Fragmentation binding energies and cross sections of isotopes near the proton dripline

Yi-Dan Song (宋一丹),[1] Hui-Ling Wei (魏慧玲),[1] and Chun-Wang Ma (马春旺)[1,2,3,*]
[1]*College of Physics and Materials Science, Henan Normal University, Xinxiang 453007, China*
[2]*Institute of Particle and Nuclear Physics, Henan Normal University, Xinxiang 453007, China*
[3]*Shanghai Institute of Applied Physics, Chinese Academy of Sciences, Shanghai 201800, China*

(Received 25 May 2018; published 22 August 2018)

An exponential correlation has been observed between the cross section σ and the average binding energy $\langle B' \rangle$ of proton-rich isotopes measured in the 650A MeV ^{58}Ni + ^{9}Be projectile fragmentation reaction. Based on the correlation, both the cross section and the binding energy of the isotope, which is close to the proton-drip, can be predicted. The binding energy of $Z = 21$–28 isotopes with T_z from -3 to -2, have been determined from their measured cross sections. And the cross sections for isotopes without experimental results have been predicted by adopting the evaluated binding energy in AME16. The determined binding energies of isotopes in the 650A MeV ^{58}Ni + ^{9}Be reaction have also been used to study the isotopes production in the 140A MeV ^{58}Ni + ^{9}Be reaction. The determined binding energies of the isotopes are verified to obey a scaling phenomenon of mirror nuclei.

DOI: 10.1103/PhysRevC.98.024620

I. INTRODUCTION

The nuclei at the proton and neutron drip lines are important to test nuclear models in extreme conditions. Many new phenomena, which are very different to the β-stable nucleus, have been found in the nuclei around the proton drip line, including the exotic proton-halo or proton-skin structure, the β-delayed one-, two-, or multi-proton emission, shell evolution, etc. [1–13]. Besides, the nuclei that lie on the proton-drip line are very important in nuclear astrophysics because of their importance in the study of the energy and mass production and the astrophysical nucleosynthesis process including the rp process [14]. The properties of the proton-rich nucleus near the proton drip line, such as the binding energy B, the one-proton S_p or two-proton S_{2p} separation energy, the particle emission [12,13], and the deformation, continuously attract interest. However, it is difficult to perform experimental studies because of their extremely short lifetimes and low production probabilities. The improved radioactive-ion-beam techniques with highly improved quality provide us the new opportunity to learn about nuclei near the proton drip line [15–20].

It is believed that the isotopic cross-section distribution is correlated to its average binding energy in a quite simple manner [21]. This correlation was further developed to study the neutron separation energy [22]. An experimental study of the near-proton-drip-line isotopes was carried out by Blank *et al.* at the projectile-fragment separator (FRS), GSI, in which the 650A MeV ^{58}Ni projectile was bombarded on the ^{9}Be target [23]. The fragments of the $Z = 21$–28 isotopes have been identified by the $\Delta E - \text{TOF} - B\rho$ techniques. The cross sections for the nuclides with neutron excess $I (\equiv N - Z)$ from -6 to 2 (or isospin T_z from -3 to 1) of the $Z = 21$–28 elements have been measured, which makes it possible to study their properties and may extend our knowledge to more proton-rich isotopes. The measured cross sections of the $Z = 21$–28 proton-rich isotopes in the 650A MeV ^{58}Ni + ^{9}Be reaction provide the chance to determine their binding energy, which are not experimentally reported in the latest atomic mass evaluation (AME16) table [24]. In this article, the binding energy of the proton-rich $Z = 21$–28 isotopes will be determined from the measured cross sections, and the cross sections for the more proton-rich isotopes will also be predicted from the correlation obtained.

