

## Beam-energy dependence of the relativistic retardation effects of electrical fields on the $\pi^-/\pi^+$ ratio in heavy-ion collisions

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In this article we investigate the beam-energy dependence of relativistic retardation effects of electrical fields on the single and double  $\pi^-/\pi^+$  ratios in three heavy-ion reactions with an isospin- and momentum-dependent transport model IBUU11. With the beam energy increasing from 200 to 400 MeV/nucleon, effects of the relativistically retarded electrical fields on the  $\pi^-/\pi^+$  ratio are found to increase gradually from negligible to considerably significant as expected; it is, however, an interesting observation that the relativistic retardation effects of electrical fields on the  $\pi^-/\pi^+$  ratio become gradually insignificant as the beam energy further increases from 400 to 800 MeV/nucleon. That is to say, with the beam energy increasing, the competition of enhanced anisotropic features of retarded electrical fields and reduced duration time of the reactions gets effects of the relativistically retarded electrical fields on the  $\pi^-/\pi^+$  ratio to be maximum around 400 MeV/nucleon. Therefore, the relativistic retardation effects of electrical fields should be carefully considered in heavy-ion collisions at intermediate energy especially around 400 MeV/nucleon when using the  $\pi^-/\pi^+$  ratio as the probe of nuclear symmetry energy. Moreover, we also investigate the isospin dependence of relativistic retardation effects of electrical fields on the  $\pi^-/\pi^+$  ratio in two isobar reaction systems of  $^{96}\text{Ru} + ^{96}\text{Ru}$  and  $^{96}\text{Zr} + ^{96}\text{Zr}$  at the beam energies from 200 to 800 MeV/nucleon. It is shown that the relativistic retardation effects of electrical fields on the  $\pi^-/\pi^+$  ratio are independent of the isospin of reaction. Furthermore, we also examine the double  $\pi^-/\pi^+$  ratio in reactions of  $^{96}\text{Zr} + ^{96}\text{Zr}$  over  $^{96}\text{Ru} + ^{96}\text{Ru}$  at the beam energies from 200 to 800 MeV/nucleon with the static field and retarded field, respectively. It is shown that the double  $\pi^-/\pi^+$  ratio from two reactions is still an effective observable of symmetry energy without the interference of electrical field due to using the relativistic calculation compared to the nonrelativistic calculation.

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### I. INTRODUCTION

The determination of equation of state (EoS) of isospin asymmetric nucleonic matter has been a long-standing interest in nuclear physics and nuclear astrophysics due to its importance in understanding the properties of radioactive nuclei and evolution of supernova and neutron stars [1–4]. As an important tool in terrestrial laboratories, heavy-ion collisions (HICs) with neutron-rich nuclei provide a unique opportunity to form directly the dense and isospin asymmetric nucleonic matter, and thus comparisons of the theoretical simulations of isospin observables with the corresponding experimental measurement enable us to extract the EoS of isospin asymmetric nucleonic matter. Actually, many observables have been proposed as the promising isospin signals to detect the isospin ingredients of nuclear effective interactions. These

include the isospin fractionation [5,6], the isospin diffusion [7,8], the neutron-proton transverse difference flow [9], the neutron-proton correlation function [10], the  $\pi^-/\pi^+$  ratio [11,12], the  $K^0/K^+$  ratio [13], the elliptic flow [14,15], the dynamical dipole oscillators [16,17], etc. However, compared to the achieved significant progress on the EoS of symmetric nucleonic matter [15], the knowledge on the EoS of isospin asymmetric nucleonic matter is still rather little, especially for the isospin asymmetric nucleonic matter at high baryonic density [18–20].

Presently, the most uncertain part of EoS of isospin asymmetric nucleonic matter is the nuclear symmetry energy at high densities. For example, through comparing the pion observable with the FOPI data [21], the isospin-dependent Boltzmann-Uehling-Uhlenbeck (IBUU) [22] model and isospin-dependent Boltzmann-Langevin (IBL) [23] model favor a supersoft prediction for the high-density behavior of nuclear symmetry energy, but a completely opposite prediction, i.e., a superstiff symmetry energy at high densities is supported by the

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Lanzhou quantum molecular dynamics (LQMD) model [24]. While comparing the elliptic flow with the data from the FOPI-LAND collaborators [25] and/or ASY-EOS experiments [26], a moderately soft symmetry energy at high densities is concluded from predictions using both the ultrarelativistic quantum molecular dynamics (UrQMD) [26–28] model and the Tübingen quantum molecular dynamics (TüQMD) [29] model. This is mainly due to the fact that the isovector potential of nuclear effective interactions is much weaker than that of the isoscalar one, and thus the corresponding isospin signals are usually interfered with by other factors during experimental measurement and theoretical simulations. It is therefore the covariance analysis of these isospin observables [30] and using some strategies [31] to maximize the effects of symmetry energy on them are necessary, except for the experimental efforts [21,25,26,32]. Certainly, to better constrain the high-density behavior of nuclear symmetry energy, the transport reaction theory communities have been making efforts to compare their codes under controlled conditions to better understand the origins of discrepancies among widely used transport models, and expect to extract reliable information about the EoS of isospin asymmetric matter using these isospin observables from HICs at the intermediate energy [33–36].

