Matter radii of $^{16-23}$N

H. T. Fortune
Department of Physics and Astronomy, University of Pennsylvania, Philadelphia, Pennsylvania 19104, USA

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I have used a simple model to compute matter radii for $^{16-23}$N, paying special attention to the configurations of the valence neutron(s). I compare results with recent predictions and with earlier matter radii extracted from measurements of interaction cross sections. The present calculations are closer to the experimental radii than any of the results from the other procedures.

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I. INTRODUCTION

The topic of matter radii for nuclei near the neutron dripline is of current interest. These are usually extracted from measured interaction cross sections, using some form of Glauber model. In many light nuclei, nuclei near the end of the stability line, e.g., ^{21,22}C, ^{22,23}N, ^{23,24}O, exhibit R_m values that are larger than those for their neighbors. However, those all correspond to $2s_{1/2}$ neutrons that are loosely bound. One question is whether the loosely bound nature and the low ℓ value are sufficient to explain the observations, or is something else going on?

Sherr and I have used a simple model to compute matter radii in several series of nuclei, including $^{15-22}$C [1] and $^{17-24}$O [2]. In those cases, and for several other so-called halo nuclei [3], the model calculations were able to reproduce most of the effect. In most cases [1–4], our computed radii agreed with the experimental ones within their uncertainties.

II. ANALYSIS

Ozawa *et al.* [5] used beams of 920A- to 1020A-MeV $^{14-16}$N and $^{18-23}$N incident on a carbon target and measured interaction cross sections. They used a Glauber-model analysis in the optical limit and deduced effective matter radii of these nuclei. They found a large increase from ^{21}N to ^{22}N and a similar increase from ^{22}N to ^{23}N, but with large uncertainties in both cases. Standard calculations were unable to reproduce these results. They concluded that ^{22}N is a one-neutron halo nucleus, and they discussed the possibility that ^{23}N might be a two-neutron halo. Their results are listed in Table I. They did not provide a value for ^{17}N in their table, but I have read it from their graph.

Beginning with ^{15}N, any nucleus ^{A}N is predominantly of the structure $^{A+1}\text{O} \otimes (\pi 1 p_{1/2})^{-1}$, where π stands for proton. Thus, one might expect N and O nuclei [5,6] of the same neutron number to have similar matter radii. This comparison is made in Fig. 1. Of course, the matter radius depends on the configuration of the last one or two neutrons and on the neutron separation energy. This dependence is slight for d neutrons (except near zero binding), but strong for s neutrons. It can be noted from Fig. 1 that the overall trends for N and O are similar.

The case of ^{18}N merits special mention. Its 1^- ground state (g.s.) is not a proton hole in the $5/2^+$ g.s. of ^{19}O, but rather in the $3/2^+$ first-excited state at 0.096 MeV. That energy is small enough that the difference in radius caused by it is negligible.

For the oxygen results, more recent measurements [6] for ^{22,23}O produced smaller radii than the earlier ones [5]. With these new values, at given neutron number, the N and O radii agree within the uncertainties for all except $N = 13$ (^{20}N and ^{21}O). Using the same simple model that was successful for carbon and oxygen, I have computed matter radii for $^{16-23}$N, and these are plotted with the data in Fig. 2. For a single valence neutron, the relevant equation is [7–12]

$$A R_m^2 = (A\text{–}1)\left(R_c^2 + R_v^2/A\right), \tag{1}$$

and the $2n$ form of the equation is [8,10,12–17]

$$A R_m^2 = (A\text{–}2)\left(R_c^2 + 2R_v^2/A\right). \tag{2}$$

Here, R_m is the matter radius of nucleus A, R_c is the matter radius of the A-1 (or A-2) core, and $R_v{}^2$ is the expectation value of r^2 for the valence neutron. The calculation of $R_v{}^2$ uses a Woods-Saxon potential with geometric parameters r_0, $a = 1.25, 0.65$ fm. The well depth is adjusted to reproduce the experimental separation energy.

For a series of nuclei all having the same Z, the calculations contain only one adjustable parameter—the radius of one of the nuclei. It is customary (but not required) to choose the lightest of the nuclei being treated. In the present case, it would be natural to choose ^{15}N as the one to be input. Its experimental radius is 2.42(10) fm, but the radius computed for it using the measured ^{14}N radius of 2.47(3) fm [5] is 2.50 fm. Therefore, I have chosen to display two sets of computed radii—one using a core radius of 2.42 fm for ^{15}N, the other using 2.50 fm. These are plotted as open squares and open triangles, respectively.

The 2^- g.s. of ^{16}N is predominantly of the structure ^{15}N (g.s.) $\times\ d$. A recent measurement of its spectroscopic factor reported $S = 0.96(9)$ [18]. Both calculations of this matter radius agree with the experimental value within the uncertainty.