II. METHODS

In extracting the binding energy of the neutron-rich $^{76-79}$Cu isotopes, which have been measured in the 64A MeV ^{86}Kr + ^{9}Be reaction, the isotopic cross-section distributions for the neutron-rich isotope were found to exponentially depend on the average binding energy per nucleon $\langle B' \rangle$ [21,25],

$$\sigma = C\exp[(\langle B' \rangle - 8)/\tau], \quad (1)$$

where C and τ are free parameters. $\langle B' \rangle = (B - \varepsilon_p)/A$, with ε_p being the pairing energy,

$$\varepsilon_p = 0.5[(-1)^N + (-1)^Z]\varepsilon_0 A^{-3/4}. \quad (2)$$

ε_p is introduced to minimize the odd-even staggering in the isotopic distribution, and $\varepsilon_0 = 30$ MeV has been chosen [21]. The value is not adjusted to keep its consistence in theory. In the next section, it will be first verified that the correlation in Eq. (1) can be applied to the $Z = 21$ to 28 proton-rich isotopes measured in the 650A MeV ^{58}Ni + ^{9}Be reaction.

*machunwang@126.com

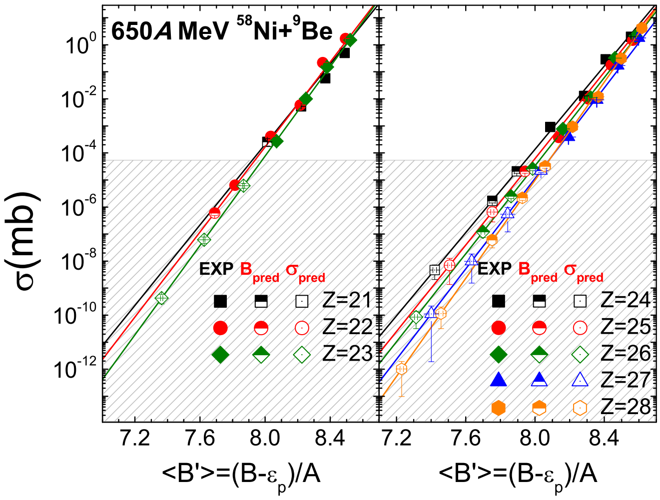

FIG. 1. The correlation between σ and $\langle B'\rangle$ for the proton-rich $Z = 21$–28 isotopes in the 650A MeV ^{58}Ni + ^{9}Be reaction. The lines are the fitting results using Eq. (1). The full symbols denote the experimental σ and $\langle B'\rangle$. The half-full symbols denote the calculated $\langle B'\rangle$ using the experimental σ, and the open symbols denote the calculated σ using the theoretical $\langle B'\rangle$.

III. RESULTS AND DISCUSSION

In Fig. 1, the correlation between the isotopic cross section and the average binding energy is shown, for which the measured isotopes of $Z = 21$ to 28 in the 650A MeV ^{58}Ni + ^{9}Be [23] are denoted by the solid and open symbols. The fitting results show that the correlation in Eq. (1) is well obeyed in the proton-rich $Z = 21$ to 28 isotopes. This makes it possible to predict the binding energy and cross section for unknown isotopes once C and τ are determined [26]. For the $T_z = -2$ to -3 nuclides, of which the binding energy is not experimentally determined yet, the values of $\langle B'\rangle$ (and thus B) are predicted by the experimental cross sections (see the half-full symbols). For the isotopes unmeasured in the reaction, with T_z from $-3/2$ to -4, the cross sections are also predicted (denoted by the open symbols) by adopting the systematically evaluated binding energy in AME16 [24].

The extracted values for B/A and B, together with the evaluated binding energy B^* in AME16 [24] are listed in Table I. A quantity $\delta B \equiv B - B^*$ is defined to show the difference between the binding energy extracted in this work and that in AME16. It is seen that the extracted binding energy of isotopes is similar to the evaluated binding energy in AME16 ($\delta B < 1$ MeV), but some results have relatively large differences with respect to the results in AME16 ($\delta B > 2$ MeV).

