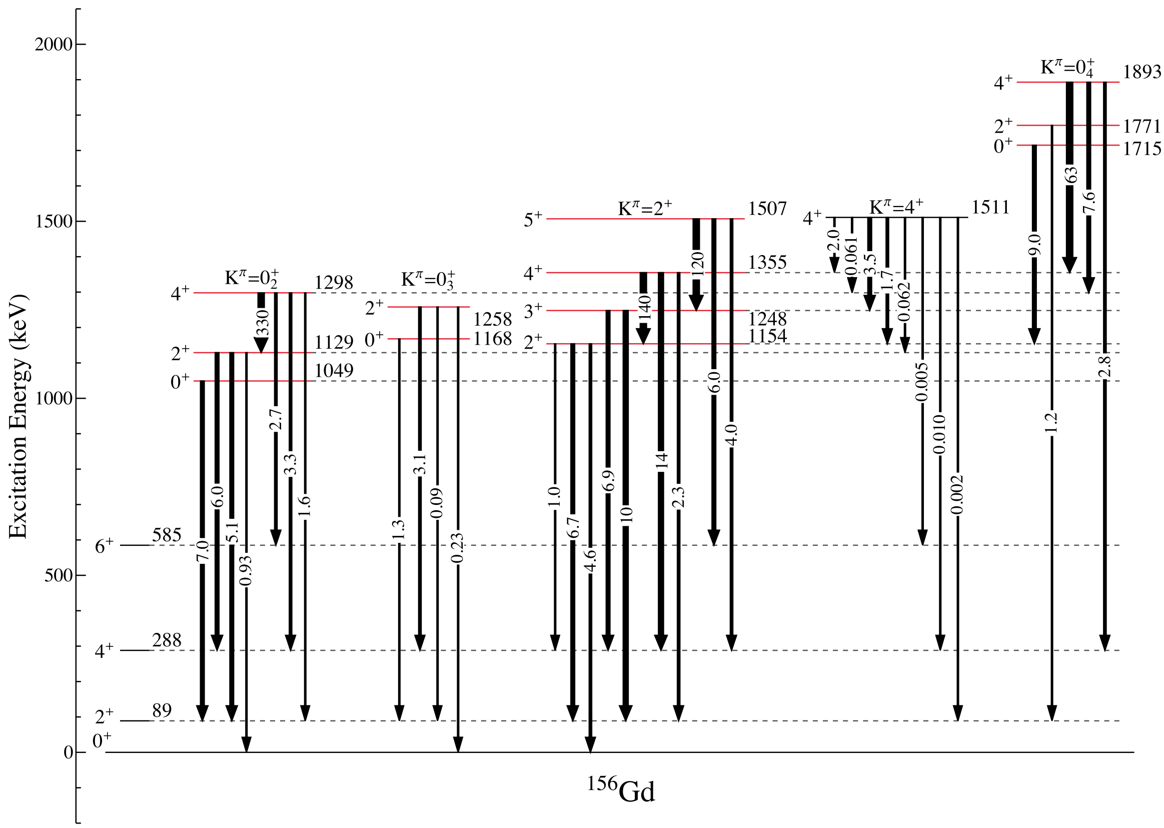

FIG. 2. A partial level scheme for ^{156}Gd showing levels for which lifetimes were measured using $0.6 \times \tau_{\max}$ of the GRID range as described in the text. The $K^\pi = 4^+$, $J^\pi = 4^+$ level was previously measured by Coulomb excitation. The width of the transition lines depopulating the levels of interest are in proportion to the transition probability in W.u. Absolute $B(E2)$ values are presented in Table III.

ranges of lifetimes as described in the experimental section. Table III shows the levels of interest including lifetimes and their depopulating transitions for the first three excited $K^\pi = 0^+$ bands and from the $K^\pi = 2^+$ γ band. This paper presents the lifetime measurements of all four excited states of the first $K^\pi = 0^+$ band, the $K^\pi = 2^+$ band, the $K^\pi = 0_2^+$ band, as well as two additional $K^\pi = 0^+$ bands. In each case, the measurements of the $K^\pi = 0^+$ bands include the lifetimes of the $J^\pi = 0^+$ bandheads. In the deformed rare-earth region of the chart of nuclides, there are very few lifetimes for the $J^\pi = 0^+$ levels, whereas there are numerous lifetime measurements for the $J^\pi = 2^+$ and $J^\pi = 4^+$ levels of $K^\pi = 0^+$ bands. This presents a unique opportunity to test predictions regarding the characters of these excited $K^\pi = 0^+$ bands. In addition, we have included lifetimes and depopulating transition probabilities to and from the $K^\pi = 4^+$ banded state at 1510.6 keV to allow for comparison with the $K^\pi = 2^+$ γ band. These $K^\pi = 4^+$ lifetimes had previously been measured by Coulomb excitation.

$K^\pi = 0_2^+$ *band at 1049.479 keV*. The lowest-energy excited band in ^{156}Gd is the $K^\pi = 0^+$ band at 1049.5 keV. Lifetimes were extracted for the bandhead and first two excited states. The $J^\pi = 0^+$ level was previously measured using GRID with a lifetime range of $1.39 < \tau < 12.97$ [47]. In the current measurement, using the same technique, the new lifetime range of $1.81 \rightarrow 4.75$ ps is in excellent agreement but with a smaller range of values yielding $B(E2\colon 0^+_{K^\pi=0_2^+} \rightarrow 2^+_{\text{g.s.}}) = 4.20 \rightarrow 11.0$ W.u. The $J^\pi = 2^+$ level at the 1129.4-keV lifetime range was measured at $1.26 \rightarrow 2.58$ ps and compares well with the previously reported value of 2.27 ± 0.17 ps from Coulomb excitation [60]. The dominant transition probability from this level is $E_\gamma = 841.2$ keV to the $J^\pi = 4^+_{\text{g.s.}}$ level with a $B(E2\colon 2^+_{K^\pi=0_2^+} \rightarrow 4^+_{\text{g.s.}})$ range of $3.61 \rightarrow 7.42$ W.u. The $J^+ = 4^+$ level at 1297.8 keV has a measured lifetime range of $1.49 \rightarrow 3.36$ ps yielding a $B(E2\colon 4^+_{K^\pi=0_2^+} \rightarrow 6^+_{\text{g.s.}})$ range of $1.60 \rightarrow 3.60$ W.u. Also calculated is an intraband transition from the $J^+ = 4^+$ level to the $J^\pi = 2^+$ level of the same band yielding a $B(E2\colon 4^+_{K^\pi=0_2^+} \rightarrow 2^+_{K^\pi=0_2^+})$ range of $200 \rightarrow 440$ W.u. typical of intraband rotational transition strengths.