The ratio  $\pi^-/\pi^+$  of charged pions is found to be one of the most promising isospin observables, especially concerning the aspects of probing the high-density behavior of nuclear symmetry energy using HICs [11,12,21–24,31,37–39]. Nevertheless, some recent studies on the pion observables have shown clearly that the pion potential [40–45], the uncertainty of the  $\Delta$  isovector potential [46,47], the neutron skins of colliding nuclei [48], and the isospin-dependent high-momentum tails of nucleons induced by the short-range correlations [49–52] can all interfere appreciably the sensitivities of the  $\pi^-/\pi^+$  ratio of charged pions in probing the nuclear symmetry energy using HICs. Therefore, before making a final conclusion about the high-density behavior of nuclear symmetry energy through comparing theoretical simulations of the pion observable with the data, one should check all of the possible uncertain factors in theoretical simulations of this observable, and thus to get it clean as much as possible in probing the symmetry energy using HICs.

Very recently, we have shown that the relativistically retarded electrical fields created by fast-moving charged particles are anisotropic compared to the isotropic static Coulomb fields as normally used in most of the transport models, and affect significantly the  $\pi^-/\pi^+$  ratio in a typical reaction of  $^{197}\text{Au} + ^{197}\text{Au}$  collision at a beam energy of 400 MeV/nucleon available in several laboratories [53]. As a subsequent consideration, one naturally assumes that the relativistic retardation effects of electrical fields created by these fast-moving charged particles on the  $\pi^-/\pi^+$  ratio may increase with increasing the beam energy of HICs. To answer this question precisely, in this article we perform the Au + Au collisions at the beam energies from 200 to 800 MeV/nucleon to show the beam-energy dependence of relativistic retardation effects of electrical field on the  $\pi^-/\pi^+$  ratio. As expected, with the beam energy increasing from 200 to 400 MeV/nucleon, effects of the relativistically retarded electrical fields on the  $\pi^-/\pi^+$  ratio increase gradually from negligible to considerably significant; it is however, an

interesting observation that the relativistic retardation effects of electrical fields on the  $\pi^-/\pi^+$  ratio become gradually insignificant as the beam energy further increasing from 400 to 800 MeV/nucleon due to the competition of reduced duration time of the reaction and the enhanced anisotropic features of retarded electrical fields. Moreover, we also investigate the isospin dependence of relativistic retardation effects of electrical fields on the  $\pi^-/\pi^+$  ratio in two isobar reaction systems of  $^{96}\text{Ru} + ^{96}\text{Ru}$  and  $^{96}\text{Zr} + ^{96}\text{Zr}$  at the beam energies from 200 to 800 MeV/nucleon. It is shown that the relativistic retardation effects of electrical fields on the  $\pi^-/\pi^+$  ratio are independent of the isospin of reaction. Furthermore, we also examine the double  $\pi^-/\pi^+$  ratio from reactions of  $^{96}\text{Zr} + ^{96}\text{Zr}$  over  $^{96}\text{Ru} + ^{96}\text{Ru}$  at the beam energies from 200 to 800 MeV/nucleon with the static field and retarded field, respectively. It is shown that the double  $\pi^-/\pi^+$  ratio in two reactions can not only effectively eliminate effects of the electrical fields with the relativistic calculation compared to the nonrelativistic calculation but also sustain the sensitivities to symmetry energy, and thus is still an effective observable of symmetry energy in HICs at intermediate energy.

## II. THE MODEL

Similar to our recent work [53], the present study is carried out within the isospin- and momentum-dependent Boltzmann-Uehling-Uhlenbeck transport model [54,55] of the IBUU11 version. In this model, the nuclear mean-field interaction is expressed as

$$\begin{aligned}
 U(\rho, \delta, \vec{p}, \tau) &= A_u(x) \frac{\rho_{-\tau}}{\rho_0} + A_l(x) \frac{\rho_\tau}{\rho_0} + \frac{B}{2} \left( \frac{2\rho_\tau}{\rho_0} \right)^\sigma (1-x) \\
 &+ \frac{2B}{\sigma+1} \left( \frac{\rho}{\rho_0} \right)^\sigma (1+x) \frac{\rho_{-\tau}}{\rho} \left( 1 + (\sigma-1) \frac{\rho_\tau}{\rho} \right) \\
 &+ \frac{2C_{\tau,\tau}}{\rho_0} \int d^3 p' \frac{f_\tau(\vec{p}')}{1 + (\vec{p} - \vec{p}')^2 / \Lambda^2} \\
 &+ \frac{2C_{\tau,-\tau}}{\rho_0} \int d^3 p' \frac{f_{-\tau}(\vec{p}')}{1 + (\vec{p} - \vec{p}')^2 / \Lambda^2}, \quad (1)
 \end{aligned}$$

and the parameters  $A_l(x)$ ,  $A_u(x)$  are expressed in the forms of

$$A_l(x) = A_{l0} - \frac{2B}{\sigma+1} \left( \frac{(1-x)\sigma(\sigma+1) - \frac{1+x}{2}}{4} \right), \quad (2)$$

$$A_u(x) = A_{u0} + \frac{2B}{\sigma+1} \left( \frac{(1-x)\sigma(\sigma+1) - \frac{1+x}{2}}{4} \right). \quad (3)$$

