Low-energy process $e^+e^- \to K^+K^-$ in the extended Nambu–Jona-Lasinio model

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The process $e^+e^- \to K^+K^-$ is described in the framework of the Nambu–Jona-Lasinio model in the energy region 1–1.7 GeV. The contact terms with intermediate photon and terms with intermediate ρ, ω, and ϕ vector mesons in the ground and first radially excited states are taken into account. Our results are in satisfactory agreement with experimental data recently obtained at VEPP-2000 and SLAC.

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I. INTRODUCTION

Recently, great attention has been paid to the experimental study of the production of charged kaon pairs in colliding electron-positron beams in the energy range 1–2 GeV at the Budker Institute of Nuclear Research (Novosibirisk) [1–5] and in SLAC National Accelerator Laboratory (Stanford) [6]. For theoretical description of this process in the low-energy region, unfortunately, we can not use the QCD perturbation theory, but here we can successfully apply various phenomenological models based, as a rule, on the chiral symmetry of strong interactions. Among them, we would like to specify the chiral perturbation theory (ChPT) [7,8], since this model is preferable only at lower energies not grafting the mass of the ρ meson. At present, a number of extended versions of the ChPT have appeared, for example the ChPT with resonances [9] and the extended vector-meson-dominance (VMD) model [10]. In the extended VMD model, the experimental results of SND [1] have been recently described, but in this model in order to include the radially excited intermediate states, it was necessary to use a number of additional arbitrary parameters.

A well-known model is also the Nambu–Jona-Lasinio (NJL) model. In addition to the standard NJL model [11–20] that is used to describe mesons in the ground states and their interactions at low energies, there is also an extended version of the NJL model [21–26], which allows one to describe not only the ground states but also the first radially excited meson states, without violating the U(3)×U(3) chiral symmetry. This is achieved by using the simplest form factors that have the form of the first-order polynomials in the square of the quark momenta.

The form factor:

$$F(|k_\perp|^2) = cf(|k_\perp|^2), f(|k_\perp|^2) = 1 + d|k_\perp|^2, \tag{1}$$

$$|k_\perp| = \sqrt{-k_\perp{}^2}, \tag{2}$$

being a Lorentz scalar, can be calculated in any convenient reference frame. The constant c only affects the radially excited meson masses,

$$k_\perp = k - \frac{(kp)p}{p^2} \tag{3}$$

is the transverse part of the relative momentum k of the quark-antiquark pair and p is the four-momentum of their center of mass reference frame, i.e., the meson momentum. In the following we use the rest frame of the meson. In this case

$$k_\perp = (0,\vec{k}), f(k_\perp{}^2) = 1 + d\vec{k}^2 \equiv f(\vec{k}^2). \tag{4}$$

The slope parameter d is chosen so that the radially excited state does not influence the quark condensate and, hence, the values of the constituting quark masses. These masses are the main parameters of the NJL model. In this case, the quark loop having the form of a tadpole containing the form factor $f(\vec{k}^2)$ should be equated to zero. After the introduction of the first radially excited meson states, nondiagonal terms appear in the free Lagrangian in the NJL model. These terms correspond to mixing between the meson states with and without form factors. The free Lagrangian is diagonalized by introducing mixing angles [22,24,26]. After diagonalizing the free Lagrangian, we can describe various meson interactions in both the ground and first radially excited states without introducing any additional arbitrary parameters.

Recently, by using the extended NJL model, the following processes have been described: $e^+e^- \to \gamma[\eta, \eta'(958), \eta(1295), \eta(1475)]$, $e^+e^- \to \gamma[\pi, \pi(1300)]$, $e^+e^- \to \pi[\pi, \pi(1300)]$, $e^+e^- \to 2\pi[\eta, \eta'(958)]$, $e^+e^- \to \pi\omega$ (see in Refs. [26,27]), $e^+e^- \to \eta[\phi(1020), \rho]$, $e^+e^- \to K^\pm [K^{*\mp}(892)]$ [28,29]. The process $e^+e^- \to K^+K^-$, considered

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in the present paper, naturally supplements the previously considered similar processes. In the review [27], it is shown that the extended NJL model makes it possible to successfully describe not only the numerous meson production processes in colliding electron-positron beams but also many decay modes of τ lepton.