It will be verified whether the determined binding energies are accurate. A strong scaling phenomenon in the difference between the binding energy of mirror nuclei [$|I|$ ($|T_z|$) from 1 (1/2) to 4 (2)] has been found in Ref. [27] (see the open symbols in Fig. 2), which shows that the difference between the mass of mirror nuclei is linearly correlated to their difference in Coulomb energy,

$$M(N-|I|,Z)-M(N,Z-|I|) = a_c\delta_{\text{coul}} + a_{sh}\delta_{sh} + |I|(M_p - M_n), \qquad (3)$$

where M denotes the mass, $a_{sh}\delta_{sh}$ is defined to modify the shell effect on the separation energy, and M_p (M_n) is the mass of a proton (neutron). a_c is the Coulomb energy coefficient, and $\delta_{\text{coul}} = |I|(A-|I|)^{2/3}$. The scaling phenomenon provides a chance to check the determined binding energy of the more asymmetric nuclei. The correlations between $M(N-|I|,Z) - M(N,Z-|I|)$ and δ_{coul} for the predicted $|I|$ ($|T_z|$) from 4 (2) to 6 (3), are plotted in Fig. 2 as the solid symbols. For the $|I| = 4$ ($|T_z| = 2$) nuclei, the predicted results agree well with the correlation obtained from the experimental binding energy in AME12 [28]. For the $|I| = 5$ ($|T_z| = 5/2$) and $|I| = 6$ ($|T_z| = 3$) nuclei, the predicted results also obey the linear

TABLE I. The values of B/A and B (in MeV) of the $Z = 21$–28 isotopes with $T_z = -2$ and -3 determined by the measured cross section in the 650A MeV ^{58}Ni + ^{9}Be reaction. B^* denotes the evaluated B in AME16 [24]. $\delta B = B - B^*$.

^AZ	T_z	B/A	B	B^*	δB
$^{39}_{22}$Ti	$-5/2$	7.6895 ± 0.0214	299.8909		
$^{43}_{24}$Cr	$-5/2$	7.7531 ± 0.0177	333.3837	330.240	3.1437
$^{44}_{24}$Cr	-2	7.9347 ± 0.0146	349.1268	349.712	-0.5852
$^{46}_{25}$Mn	-2	7.9047 ± 0.0245	363.5931	364.274	-0.6809
$^{46}_{26}$Fe	-3	7.7363 ± 0.0109	355.8709	350.014	5.8569
$^{47}_{26}$Fe	$-5/2$	7.8613 ± 0.0161	369.4788	365.895	3.5838
$^{48}_{26}$Fe	-2	8.0203 ± 0.0162	384.9765	385.104	-0.1276
$^{50}_{27}$Co	-2	8.0022 ± 0.0320	400.1095	400.050	0.0595
$^{50}_{28}$Ni	-3	7.7839 ± 0.0221	389.1940	385.800	3.3940
$^{51}_{28}$Ni	$-5/2$	7.9271 ± 0.0190	404.2806	401.625	2.6556
$^{52}_{28}$Ni	-2	8.0876 ± 0.0320	420.5564	420.108	0.4484

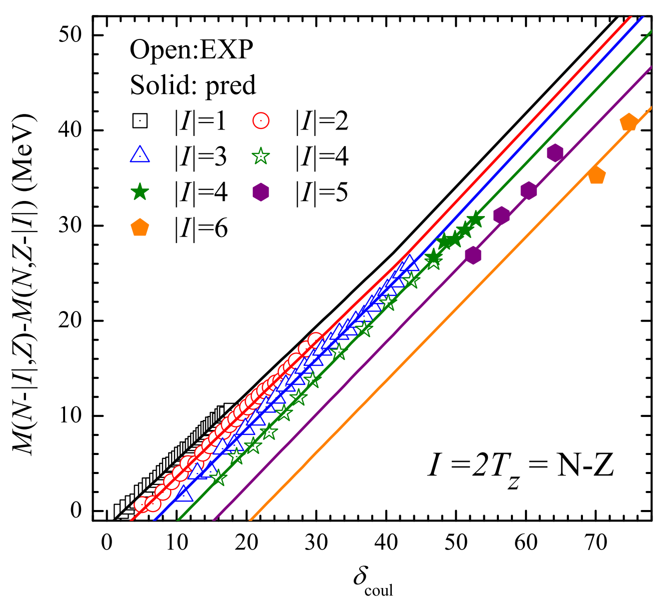

FIG. 2. The correlation between $M(N-|I|,Z) - M(N,Z-|I|)$ and δ_{coul} for the mirror nuclei. The open symbols denote the same results in Ref. [27] obtained from the experimental binding energy in AME12 [28]. The solid symbols denote the $|I| = 4$, 5, and 6 nuclei predicted in this work.