It should be mentioned that in the IBUU11 version of this model we have updated the expression of nuclear mean-field interaction and/or readjusted the corresponding parameters to consider more accurately the spin-isospin dependence of in-medium effective many-body forces by distinguishing the density dependencies of  $nn$ ,  $pp$ , and  $np$  interactions in the effective three-body force term [56,57], as well as the high-momentum behaviors of the nucleon optical potential extracted from nucleon-nucleus scattering experiments [58]. The values of these parameters used in this study are determined as  $A_{l0}(x) = -76.963$  MeV,  $A_{u0}(x) = -56.963$  MeV,  $B = 141.963$  MeV,

$C_{\tau,\tau} = -57.209$  MeV,  $C_{\tau,-\tau} = -102.979$  MeV,  $\sigma = 1.2652$ , and  $\Lambda = 2.424 p_{f0}$  where  $p_{f0}$  is the nucleon Fermi momentum in symmetric nuclear matter at normal density  $\rho_0 = 0.16$  fm $^{-3}$ . Using the above single particle potential, one can derive the symmetry energy for a specific parameter  $x$ , which is introduced to mimic the different forms of symmetry energy predicted by various many-body theories without changing any property of symmetric nuclear matter and the value of symmetry energy  $E_{\text{sym}}(\rho_0) = 30.0$  MeV at saturation density.

On the other hand, the relativistically retarded electric fields created by fast-moving charged particles in HICs are calculated according to the well-known Liénard-Wiechert expression

$$e\vec{E}(\vec{r}, t) = \frac{e^2}{4\pi\epsilon_0} \sum_n Z_n \frac{c^2 - v_n^2}{(cR_n - \vec{R}_n \cdot \vec{v}_n)^3} (c\vec{R}_n - R_n\vec{v}_n), \quad (4)$$

where  $Z_n$  is the charge number of the  $n$ th particle, and  $\vec{R}_n = \vec{r} - \vec{r}_n$  is the position of the field point  $\vec{r}$  relative to the source point  $\vec{r}_n$  where the  $n$ th particle is moving with velocity  $\vec{v}_n$  at the retarded time of  $t_n = t - |\vec{r} - \vec{r}_n|/c$ . It is necessary to mention that to calculate the retarded electric fields  $e\vec{E}(\vec{r}, t)$ , the phase space histories of all charged particles before the moment  $t$  have to be saved in transport model simulations. Moreover, a precollision phase space history for all nucleons is made assuming that they are frozen in the projectile and target moving along their Coulomb trajectories. More technical details about calculating the  $e\vec{E}(\vec{r}, t)$  and numerical checks can be found in Ref. [59], whereas, in the nonrelativistic limit  $v_n \ll c$ , Eq. (4) naturally reduces to the static Coulomb field of the form

$$e\vec{E}(\vec{r}, t) = \frac{e^2}{4\pi\epsilon_0} \sum_n Z_n \frac{\vec{R}_n}{R_n^3}. \quad (5)$$

As has been shown in our recent work [53] in Au + Au collision at 400 MeV/nucleon the maximum difference between the retarded and static fields is located in the reaction ( $X$ - $o$ - $Z$ ) plane at the maximum compression stage. Therefore, we plot in the upper window of Fig. 1 the strength  $|e\vec{E}|$  contours of the static and retarded electric fields at the maximum compression stage in the  $X$ - $o$ - $Z$  reaction plane at three beam energies of 200, 400, and 800 MeV/nucleon. It is obvious to see the most important difference between them is the nonisotropic nature of the retarded electrical fields created by fast-moving charged particles, i.e., transversely enhanced by a factor  $\gamma = 1/(1 - \beta^2)^{1/2}$  while longitudinally reduced by  $1/\gamma^2$ . Certainly, the retardation effect depends on the reduced velocity  $\beta = v/c$  of charged particles and thus depends on the beam energies. In fact, in the relativistic case, all charged particles with velocity  $\vec{v}_n$  at the retarded time  $t_n$  contribute to the  $e\vec{E}(\vec{r}, t)$  at the instant  $t$  and location  $\vec{r}$ ; whereas in the nonrelativistic case, those charged particles contribute to the  $e\vec{E}(\vec{r}, t)$  just at the same moment  $t$ . More specifically, due to the longer retardation time of the reaction at the lower beam energy compared to the case at the higher beam energy, the retarded electrical field shows more weaker strength than those of static field at 200 MeV/nucleon; whereas in the higher beam energy, the stronger a Lorentz contraction gets the retarded