II. EFFECTIVE QUARK-MESON LAGRANGIAN

In the extended NJL model, for the description of the process $e^+e^- \to K^+K^-$, we will take into account the contact term and channels with intermediate ρ, ω, ϕ, $\rho(1450)$, $\omega(1420)$, $\phi(1680)$ vector mesons. The corresponding interaction Lagrangians read

$$\mathcal{L}_K = \bar{q}\left[i\gamma^5 \sum_{j=\pm} \lambda_j (a_K K^j + b_K K'^j)\right] q, \tag{5}$$

$$\mathcal{L}_\rho = \bar{q}\left[\frac{1}{2}\gamma^\mu \lambda_\rho (a_\rho \rho_\mu + b_\rho \rho'_\mu)\right] q, \tag{6}$$

$$\mathcal{L}_\omega = \bar{q}\left[\frac{1}{2}\gamma^\mu \lambda_\omega (a_\omega \omega_\mu + b_\omega \omega'_\mu)\right] q, \tag{7}$$

$$\mathcal{L}_\phi = \bar{q}\left[\frac{1}{2}\gamma^\mu \lambda_\phi (a_\phi \phi_\mu + b_\phi \phi'_\mu)\right] q, \tag{8}$$

where q and $\bar{q}$ are the u, d and s constituent quark fields with masses $m_u = m_d = 280$ MeV, $m_s = 420$ MeV [27,30],[1] $K^\pm$, ρ, ω, and ϕ are the pseudoscalar and vector mesons; the excited states are marked with prime. The coefficients are

$$\begin{aligned} a_a &= \frac{1}{\sin\left(2\theta_a^0\right)}\left[g_a \sin\left(\theta_a + \theta_a^0\right) + g_a' f_a(\vec{k}^2) \sin\left(\theta_a - \theta_a^0\right)\right], \\ b_a &= \frac{-1}{\sin\left(2\theta_a^0\right)}\left[g_a \cos\left(\theta_a + \theta_a^0\right) + g_a' f_a(\vec{k}^2) \cos\left(\theta_a - \theta_a^0\right)\right], \end{aligned} \tag{9}$$

$f(\vec{k}^2)$ is the form factor, θ_a and θ_a^0 are the mixing angles for the mesons in the ground and excited states [26,27].

The slope parameters and mixing angles are

$$d_{uu} = -1.784\,\mathrm{GeV}^{-2}, \quad d_{ss} = -1.737\,\mathrm{GeV}^{-2},$$

$$\begin{aligned} &\theta_K = 58.11^\circ, \quad \theta_\rho = \theta_\omega = 81.8^\circ, \quad \theta_\phi = 68.4^\circ, \\ &\theta_K^0 = 55.52^\circ, \quad \theta_\rho^0 = \theta_\omega^0 = 61.5^\circ, \quad \theta_\phi^0 = 57.13^\circ. \end{aligned} \tag{10}$$

The matrices

$$\lambda_\rho = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_\omega = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\lambda_\phi = -\sqrt{2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \lambda_+ = \sqrt{2}\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\lambda_- = \sqrt{2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix},$$

The coupling constants:

$$\begin{aligned} g_K &= \left(\frac{4}{Z_K} I_2(m_u,m_s)\right)^{-1/2} \approx 3.77, \\ g_K' &= \left(4 I_2^{f_{us}^2}(m_u,m_s)\right)^{-1/2} \approx 4.69, \\ g_\rho = g_\omega &= \left(\frac{2}{3} I_2(m_u,m_u)\right)^{-1/2} \approx 6.14, \\ g_\rho' = g_\omega' &= \left(\frac{2}{3} I_2^{f_{uu}^2}(m_u,m_u)\right)^{-1/2} \approx 9.87, \\ g_\phi &= \left(\frac{2}{3} I_2(m_s,m_s)\right)^{-1/2} \approx 7.5, \\ g_\phi' &= \left(\frac{2}{3} I_2^{f_{ss}^2}(m_s,m_s)\right)^{-1/2} \approx 13.19, \end{aligned}$$

where

$$Z_K = \left(1 - \frac{3}{2}\frac{(m_u + m_s)^2}{M_{K_1}^2}\right)^{-1} \approx 1.83, \tag{11}$$

Z_K is the factor corresponding to the K-K_1 transitions, $M_{K_1} = 1272$ MeV [31] is the mass of the axial-vector K_1 meson, and the integral I_2 has the following form:

$$\begin{aligned} I_2^{f^n}(m_1,m_2) = -i\frac{N_c}{(2\pi)^4}\int \frac{f^n(\vec{k}^2)}{\left(m_1^2 - k^2\right)\left(m_2^2 - k^2\right)} \\ \times \theta\left(\Lambda_3^2 - \vec{k}^2\right) d^4k, \end{aligned} \tag{12}$$

$\Lambda_3 = 1.03$ GeV is the cut-off parameter [22,24,26]. All these parameters were defined earlier and are standard for the extended NJL model.