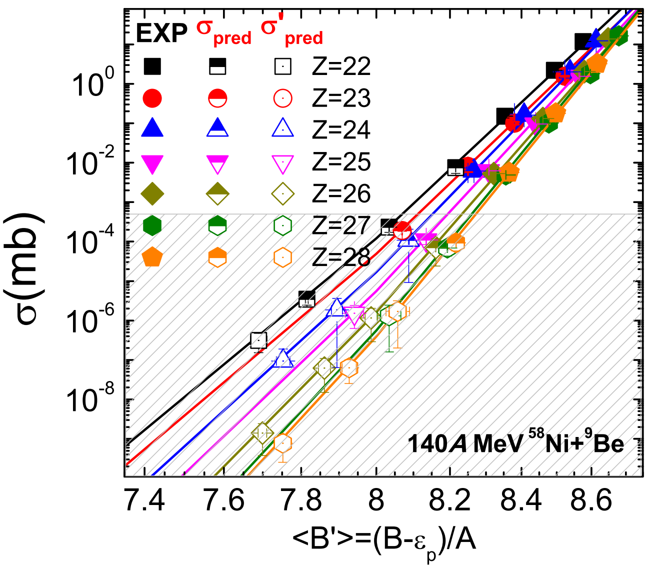

FIG. 3. The predicted σ for the proton-rich isotopes in the 140A MeV ^{58}Ni + ^{9}Be reaction. The full symbols denote that both the σ and B are experimentally determined. The half-full ones denote that σ are predicted by experimental $\langle B'\rangle$ by using Eq. (1). The open ones denote that σ are predicted by $\langle B'\rangle$ determined in the 650A MeV ^{58}Ni + ^{9}Be reaction as listed in Table I. The lines denote the fitting by Eq. (1).

correlation. It thus indicates that the scaling phenomenon for the mirror nuclei is well obeyed even by the very asymmetric $I = -6$ nuclei, and also indicates that the extracted binding energy of the $I = -4$ ($T_z = -2$), $I = -5$ ($T_z = -5/2$), and $I = -6$ ($T_z = -3$) isotopes are reasonable.

To further verify the predicted binding energy of isotopes in the 650A MeV ^{58}Ni + ^{9}Be reaction, they are applied to the measured fragments in the 140A MeV ^{58}Ni + ^{9}Be reaction [29,30]. In Fig. 3, it shows that the isotopic distributions in the 140A MeV ^{58}Ni + ^{9}Be reaction well obey Eq. (1). The correlation between σ and $\langle B'\rangle$ for the $Z = 22$–28 isotopes includes the following data:

(1) The full symbols, denoting fragments of T_z from -1 to 1/2, indicate that both the σ and $\langle B'\rangle$ are experimental data.
(2) The half-full symbols, denoting the fragments of $T_z = -1$ to -2, indicate that the experimental binding energy in AME16 are adopted and σ is predicted.
(3) The open symbols denote that σ is predicted using $\langle B'\rangle$ that have been determined in the 650A MeV ^{58}Ni + ^{9}Be reaction.

The predicted cross section for the isotopes in the 140A MeV ^{58}Ni + ^{9}Be reaction have also been listed in Table II.