Sherr and I [19] computed the matter radius of ^{17}N with both the $1n$ and $2n$ procedures. In the $1n$ method, ^{17}N (g.s.) contains s and d neutrons coupled to the first four states of ^{16}N. Results of this calculation are labeled "full" in Table I. In

TABLE I. Matter radii (fm) for $^{16-23}$N.

A	Experimental[a]	Configuration	Calculated-1[e]	Calculated-2[f]
16	2.50(10)	d	2.50	2.57
17	2.48(5)[b]	s^2	2.627	2.697
		d^2	2.534	2.604
		Mixed[c]	2.56	2.63
		full	2.58	2.65
18	2.65(2)	full	2.643	2.71
		$3/2^- \times d$	2.640	2.71
		$5/2^- \times d$	2.650	2.72
19	2.71(3)	s^2	2.734	2.804
		d^2	2.647	2.717
		Mixed[d]	2.656	2.736
20	2.81(4)	d	2.712	2.79
21	2.75(3)	d	2.747	2.82
22	3.07(13)	s	2.888	2.954
23	3.41(23)	s	3.03	3.12

[a]Ref. [5].
[b]Not in table, read from graph.
[c]With 0.24–0.29 s^2, rest d^2 [20].
[d]With 0.10 s^2, 0.90 d^2.
[e]Using $R_c = 2.42$ fm [5] (from experiment for ^{15}N) for ^{16}N calculation.
[f]Using $R_c = 2.50$ fm (from present calculation for ^{15}N) for ^{16}N calculation.

the 2n approach, the structure is ^{15}N (g.s.) $\otimes$ ($a\, s^2 + b\, d^2$). For the latter, we took a and b from a shell-model calculation [20]. Calculated radii for pure s^2 and d^2 and the shell-model mixture are listed in the table. Results of the two procedures differed by only 0.02 fm. All the calculated radii are significantly larger than the experimental value.

As noted above, the 1^- ground state (g.s.) of ^{18}N is primarily a proton hole in the $3/2^+$ first-excited state of ^{19}O at 0.096 MeV.

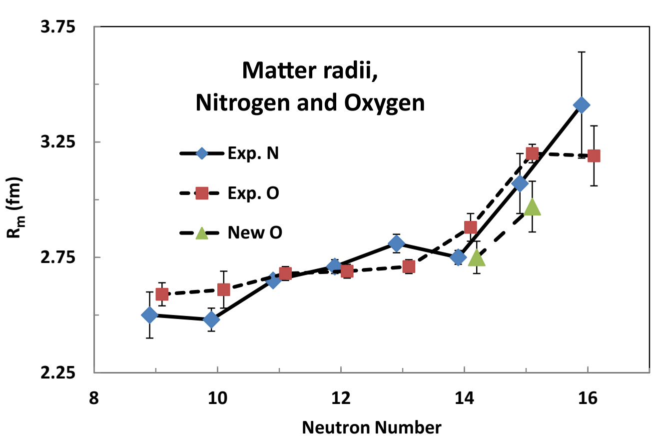

FIG. 1. Comparison of matter radii extracted from measured interaction cross sections for nitrogen and oxygen nuclei: squares (O) and diamonds (N) from Ref. [5]; triangles are more recent O results for ^{22,23}O [6].

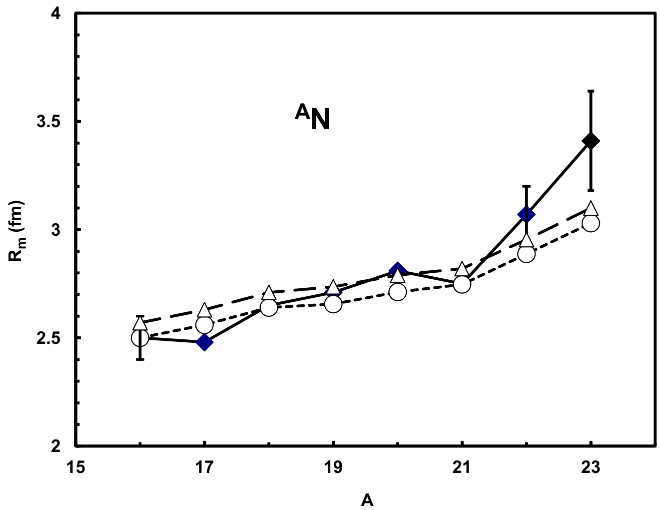

FIG. 2. Experimental [5] (closed diamonds) matter radii for N nuclei, compared with two sets of calculated values (open circles and open triangles).