Reduced matrix elements are also listed in Table III and are defined as the square roots of the $B(E2)$ values divided by the Clebsch-Gordon coefficient. The reduced matrix elements for those transitions depopulating the first excited $K^\pi = 0^+$ band average between 0.163 and 0.241 eb. These numbers are deduced from the averages of the lower(upper) limits for all the transitions depopulating this band. The measured $B(E2\colon 0^+_{K^\pi=0_2^+} \rightarrow 2^+_{\text{g.s.}})$ value is in the range of $0.021 \rightarrow 0.055$ $e^2\text{b}^2$. This value is divided by $2J_i + 1$ to yield a $B(E2\colon 2^+_{\text{g.s.}} \rightarrow 0^+_{K^\pi=0_2^+})$ range of 0.0042–0.011 $e^2\text{b}^2$. The intraband reduced matrix element is between 1.87 and 2.77 eb in agreement with values from Coulomb excitation [59].

$K^\pi = 2_1^+$ *band at 1154.151 keV.* Our results confirm previous reports of the $K^\pi = 2^+$ band as the γ band. The

TABLE III. Measured level lifetimes in the ^{156}Gd nucleus and the extracted $B(E2)$ values. All of the level lifetimes are measured in this paper with the exception of the $K^\pi = 4^+$ bandhead at 1510.595 keV [67]. Transition intensities and conversion coefficients are from Klora *et al.* [47]. The last column is the square root of $B(E)\lambda$ divided by the Clebsch-Gordan (CG) CG^2 ($\sqrt{\frac{B(E)\lambda}{\mathrm{CG}^2}}$). The CG^2 coefficients are the standard Alaga rules $(J_i K_i 2\,\Delta K | J_f K_f)^2$ values. If the exact multipole admixture is unknown, 100% $E2$ is calculated. The level lifetime for the $K^\pi = 4^+$ bandhead state at 1510.6 keV was not measured in this paper, but it is included to allow discussion and comparison.

K_i^π, J_i^π	E_x (keV)	τ (ps)	E_γ (keV)	K_f^π, J_f^π	I_γ	α	Multipolarity	$B(E1)$ or $B(E2)$ mW.u. or W.u.	$B(E\lambda)$ (e^nb^n)[a]	CG^2	Reduced matrix element (eb)
$0_2^+, 0^+$	1049.479	$1.81 < \tau < 4.75$	960.510	$0^+_{\mathrm{g.s.}}, 2^+$	138	0.00259	$E2$	$4.20 \rightarrow 11.0$	$0.021 \rightarrow 0.055$	1.000	$0.145 \rightarrow 0.234$
$0_2^+, 2^+$	1129.440	$1.26 < \tau < 2.58$	1129.423	$0^+_{\mathrm{g.s.}}, 0^+$	80	0.00197	$E2$	$0.56 \rightarrow 1.14$	$0.0028 \rightarrow 0.0057$	0.200	$0.118 \rightarrow 0.169$
			1040.474	$0^+_{\mathrm{g.s.}}, 2^+$	294	0.011	$E2 + M1 + E0$	$3.10 \rightarrow 6.28$	$0.016 \rightarrow 0.031$	0.286	$0.233 \rightarrow 0.331$
			841.243	$0^+_{\mathrm{g.s.}}, 4^+$	119	0.0032	$E2$	$3.61 \rightarrow 7.42$	$0.018 \rightarrow 0.037$	0.515	$0.187 \rightarrow 0.268$
$0_2^+, 4^+$	1297.825	$1.49 < \tau < 3.36$	1208.875	$0^+_{\mathrm{g.s.}}, 2^+$	97	0.00159	$E2$	$0.95 \rightarrow 2.14$	$0.0047 \rightarrow 0.011$	0.286	$0.129 \rightarrow 0.196$
			1009.622	$0^+_{\mathrm{g.s.}}, 4^+$	83	0.0168	$f\,E2 + M1 + E0$	$1.96 \rightarrow 4.43$	$0.0098 \rightarrow 0.022$	0.260	$0.194 \rightarrow 0.291$
			713.104	$0^+_{\mathrm{g.s.}}, 6^+$	11.7	0.0044	$E2$	$1.60 \rightarrow 3.60$	$0.0080 \rightarrow 0.018$	0.455	$0.132 \rightarrow 0.199$
			168.382	$0_2^+, 2^+$	1.08	0.0234	$E2$	$200 \rightarrow 440$	$1.00 \rightarrow 2.19$	0.286	$1.87 \rightarrow 2.77$
$2_1^+, 2^+$	1154.151	$0.87 < \tau < 1.35$	1154.151	$0^+_{\mathrm{g.s.}}, 0^+$	498	0.00192	$E2$	$2.74 \rightarrow 4.25$	$0.014 \rightarrow 0.021$	0.200	$0.265 \rightarrow 0.324$
			1065.182	$0^+_{\mathrm{g.s.}}, 2^+$	549	0.00219	89% $E2$	$4.01 \rightarrow 6.23$	$0.020 \rightarrow 0.031$	0.286	$0.264 \rightarrow 0.329$
			865.971	$0^+_{\mathrm{g.s.}}, 4^+$	26.9	0.00272	$E2$	$0.62 \rightarrow 0.96$	$0.0031 \rightarrow 0.0048$	0.015	$0.454 \rightarrow 0.565$
$2_1^+, 3^+$	1248.008	$0.68 < \tau < 1.01$	1159.000	$0^+_{\mathrm{g.s.}}, 2^+$	659	0.00178	$E2$	$6.12 \rightarrow 9.10$	$0.031 \rightarrow 0.046$	0.358	$0.294 \rightarrow 0.355$
			959.823	$0^+_{\mathrm{g.s.}}, 4^+$	173	0.00265	$E2$	$4.12 \rightarrow 6.12$	$0.021 \rightarrow 0.030$	0.143	$0.383 \rightarrow 0.458$
$2_1^+, 4^+$	1355.425	$0.58 < \tau < 1.02$	1266.451	$0^+_{\mathrm{g.s.}}, 2^+$	94	0.0014	$E2$	$1.40 \rightarrow 2.46$	$0.007 \rightarrow 0.012$	0.120	$0.241 \rightarrow 0.316$
			1067.236	$0^+_{\mathrm{g.s.}}, 4^+$	235	0.00206	$E2$	$8.24 \rightarrow 14.5$	$0.041 \rightarrow 0.072$	0.351	$0.342 \rightarrow 0.453$
			201.269	$2_1^+, 2_1^+$	0.63	0.078	$E2$	$86.1 \rightarrow 150$	$0.429 \rightarrow 0.748$	0.120	$1.89 \rightarrow 2.60$
$2_1^+, 5^+$	1506.868	$0.25 < \tau < 1.82$	1218.710	$0^+_{\mathrm{g.s.}}, 4^+$	81	0.00168	$E2$	$2.41 \rightarrow 17.6$	$0.012 \rightarrow 0.088$	0.319	$0.194 \rightarrow 0.525$
			922.186	$0^+_{\mathrm{g.s.}}, 6^+$	30	0.00268	$E2$	$3.60 \rightarrow 26.2$	$0.018 \rightarrow 0.131$	0.182	$0.314 \rightarrow 0.848$
			258.860	$2_1^+, 3^+$	1.15	0.073	$E2$	$74 \rightarrow 538$	$0.370 \rightarrow 2.69$	0.191	$1.39 \rightarrow 3.75$
$0_3^+, 0^+$	1168.190	$3.00 < \tau < 14.7$	1079.230	$0^+_{\mathrm{g.s.}}, 2^+$	111	0.00215	$E2$	$0.76 \rightarrow 3.72$	$0.0038 \rightarrow 0.019$	1.000	$0.062 \rightarrow 0.138$
$0_3^+, 2^+$	1258.075	$2.31 < \tau < 5.06$	1258.092	$0^+_{\mathrm{g.s.}}, 0^+$	68	0.00145	$E2$	$0.14 \rightarrow 0.31$	$0.00070 \rightarrow 0.0015$	0.200	$0.059 \rightarrow 0.088$
			1169.092	$0^+_{\mathrm{g.s.}}, 2^+$	180	0.00272	10% $E2$	$0.053 \rightarrow 0.12$	$0.00026 \rightarrow 0.00060$	0.286	$0.030 \rightarrow 0.046$
			969.868	$0^+_{\mathrm{g.s.}}, 4^+$	250	0.0024	$E2$	$1.90 \rightarrow 4.13$	$0.0095 \rightarrow 0.021$	0.515	$0.136 \rightarrow 0.202$

