Editors' Suggestion

Structure of even-even cadmium isotopes from the beyond-mean-field interacting boson model

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The structure of even-even $^{108-116}$Cd isotopes is investigated based on the self-consistent mean-field approach. By mapping the quadrupole-(β,γ) deformation energy surface, obtained from the constrained self-consistent mean-field calculations with a choice of the Skyrme force and pairing property, onto the Hamiltonian of the interacting boson model with configuration mixing, the strength parameters of the Hamiltonian are determined. The low-lying excitation spectra and electric quadrupole and monopole transition rates for the considered Cd nuclei are computed by the resultant Hamiltonian, and are compared in detail with the experimental data. Our semimicroscopic prediction identifies several intruder states as suggested empirically, and overall, provides a reasonable qualitative description of the experimental energy levels and transition rates.

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I. INTRODUCTION

The $Z = 50$ mass region is very favorable for nuclear structure studies due to the large abundance of stable isotopes combined with the interesting features of the nearby $Z = 50$ proton shell closure and with the occurrence of neutrons in the middle of the $N = 50$–82 shell.

The earliest work started with the observation by Schraff-Goldhaber and Wesener [1] that the Cd isotopes exhibit low-lying states that resemble the quadrupole-vibrational excitations of a surface with spherical equilibrium as predicted by the collective model of Bohr and Mottelson [2]. Besides the one-phonon quadrupole state, candidates for two-phonon quadrupole states were observed around 1.2 MeV, twice the energy of the first-excited 2^+ state. The two-phonon states were found nondegenerate, indicating the need to include anharmonic effects in the phonon-phonon interactions. Furthermore, additional 0^+ and 2^+ states were observed in transfer studies [3]. Attempts by Bes and Dussel to explain these as strongly anharmonic three-phonon states failed [4]. The explanation of the additional states was then related to two-particle–two-hole (2p-2h) excitations of the protons across the $Z = 50$ closed shell. Evidence that the extra 0^+ and 2^+ states were indeed 2p-4h states was obtained from (^{3}He,n) two-proton transfer experiments [5]. By the early 1990s intruder bands were identified in most even-even Cd isotopes. Strong support for the intruder interpretation came from the systematic behavior of these states as a function of the number of valence neutrons. Due to the increase in neutron-proton quadrupole interaction, intruder states decrease in energy proportional to the number of neutrons, reaching a lowest value near midshell [6]. In the stable Cd isotopes this mechanism gives rise to intruder and two-phonon states close in energy, resulting in complex spectra.

The even-even Cd isotopes have been studied intensively from $N = 46$ [7] till $N = 86$ [8]. The most detailed information is obtained for the stable isotopes with $A = 106$ till $A = 116$ [9–59]. The systematics for those data are discussed in Refs. [60–62]. A recent review on the structure of ^{100}Sn and neighboring nuclei including the light Cd isotopes below $N = 50$ is given in [63].

Shell-model calculations have been performed for the lighter Cd isotopes [33,64–72] up to ^{108}Cd [73]. The used model spaces, however, do not consider proton 2p-2h excitation across $Z = 50$. Different calculation that include the intruder states have been performed using the interacting boson model in its simplest version with s and d bosons (IBM-1) or more elaborated versions with s, p, d, and f bosons, broken pairs or using the neutron-proton version (IBM-2) [14,19,20,28,33,45,53,74–82]. Besides that, other studies, starting from a general collective Bohr Hamiltonian, derived from a microscopic starting point using a Skyrme force, calculations using the adiabatic time-dependent Hartree-Fock-Bogoliubov (ATDHFB) method (for the nuclei $^{106-116}$Cd) [83], as well as using a self-consistent HFB approach, starting from the finite range Gogny interaction [84] have been carried out.

In this work we want to use the results of the self-consistent mean-field approach to perform IBM-2 calculations with normal and intruder states for the $A = 108$–116 even-even Cd isotopes using the approach introduced in [85]. A detailed comparison with the extensive experimental data set on energy levels and electric quadrupole and monopole transitions is then made.

II. THEORETICAL FRAMEWORK

In this section we briefly outline the theoretical scheme used in the present work to study the even-even $^{108-116}$Cd isotopes. For the detailed accounts of the method, the reader is referred to Refs. [85,86].

We have first carried out, for each Cd nucleus, the constrained self-consistent mean-field (SCMF) calculations to obtain the deformation energy surface in the (β,γ) quadrupole

deformation space. The constraints imposed are on mass quadrupole moments Q_{20} and Q_{22}, which can be associated with the axial β and triaxial γ deformation parameters of the collective model [2]. For the SCMF calculations, we have employed the Hartree-Fock+BCS method [87,88], where the particle-hole interaction is modeled by the SLy6 parametrization [89] of the Skyrme force and the particle-particle channel is described by the density-dependent zero-range pairing force with the strength of $V_0 = 1000$ MeV fm^3 truncated below and above the Fermi surface by 5 MeV, for both protons and neutrons. More details about the HF+BCS calculation can be found in Refs. [87,88].

On the left-hand side of Fig. 1 we draw contour plots of the (β,γ)-deformation energy surfaces for the $^{108-116}$Cd isotopes, obtained from the above-mentioned SCMF calculation. For all the considered Cd nuclei, a prolate global minimum is found with moderate axial deformation $\beta \approx 0.15$. We also observe on the oblate side ($\gamma \approx 60^\circ$) a much less pronounced local minimum between $\beta = 0.2$ and 0.3. In order to examine the sensitivity of the SCMF result to the pairing property, we depict, on the left-hand side of Fig. 2, the SCMF energy surface for the ^{112}Cd nucleus, obtained with the same Skyrme force but with the pairing strength of $V_0 = 1250$ MeV fm^3. By comparing it with the corresponding SCMF energy surface in Fig. 1 in the case of $V_0 = 1000$ MeV fm^3, one could notice that, when the pairing strength is increased, the surface has a less pronounced prolate minimum and more resembles the potential typical of spherical vibrator. In addition, local minimum is no longer visible on the oblate side of the SCMF energy surface with $V_0 = 1250$ MeV fm^3. Later we show how the difference in the SCMF energy surface between the different pairing strengths influences the energy spectra.

The next step is to construct from those SCMF results configuration mixing IBM-2 Hamiltonian by using the procedure of Ref. [85]. It is based on the method developed in [86], in which the SCMF energy surface is mapped onto the expectation value of the IBM-2 Hamiltonian in the boson coherent state [90] so as to determine the strength parameters of the Hamiltonian. In contrast to many phenomenological IBM studies, there is no adjustment of the parameters to experimental data in this procedure. Diagonalization of the resulting Hamiltonian in the laboratory frame provides one with excitation spectra and electromagnetic transition rates.