III. AMPLITUDE OF THE PROCESS $e^+e^- \to K^+K^-$

The diagrams of the process $e^+e^- \to K^+K^-$ are shown in Figs. 1 and 2. This process contains the contributions of the following amplitudes:

$$\begin{aligned} T = \frac{16\pi\alpha_{em}}{s} l^\mu \left[B_{(\gamma)} + B_{(\rho+\rho')} + B_{(\omega+\omega')} + e^{i\pi} B_{(\phi+\phi')}\right]_{\mu\nu} \\ \times (p_{K^+} - p_{K^-})^\nu, \end{aligned} \tag{13}$$

where $s = [p(e^-) + p(e^+)]^2$, $l^\mu = \bar{e}\gamma^\mu e$ is the lepton current. Unfortunately, the NJL model can not describe a relative phase between different states. Thus, we take a phase from e^+e^- annihilation experiments [1,2] ($e^{i\pi}$ factor in the ϕ mesons). The contribution of the contact diagram is

$$B_{(\gamma)\mu\nu} = g_{\mu\nu} I_2^{a_K a_K}, \tag{14}$$

[1]Note that in different models the parameters m_u, m_d, and m_s being constituent quark masses can differ noticeably, for example, $m_u = m_d = 315$ MeV, $m_s = 510$ MeV [15], $m_u = m_d = 364$ MeV, $m_s = 522$ MeV [16,17], $m_u = 250$ MeV, $m_s = 500$ MeV [33].

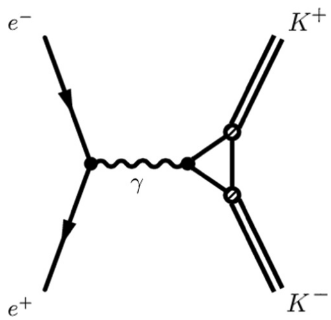

FIG. 1. The Feynman diagram with photon exchanges (contact diagram).

integrals $I_2^{a...,b...}$ are defined at the end of this section. The sum of V and V' vector meson contributions reads

$$B_{(V+V')\mu\nu} = r_V\left[\frac{C_V}{g_V}\frac{g_{\mu\nu}s - p_\mu p_\nu}{M_V^2 - s - i\sqrt{s}\Gamma_V(s)} I_2^{a_V a_K a_K} + \frac{C_{V'}}{g_V}\frac{g_{\mu\nu}s - p_\mu p_\nu}{M_{V'}^2 - s - i\sqrt{s}\Gamma_{V'}(s)} I_2^{b_V a_K a_K}\right], \quad (15)$$

here $V = \rho,\omega,\phi$ and $V' = \rho',\omega',\phi'$ are vector mesons. The numerical coefficients are $r_\rho = r_{\rho'} = 1/2$, $r_\omega = r_{\omega'} = 1/6$, $r_\phi = r_{\phi'} = 1/3$, and $M_\rho = 775$ MeV, $M_{\rho'} = 1465$ MeV, $M_\omega = 783$ MeV, $M_{\omega'} = 1420$ MeV, $M_\phi = 1019$ MeV, $M_{\phi'} = 1680$ MeV are the masses of the intermediate vector mesons [31]. Here, instead of the constant decay width Γ_V, we used $\Gamma(s)$ like in Ref. [6]:

$$\Gamma_V(s) = \Gamma_V \frac{s}{M_V^2}\left(\frac{\beta(s,M_K)}{\beta(M_V^2,M_K)}\right)^3, \quad (16)$$

where $\beta(s,M_K) = \sqrt{1 - 4{M_K}^2/s}$.

The numerical coefficients C_V are obtained from the quark loops in the transitions of the photon into the intermediate vector mesons:

$$C_V = \frac{1}{\sin\left(2\theta_a^0\right)}\left[\sin\left(\theta_a + \theta_a^0\right) + R_V \sin\left(\theta_a - \theta_a^0\right)\right], \quad (17)$$

$$R_V = \frac{I_2^f(m_1,m_2)}{\sqrt{I_2(m_1,m_2) I_2^{f^2}(m_1,m_2)}},$$

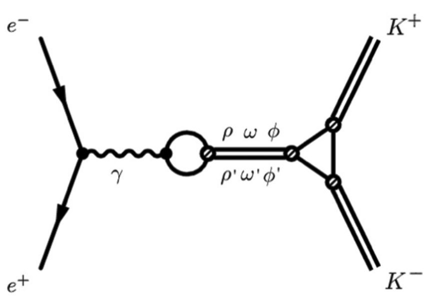

FIG. 2. Feynman diagram(s) with intermediate vector meson exchange.