K_i^π, J_i^π	E_x (keV)	τ (ps)	E_γ (keV)	K_f^π, J_f^π	I_γ	α	Multipolarity	$B(E1)$ or $B(E2)$ mW.u. or W.u.	$B(E\lambda)$ (e^nb^n)[b]	CG^2	Reduced matrix element (eb)
$4_1^+, 4^+$	1510.595	274 ± 7	1421.601	$0^+_{\mathrm{g.s.}}, 2^+$	79	0.00117	$E2$	0.0020	10×10^{-6}		
			1222.432	$0^+_{\mathrm{g.s.}}, 4^+$	189	0.00191	$E2$	0.0102	50×10^{-6}		
			925.920	$0^+_{\mathrm{g.s.}}, 6^+$	22.6	0.00284	$E2$	0.0049	24×10^{-6}		
			381.155	$0_2^+, 2^+$	3.47	0.0241	$E2$	0.062	0.00031		
			356.466	$2_1^+, 2^+$	68	0.0252	$E2$	1.71	0.0085	0.556	0.124
			262.589	$2_1^+, 3^+$	31.9	0.069	$E2$	3.54	0.018	0.312	0.240
			212.771	$0_2^+, 4^+$	0.21	0.166	$E2 + M1$	0.061	0.00030		
			155.168	$2_1^+, 4^+$	9.0	0.505	20% $E2$	1.97	0.0098	0.110	0.298
$0_4^+, 0^+$	1715.181	$0.02 < \tau < 7.08$	561.024	$2_1^+, 2^+$	4.81	0.0082	$E2$	$5.4 \rightarrow 1890$	$0.027 \rightarrow 9.48$	1.000	$0.164 \rightarrow 3.07$
			472.700	$1^-, 1^-$	28.6	0.0058	$E1$	$0.35 \rightarrow 120$	$(1.75 \rightarrow 600) \times 10^{-6}$	1.000	$0.0013 \rightarrow 0.025$
			348.726	$0^-, 1^-$	3.1	0.006	$E1$	$0.095 \rightarrow 33$	$(0.48 \rightarrow 165) \times 10^{-6}$	1.000	$0.0007 \rightarrow 0.013$

TABLE III. (*Continued.*)