The IBM-2 is comprised of the neutron (proton) s_ν (s_π) and d_ν (d_π) bosons, which represent, respectively, the collective $J = 0^+$ and 2^+ pairs of valence neutrons (protons) [91]. The number of neutron (proton) bosons, denoted as N_ν (N_π), is equal to that of the valence neutron particles/holes (proton holes). Here we take the doubly magic nuclei ^{100}Sn and ^{132}Sn (for ^{116}Cd) as inert cores for the bosons. In addition, by using the procedure proposed by Duval and Barrett to incorporate intruder states in the IBM [92], we take into account the proton 2p-2h intruder excitation across the $Z = 50$ shell closure. In this proposal [92] particle-like and hole-like bosons are not distinguished and, as the excitation of a pair (or boson) increases the boson number by two, the 0p-0h and 2p-2h configurations differ in boson number by two. Applying this boson-number counting rule to the considered $^{106-116}$Cd nuclei, $N_\pi = 1$ and 3 for the normal (0p-0h) and intruder

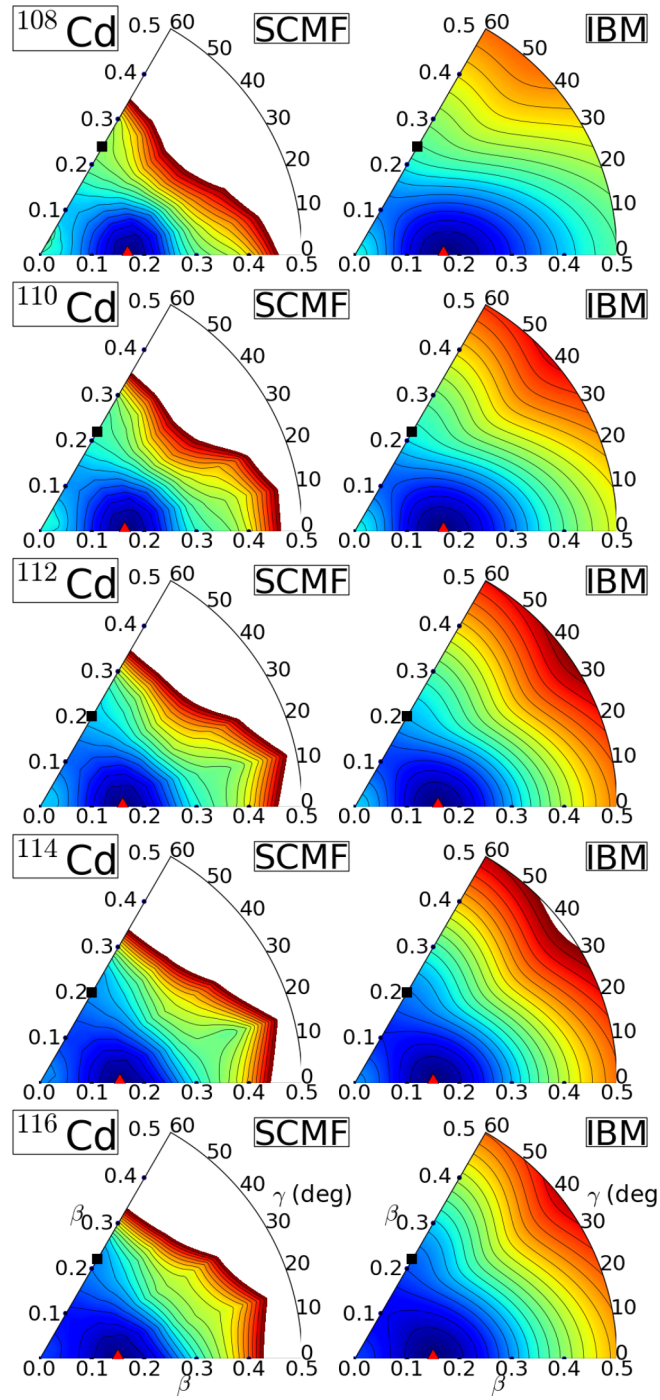

FIG. 1. Left column: Contour plots for the $\beta\gamma$-deformation energy surfaces for $^{108-116}$Cd, that have been obtained from the self-consistent mean-field (SCMF) calculation using the Skyrme SLy6 parametrization with the density-dependent zero-range pairing interaction with the strength $V_0 = 1000$ MeV fm^3. Right column: the corresponding IBM-2 energy surfaces. Energy difference between neighboring contours is 250 keV. The global minimum is indicated by solid triangle, while the local minimum is identified by solid squares.

(2p-2h) configurations, respectively, while $5 \leqslant N_\nu \leqslant 8$. The configuration mixing IBM-2 Hamiltonian is then written as

$$\hat{H} = \hat{\mathcal{P}}_1 \hat{H}_1 \hat{\mathcal{P}}_1 + \hat{\mathcal{P}}_3 (\hat{H}_3 + \Delta) \hat{\mathcal{P}}_3 + \hat{H}_{\rm mix}, \qquad (1)$$

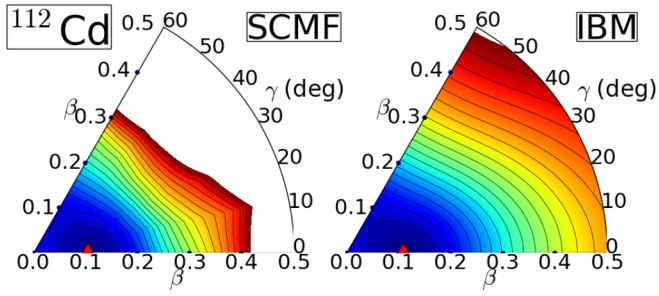

FIG. 2. Same as Fig. 1, but for the calculation with the pairing strength of $V_0 = 1250$ MeV fm^3 for the ^{112}Cd.

where $\hat{H}_1$ ($\hat{H}_3$) and $\hat{\mathcal{P}}_1$ ($\hat{\mathcal{P}}_3$) are the Hamiltonian of and the projection operator onto the normal and intruder configuration spaces, respectively. Δ stands for the energy needed to promote a proton boson across the shell closure. $\hat{H}_{\rm mix}$ in the above equation is the term that is allowed to mix the two configurations. Here we employ the same Hamiltonian as in [93], but the three-body boson term is not included here.

The coherent state for the configuration mixing IBM was introduced in [94] as the direct sum of the coherent state for each unperturbed configuration. The energy surface of the configuration-mixing IBM-2 is obtained as the lower eigenvalue of the 2×2 coherent-state matrix [94]. The analytical expressions for each component of the coherent-state matrix are found in [93].

The parameters for the Hamiltonian for each configuration are determined by associating the Hamiltonian with each mean-field minimum, i.e., 0p-0h Hamiltonian for the prolate minimum, and 2p-2h one for the oblate local minimum, and the energy offset Δ and the mixing strength in $\hat{H}_{\rm mix}$ are determined so that the energy difference between the two mean-field minima and the barrier height for these minima, respectively, are reproduced. The derived strength parameters for $^{108-116}$Cd are listed in Table I.

On the right column of Figs. 1 and 2 we show the mapped IBM-2 energy surfaces. Note the intruder configuration has not been included for the calculation of ^{112}Cd with the pairing strength of $V_0 = 1250$ MeV fm^3, because there is no additional

TABLE I. The parameters of the configuration mixing IBM-2 Hamiltonian in Eq. (1) employed in the calculation (in MeV units). Their definitions are given in Ref. [93]. For the mixing strength ω in $\hat{H}_{\rm mix}$ constant value of $\omega = 0.15$ MeV is used.

