Energies and widths in ^{11}N

H. T. Fortune
Department of Physics and Astronomy, University of Pennsylvania, Philadelphia, Pennsylvania 19104, USA

(Received 17 April 2018; published 5 July 2018)

Using experimental information from ^{11}Be and a simple potential model, I have computed energies and widths expected for several resonances in ^{11}N. The conclusion is that two or three expected resonances have never been observed.

DOI: 10.1103/PhysRevC.98.014307

I. INTRODUCTION AND HISTORY

Several experimental groups [1–8] have investigated resonances in ^{11}N, which has no bound states [9]. These experiments (listed in Table I) have included $^{10}C + p$ elastic scattering, plus several different reactions that made use of both stable and radioactive beams. Despite these many investigations, making a one-to-one correspondence for resonances reported in different reactions is frequently not easy. I illustrate this point in Figs. 1 and 2, where I have plotted data from two different experiments. In Fig. 1, the resonance energies from Guimaraes *et al.* [7] in the reaction $^{14}N(^3He, ^6He)$ are plotted vs those from Oliveira *et al.* [5] in the reaction $^{10}B(^{14}N, ^{13}B)$. The straight line corresponds to $E2 = E1$. For the lowest resonance, the energy reported by Guimaraes *et al.* is significantly below that of Oliveira *et al.*, but for all the others the former ones are very slightly larger. This plot would make it appear that one-to-one correspondences are trivial. However, the situation is quite different for the widths, as can be seen in Fig. 2. Here, three widths agree, two disagree badly, and one disagrees by about 2σ. This plot would make it appear that, for at least two or three cases, the two experiments are populating different resonances. Furthermore, most of the J^π suggestions made in the various papers are based only on the assumption of assigned mirror correspondence and J^π information from ^{11}Be. However, comparison of the energy levels of ^{11}Be [9] and the resonances in ^{11}N demonstrates that a few ^{11}Be states have no apparent counterparts. The aim of the present exercise is to use the available information in ^{11}Be to predict energies and widths in ^{11}N. But, first I summarize a little history.

Benenson, *et al.* [1] used the $^{14}N(^3He, ^6He)$ reaction, at $E(^3He) = 70$ MeV to populate the $1/2^-$ resonance that is the mirror of the first excited state of ^{11}Be. Failure to excite the $1/2^+$ and $5/2^+$ states was understandable on the basis of the scarcity of $2s1d$ neutrons in ^{14}N. Co-workers and I [10] used a potential model, plus mirror symmetry, to compute energies and widths for the first three resonances. We concluded that the ground state (g.s.) resonance was so broad that "the state is nearly too unbound to compute," and we assigned an uncertainty of 0.22 MeV to our predicted energy. We used the energy and width from Benenson *et al.* [1] together with our computed single-particle width to extract a spectroscopic factor for the $1/2^-$ state. The result was 0.64(12), in good agreement with Auton's ^{11}Be value of 0.63(15) [11] from the $^{10}Be(d, p)$ reaction, but in disagreement with Zwieglinski's $S = 0.96$ from the same reaction [12]. Benenson *et al.* had reported that their width corresponded to $S = 0.7(1)$.

Sherr and I [13] analyzed the first three states in all four members of the $A = 11$ isospin quartet. We concluded that something was wrong with the energies and widths of the $1/2^+$, $T = 3/2$ states in both ^{11}B and ^{11}C. Later, I reanalyzed data for the $^{10}Be(p, \gamma)$ reaction [14] and found a much larger width: 600 to 640 keV [15], compared to 230(65) keV in the original analysis and 210(20) keV in the compilation. Barker disagreed with my results and also refitted the data. His results were "a width of order 400 keV" and "a width of order 600 keV" [16]. Still later, by requiring self-consistency between energies of $1/2^+$, $T = 3/2$ states and those having $J^\pi = 0^+$, $T = 2$, I demonstrated [17] that the energy in ^{11}C required a downward correction of about 200 keV. MacCormick and Audi [18] came to a similar conclusion.

After several experiments had been performed to measure the energy of the ^{11}N g.s. resonance, it was obvious that the various results differed by more than the stated uncertainties, prompting three independent unweighted averages [9,17,19] of both energy and width. In the discussion that follows, I use resonance energies rather than excitation energies in ^{11}N. Some old-timers might think my E_p should correspond to energies in the laboratory frame, but they do not. All my energies—E_n, E_p, etc.—represent center-of-mass energies. My E_p is sometimes called E_r or E_{res}, or $-S_p$.