K_i^π, J_i^π	E_x (keV)	τ (ps)	E_γ (keV)	K_f^π, J_f^π	I_γ	α	Multipolarity	$B(E1)$ or $B(E2)$ mW.u. or W.u.	$B(E\lambda)$ $(e^n\text{b}^n)$[b]	CG^2	Reduced matrix element (eb)
$0_4^+, 2^+$	1771.089	$0.13 < \tau < 0.73$	1682.184	$0_{\text{g.s.}}^+, 2^+$	158	0.00103	50% $E2$	$0.74 \rightarrow 4.18$	$0.0037 \rightarrow 0.021$	0.286	$0.114 \rightarrow 0.271$
			528.627	$1^-, 1^-$	1.29	0.0032	$E1$	$0.023 \rightarrow 0.13$	$(0.12 \rightarrow 0.65) \times 10^{-6}$	0.100	$0.001 \rightarrow 0.002$
			513.021	$0_3^+, 2^+$	3.3	0.024	$M1$				
			494.942	$1^-, 3^-$	11.9	0.0053	$E1$	$0.26 \rightarrow 1.43$	$(1.3 \rightarrow 7.2) \times 10^{-6}$	0.400	$0.0018 \rightarrow 0.0042$
			404.633	$0^-, 1^-$	1.78	0.02	$E1$	$0.069 \rightarrow 0.39$	$(0.34 \rightarrow 1.9) \times 10^{-6}$	0.400	$0.0009 \rightarrow 0.0022$
$0_4^+, 4^+$	1893.395	$0.00015 < \tau < 0.45$	1605.217	$0_{\text{g.s.}}^+, 4^+$	59	0.0011	59% $E2$	$1.70 \rightarrow 5109$	$0.0085 \rightarrow 25$	0.286	$0.17 \rightarrow 9.44$
			617.24	$1^-, 3^-$	1.4	0.0046	$E1$	$0.064 \rightarrow 190$	$(0.32 \rightarrow 950) \times 10^{-6}$	0.351	$0.0010 \rightarrow 0.052$
			595.580	$0_2^+, 4^+$	0.67	0.0184	$M1 + E2$	$4.6 \rightarrow 13\,800$	$0.023 \rightarrow 70$	0.260	$0.297 \rightarrow 16$
			537.954	$2_1^+, 4^+$	5.6	0.0142	56% $E2$	$36 \rightarrow 107\,000$	$0.18 \rightarrow 540$	0.351	$0.715 \rightarrow 39$
			485.274	$1^-, 5^-$	2.4	0.0062	$E1$	$0.224 \rightarrow 673$	$(1.12 \rightarrow 3360) \times 10^{-6}$	0.334	$0.0018 \rightarrow 0.100$
			431.123	$0_3^+, 4^+$	0.58	0.043	$M1$				

[a] $E1$ transitions calculated in units of e^2b, and $E2$ transitions calculated in units of $e^2\text{b}^2$.
[b] $E1$ transitions calculated in units of e^2b, and $E2$ transitions calculated in units of $e^2\text{b}^2$.

lifetimes of the four lowest-energy levels ($2^+, 3^+, 4^+, 5^+$) of this band were measured. The $J^\pi = 2^+$ level was measured in this experiment to be $0.87 \rightarrow 1.35$ ps in this experiment, in agreement with previously results of 0.82(3) from Coulomb excitation [59] and a previous GRID measurement of $0.87 < \tau < 1.44$ ps [47].

All states in this band are connected to the ground state yielding reduced matrix element average values between 0.305 and 0.465 eb, whereas the intraband transitions are between 1.64 and 3.12 eb. The $B(E2)$ values of transitions from the measured levels in this band to the ground-state band are sufficiently strong to be considered collective transitions as calculated in Table III and shown in Fig. 2. There are also several intraband transitions known, and the measured lifetimes result in $B(E2)$ strengths of tens to hundreds of W.u. providing evidence that these levels are members of the same band.

$K^\pi = 0_3^+$ *band at 1168.190 keV.* The $J^\pi = 0^+$ bandhead located at 1168.190 keV and the $J^\pi = 2^+$ level located at 1258.075 keV resulted in lifetimes of $3.00 \rightarrow 14.7$ and $2.31 \rightarrow 5.06$ ps, respectively. The $B(E2)$ values from these levels to the ground state are given in Table III and illustrated in Fig. 2. The average matrix elements for the transitions depopulating the two levels of this band are smaller and range from 0.030 to 0.202 eb. An examination of the other $K^\pi = 0^+$ bands in comparison with this one reveals somewhat weaker $B(E2)$ transition probabilities and therefore would not be chosen as the collective excitation built on the ground-state band. Further evidence comes from the dynamic moments of inertia for all the $K^\pi = 0^+$ bands of interest in this nucleus. Figure 3 shows the dynamic moments of inertia for the $K^\pi = 0^+$ bands, including the $K^\pi = 0_1^+$ ground-state band, the first excited $K^\pi = 0_2^+$ band at 1049.479 keV, the $K^\pi = 0_3^+$ band at 1168.190 keV, and the $K^\pi = 0_4^+$ band at 1715.181 keV. The only band with some significant variation in the dynamic moment of inertia is the 1168.190-keV band. All the others have identical dynamic moments of inertia similar to the g.s. within a 7% variation in the slopes. The $K^\pi = 0_3^+$ band at 1168.2 keV is significantly different from the other $K^\pi = 0^+$ bands. Reference [25] reports on a study of quantum fluctuations around the equilibrium deformed shape of rare-earth nuclei with respect to the nature of collective 0^+ states. The work includes studies of the Gd isotopes where they show wave functions for 0^+ states built on the equilibrium-deformed shape with one and two nodes (one and two phonon) oscillations. Their work does not report on a third excited 0^+ state. The deviation of the $K^\pi = 0_3^+$ states in the dynamic moment of inertia indicates a 0^+ state of a different nature whereas the $K^\pi = 0_2^+$ and the $K^\pi = 0_4^+$ do indeed show themselves as oscillations or fluctuations around an equilibrium shape in this ^{156}Gd nucleus. A similar study [28] on the ^{178}Hf nucleus where there are four excited $K^\pi = 0^+$ bands below 2 MeV showed that two of the $K^\pi = 0^+$ bands