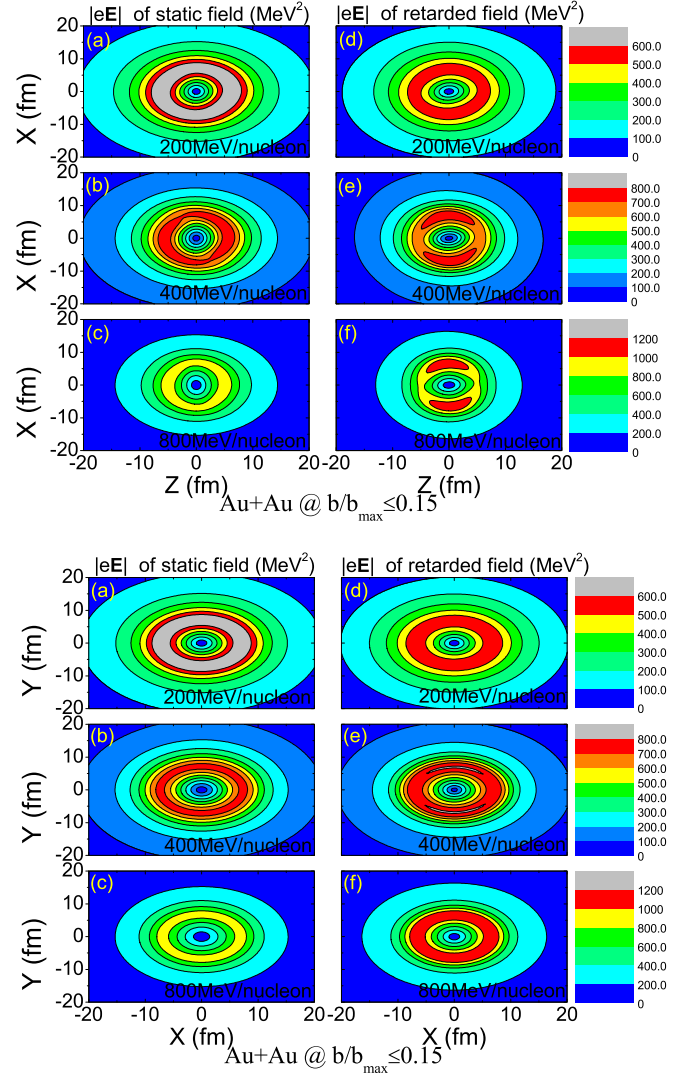


FIG. 1. Contours of the electric fields in the  $X$ - $o$ - $Z$  reaction plane (upper) and  $X$ - $o$ - $Y$  plane (lower) at the maximum compression stage in central Au + Au collisions at the beam energies of 200, 400, and 800 MeV/nucleon. Panels (a)–(c) are for the static fields while panels (d)–(f) are for the retarded fields.

electrical fields show more clearly a nonisotropic feature at 800 MeV/nucleon. However, in the  $X$ - $o$ - $Y$  plane, which is perpendicular to the beam direction and thus has the symmetric electrical components in  $X$  and  $Y$  directions, the nonisotropic feature is invisible in retarded electrical fields as shown in the lower window of Fig. 1. Certainly, the electrical fields in the relativistic calculations stem from all the charged particles at the retarded time  $t_n$ , whereas those in the nonrelativistic calculations are just from those of charged particles at the same moment  $t$ ; it is thus the weaker (stronger) strength in retarded field than static field at lower (higher) beam energy of 200 (800) MeV/nucleon that is observed in  $X$ - $o$ - $Y$  plane as shown in the lower window of Fig. 1.

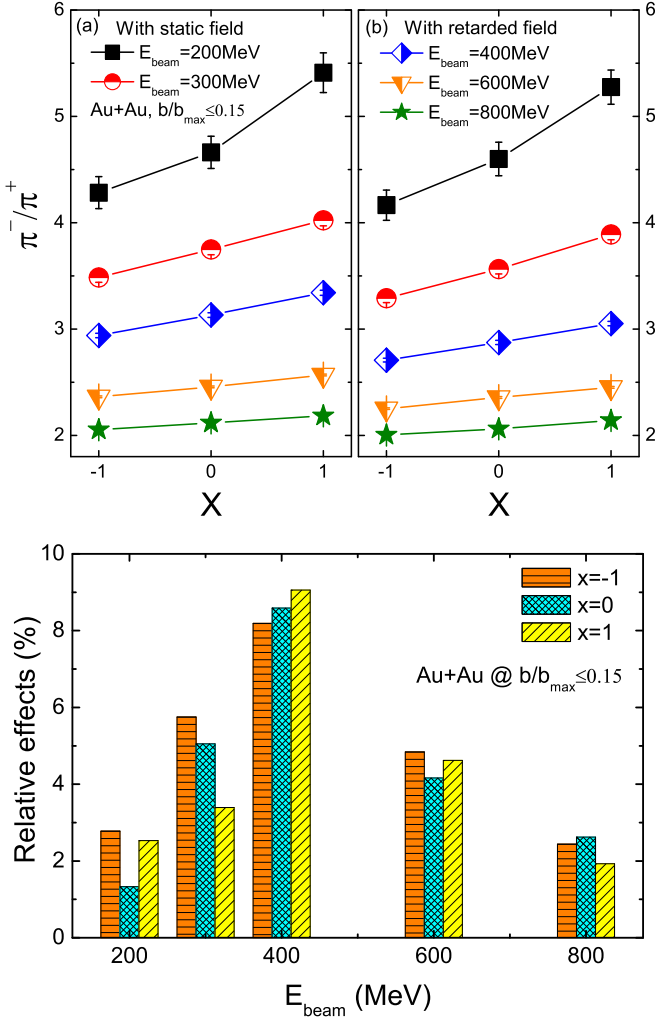


FIG. 2. Upper: The final  $\pi^-/\pi^+$  ratio in central Au + Au collisions as a function of beam energy with three symmetry energy settings from the hard  $x = -1$  to soft  $x = 1$ . Lower: The relative effects of relativistically retarded electrical fields on the final  $\pi^-/\pi^+$  ratio shown in the same reactions.