II. CALCULATIONS AND RESULTS

I have used a simple potential model to compute resonance energies in ^{11}N, based on the known excitation energies in ^{11}Be. Briefly, the depth of a Woods-Saxon well with geometrical parameters r_0, $a = 1.26$, 0.60 fm is adjusted to reproduce the separation energy of a given state in ^{11}Be. Then, this potential is used, with the addition of the Coulomb potential of a uniform sphere having $r_{0c} = 1.40$ fm, to compute the proton energy in ^{11}N. Because most of these resonances are very broad, uncertainties in the calculation arising from subtleties in configuration mixing are swamped by uncertainty in the definition of the energy of a resonance. Of course, it is still important to pay attention to whether a given state is primarily

TABLE I. Reactions that have been used to investigate resonances in ^{11}N.

Year	Reaction	E_{max}(MeV)	Ref.
1974	^{14}N(^{3}He, ^{6}He)	∼5	Benenson *et al.* [1]
1996	^{10}C + *p*	4+	Axelsson *et al.* [2]
1998	^{12}C(^{14}N, ^{15}C)	5.87 (no g.s.)	Lepine-Szily *et al.* [3]
1998	^{9}Be(^{12}N, ^{11}N),	4+	Azhari *et al.* [4]
2000	^{10}B(^{14}N, ^{13}B)	6.54	Oliveira *et al.* [5]
2000	^{10}C + *p*	5	Markenroth [6]
2003	^{14}N(^{3}He, ^{6}He)	6.81	Guimaraes *et al.* [7]
2006	^{10}C + *p*	2.7	Casarejos *et al.* [8]

a *p*-shell excitation or has one or two nucleons in the *sd* shell. For the latter, it is important to keep track of the *s*/*d* ratio, because the energy shifts depend significantly on this mixing.

I have not considered the first three states because there is no question about their identity, even though experimental results from various experiments are somewhat different. Beginning with the fourth state of ^{11}Be, states with known (or suspected) J^{π} assignments [9,20,21] are listed in Table II, along with experimental widths. Beginning with the $5/2^-$ state at 3.89 MeV, all states can decay to the 2^+ first excited state of ^{10}Be as well as to the g.s. Of course, all the ^{11}N resonances in this table can decay to the 2^+ of ^{10}C. For each state, the widths on the first line refer to g.s. decays, and those on the second line are for decays to the 2^+. Looking first at the predicted energies, large differences can be seen for different states, because of the question of *p* vs *s* vs *d* configurations. For each state in ^{11}Be, I have computed single-particle decay widths for the appropriate ℓ value and energy. For resonances in ^{11}N, I have used the calculated energies for this purpose. Of course, experimental energies should be used, but they are not generally well enough known. Many of these widths are very large, and thus difficult to compute, so I have estimated them. In ^{11}Be, I have computed the spectroscopic factors from the expression $S = \Gamma_{exp}/\Gamma_{sp}$. In some cases, I also give the shell-model values of *S*. For the ^{11}N resonances, I define $\Gamma_{calc} = S\,\Gamma_{sp}$, where I have used the *S*'s from the mirrors in ^{11}Be. The last column labeled Γ_{tot} contains the sum of calculated widths for decays to the g.s. and 2^+ state of ^{10}C.

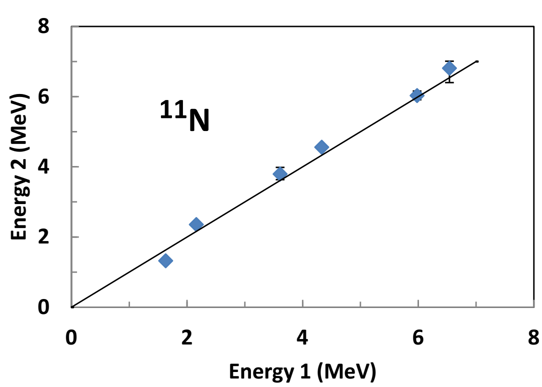

FIG. 1. Resonance energies from Guimaraes *et al.* [7] in the reaction ^{14}N(^{3}He, ^{6}He) are plotted vs those from Oliveira *et al.* [5] in the reaction ^{10}B(^{14}N, ^{13}B). The straight line corresponds to $E2 = E1$.