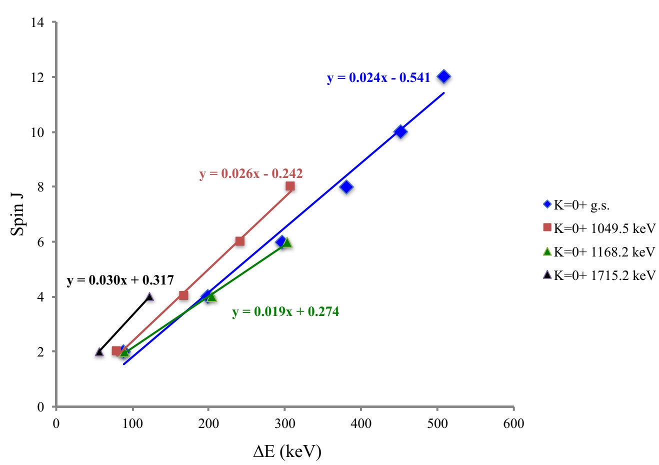

FIG. 3. Dynamic moments of inertia for the four $K^\pi = 0^+$ bands, including the $K^\pi = 0_1^+$ ground-state band, the first excited $K^\pi = 0_2^+$ band at 1049.479 keV, the $K^\pi = 0_3^+$ band at 1168.190 keV, and the $K^\pi = 0_4^+$ band at 1715.181 keV. Only the 1168.190-keV band has some significant variation in the dynamic moment of inertia.

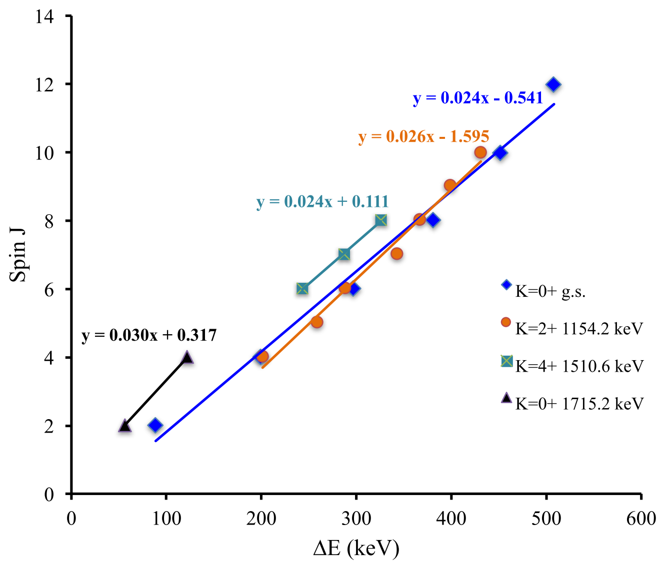

FIG. 4. Dynamic moments of inertia for the four bands considered including the g.s. band, the $K^\pi = 2^+$ γ band, the $K^\pi = 4^+$ band, and the $K^\pi = 0^+$ band at 1715.181 keV. These four bands have dynamic moments of inertia with less than 5% variation in slope. This is an indication that all of them are excitations built on the g.s. band.

that were strongly connected by collective $B(E2)$ transitions had identical dynamic moments of inertia and indicated that they were collective oscillations built on the ground-state equilibrium shape whereas the two intermediate $K^\pi = 0^+$ bands showed significantly different moments of inertia.

$K^\pi = 4_1^+$ band at 1510.595 keV. A lifetime of 274(7) ps was previously measured by Coulomb excitation [59,60] and not remeasured in this paper. $B(E2)$ transitions from this $J^\pi = 4^+$ state to the g.s. are K forbidden and show very weak transition probabilities for the g.s. band. The transitions depopulating to the 2^+, 3^+, and 4^+ members of the $K^\pi = 2_1^+$ γ band however are much stronger yielding matrix elements on the average of 0.221 eb. The energy ratio of the $(K^\pi = 4_1^+)/(K^\pi = 2_1^+)$ bands is 1.31, well below the expected value of 2, and the $E2$ matrix elements are of the same order as the $K^\pi = 2_1^+$ to the ground-state transitions.

$K^\pi = 0_4^+$ band at 1715.181 keV. The 1715.2-keV band is the third $K^\pi = 0^+$ band for which lifetimes were measured including the first three levels $(0^+, 2^+, 4^+)$. Low intensity transitions from these levels resulted in fewer statistics. The feeding to these transitions is not well known: The $J^\pi = 0^+$ level at 1715.181 keV, the $J^\pi = 2^+$ level at 1771.089 keV, and the $J^\pi = 4^+$ level at 1893.395 keV have known feeding intensities of 11%, 26%, and 1%, respectively. This resulted in the extracted lifetimes with larger ranges than other measurements in this experiment. $B(E2)$ transition strengths were calculated for transitions depopulating the $K^\pi = 0_4^+$ band, and despite the less than ideal lifetime ranges some conclusions concerning the nature of this band can still be made. The $E2$ transitions from this band to states within the ground-state band include the transitions and $B(E2)$ values $B(E2{:}\,2^+_{K^\pi=0_4^+} \to 2^+_{\rm g.s})$ and $B(E2{:}\,4^+_{K^\pi=0_4^+} \to 4^+_{\rm g.s.})$ range of $0.74 \to 4.18$ and >1.70 W.u., respectively. The $B(E2{:}\,0^+_{K^\pi=0_4^+} \to 2^+_{K^\pi=2_1^+})$ range of $5.4 \to 1890$ W.u. and the $B(E2{:}\,4^+_{K^\pi=0_4^+} \to 4^+_{K^\pi=2_1^+})$ range yields >36 W.u., connecting the 0^+ and 4^+ states of this band to the $K^\pi = 2_1^+$ γ band. These transitions show the $K^\pi = 0_4^+$ band to be collectively built on the $K^\pi = 2_1^+$ band and is evidence that the $K^\pi = 0_4^+$ band is a $K^\pi = 0^+$ $\gamma\gamma$ two-phonon vibrational band $(K^+ = 0^+_{\gamma\gamma})$. Figure 4 shows the dynamic moments of inertia plots with identical (within 7%) slopes for the four bands considered here, the g.s. $K^\pi = 0^+$, the $K^\pi = 2^+$, the $K^\pi = 4^+$, and the $K^\pi = 0_4^+$ band at 1715.181 keV. This is an indication that all of them are excitations built on the g.s. band. A summary of the relevant energy and $B(E2)$ ratios are shown in Table IV.