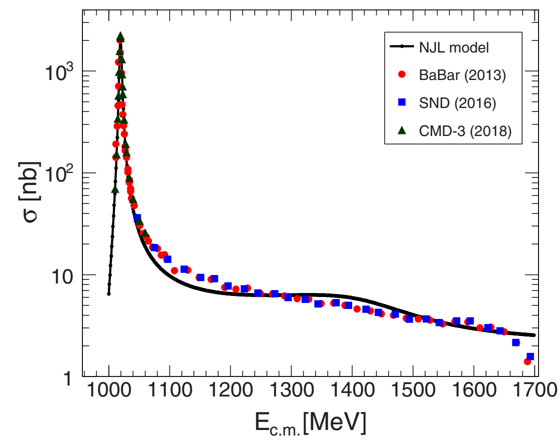

FIG. 3. The figure in the logarithmic scale shows the theoretical predictions of the NJL model for the process $e^+e^- \to K^+K^-$ with the account of six intermediate meson states. The experimental data [4–6] are shown as separate points.

where m_1 and m_2 are the masses of the u or s quarks depending on the quark structure of the intermediate vector meson. The integrals

$$I_2^{a...b...}(m_u,m_s) = -i\frac{N_c}{(2\pi)^4} \times \int \frac{a(\vec{k}^2)...b(\vec{k}^2)...}{\left(m_u^2 - k^2\right)\left(m_s^2 - k^2\right)} \times \theta\left(\Lambda_3^2 - \vec{k}^2\right) d^4k, \quad (18)$$

are obtained from the quark triangular loops, $a(\vec{k}^2)$ and $b(\vec{k}^2)$ are the coefficients for different mesons defined in (9).

IV. NUMERICAL ESTIMATIONS

The cross section of the process $e^+e^- \to K^+K^-$ as a function of energy is shown in Figs. 3 and 4.

The results are basically in satisfactory agreement with the experimental data of SND, CMD-3 [4,5], and the BaBar Collaboration [6] in the energy range 1–1.6 GeV (see Figs. 3, 4). It is also interesting to compare our results with the

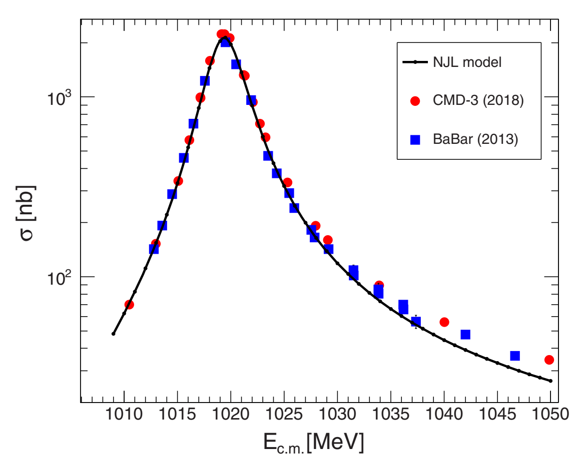

FIG. 4. The resonance region of the ϕ meson in the process $e^+e^- \to K^+K^-$.

TABLE I. The comparison of the absolute values of the factors in the numerators of the Breit-Wigner propagators, which describe the transition $\gamma \to V \to K^+K^-$.

	Our result	[10]	[32]
N_ρ	0.44	0.598	0.598
N_ω	0.147	0.171	0.199
N_ϕ	0.34	0.283	0.339
$N_{\rho'}$	0.066	0.056	0.056
$N_{\omega'}$	0.022	0.016	0.019
$N_{\phi'}$	0.0005	0.005	0.006

results obtained in other phenomenological models [10,32], in particular, the vector-meson-dominance model, which is based on the chiral perturbation theory mentioned in Sec. I. These results together with our results are summarized in Table I.

In Table I, the absolute values of the numerators of the Breit-Wigner propagators of the form factors are given. In Ref. [10] four possible different variants are compared in Table 4. We selected the third one, which is based on comparisons with Ref. [32]. In the NJL model, these numerators in the amplitude correspond to the following expression:

$$N_V = r_V \frac{4C_V}{g_\phi} I_2^{a_V(b_V)a_K a_K}, \tag{19}$$

where the coefficient C_V, as shown in (17), is obtained from the quark loops in the transitions of the photon into the intermediate vector mesons. The integral $I_2^{a_V(b_V)a_K a_K}$ is defined in (18). Also, we would like to note that there are many papers devoted to the investigation of the kaon form factors at low energy (see, for instance, Refs. [33–36]).

V. CONCLUSION

The process $e^+e^- \to K^+K^-$ is described in the framework of the extended NJL model in the energy domain 1–1.6 GeV in satisfactory agreement with experimental data. In the region of energies exceeding 1.6 GeV the second radially excited $\rho(1700)$, $\omega(1650)$ meson states play an important role. Since our model does not include these states, we can not claim to have results in this region. We would like to emphasize that the results were obtained without using any additional parameters to fit the experimental data.

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