### III. RESULTS AND DISCUSSIONS

Now, we turn to the beam-energy dependence of relativistic retardation effects of electrical fields on the  $\pi^-/\pi^+$  ratio. Shown in the upper window of Fig. 2 is the final  $\pi^-/\pi^+$  ratio in central Au + Au collisions at the beam energies from 200 to 800 MeV/nucleon with the symmetry energy settings from the hard  $x = -1$  to soft  $x = 1$ . First, consistent with the established systematics for pion production [21], the  $\pi^-/\pi^+$  ratio decreases with the beam energy increase and shows more sensitivities to the density dependent nuclear symmetry energy at the lower beam energy especially around and below the pion production threshold regardless of how the electrical fields are calculated. Second, it can be found that the final  $\pi^-/\pi^+$  ratio is smaller on the whole with the retarded electric fields than those with the static ones in collisions at all beam energies studied here. Third, with the beam energy increasing from 200 to 400 MeV/nucleon, the relativistic retardation effects of electrical fields on  $\pi^-/\pi^+$  ratio are found to increase gradually from

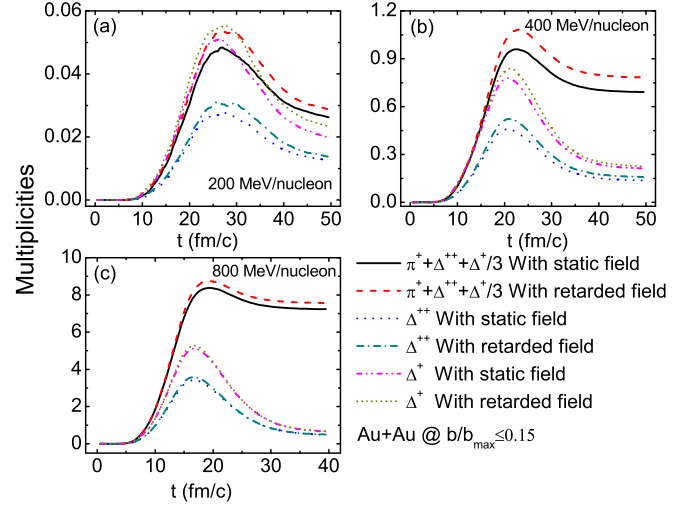


FIG. 3. Evolution of dynamical multiplicities of  $\pi^+$  (i.e.,  $\pi^+ + \Delta^{++} + \frac{1}{3}\Delta^+$ ),  $\Delta^{++}$ , and  $\Delta^+$  resonances in central Au + Au collisions at the beam energies of 200, 400, and 800 MeV/nucleon, respectively.

negligible to considerably significant as expected; however, one can also see that the relativistic retardation effects of electrical fields on the  $\pi^-/\pi^+$  ratio are becoming gradually insignificant as the beam energy further increases from 400 to 800 MeV/nucleon. More specifically, the relative effect of retarded electrical fields on the ratio  $\pi^-/\pi^+$  of charged pions can reach maximally about 8–9% at 400 MeV/nucleon but less than about 3% at both 200 and 800 MeV/nucleon as shown in the lower window of Fig. 2. Nevertheless, the more prominent relativistic effects of charged particles moving with higher speed get one naturally assuming that the effects of relativistically retarded electrical fields on the  $\pi^-/\pi^+$  ratio increase as the beam energy increases. Therefore, how to understand this interesting observation is the main task we shall discuss in the following.

For pion production in HICs at intermediate energies, almost all pions are from the decay of  $\Delta(1232)$  resonances. As a result, the multiplicities of dynamical  $\pi^-$  and  $\pi^+$  during reactions are determined as  $\pi^- + \Delta^- + \frac{1}{3}\Delta^0$  and  $\pi^+ + \Delta^{++} + \frac{1}{3}\Delta^+$  according to the decay mechanism of  $\Delta(1232)$  resonances. Of course, all the  $\Delta$  resonances will eventually decay into nucleons and pions at the end of the reaction. Naturally, the  $\pi^+ + \Delta^{++} + \frac{1}{3}\Delta^+$  and  $\pi^- + \Delta^- + \frac{1}{3}\Delta^0$  will become  $\pi^+$  and  $\pi^-$  mesons at the end of the reactions. However, to better understand how the retarded electrical fields affect the  $\Delta(1232)$  resonances during reactions and thus the multiplicities of charged pions and their ratio  $\pi^-/\pi^+$  at the end of the reactions, it is necessary to trace the dynamical production of  $\Delta(1232)$  resonances and their contributions to the multiplicities of dynamical  $\pi^-$  and  $\pi^+$ . Shown in Figs. 3 and 4 are the dynamical multiplicities of  $\pi^+$  (i.e.,  $\pi^+ + \Delta^{++} + \frac{1}{3}\Delta^+$ ) and  $\pi^-$  (i.e.,  $\pi^- + \Delta^- + \frac{1}{3}\Delta^0$ ) and the corresponding  $\Delta(1232)$  resonances with different charge states in central Au + Au collisions at three beam energies of 200, 400, and 800 MeV/nucleon. Obviously, regardless of the  $\Delta^{++}$  or  $\Delta^+$  resonances, the retarded fields can increase their multiplicities compared to those with the static field.