TABLE IV. The experimental energy and $B(E2)$ ratios for the proposed single- and double-phonon excitations.

$E(4^+_{\gamma\gamma})/E(2^+_\gamma)$	1.31
$E(0^+_{\gamma\gamma})/E(2^+_\gamma)$	1.49
$B(E2{:}\,4^+_{\gamma\gamma} \to 2^+_\gamma)/B(E2{:}\,2^+_\gamma \to 0^+_{\rm g.s})$	0.39
$B(E2{:}\,0^+_{\gamma\gamma} \to 2^+_\gamma)/B(E2{:}\,2^+_\gamma \to 0^+_{\rm g.s})$	1.96

IV. DISCUSSION

A comprehensive theoretical study of the low-energy structure of well-deformed nuclei was carried out for even-even nuclei from $Z = 10$ to $Z = 110$ using the Hartree-Fock-Bogoliubov theory extended by the generator coordinate method and mapped onto a five-dimensional collective quadrupole Hamiltonian (5DCH) by Delaroche *et al.* [62]. This approach is able to distinguish and to separate between excited 0^+ bands that are β vibrations from those that are, in fact, due to coexisting minima on the basis of relative quadrupole transition strengths. Reference [62] has a calculated data set of over 1710 even-even nuclei, and the authors find the shape coexistence phenomenon to be more prevalent but characterize the first excited $K^\pi = 0^+$ band in ^{152}Sm to be a β vibration. The 5DCH calculations for the ^{156}Gd nucleus result in a β deformation of 0.347 and a $4^+/2^+$ energy ratio of 3.27. The experimental $4^+/2^+$ energy ratio is 3.24. The calculated first excited $K^\pi = 0^+$ and $K^\pi = 2^+$ bandheads are at 1274 and 1159 keV, in comparison to the experimental values at 1049.48 and 1129.40 keV, respectively. The calculations of Delaroche *et al.* [62] place the first excited $K^\pi = 0^+$ band above the $K^\pi = 2_1^+$ band and predict $B(E2{:}\,2^+_{K^\pi=0_2^+} \to 0^+_{\rm g.s.}) = 0.0253$ e^2b^2 in comparison with the measured range of $0.0028 \to 0.0057$ e^2b^2. The theoretical predication is nine to four times larger than the experimental $B(E2)$ value range.

Perhaps the most important structure indicators are the intra- and interband $E2$-reduced transition probabilities. This paper provides measured absolute transition probabilities for both types of transitions in ^{156}Gd to allow for experimental comparison. The reduced matrix elements extracted from measurements for the transitions depopulating the $K^\pi = 0_2^+$ average between 0.163 and 0.241 eb, whereas the intraband

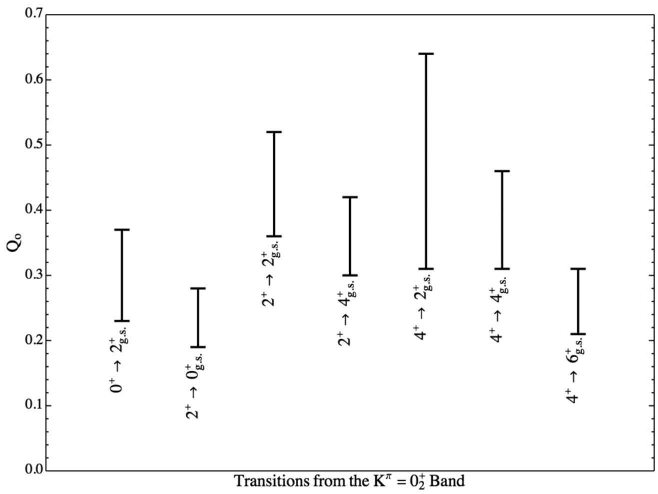

FIG. 5. The intrinsic quadrupole moment for all the transitions depopulating the first excited $K^\pi = 0^+$ band extracted from the upper and lower ranges of the $B(E2)$ values determined from the lifetime measurements in this paper shown in Table III.

reduced matrix element is between 1.87 and 2.77 eb and dividing by $\mathbf{e}$ yields a range of 1.28 → 1.94 b [63] for the matrix elements extracted from the measurements.

The ground-state $B(E2{:}\,2^+_{\rm g.s.} \to 0^+_{\rm g.s.})$ value yields a matrix element of 1.9 b from the Coulomb excitation [64]. The $B(E2)$ strength of transitions from each of the levels of this $K^\pi = 0^+_2$ band to the ground-state band is evidence that this lowest-lying $K^\pi = 0^+$ band is a collective excitation built on the ground state and perhaps, most likely, the β-vibrational one-phonon band albeit with less collectivity than expected by the 5DCH approach.

The viability of the first excited $K^\pi = 0^+$ band as a β vibration is dependent on a consistent intrinsic matrix element for the interband transitions. The $B(E2)$ values for transitions between $K_1, K_2 = 0$ bands considered here are given by the equation,

$$B(E2{:}\,J_1 \to J_2) = (J_1 020|J_2 0)^2 \frac{5}{4\pi} e^2 Q_0^2. \quad (1)$$

The measured ranges of $B(E2)$ values presented in the results table are used to plot the extracted intrinsic quadrupole moments shown in Fig. 5. Ideally, we would expect a complete overlap of the error bars for the values of the intrinsic quadrupole moment, but the results vary slightly and yield a value of approximately 0.3 b, indicating that the first excited $K^\pi = 0^+$ band is a potential β excitation built on the ground state.