At last, the predicted cross section for fragments in the 140A and 650A MeV ^{58}Ni + ^{9}Be reactions are compared in Fig. 4. It is seen that the distributions for the $T_z = -3/2$ and -2 fragments are very similar, which are in the order of 10^{-4} and 10^{-6} mb, respectively. An obvious staggering has been shown in the $T_z = -5/2$, -3 fragments. The cross section for the $T_z = -7/2$ fragments are very close to 10^{-10} mb, which are near the lower limit in experiments. The predicted cross sections for the $Z = 21$–28 isotopes suggest that it is possible to study them in high precision by using storage-ring techniques [18] since isotopes with similar cross section have been successfully studied. It should be noted that some methods have also been proposed to precisely predict the cross section of very asymmetric isotopes, for example, the FRACS [31] and its improved version for very-proton-rich isotopes [32].

TABLE II. The predicted σ (in mb) for the $Z = 21$–28 isotopes with $T_z = -1$ and -4 in the 650A MeV ^{58}Ni + ^{9}Be and 140A MeV ^{58}Ni + ^{9}Be reactions using Eq. (1). The star in $\langle B'\rangle$ denotes that the binding energy in AME16 is the evaluated data.

^{A}Z	T_z	$\langle B'\rangle$	σ(mb) $E = 650A$ MeV
$^{39}_{21}$Sc	$-3/2$	8.0135	$(2.5957 \pm 0.77) \times 10^{-4}$
$^{40}_{23}$V	-3	7.3642*	$(4.2696 \pm 1.64) \times 10^{-10}$
$^{41}_{23}$V	$-5/2$	7.6250*	$(6.1259 \pm 1.65) \times 10^{-8}$
$^{42}_{23}$V	-2	7.8673*	$(6.1739 \pm 1.27) \times 10^{-6}$
$^{42}_{24}$Cr	-3	7.4207*	$(4.6663 \pm 2.60) \times 10^{-9}$
$^{44}_{25}$Mn	-3	7.5069*	$(7.0410 \pm 5.67) \times 10^{-9}$
$^{45}_{25}$Mn	$-5/2$	7.7530*	$(6.5598 \pm 3.75) \times 10^{-7}$
$^{45}_{26}$Fe	$-7/2$	7.3130*	$(8.5275 \pm 5.37) \times 10^{-11}$
$^{47}_{27}$Co	$-7/2$	7.4010*	$(1.1006 \pm 1.08) \times 10^{-10}$
$^{48}_{27}$Co	-3	7.6343*	$(9.8801 \pm 8.34) \times 10^{-9}$
$^{49}_{27}$Co	$-5/2$	7.8420*	$(5.4201 \pm 4.21) \times 10^{-7}$
$^{48}_{28}$Ni	-4	7.2307*	$(1.0172 \pm 0.92) \times 10^{-12}$
$^{49}_{28}$Ni	$-7/2$	7.4570*	$(1.1522 \pm 0.83) \times 10^{-10}$
^{A}Z	T_z	$\langle B'\rangle$	σ(mb) $E = 140A$ MeV
$^{39}_{22}$Ti	$-5/2$	7.6895*	$(3.1396 \pm 1.61) \times 10^{-7}$
$^{40}_{22}$Ti	-2	7.8149	$(3.4662 \pm 1.01) \times 10^{-6}$
$^{41}_{22}$Ti	$-3/2$	8.0344	$(2.3273 \pm 0.62) \times 10^{-4}$
$^{42}_{22}$Ti	-1	8.2160	$(7.5400 \pm 2.16) \times 10^{-3}$
$^{43}_{23}$V	$-3/2$	8.0695	$(1.9087 \pm 0.08) \times 10^{-4}$
$^{43}_{24}$Cr	$-5/2$	7.7531*	$(9.3777 \pm 9.51) \times 10^{-8}$
$^{44}_{24}$Cr	-2	7.8948*	$(1.8537 \pm 1.79) \times 10^{-6}$
$^{45}_{24}$Cr	$-3/2$	8.0877	$(1.0778 \pm 0.99) \times 10^{-4}$
$^{46}_{25}$Mn	-2	7.9411*	$(1.5233 \pm 0.90) \times 10^{-6}$
$^{47}_{25}$Mn	$-3/2$	8.1353	$(1.1045 \pm 0.26) \times 10^{-4}$
$^{46}_{26}$Fe	-3	7.6991*	$(1.3964 \pm 1.02) \times 10^{-9}$
$^{47}_{26}$Fe	$-5/2$	7.8613*	$(6.2155 \pm 4.70) \times 10^{-8}$
$^{48}_{26}$Fe	-2	7.9861*	$(1.1609 \pm 0.87) \times 10^{-6}$
$^{49}_{26}$Fe	$-3/2$	8.1613	$(7.0718 \pm 4.65) \times 10^{-5}$
$^{50}_{27}$Co	-2	8.0341*	$(1.3020 \pm 1.14) \times 10^{-6}$
$^{51}_{27}$Co	$-3/2$	8.1933	$(6.9125 \pm 2.62) \times 10^{-5}$
$^{50}_{28}$Ni	-3	7.7520*	$(7.7218 \pm 5.17) \times 10^{-10}$
$^{51}_{28}$Ni	$-5/2$	7.9271*	$(6.3142 \pm 3.78) \times 10^{-8}$
$^{52}_{28}$Ni	-2	8.0578*	$(1.6928 \pm 1.49) \times 10^{-6}$
$^{53}_{28}$Ni	$-3/2$	8.2171	$(9.2958 \pm 3.45) \times 10^{-5}$