		ϵ	κ	χ_ν	χ_π	κ'	Δ
^{108}Cd	normal	0.297	−0.490	−0.446	−0.950	0.0383	2.572
	intruder	0.486	−0.200	0.600	0.650	0.0157	
^{110}Cd	normal	0.404	−0.513	−0.269	−0.845	0.0415	2.221
	intruder	0.368	−0.195	0.400	0.650	0.0420	
^{112}Cd	normal	0.497	−0.495	−0.127	−0.845	0.0424	1.804
	intruder	0.326	−0.180	0.050	0.650	0.0490	
^{114}Cd	normal	0.462	−0.417	−0.126	−0.844	0.0449	2.132
	intruder	0.321	−0.180	0.050	0.650	0.0498	
^{116}Cd	normal	0.626	−0.422	−0.217	−0.696	0.0340	2.811
	intruder	0.489	−0.197	0.600	0.650	0.0302	

minimum on the oblate side (see Fig. 2). One sees that the topology of the corresponding SCMF energy surfaces in the neighborhood of the minimum is well reproduced by the IBM ones. The IBM surface, however, tends to be flat in the region far from the minimum, compared to the SCMF one. Main reason is that we have paid particular attention to reproduce, as much as possible, the topology of the SCMF energy surface in the vicinity of, typically a few MeVs above, the minimum: The most relevant mean-field configurations to low-lying quadrupole collective states are those in the neighborhood of the minimum, while those very far from it tend to be dominated by noncollective, i.e., quasiparticle, degrees of freedom, which are out of the model space of the IBM-2 framework. Another reason is, of course, that the employed IBM-2 Hamiltonian and/or coherent-state formalism may have been too simple to account for every detail of the SCMF energy surface.

Using the resulting wave functions of the IBM-2 Hamiltonian, we have also computed the electric quadrupole ($E2$) and monopole ($E0$) transition rates. We used the $E2$ operator

$$\hat{T}^{E2} = \sum_{\tau,i} \hat{\mathcal{P}}_i e_i^\tau \hat{Q}_{\tau,i} \hat{\mathcal{P}}_i, \tag{2}$$

where $\tau = \nu$ (neutron) or π (proton), $i = 1$ ($0p - 0h$) or 3 ($2p - 2h$), $\hat{Q}_{\tau,i}$ is the quadrupole operator same as the one in each unperturbed Hamiltonian, and e_i^τ the boson effective charge. The $E0$ operator used in this study is given by

$$\hat{T}^{E0} = \sum_{\tau,i} \hat{\mathcal{P}}_i (\beta_{\tau,i} \hat{n}_{d\tau,i} + \gamma_{\tau,i} \hat{N}_{\tau,i}) \hat{\mathcal{P}}_i, \tag{3}$$

where $\hat{n}_{d\tau,i}$ is the neutron or proton d-boson number operator for a given configuration while $\hat{N}_{\tau,i}$ the total neutron or proton boson number. $\beta_{\tau,i}$ and $\gamma_{\tau,i}$ are parameters.

III. COMPARISON WITH EXPERIMENT

Figure 3 shows the comparison of the predicted and experimental excitation spectra for $^{108-116}$Cd. The experimental data are taken from [95–99] for $^{108-116}$Cd, respectively. Shown are the experimental levels up to an energy of about 2.5 MeV and maximally five states are shown for a given spin J^+. The corresponding theoretical levels are given up to an energy of about 2.8 MeV. The predicted energies of the intruder states can be seen in those of the third or higher 0^+ states (see, Table II). In Fig. 3, the semimicroscopic calculations based on the energy surface do predict the approximately right excitation energies and the dependence on neutron number with the lowest 0_3^+ energy at midshell in ^{114}Cd. The description of the spacing between both 0^+ states is overestimated in ^{108,110}Cd, but agrees quite well with experiment in $^{112-116}$Cd. However, phenomenological calculations assign in ^{114}Cd the experimental 0_2^+ state to the intruder configuration [12]. Therefore, the energy difference between intruder and normal states is generally overestimated. The predicted normal states are systematically too deformed as can be seen by the energies of the first 2^+, 4^+, and especially 6^+ states. Experimentally states are observed with more vibrational energies although there are serious problems with the electric quadrupole transitions [82]. The description of the energies of the 2^+ intruder state is not good. Phenomenological calculations, i.e., [19,31], identified