An interpretation by Leviatan *et al.* [46] has proposed the use of partial dynamical symmetry (PDS) as a selection criterion for states in a given nucleus that obey a specific symmetry whereas others break the symmetry strongly. They presented their calculations for ^{156}Gd and made available calculated $B(E2)$ values presuming that the first excited $K^\pi = 0^+$ band is a β vibration. These calculations, in comparison with measured values are shown in Table V. The PDS-calculated $B(E2{:}\,0^+_{K^\pi=0^+_2} \to 2^+_{\rm g.s.}) = 0.034\text{-}e^2\text{b}^2$ values in comparison with a measured range of 0.021 → 0.055 e^2b^2; and $B(E2{:}\,2^+_{K^\pi=0^+_2} \to 0^+_{\rm g.s.}) = 0.0055\ e^2\text{b}^2$ values in comparison to the measured range of 0.0028 → 0.0057 e^2b^2 are in the same order as expected for a β vibration. The transitions depopulating the $K^\pi = 0^+_4$ band at 1715.181 keV show strong connections to the $K^\pi = 2^+_\gamma$ band and are evidence that the $K^\pi = 0^+_4$ band is a collective excitation and perhaps the $K^\pi = 0^+\ \gamma\gamma$ two-phonon vibrational band. The dynamic moments of inertia for the various excitation bands in ^{156}Gd show nearly identical slopes with the exception of the $K^\pi = 0^+$ band at 1168.190 keV, indicating that the excitations are oscillations around the equilibrium-deformed shape of the nucleus. The $K^\pi = 0^+_2$, $K^\pi = 2^+$, the $K^\pi = 4^+$, and the $K^\pi = 0^+_4$ band at 1715.181 keV all have identical dynamic moments of inertia as shown in Fig. 4.

The focus on nuclear structure has been whether two-phonon excitations can occur with negative anharmonicity or a ratio of less than 2. The ^{166}Er nucleus is the only other case of a known case of a two-phonon $K^\pi = 0^+_{\gamma\gamma}$ and $K^\pi = 4^+_{\gamma\gamma}$ set of vibrations built on the γ band where the excitation energy ratios of $K^\pi = 4^+_{\gamma\gamma}$, $K^\pi = 0^+_{\gamma\gamma}$ excitations [65] to the $K^\pi = 2^+_\gamma$ excitations are 2.6 and 2.5, respectively, in comparison with the expected values of 2. In ^{156}Gd, the energy ratio of the $(K^\pi = 0^+_{\gamma\gamma})/(K^\pi = 2^+_{\rm g.s.})$ bandheads is 1.49, well below the expected value of 2.0, and the ratio of $B(E2)$ values for $B(E2{:}\,0^+_{\gamma\gamma} \to 2^+_\gamma)/B(E2{:}\,2^+_\gamma \to 0^+_{\rm g.s})$ is 1.96 well below the pure geometric expectation value of 5. In a study [66] of ^{232}Th, a $K^\pi = 4^+_{\gamma\gamma}$ band to the

TABLE V. Comparison of measured $B(E2)_{\rm exp}$ values and theoretical $B(E2)$ values; PDS-calculated values are from Ref. [46] for the first excited 0^+ band $K^\pi = 0^+_2$.

K_i^π, J_i^π	E_x (keV)	E_γ (keV)	K_i^π, J_i^π	$B(E2)_{\rm exp}$ (e^2b^2)	PDS (e^2b^2)
$0^+_2, 0^+$	1049.479	960.510	$0^+_{\rm g.s.}, 2^+$	0.021 → 0.055	0.034
$0^+_2, 2^+$	1129.440	1129.423	$0^+_{\rm g.s.}, 0^+$	0.0028 → 0.0057	0.0055
		1040.474	$0^+_{\rm g.s.}, 2^+$	0.016 → 0.031	0.0084
		841.243	$0^+_{\rm g.s.}, 4^+$	0.018 → 0.037	0.02
$0^+_2, 4^+$	1297.825	1208.875	$0^+_{\rm g.s.}, 2^+$	0.0047 → 0.011	0.0067
		1009.622	$0^+_{\rm g.s.}, 4^+$	0.0098 → 0.022	0.0067
		713.104	$0^+_{\rm g.s.}, 6^+$	0.0080 → 0.018	0.021
		168.382	$0^+_2, 2^+$	1.00 → 2.19	0.951