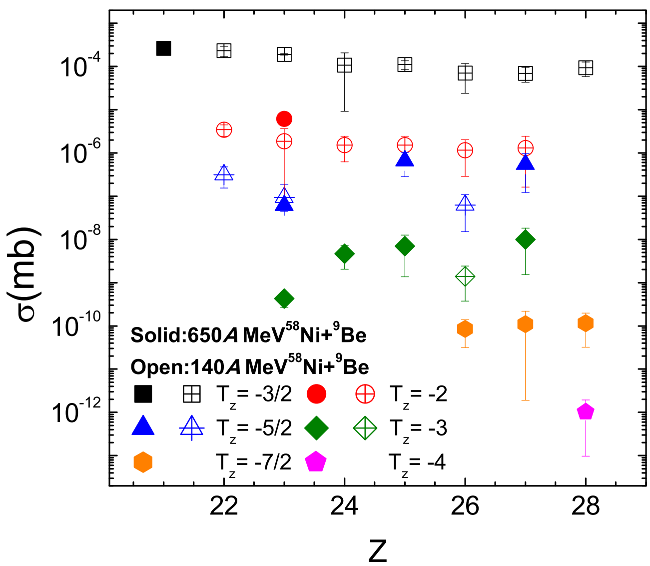

FIG. 4. The predicted σ for the proton-rich isotopes in the 140A MeV ^{58}Ni + ^{9}Be and 650A MeV ^{58}Ni + ^{9}Be reaction. The full and open symbols denote the ^{58}Ni + ^{9}Be reaction at 650A and 140A MeV, respectively.

Basing on the results both for the neutron-rich [21,22] and proton-rich isotopes, it is verified that the empirical correlation between σ and $\langle B' \rangle$ for fragment is well obeyed by fragments produced in the projectile fragmentation reactions. It thus provides a general method to predict the binding energy or cross section of fragments in experiments.

IV. SUMMARY

In this article, it is verified that the empirical formula between isotopic cross section and the average binding energy is generally obeyed in proton-rich isotopes which are produced in the projectile fragmentation reactions. The measured cross sections for proton-rich isotopes, with T_z from $T_z = -2$ to -3, have been adopted in the analysis. Basing on the predicted binding energy of the $T_z = -2$, $-5/2$, and -3 nuclides, it is proven that the scaling phenomenon in the mirror nuclei is well obeyed up to $T_z = -3$. The determined binding energy of the proton-rich nuclide in 650A MeV ^{58}Ni + ^{9}Be reaction are further checked in the 140A MeV ^{58}Ni + ^{9}Be reaction, indicating that the extracted binding energy is reasonable. The empirical formula in Eq. (1) thus provides a general method to determine the binding energy and also to predict the cross section for both very-proton-rich and very-neutron-rich isotopes (as shown in Ref. [21]).