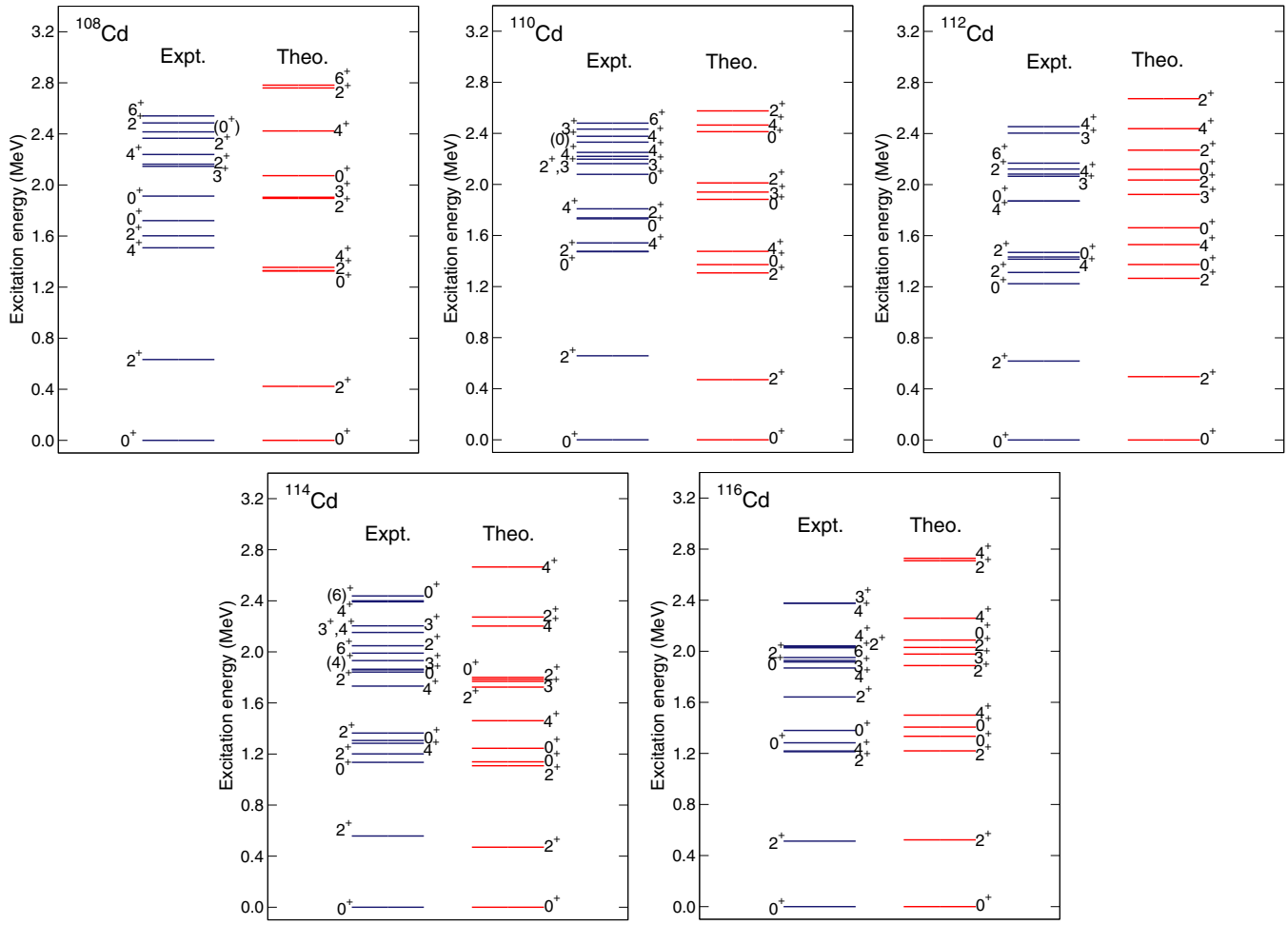

FIG. 3. Experimental and predicted excitation spectra for $^{108-116}$Cd.

in $^{110-114}$Cd the 2_3^+ state as the intruder state, while the present calculation yield as main component the much higher lying 2_4^+ state. In contrast to the normal states the spacing between the 0^+ and 2^+ intruder states is too large. As expected the semi-microscopic predictions have problems to describe the increasing level densities above 2 MeV.

TABLE II. Fraction (in units of percent) of the intruder configuration in the lowest five 0^+ and three 2^+ wave functions of the considered Cd nuclei.

	^{108}Cd	^{110}Cd	^{112}Cd	^{114}Cd	^{116}Cd
0_1^+	1	2	3	5	4
0_2^+	2	3	6	29	3
0_3^+	5	26	55	52	82
0_4^+	5	74	46	26	20
0_5^+	91	3	7	8	40
2_1^+	1	2	3	4	4
2_2^+	3	5	8	16	17
2_3^+	1	2	4	9	74
2_4^+	6	55	81	74	10

In order to investigate the influence of the density-dependent pairing interaction we have performed the IBM-2 calculations using the energy surface shown in Fig. 2. The corresponding excitation spectrum is shown in Fig. 4. As there is no second minimum we only have states corresponding to the normal states. This is clearly not the case experimentally as reflected, i.e., by the excited 0^+ states. On the other hand the comparison for the energies of the normal states is substantially improved, i.e., for the 6^+ states.

The theoretical $B(E2; J_i^+ \rightarrow J_f^+)$ values were calculated for $^{108-116}$Cd using the transition operator given in Eq. (2) with fixed values $e_1^\nu = e_1^\pi = 0.084\,e$b and $e_3^\nu = e_3^\pi = 0.113\,e$b, which are taken from [19]. Table III compares them in Weisskopf units (W.u.) with the experimental data, when available. In view of the absence of fitting the data, there is good agreement and changing trends are well described. However, there are exceptions, mostly involving the 0_2^+, 0_3^+, 2_2^+, and 2_3^+ states as could be expected from the different nature of these states compared to the phenomenological IBM-2 calculations. As an example the $B(E2; 0_2^+ \rightarrow 2_1^+)$ are ten times underpredicted in ^{112,114}Cd and ten times overpredicted in ^{116}Cd. This confirms that these states, which are the lowest where normal and intruder states mix, are poorly described. Another reason could be, again, that at the SCMF level the

TABLE III. Comparison between experimental and theoretical $B(E2; J_i^+ \rightarrow J_f^+)$ values in Weisskopf units.

J_i^+	J_f^+	^{108}Cd		^{110}Cd		^{112}Cd		^{114}Cd		^{116}Cd	
		Exp[a]	Theory	Exp[b]	Theory	Exp[c]	Theory	Exp[d]	Theory	Exp[e]	Theory
2_1	0_1	26.6(3)	29	*27.0(8)*	33	30.3(2)	39	31.1(19)	46	33.5(12)	36
0_2	2_1	–	1.4	<40	2.8	51(14)	4.5	27.4(17)	2.9	0.79(22)	9.5
2_2	0_1	1.8(3)	1.1	0.68(14)	1.7	0.65(11)	2.4	0.48(6)	3.2	1.11(18)	1.9
2_2	2_1	17(5)	6	19(4)or *30(5)*	11	39(7)	18	22(6)	21	25(10)	27
2_2	0_2	–	1.7	*1.35(20)*	1.2	–	2.6	3.4(7)	11	–	1.7
4_1	2_1	41(6)	39	*42(9)*	47	63(8)	55	62(4)	65	56(14)	51
0_3	2_1	–	0.003	<7.9	0.10	0.0121(17)	0.82	0.0026(4)	4.4	30(6)	1.6
0_3	2_2	–	13	<1680	29	99(16)	42	127(16)	39	–	96
2_3	0_1	–	0.02	0.28(4)	0.051	0.88(17)	0.085	0.33(4)	0.072	1.11(18)	0.25
2_3	2_1	–	0.02	0.7^{+3}_{-4}	0.068	*0.12(7)*	0.14	<0.045	0.17	6.2^{+22}_{-26}	0.0083
2_3	0_2	–	16	29(5)	20	120(50)	25	65(9)	32	–	2.8
2_3	2_2	–	0.17	<8	0.46	–	0.79	–	0.22	–	7.8
2_3	0_3	–	0.56	–	0.43	–	0.98	–	1.9	86^{+24}_{-30}	76
3_1	2_1	–	1.5	0.85(25)	2.5	1.8(5)	3.3	–	4.2	*2.6(7)*	2.0
3_1	2_2	–	30	22.7(69)	38	64(18)	47	–	55	*61(17)*	39
3_1	4_1	–	3.9	2.4^{+9}_{-8}	6.8	25(8)	10	–	12	*18(10)*	11
3_1	2_3	–	2.3	<5	1.9	–	1.6	–	1.9	–	3.6
4_2	2_1	–	0.035	0.14(6)	0.083	–	0.14	0.50(5)	0.32	*3.0(7)*	0.22
4_2	2_2	–	15	22(10)	23	–	31	32(4)	45	*230(130)*	44
4_2	4_1	–	4.8	10.7^{+49}_{-48}	8.6	–	13	17(6)	16	*150(90)*	18
4_2	2_3	–	1.4	<0.5	0.98	–	0.79	119(12)	5.9	–	31
2_4	0_1	–	0.011	–	0.10	0.017(5)	0.19	0.19(4)	0.28	*0.13(4)*	0.011
2_4	2_1	–	0.021	0.28^{+6}_{-10}	0.032	2.2(6)	0.0011	0.84(17)	0.059	*1.23(44)*	0.16
2_4	0_2	–	0.28	–	0.88	5.3(15)	2.2	42(9)	15	–	21
2_4	2_2	–	0.0011	–	1.6	*<2.8*	2.7	5.6(11)	5.8	*31(9)*	0.27
2_4	0_3	–	13	–	35	25(7)	62	34(8)	68	–	5.6
2_4	2_3	–	0.87	–	0.97	–	0.83	70(40)	16	–	21
0_4	2_1	–	0.0049	–	0.32	*<1.4*	0.50	2.4(6)	0.44	–	0.25
0_4	2_3	–	1.3	–	0.17	<23	1.0	18(6)	10	–	64
4_3	2_1	–	0.0044	0.14(4)	0.0019	0.9(3)	0.012	–	0.057	–	0.11
4_3	2_2	–	0.0000	1.2(4)	1.7	58(17)	2.8	–	4.8	–	0.64
4_3	4_1	–	0.0038	1.8^{+10}_{-15}	0.032	24(8)	0.13	–	0.35	–	0.67
4_3	2_3	–	18	115(35)	0.99	59(20)	0.14	–	3.2	–	86
6_1	4_1	–	39	62(18)	49	–	59	119(15)	72	*110(46)*	58
6_1	4_3	–	0.47	36(11)	0.11	–	0.064	–	0.011	–	0.90
2_5	0_1	–	0.0036	–	0.093	*0.136(16)*	0.038	0.08(3)	0.012	–	0.0048
2_5	2_1	–	0.021	3.2(3)	0.077	0.060^{+27}_{40}	0.063	–	0.19	–	0.079
2_5	2_5	–	0.055	0.7^{+5}_{-6}	0.087	–	0.0098	<1.9	0.018	–	0.014
2_5	0_3	–	0.99	24.2(22)	0.15	–	0.42	17(5)	0.15	–	0.71
2_5	2_3	–	4.2	<5	0.23	22^{+6}_{-19}	0.37	–	0.71	–	13
8_1	6_1	–	34	80(22)[f]	45	–	58	*86(28)*	73	–	61
6_2	4_2	–	23	–	33	<77	49	*129(42)*	68	–	88