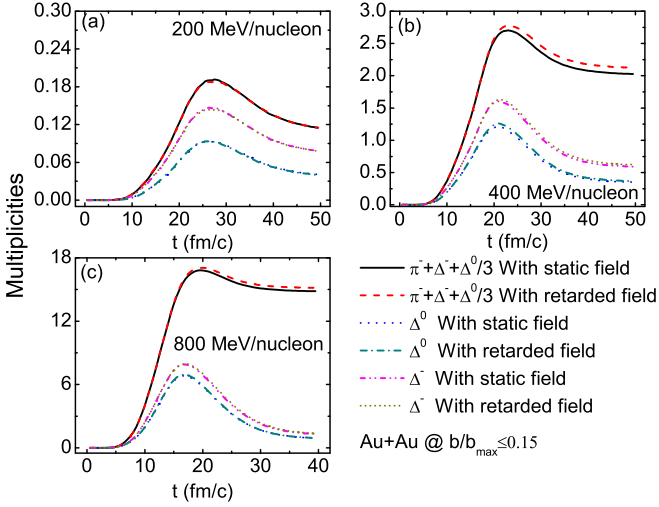


FIG. 4. Evolution of dynamical multiplicities of  $\pi^-$  (i.e.,  $\pi^- + \Delta^- + \frac{1}{3}\Delta^0$ ),  $\Delta^-$ , and  $\Delta^0$  resonances in central Au + Au collisions at the beam energies of 200, 400, and 800 MeV/nucleon, respectively.

Naturally, the corresponding dynamical multiplicities of  $\pi^+$  (i.e.,  $\pi^+ + \Delta^{++} + \frac{1}{3}\Delta^+$ ) are larger with the retarded field than those with the static field. However, the increasing of multiplicities of  $\Delta^-$  and  $\Delta^0$  resonances is very little and only obviously visible at the beam energy of 400 MeV/nucleon. As a result, the corresponding dynamical multiplicities of  $\pi^-$  (i.e.,  $\pi^- + \Delta^- + \frac{1}{3}\Delta^0$ ) are tiny larger with the retarded field than those with the static field. This is the reason why we see in Fig. 2 that the ratio  $\pi^-/\pi^+$  of charged pions is smaller on the whole with the retarded electrical fields than those with the static ones. On the other hand, while the nonisotropic feature of retarded electrical fields and thus their instantaneous effects on both  $\Delta^{++}$  and  $\Delta^+$  resonances at the compression stage increase with the beam energy increasing, the corresponding duration time of compression stage of the reaction decreases. Consequently, the competition of these two factors gets the effects of retarded fields on the final multiplicities of  $\pi^+$  and thus the  $\pi^-/\pi^+$  ratio to be maximum around 400 MeV/nucleon.

Certainly, one may ask why and how the multiplicities of the  $\Delta(1232)$  resonances, especially the  $\Delta^{++}$  and  $\Delta^+$  resonances, are affected by the retarded fields. To answer this question, we investigate the relative change in nucleon kinetic energy distributions due to using the retarded electrical fields compared to the static ones. For this purpose, we examine the ratio

$$R^i = \frac{\text{number}(i)_R}{\text{number}(i)_S}, \quad i \equiv \text{neutron or proton} \quad (6)$$

of nucleons with local densities higher than  $\rho_0$  at the maximum compression stage in the Au + Au reactions with the retarded ( $R$ ) and static ( $S$ ) electrical fields. Shown in Fig. 5 are  $R^n$  and  $R^p$  as a function of nucleon kinetic energy in Au + Au collisions at three beam energies of 200, 400, and 800 MeV/nucleon. Obviously, for the energetic nucleons above a certain kinetic energy depending on the beam energy of reactions, the values of both  $R^n$  and  $R^p$  are larger than 1, indi-

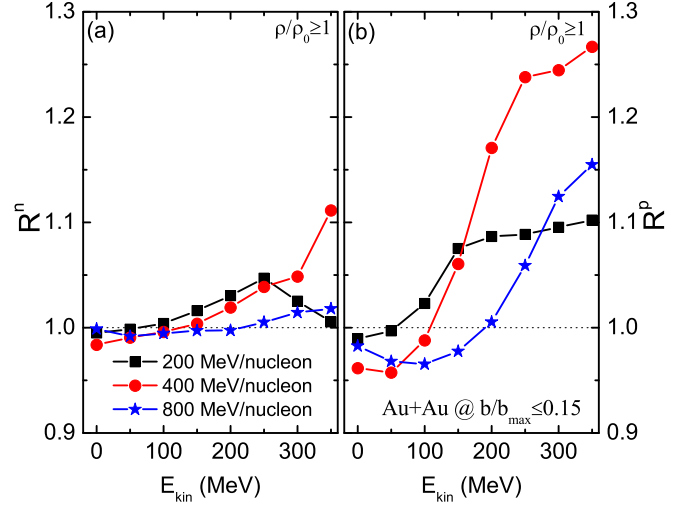


FIG. 5. The ratio  $R^n$  for neutrons and  $R^p$  for protons at suprasaturation densities at the maximum compression stage as a function of nucleon kinetic energy in central Au + Au collisions at the beam energies of 200, 400, and 800 MeV/nucleon, respectively.