Uncoupling and recoupling the angular momenta produces largest spectroscopic factors for $3/2^-{}_1 \otimes d$ and $5/2^-{}_1 \otimes d$. Results for those two configurations are listed in the table. They are very similar and quite similar to the calculation labeled “full”, which used spectroscopic factors for 10 core states in ^{17}N [21]. Calculation 1 is in excellent agreement with the experimental R_m. This comparison suggests that a remeasurement for ^{17}N might produce a larger radius. Comparison between N and O reinforces this possibility.

The g.s. of ^{19}N is primarily a $p_{1/2}$ proton hole in the ^{20}O g.s., or perhaps better described as ^{17}N (g.s.) $\otimes$ ($c\, s^2 + d\, d^2$). I have performed calculations for pure s^2 and d^2, and an admixture. From ^{20}O, it is known that d^2 should dominate [2]. The experimental radius is between the two calculated radii, but closer to the result of calculation 2.

For $^{20-23}$N, I have used the same neutron configurations as for $^{21-24}$O. Overall agreement is good, but for the heaviest two nuclei, the calculated radii are smaller than the results of experiment—albeit with large uncertainties. For ^{22,23}N, the disagreements with calculation 2 are 0.89 σ and 1.3 σ, respectively. Present results are certainly consistent with a pure s configuration in both ^{22}N and ^{23}N.

Of course, I am not free to choose calculation 1 for some nuclei and calculation 2 for others. One set should be applied to all the nuclei. For ^{19}N and upward, calculation 2 is in much better agreement with experimental radii. Recall that it is the set based on a calculated value for the radius of ^{15}N of 2.50 fm, rather than the experimental value of 2.42, which has an uncertainty of 0.10 fm. Thus, it would appear that calculation 2 is to be preferred. New measurements for the lighter N nuclei would be helpful.

Recently, Ahmad *et al.* [22] considered densities of $^{14-23}$N using two separate approaches: the relativistic mean-field (RMF) method, and with Slater determinants consisting of harmonic oscillator single-particle wave functions (SDHO). The RMF were taken from earlier work [23,24]. The SDHO

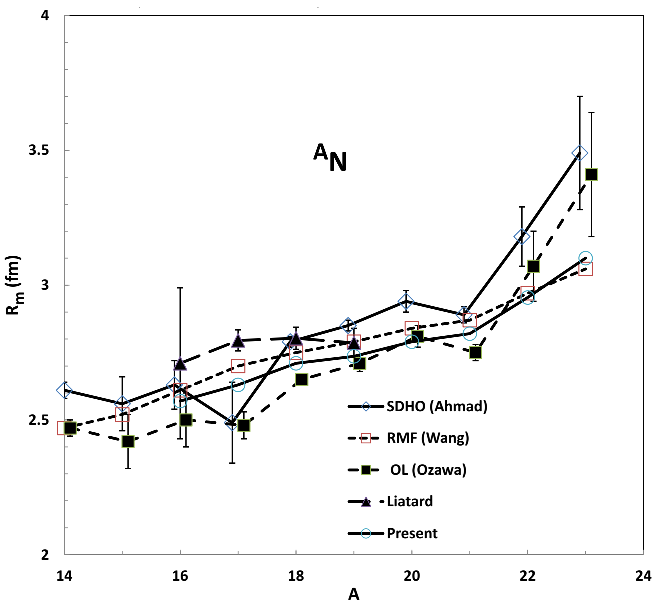

FIG. 3. Matter radii for $^{14-23}$N: theoretical results from RMF (Wang *et al.* [23]), open squares connected by short-dashed line, and closed triangles connected by long-dashed line (Liatard *et al.* [24]); SDHO (Ahmad *et al.* [22]), open diamonds connected by solid line; simple-model calculations [present], open circles connected by solid line; experimental results (Ozawa *et al.* [5]), closed squares connected by medium-dashed line.

technique contains two variable parameters for each nucleus (one for protons, one for neutrons) that can be adjusted in order to reproduce interaction cross sections, and thus extract matter radii. They compared the two sets of results with radii extracted with a Glauber model in the optical limit [5]. Their results are compared with present calculations and with experimental matter radii [5] in Fig. 3. The RMF calculations are larger than the experimental values for all but the heaviest nuclei. In all cases except ^{17}N, the present calculations are closer to the experimental values than either RMF or SDHO. This comparison emphasizes the point that ^{23}N should be remeasured.

III. SUMMARY

A simple model for computing matter radii for neutron-excess nuclei has been successful for a number of isotopic chains. Results are sensitive to the ℓ value of the valence neutron(s) and to the separation energy. I have used this simple model to compute matter radii for $^{16-23}$N, paying special attention to the configurations of the valence neutron(s). I have compared present results with other recent predictions and with earlier matter radii extracted from measurements of interaction cross sections. The present calculations are closer to the experimental radii than any of the results from the other procedures. The analysis suggests that ^{23}N should be remeasured.

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