[a]All from [95].
[b]From P. E. Garrett *et al.*, [53] except italic from [96].
[c]From [97], except italic from P. E. Garrett *et al.* [45].
[d]All from [98], except italic from M. Délèze *et al.* [19].
[e]All from [99] except italic from M. Kadi *et al.*[38].
[f]Assuming that the second 8^+ state corresponds to the first theoretical 8^+ state.

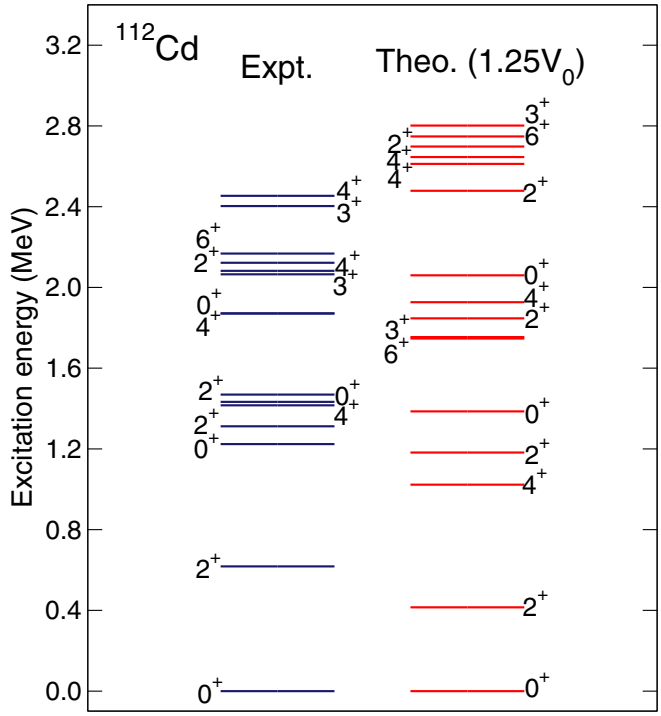

FIG. 4. Same as Fig. 3, but for the calculation on ^{112}Cd based on the pairing strength increased by 25%.

prolate minimum, from which the normal states are mainly constructed, is predicted to be too deformed (see, Fig. 1).

When using the increased pairing for ^{112}Cd and assuming that the 0_3^+ and 2_3^+ are outside the model space the $B(E2; 0_2^+ \rightarrow 2_1^+) = 15$ W.u. is still underpredicted by a factor three. We note that there is, overall, no striking difference between the predictions of the $B(E2)$ rates with different pairing strengths. Exceptions are perhaps $B(E2; 2_4^+ \rightarrow 0_3^+) = 2.3$ and $B(E2; 2_5^+ \rightarrow 0_3^+) = 15$ W.u., which were obtained with the increased pairing and are by about a factor ten different from those results with the pairing strength of $V_0 = 1000$ MeV fm^3 (see Table III). Not surprisingly, such a difference occurs due to whether or not the intruder configuration is included. For instance, the transition $2_4^+ \rightarrow 0_3^+$ is between the states mainly made of the intruder components in the configuration mixing calculation (see Table II), while it is between normal states with the pairing strength of $V_0 = 1250$ MeV fm^3.

By using the $E0$ transition operator in Eq. (3) we have calculated the theoretical $\rho^2(E0; J_i \rightarrow J_f)$ values

$$\rho^2(E0; J_i \rightarrow J_f) = \frac{Z^2}{e^2 R^4} |\langle J_f | \hat{T}^{E0} | J_i \rangle|^2, \qquad (4)$$

where $R = 1.2A^{1/3}$ fm, and for the parameters in the $E0$ operator we used the fixed values $\beta_{\nu,1} = \beta_{\nu,3} = 0.10$ fm^2, $\beta_{\pi,1} = \beta_{\pi,3} = 0.60$ fm^2, taken from Ref. [100]. We have also set $\gamma_{\nu,1} = \gamma_{\nu,3} = 0$ fm^2. In Table IV the theoretical $\rho^2(E0)$ values are compared with the available experimental data. The large experimental error bars make a clear comparison in ^{110}Cd difficult. However, in ^{112}Cd were the values are better defined the agreement is very good with one exception (the 0_3^+ to 0_1^+ transition). In ^{114}Cd, where the mixing is the strongest the $E0$ transitions are instead very poorly described.

TABLE IV. Comparison between experimental and theoretical $\rho^2(E0; J_i^+ \rightarrow J_f^+)$ values. The experimental $\rho^2(E0)$ values are not known for ^{108}Cd and ^{116}Cd.

	J_i^+	J_f^+	$\rho^2(E0) \times 10^3$	
			Exp	Theory
^{110}Cd	0_2	0_1	<31(5)[a]	37
	0_3	0_1	<11[b]	1.1
	2_2	2_1	20(15)[c]	1.1
	2_3	2_1	9(8)[a]	26
	4_3	4_1	106^{+98}_{-91}[a]	0.44
^{112}Cd	0_2	0_1	34(9)[d]	36
	0_3	0_1	0.87(5)[d]	8.6
	0_3	0_2	10.7(6)[d]	12
	2_3	2_1	31(20)[c]	27
^{114}Cd	0_2	0_1	19(2)[d]	12
	0_3	0_1	1.83(13)[d]	44
	0_3	0_2	0.65(5)[d]	100
	0_4	0_1	0.9(4)[d]	8.8
	2_2	2_1	<28[c]	0.25
	2_3	2_1	38(5)[e]	22
	2_3	2_2	22(6)[e]	1.1
	2_4	2_2	<20[e]	57
	3_2	3_1	<130[e]	35
	4_2	4_1	67(10)[e]	0.38

[a]Reference [101].
[b]Reference [102].
[c]Reference [100].
[d]Reference [103].
[e]Reference [104].