ating that the retarded fields increase (decrease) the number of high (low) energy nucleons. These increased energetic protons (neutrons) naturally increase some  $pp$  ( $nn$ ) inelastic collisions and thus increase the multiplicities of  $\Delta^{++}$  and  $\Delta^+$  ( $\Delta^-$  and  $\Delta^0$ ) through  $p + p \rightarrow \Delta^{++} + n$  and  $p + p \rightarrow \Delta^+ + p$  ( $n + n \rightarrow \Delta^- + p$  and  $n + n \rightarrow \Delta^0 + n$ ) reaction channels, thus leading to more  $\pi^+$  ( $\pi^-$ ) mesons through the decay of these resonances. Certainly, one may suspect why the kinetic energy distribution of neutrons is also affected slightly by the retarded electrical fields as shown in Fig. 5. Actually, the neutrons are not affected directly by the electrical fields; however, secondary collisions between neutrons and energetic protons provide the chance for these neutrons to increase their kinetic energies at a lower level compared to those of protons. Nevertheless, as the secondary effects the increased  $\Delta^-$  and  $\Delta^0$  and thus the  $\pi^-$  at the final reaction stage are relative fewer than those of  $\pi^+$ , and thus the final  $\pi^-/\pi^+$  ratio is smaller on the whole with the retarded fields than those with the static ones. Again, with the beam energy increasing, the competition between the enhanced nonisotropic feature of retarded electrical fields and reduced duration time of compression stage of the reaction gets the relativistic retardation effects of electrical field on nucleon kinetic energy distribution to be maximum around 400 MeV/nucleon as shown in Fig. 5.

Next, we investigate the isospin dependence of relativistic retardation effects of electrical fields on the  $\pi^-/\pi^+$  ratio. For this purpose, we show in Fig. 6 the final  $\pi^-/\pi^+$  ratio in two isobar reaction systems of  $^{96}\text{Ru} + ^{96}\text{Ru}$  and  $^{96}\text{Zr} + ^{96}\text{Zr}$  at beam energies from 200 to 800 MeV/nucleon. Again, similar to the above in the Au + Au reaction, two observations can be found in reactions of  $^{96}\text{Ru} + ^{96}\text{Ru}$  and  $^{96}\text{Zr} + ^{96}\text{Zr}$ . The first is that the  $\pi^-/\pi^+$  ratio shows more sensitivities to the density dependent symmetry energy at the lower beam energy, especially around and below the pion production threshold, regardless of how the electrical fields are calculated.

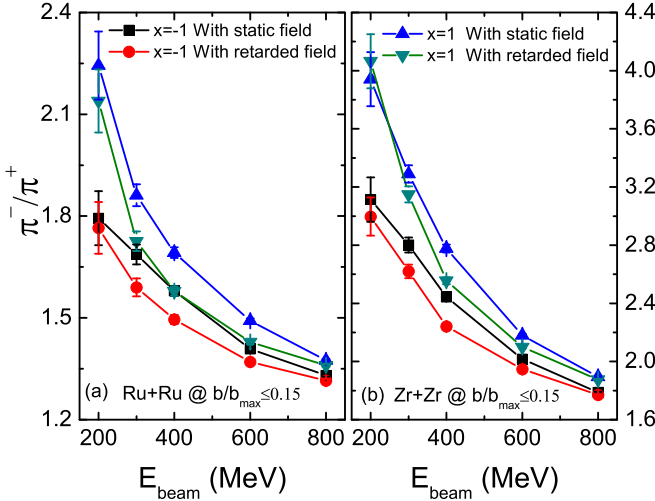


FIG. 6. The final  $\pi^-/\pi^+$  ratio in two isobar reaction systems of  $^{96}\text{Ru} + ^{96}\text{Ru}$  (a) and  $^{96}\text{Zr} + ^{96}\text{Zr}$  (b) at the beam energies from 200 to 800 MeV/nucleon. The hard symmetry energy with parameter  $x = -1$  and the soft one with parameter  $x = 1$  are used.

The second is that the influences of relativistically retarded electrical fields on the  $\pi^-/\pi^+$  ratio are more apparent around 400 MeV/nucleon. Moreover, it is seen that the retardation effects of electrical fields on the  $\pi^-/\pi^+$  ratio are independent of the isospin of reaction.