ACKNOWLEDGMENTS

The discussions with Prof. Z. G. Xiao, Y. M. Zhao, S. G. Zhou, and H. Jiang are acknowledged. We thank M. Bao for providing us her data for mirror nuclei. This work was supported by the National Natural Science Foundation of China (Grants No. 11421505 and No. U1732135), Major State Basic Research Development Program in China (Grant No. 2014CB845401), Natural Science Foundation of Henan Province (Grant No. 162300410179), and Henan Normal University for the Excellent Youth (Grant No. 154100510007).

[1] D. Q. Fang *et al.*, Phys. Rev. C **76**, 031601(R) (2007).
[2] Y. G. Ma *et al.*, Phys. Lett. B **743**, 306 (2015).
[3] B. Blank and M. J. G. Borge, Prog. Part. Nucl. Phys. **60**, 403 (2008).
[4] M. Pomorski *et al.*, Phys. Rev. C **83**, 014306 (2011).
[5] M. Pomorski *et al.*, Phys. Rev. C **90**, 014311 (2014).
[6] L. Audirac *et al.*, Eur. Phys. J. A **48**, 179 (2012).
[7] B. Blank and M. Płoscajczak, Rep. Prog. Phys. **71**, 046301 (2008).
[8] B. A. Brown, R. R. C. Clement, H. Schatz, A. Volya, and W. A. Richter, Phys. Rev. C **65**, 045802 (2002).
[9] N. Paar *et al.*, Phys. Lett. B **624**, 195 (2005).
[10] V. N. Tarasov *et al.*, Int. J. Mod. Phys. E **17**, 1273 (2008).
[11] Z. Dlouhý, Int. J. Mod. Phys. E **15**, 1471 (2006).
[12] T. Goigoux *et al.*, Phys. Rev. Lett. **117**, 162501 (2016).
[13] L. V. Grigorenko, T. A. Golubkova, J. S. Vaagen, and M. V. Zhukov, Phys. Rev. C **95**, 021601(R) (2017).
[14] H. Schatz *et al.*, Phys. Rep. **294**, 167 (1998).
[15] Y. H. Zhang *et al.*, Phys. Rev. Lett. **109**, 102501 (2012).
[16] X. Xu *et al.*, Phys. Rev. Lett. **117**, 182503 (2016).
[17] F. Bosch, Prog. Part. Nucl. Phys. **73**, 84 (2013).
[18] P. Shuai *et al.*, Phys. Lett. B **735**, 327 (2014).
[19] L. V. Grigorenko *et al.*, Phys. Usp. **59**, 321 (2016).
[20] C. W. Ma and Y. G. Ma, Prog. Part. Nucl. Phys. **99**, 120 (2018).
[21] M. B. Tsang *et al.*, Phys. Rev. C **76**, 041302(R) (2007).
[22] M. Yu, H.-L. Wei, Y.-D. Song, and C.-W. Ma, Chin. Phys. C **41**, 094001 (2017).
[23] B. Blank, S. Andriamonje, R. Del Moral, *et al.*, Phys. Rev. C **50**, 2398 (1994).
[24] M. Wang *et al.*, Chin. Phys. C **41**, 030003 (2017).
[25] M. Mocko *et al.*, Europhys. Lett. **79**, 12001 (2007).
[26] Y.-D Song, H.-L. Wei, and C.-W. Ma, Chin. Phys. C **42**, 074102 (2018).
[27] M. Bao, Y. Lu, Y. M. Zhao, and A. Arima, Phys. Rev. C **94**, 044323 (2016).
[28] M. Wang, G. Audi, A. H. Wapstra *et al.*, Chin. Phys. C **36**, 1603 (2012).
[29] M. Mocko *et al.*, Phys. Rev. C **74**, 054612 (2006).
[30] M. Mocko, Ph.D. thesis, Michigan State University, 2006 (unpublished).
[31] B. Mei, Phys. Rev. C **95**, 034608 (2017).
[32] Y.-D. Song, H.-L. Wei, C.-W. Ma *et al.*, Nucl. Sci. Tech. **29**, 96 (2018).