IV. CONCLUSION

Based on constrained self-consistent mean-field calculations, deformation energy surfaces were calculated for the even-even $^{108-116}$Cd isotopes. The energy surfaces yielded both a prolate and a minor oblate minimum which were consider to be associated to proton 0p-0h normal excitations and 2p-2h intruder excitations. They were used to fit the parameters of an IBM-2 Hamiltonian, which involves normal and intruder states and their mixing. The resulting energy spectra, and $B(E2)$ and $\rho^2(E0)$ values were compared with experiment. An overall reasonable agreement was found in view of the fact that no additional fitting to the experiment was done. The calculations describe the energy dependence of the lowest intruder state with the expected minimum at mid shell, but predict these states at higher energies than phenomenological calculations. Therefore the mixing of the lowest two intruder states with the normal states is not reproduced correctly leading to large discrepancies for the $B(E2)$ and $\rho^2(E0)$ values. Moreover, the normal states are predicted to be too deformed. We also studied the effect of the pairing interaction in the case of ^{112}Cd and found that an increase of the strength leads to a disappearance of the intruder states.

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[1] G. Schraff-Goldhaber and J. Wesener, Phys. Rev. **98**, 212 (1955).
[2] A. Bohr and B. Mottelson, Mat. Fys. Medd. Dan. Vid. Selsk. **27**, No 16 (1953).
[3] B. L. Cohen and R. E. Price, Phys. Rev. **121**, 1441 (1961).
[4] D. R. Bès and G. G. Dussel, Nucl. Phys. A **135**, 1 (1969).
[5] R. E. Anderson *et al.*, Nucl. Phys. A **281**, 389 (1977).
[6] K. L. G. Heyde, J. Jolie, J. Moreau, J. Ryckebusch, M. Waroquier, P. Van Duppen, M. Huyse, and J. L. Wood, Nucl. Phys. A **466**, 189 (1987).
[7] P. J. Davies *et al.*, Phys. Lett. B **767**, 474 (2017).
[8] G. Lorusso *et al.*, Phys. Rev. Lett. **114**, 192501 (2015).
[9] W. T. Milner, F. K. McGowan, P. H. Stelson, R. L. Robinson, and R. O. Sayer, Nucl. Phys. A **129**, 687 (1969).
[10] M. T. Esat, D. C. Kean, and R. H. Spear, Nucl. Phys. A **274**, 237 (1976).
[11] K. Schreckenbach, A. Mheemeed, G. Barreau, T. von Egidy, H. R. Faust, H. G. Börner, R. Brissot, M. L. Stelts, K. L. G. Heyde, P. Van Isacker *et al.*, Phys. Lett. B **110**, 364 (1982).
[12] A. Mheemeed, K. Schreckenbach, G. Barreau, H. R. Faust, H. G. Börner, R. Brissot, P. Hungerford, H. H. Schmidt, H. J. Scheerer, T. Von Egidy *et al.*, Nucl. Phys. A **412**, 113 (1984).
[13] A. Aprahamian, D. S. Brenner, R. F. Casten, R. L. Gill, A. Piotrowski, and K. L. G. Heyde, Phys. Lett. B **140**, 22 (1984).
[14] D. Kusnezov, A. Bruder, V. Ionescu, J. Kern, M. Rast, K. L. G. Heyde, P. Van Isacker, J. Moreau, M. Waroquier, and R. A. Meyer, Helv. Phys. Acta. **60**, 456 (1987).
[15] C. Fahlander *et al.*, Nucl. Phys. A **485**, 327 (1988).
[16] H. Mach, M. Moszyński, R. F. Casten, R. L. Gill, D. S. Brenner, J. A. Winger, W. Krips, C. Wesselborg, M. Büscher, F. K. Wohn *et al.*, Phys. Rev. Lett. **63**, 143 (1989).
[17] J. Kumpulainen, R. Julin, J. Kantele, A. Passoja, W. H. Trzaska, E. Verho, J. Väärämäki, D. Cutoiu, and M. Ivascu, Phys. Rev. C **45**, 640 (1992).
[18] R. F. Casten, J. Jolie, H. G. Börner, D. S. Brenner, N. V. Zamfir, W. T. Chou, and A. Aprahamian, Phys. Lett. B **297**, 19 (1992).
[19] M. Délèze, S. Drissi, J. Kern, P. A. Tercier, J. P. Vorlet, J. Rikovska, T. Otsuka, S. Judge, and A. Williams, Nucl. Phys. A **551**, 269 (1993).
[20] M. Délèze, S. Drissi, J. Jolie, J. Kern, and J. P. Vorlet, Nucl. Phys. A **554**, 1 (1993).
[21] M. Bertschy, S. Drissi, P. E. Garrett, J. Jolie, J. Kern, S. J. Mannanal, J. P. Vorlet, N. Warr, and J. Suhonen, Phys. Rev. C **51**, 103 (1995); **52**, 1148(E) (1995).
[22] S. Juutinen, P. Jones, A. Lampinen, G. Lhersonneau, E. Mäkelä, M. Piiparinen, A. Savelius, and S. Törmänen, Phys. Lett. B **386**, 80 (1996).
[23] H. Lehmann, P. E. Garrett, J. Jolie, C. A. McGrath, M. Yeh, and S. W. Yates, Phys. Lett. B **387**, 259 (1996).
[24] T. Kautzsch, W. B. Walters, V. N. Fedoseyev, Y. Jading, A. Jokinen, I. Klöckl, K.-L. Kratz, V. I. Mishin, H. L. Ravn, P. Van Duppen *et al.*, Phys. Rev. C **54**, R2811 (1996).
[25] N. Warr, S. Drissi, P. E. Garrett, J. Jolie, J. Kern, S. J. Mannanal, J.-L. Schenker, and J. P. Vorlet, Nucl. Phys. A **620**, 127 (1997).
[26] P. E. Garrett, H. Lehmann, J. Jolie, C. A. McGrath, M. Yeh, and S. W. Yates, Phys. Rev. C **59**, 2455 (1999).
[27] H. Lehmann, A. Nord, A. E. De Almeida Pinto, O. Beck, J. Besserer, P. von Brentano, S. Drissi, T. Eckert, R.-D. Herzberg, D. Jager *et al.*, Phys. Rev C **60**, 024308 (1999).
[28] G. de Angelis, C. Fahlander, D. Vretenar, S. Brant, A. Gadea, A. Algora, Y. Li, Q. Pan, E. Farnea, D. Bazzacco *et al.*, Phys. Rev. C **60**, 014313 (1999).
[29] F. Corminboeuf, T. B. Brown, L. Genilloud, C. D. Hannant, J. Jolie, J. Kern, N. Warr, and S. W. Yates, Phys. Rev. Lett. **84**, 4060 (2000).
[30] T. Kautzsch, W. B. Walters, M. Hannawald, K.-L. Kratz, V. I. Mishin, V. N. Fedoseyev, W. Böhmer, Y. Jading, P. Van Duppen, B. Pfeiffer *et al.*, Eur. Phys. J. A **9**, 201 (2000).
[31] F. Corminboeuf, T. B. Brown, L. Genilloud, C. D. Hannant, J. Jolie, J. Kern, N. Warr, and S. W. Yates, Phys. Rev. C. **63**, 014305 (2000).
[32] P. E. Garrett, H. Lehmann, J. Jolie, C. A. McGrath, M. Yeh, W. Younes, and S. W. Yates, Phys. Rev. C **64**, 024316 (2001).
[33] G. A. Müller, A. Jungclaus, O. Yordanov, E. Galindo, M. Hausmann, D. Kast, K. P. Lieb, S. Brant, V. Krstić, D. Vretenar *et al.*, Phys. Rev. C **64**, 014305 (2001).
[34] Y. Wang, P. Dendooven, J. Huikari, A. Jokinen, V. S. Kolhinen, G. Lhersonneau, A. Nieminen, S. Nummela, H. Penttilä, K. Peräjärvi *et al.*, Phys. Rev. C **64**, 054315 (2001).
[35] A. Gade, J. Jolie, and P. von Brentano, Phys. Rev. C **65**, 041305(R) (2002).
[36] A. Gade, A. Fitzler, C. Fransen, J. Jolie, S. Kasemann, H. Klein, A. Linnemann, V. Werner, and P. von Brentano, Phys. Rev. C **66**, 034311 (2002).
[37] I. Dillmann, K.-L. Kratz, A. Wöhr, O. Arndt, B. A. Brown, P. Hoff, M. Hjorth-Jensen, U. Köster, A. N. Ostrowski, B. Pfeiffer *et al.*, Phys. Rev. Lett. **91**, 162503 (2003).
[38] M. Kadi, N. Warr, P. E. Garrett, J. Jolie, and S. W. Yates, Phys. Rev. C **68**, 031306(R) (2003).
[39] Y. Wang, S. Rinta-Antila, P. Dendooven, J. Huikari, A. Jokinen, V. S. Kolhinen, G. Lhersonneau, A. Nieminen, S. Nummela, H. Penttilä *et al.*, Phys. Rev. C **67**, 064303 (2003).
[40] C. Kohstall, D. Belic, P. von Brentano, C. Fransen, A. Gade, R.-D. Herzberg, J. Jolie, U. Kneissl, A. Linnemann, A. Nord *et al.*, Phys. Rev. C **72**, 034302 (2005).
[41] P. Datta, S. Chattopadhyay, S. Bhattacharya, T. K. Ghosh, A. Goswami, S. Pal, M. S. Sarkar, H. C. Jain, P. K. Joshi, R. K. Bhowmik *et al.*, Phys. Rev. C **71**, 041305(R) (2005).