Actually, the relativistic retardation effects of electrical field on the ratio  $\pi^-/\pi^+$  of charged pions are not desired to be seen when using this ratio as the probe of symmetry energy in HICs since a key step in determining the nuclear symmetry energy is the determination of experimental observables which can serve as clean and sensitive probes [60,61]. Moreover, considering the fact that the isovector potentials are generally smaller than the isoscalar potential at the same nuclear density, the search of experimental observables or their combination which have the maximum effects of isovector potentials without other interferences, such as the Coulomb effects and/or isoscalar effects, etc., is a crucial task in probing the nuclear symmetry energy using HICs. The double ratio of two reactions as the candidate of such kind of observables, such as the double  $n/p$  ratio [62,63] and double  $\pi^-/\pi^+$  ratio [64], is naturally expected to disentangle the effects of symmetry energy from those of electrical fields due to using the relativistic calculation compared to the nonrelativistic calculation, because the double ratio of two reactions has the advantage of reducing the influence of both the Coulomb fields and the systematic errors, etc. [62]. Shown in Fig. 7 is the double  $\pi^-/\pi^+$  ratio of reactions  $^{96}\text{Zr} + ^{96}\text{Zr}$  over  $^{96}\text{Ru} + ^{96}\text{Ru}$  at the beam energies from 200 to 800 MeV/nucleon with the static field and retarded field, respectively. It is seen that the double  $\pi^-/\pi^+$  ratio can indeed eliminate effectively the effects of electrical fields due to using the relativistic calculation compared to the nonrelativistic calculation, but also sustain the sensitivities to symmetry energy. It can thus be concluded that the double  $\pi^-/\pi^+$  ratio in two reactions is still an effective observable of symmetry energy in HICs at intermediate energy.

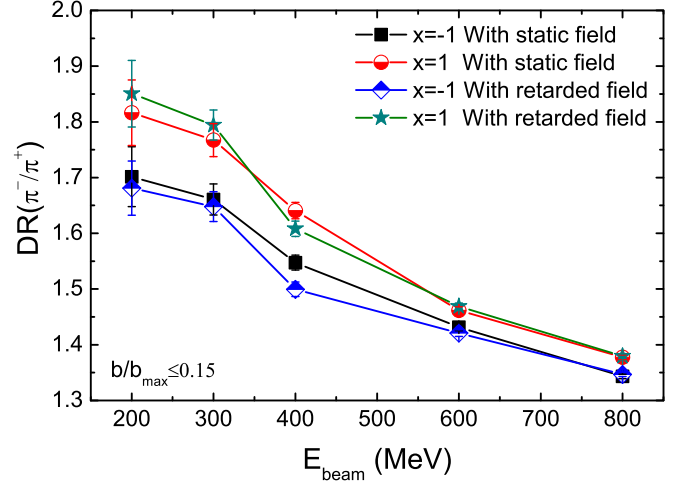


FIG. 7. The double  $\pi^-/\pi^+$  ratio [i.e.,  $\text{DR}(\pi^-/\pi^+)$ ] of reactions  $^{96}\text{Zr} + ^{96}\text{Zr}$  over  $^{96}\text{Ru} + ^{96}\text{Ru}$  at the beam energies from 200 to 800 MeV/nucleon with the static field and retarded field, respectively.

#### IV. SUMMARY

In summary, we have investigated the beam-energy dependence of relativistic retardation effects of electrical fields on the single and double  $\pi^-/\pi^+$  ratios in three heavy-ion reactions. It is shown that the relativistic retardation effects of electrical fields on the  $\pi^-/\pi^+$  ratio increase first and then decrease gradually with the beam energy increasing from 200 to 800 MeV/nucleon. Specifically, the relativistic retardation effects of electrical fields on the  $\pi^-/\pi^+$  ratio are found to be maximum at the beam energy around 400 MeV/nucleon due to the competition of anisotropic features of retarded electrical fields and duration time of the reactions. Therefore, the relativistic retardation effects of electrical fields should be carefully considered in HICs at intermediate energy especially around 400 MeV/nucleon when using the  $\pi^-/\pi^+$  ratio as the probe of nuclear symmetry energy. Moreover, we have also studied the isospin dependence of relativistic retardation effects of electrical fields on the ratio  $\pi^-/\pi^+$  of charged pions in two isobar reactions of  $^{96}\text{Ru} + ^{96}\text{Ru}$  and  $^{96}\text{Zr} + ^{96}\text{Zr}$  at the beam energies from 200 to 800 MeV/nucleon. It is found that the relativistic retardation effects of electrical fields on the  $\pi^-/\pi^+$  ratio are independent of the isospin of reaction. Furthermore, we found that the double  $\pi^-/\pi^+$  ratio of reactions  $^{96}\text{Zr} + ^{96}\text{Zr}$  over  $^{96}\text{Ru} + ^{96}\text{Ru}$  at the beam energies from 200 to 800 MeV/nucleon can not only eliminate effectively the effects of electrical fields due to using the relativistic calculation compared to the nonrelativistic calculation, but also can sustain the sensitivities to symmetry energy, and thus can still be as an effective observable of nuclear symmetry energy in HICs at intermediate energy. Before ending this work, we give a useful remark here. In full covariant transport theories, it has been confirmed that the relativistic mechanisms related to the covariant nature of the nuclear fields can also contribute to the isovector channel and thus affect the sensitivities of collective flows of nucleons in probing the symmetry energy using HICs [65]. Therefore, it is also deserved to study how these relativistic mechanisms related to the covariant nature of

the nuclear fields affect the sensitivities of a pion observable in probing the nuclear symmetry energy in HICs at intermediate energies.

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