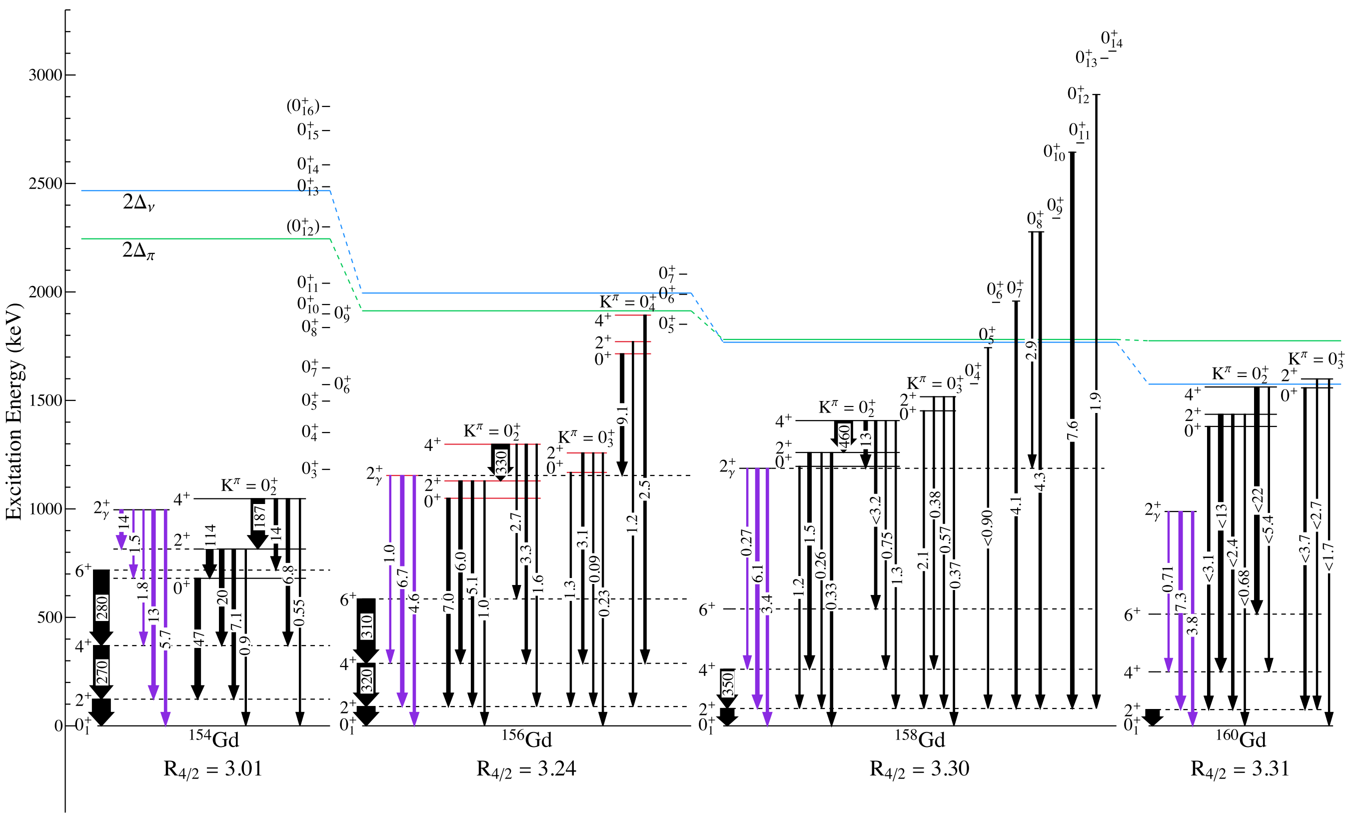

FIG. 6. Partial level schemes for $^{154-160}$Gd showing the lowest $K^\pi = 2^+$ bands with the known 0^+ states, transition probabilities, and the two-proton ($\Delta\pi$) and two-neutron ($\Delta\nu$) pairing gaps shown as horizontal lines. The width of the transition lines depopulating the levels of interest are in proportion to the transition probability in W.u.

γ band ratio of 1.8 was observed with a $B(E2)$ ratio of 3.1 ± 0.4. A similar observation [28] was made in the two-phonon $K^\pi = 0^+_{\beta\beta}$ to the one-phonon $K^\pi = 0^+_\beta$ energy ratio for the ^{178}Hf nucleus where the energy ratio is 1.5 instead of 2. The growing number of negative anharmonicity (less than 2) for two-phonon oscillations warrants further examination.

V. SUMMARY

To summarize, 12 lifetimes were measured from levels in ^{156}Gd. Six of these are new measurements, whereas the other six were previously measured lifetimes that agree with our data. The $B(E2)$ values have been calculated using the intensities and electron conversion coefficients of Klora *et al.* [47]. The results show that the $K^\pi = 0^+_2$ band is strongly connected to the ground-state band with matrix elements of the same size as those of the $K^\pi = 2^+_1$ band showing that the first excited $K^\pi = 0^+_2$ band is collectively built on the ground-state band and may, in fact, be the β-vibrational band. The transitions from the $K^\pi = 0^+_3$ band to the ground-state band are less strong, and the dynamic moment of inertia for this band varies fairly significantly from the other excitations built on the g.s. band as shown in Fig. 3. Further evidence comes from the slopes of the dynamic moments of inertia of all the four $K^\pi = 0^+$ bands. PDS calculations indicate that the first excited 0^+ is the collective β-vibrational excitation and the measured $B(E2)$ values for transitions from the first excited $K^\pi = 0^+_2$ band are in good agreement with calculated values as shown in Table V. The 5DCH calculations [62] predict greater collectivity for the β vibration as discussed in the discussion section. Quantum fluctuation studies [25] of deformed nuclei and collective 0^+ excitations indicate two excited $K^\pi = 0^+$ bands built on the ground-state equilibrium shape with one- and two-node wave functions that point to one- and two-phonon excitations in ^{156}Gd. The two-phonon excitation in this case is the $\gamma\gamma$ $K^\pi = 0^+$ band.

The $K^\pi = 4^+_1$ band is lower in excitation energy than would be expected of a $\gamma\gamma$ two-phonon excitation, but it does show considerable strength for the $K^\pi = 2^+_1$ band. This can, of course, just be a reflection of ΔK preference. The excitation energy ratio of the $K^\pi = 4^+_1$ band with the single $K^\pi = 2^+_1$ γ band is significantly less than the expected value of 2. The $K^\pi = 0^+_4$ band also does not show strong transitions to the ground-state band but does have very collective transitions to the $K^\pi = 2^+_1$, γ band. Furthermore, the slopes of the dynamic moments of inertia for the $K^\pi = 4^+_1$ band, the $K^\pi = 2^+_1$ band, and the $K^\pi = 0^+_4$ band are quite similar indicating a common origin. The $K^\pi = 0^+_4$ band is most likely the $K^\pi = 0^+_{\gamma\gamma}$ multiphonon excitation or at the very least the $\gamma\gamma$ multiphonon excitation is a significant element of the band's character. Figure 6 shows the systematics of the deformed Gd nuclei from $A = 154$–160. In ^{154}Gd, the first excited $K^\pi = 0^+$ band is much lower than the $K^\pi = 2^+$ band with strong $B(E2)$ values connecting it to the ground state. There are numerous 0^+ states identified in this nucleus, but they lack lifetime measurements. The ^{156}Gd presented in this paper shows a low-lying first excited $K^\pi = 0^+$ band near the $K^\pi = 2^+$ band with equivalent collectivity in $B(E2)$ values connecting it to the g.s. band. In ^{158}Gd, the collective 0^+ state was shown to be higher than the pairing gap, and, in ^{160}Gd, there are only limits for the lifetimes of the known 0^+ states. The indications point to the first excited $K^\pi = 0^+$ band in ^{156}Gd to be a β vibration of the deformed ground state.

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