[42] N. Boelaert, A. Dewald, C. Fransen, J. Jolie, A. Linnemann, B. Melon, O. Möller, N. Smirnova, and K. L. G. Heyde, Phys. Rev. C **75**, 054311 (2007).

[43] A. Linnemann, C. Fransen, J. Jolie, U. Kneissl, P. Knoch, C. Kohstall, D. Mücher, H. H. Pitz, M. Scheck, C. Scholl *et al.*, Phys. Rev. C **75**, 024310 (2007).

[44] S. F. Ashley, P. H. Regan, K. Andgren, E. A. McCutchan, N. V. Zamfir, L. Amon, R. B. Cakirli, R. F. Casten, R. M. Clark, W. Gelletly *et al.*, Phys. Rev. C **76**, 064302 (2007).

[45] P. E. Garrett, K. L. Green, H. Lehmann, J. Jolie, C. A. McGrath, M. Yeh, and S. W. Yates, Phys. Rev. C **75**, 054310 (2007).

[46] D. Bandyopadhyay, S. R. Lesher, C. Fransen, N. Boukharouba, P. E. Garrett, K. L. Green, M. T. McEllistrem, and S. W. Yates, Phys. Rev. C **76**, 054308 (2007).

[47] N. Hoteling, W. B. Walters, B. E. Tomlin, P. F. Mantica, J. Pereira, A. Becerril, T. Fleckenstein, A. A. Hecht, G. Lorusso, M. Quinn *et al.*, Phys. Rev. C **76**, 044324 (2007).

[48] A. Jungclaus, L. Cáceres, M. Górska, M. Pfützner, S. Pietri, E. Werner-Malento, H. Grawe, K. Langanke, G. Martínez-Pinedo, F. Nowacki *et al.*, Phys. Rev. Lett. **99**, 132501 (2007).

[49] L. Cáceres, M. Górska, A. Jungclaus, M. Pfützner, H. Grawe, F. Nowacki, K. Sieja, S. Pietri, D. Rudolph, Z. Podolyák *et al.*, Phys. Rev. C **79**, 011301(R) (2009).

[50] K. L. Green, P. E. Garrett, R. A. E. Austin, G. C. Ball, D. S. Bandyopadhyay, S. Colosimo, D. Cross, G. A. Demand, G. F. Grinyer, G. Hackman *et al.*, Phys. Rev. C **80**, 032502 (2009).

[51] J. C. Batchelder, J. L. Wood, P. E. Garrett, K. L. Green, K. P. Rykaczewski, J. C. Bilheux, C. R. Bingham, H. K. Carter, D. Fong, R. Grzywacz *et al.*, Phys. Rev. C **80**, 054318 (2009).

[52] M. Breitenfeldt, C. Borgmann, G. Audi, S. Baruah, D. Beck, K. Blaum, C. Böhm, R. B. Cakirli, R. F. Casten, P. Delahaye *et al.*, Phys. Rev. C **81**, 034313 (2010).

[53] P. E. Garrett, J. Bangay, A. Diaz Varela, G. C. Ball, D. S. Cross, G. A. Demand, P. Finlay, A. B. Garnsworthy, K. L. Green, G. Hackman *et al.*, Phys. Rev. C **86**, 044304 (2012).

[54] Y. X. Luo, J. O. Rasmussen, C. S. Nelson, J. H. Hamilton, A. V. Ramayya, J. K. Hwang, S. H. Liu, C. Goodin, N. J. Stone, S. J. Zhu *et al.*, Nucl. Phys. A **874**, 32 (2012).

[55] J. C. Batchelder, N. T. Brewer, R. E. Goans, R. Grzywacz, B. O. Griffith, C. Jost, A. Korgul, S. H. Liu, S. V. Paulauskas, E. H. Spejewski *et al.*, Phys. Rev. C **86**, 064311 (2012).

[56] D. T. Yordanov, D. L. Balabanski, J. Bieroń, M. L. Bissell, K. Blaum, I. Budinčević, S. Fritzsche, N. Frömmgen, G. Georgiev, C. Geppert *et al.*, Phys. Rev. Lett. **110**, 192501 (2013).

[57] S. Ilieva, M. Thürauf, T. Kröll, R. Krücken, T. Behrens, V. Bildstein, A. Blazhev, S. Bönig, P. A. Butler, J. Cederkäll *et al.*, Phys. Rev. C **89**, 014313 (2014).

[58] J. C. Batchelder, N. T. Brewer, C. J. Gross, R. Grywacz, J. H. Hamilton, M. Karmy, A. Fijalkowska, S. H. Liu, K. Miernik, S. W. Padgett *et al.*, Phys. Rev. C **89**, 054321 (2014).

[59] H.-K. Wang, K. Kaneko, and Y. Sun, Phys. Rev. C **89**, 064311 (2014).

[60] P. E. Garrett and J. L. Wood, J. Phys. G **37**, 064028 (2010); **37**, 069701 (2010).

[61] K. L. G. Heyde and J. L. Wood, Rev. Mod. Phys. **83**, 1467 (2011).

[62] National Nuclear Data Center (NNDC), Brookhaven National Laboratory, http://www.nndc.bnl.gov/ensdf/ (May 2016).

[63] T. Faestermann, M. Górska, and H. Grawe, Prog. Part. Nucl. Phys. **69**, 85 (2013).

[64] M. Górska, R. Schubart, H. Grawe, J. B. Fitzgerald, D. B. Fossan, J. Heese, K. H. Maier, M. Rejmund, K. Spohr, and T. Rzaca-Urban, Z. Phys. A **350**, 181 (1994).

[65] M. Górska, M. Lipoglavšek, H. Grawe, J. Nyberg, A. Atac., A. Axelsson, R. Bark, J. Blomqvist, J. Cederkäll, B. Cederwall *et al.*, Phys. Rev. Lett. **79**, 2415 (1997).

[66] A. Blazhev, M. Górska, H. Grawe, J. Nyberg, M. Palacz, E. Caurier, O. Dorvaux, A. Gadea, F. Nowacki, C. Andreoiu *et al.*, Phys. Rev. C **69**, 064304 (2004).

[67] A. Blazhev, N. Braun, H. Grawe, P. Boutachkov, B. S. Nara Singh, T. Brock, Z. Liu, R. Wadsworth, M. Górska, J. Jolie *et al.*, J. Phys. Conf. Series G **205**, 012035 (2010).

[68] R. M. Clark, J. N. Wilson, D. Appelbe, M. P. Carpenter, C. J. Chiara, M. Cromaz, M. A. Deleplanque, M. Devlin, R. M. Diamond, P. Fallon *et al.*, Phys. Rev. C **61**, 044311 (2000).

[69] K. P. Lieb, D. Kast, A. Jungclaus, I. P. Johnstone, G. de Angelis, C. Fahlander, M. de Poli, P. G. Bizzeti, A. Dewald, R. Peusquens *et al.*, Phys. Rev. C **63**, 054304 (2001).

[70] N. Boelaert, N. Smirnova, K. L. G. Heyde, and J. Jolie, Phys. Rev. C **75**, 014316 (2007).

[71] A. Ekström, J. Cederkäll, D. D. DiJulio, C. Fahlander, M. Hjorth-Jensen, A. Blazhev, B. Bruyneel, P. A. Butler, T. Davinson, J. Eberth *et al.*, Phys. Rev. C **80**, 054302 (2009).

[72] B. A. Brown and K. Rykaczewski, Phys. Rev. C **50**, 2270(R) (1994).

[73] T. Schmidt, K. L. G. Heyde, A. Blazhev, and J. Jolie, Phys. Rev. C **96**, 014302 (2017).

[74] K. L. G. Heyde, C. De Coster, J. Jolie, and J. L. Wood, Phys. Rev. C **46**, 541 (1992).

[75] J. Jolie and H. Lehmann, Phys. Lett. B **342**, 1 (1995).

[76] K. L. G. Heyde, J. Jolie, H. Lehmann, C. De Coster, and J. L. Wood, Nucl. Phys. A **586**, 1 (1995).

[77] H. Lehmann and J. Jolie, Nucl. Phys. A **588**, 623 (1995).

[78] C. De Coster, K. L. G. Heyde, B. Decroix, P. Van Isacker, J. Jolie, H. Lehmann, and J. L. Wood, Nucl. Phys. A **600**, 251 (1996).

[79] H. Lehmann, J. Jolie, C. De Coster, B. Decroix, K. L. G. Heyde, and J. L. Wood, Nucl. Phys. A **621**, 767 (1997).

[80] C. De Coster, B. Decroix, K. L. G. Heyde, J. L. Wood, J. Jolie, and H. Lehmann, Nucl. Phys. A **621**, 802 (1997).

[81] C. De Coster, B. Decroix, K. L. G. Heyde, J. Jolie, H. Lehmann, and J. L. Wood, Nucl. Phys. A **651**, 31 (1999).

[82] P. E. Garrett, K. L. Green, and J. L. Wood, Phys. Rev. C **78**, 044307 (2008).

[83] L. Prochniak, P. Quentin, and M. Imadalou, Mod. Phys. E **21**, 1250036 (2012).

[84] T. R. Rodriguez, J. L. Egido, and A. Jungclaus, Phys. Lett. B **668**, 410 (2008).

[85] K. Nomura, R. Rodríguez-Guzmán, L.M. Robledo, and N. Shimizu, Phys. Rev. C **86**, 034322 (2012).

[86] K. Nomura, N. Shimizu, and T. Otsuka, Phys. Rev. Lett. **101**, 142501 (2008).

[87] P. Bonche, H. Flocard, and P.-H. Heenen, Comput. Phys. Commun. **171**, 49 (2005).

[88] W. Ryssens, V. Hellemans, M. Bender, and P.-H. Heenen, Comput. Phys. Commun. **187**, 175 (2015).

[89] E. Chabanat, P. Bonche, P. Haensel, J. Meyer, and R. Schaeffer, Nucl. Phys. A **635**, 231 (1998).

[90] J. N. Ginocchio and M. W. Kirson, Nucl. Phys. A **350**, 31 (1980).

[91] T. Otsuka, A. Arima, and F. Iachello, Nucl. Phys. A **309**, 1 (1978).
[92] P. D. Duval and B. R. Barrett, Phys. Lett. B **100**, 223 (1981).
[93] K. Nomura, R. Rodríguez-Guzmán, and L.M. Robledo, Phys. Rev. C **87**, 064313 (2013).
[94] A. Frank, P. Van Isacker, and C.E. Vargas, Phys. Rev. C **69**, 034323 (2004).
[95] J. Blachot, Nucl. Data Sheets **91**, 135 (2000).
[96] G. Gurdal and F.G. Kondev, Nucl. Data Sheets **113**, 1315 (2012).
[97] S. Lalkovski and F. G. Kondev, Nucl. Data Sheets **124**, 157 (2015).
[98] J. Blachot, Nucl. Data Sheets **113**, 515 (2012).
[99] J. Blachot, Nucl. Data Sheets **111**, 717 (2010).
[100] A. Giannatiempo, A. Nannini, A. Perego, and P. Sona, Phys. Rev. C **44**, 1844 (1991).
[101] A. Giannatiempo *et al.*, Eur. Phys. J. A **52**, 36 (2016).
[102] P. E. Garrett, private communication (2018).
[103] T. Kibedi and R. H. Spear, At. Data Nucl. Data. Tabl. **89**, 77 (2005).
[104] P. E. Garrett, J. Phys. G **43**, 084002 (2016).