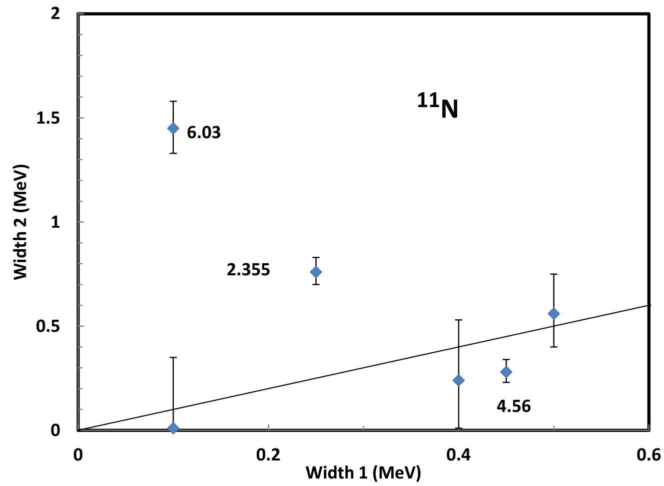

FIG. 2. As Fig. 1, but for widths.

I now discuss each state in turn. I leave aside any discussion of experimental comparisons until later. The $3/2^-$ state at 2.69 MeV in ^{11}Be is predominantly a *p*-shell state. It is reasonably broad, having a width of 213(5) keV, all of which corresponds to g.s. decay because decay to the 2^+ state is energetically forbidden. However, shell-model calculations predict a large spectroscopic factor for the 2^+ coupling, $S = 0.864$ [22]. The g.s. decay corresponds to $S = 0.12$, to be compared with shell-model values of 0.106 [22] or 0.168 [23]. In ^{11}N, the single-particle widths for decay to 2^+ and 0^+ states are 295 keV and about 3 MeV, respectively, producing calculated widths of 255 and 360 keV, totaling 615 keV.

The next state in ^{11}Be has $J = 3/2$, with uncertain parity. Consensus appears to favor positive parity, with the dominant

TABLE II. Experimental energies (MeV) and widths (keV) in ^{11}Be and calculated quantities for the mirrors in ^{11}N.

	^{11}Be					^{11}N (calculated)			
J^{π}	E_x	Final	Γ_{exp}	Γ_{sp}	S[a]	E_p	Γ_{sp}	Γ_{calc}[f]	Γ_{tot}
$3/2^-$	2.69	0^+	213(5)	1800	0.12	4.56	3000	360	615
		2^+			0.864[b]		295	255	
$3/2^{(+)}$	3.41	0^+	113(13)	1100	0.10(1)	4.77	980	98	1720
		2^+			0.9[c]		1800	1620	
$5/2^-$	3.89	0^+	1.2(5)	170	0.007(3)	5.93	230?	2	1520
		2^+	2.0(8)	3.4	0.59(24)[d]		2300	1520[e]	
$3/2^-$	3.96	0^+	1.7(3)	2300	0.7×10^{-3}	4.79	3000	2	74
		2^+	6.2(6)	30	0.21(2)		350	72	
$5/2^-$	5.24	0^+				6.55			∼70
		2^+	37(7)	1450	0.026(5)		∼2600	∼70	

[a] $S = \Gamma_{exp}/\Gamma_{sp}$, unless otherwise noted.
[b] Theoretical value.
[c] Assumed nearly pure ^{10}Be(2^+) $\otimes$ *s*.
[d] Theoretical value is 0.66.
[e] Used theoretical $S = 0.66$.
[f] $\Gamma_{calc} = S\Gamma_{sp}$.

TABLE III. Comparison of calculated energies and widths (both in MeV) with experimental ones in ^{11}N.

	^{11}Be	^{11}N calc.		^{11}N exp. Guimaraes *et al.*		^{11}N exp. Oliveira *et al.*	
J^π	E_x	E_p	Γ_{tot}	E_p	Γ	E_p	Γ
$3/2^-$	2.69	4.56	0.615	$4.559^{+0.011}_{-0.012}$	$0.28^{+0.06}_{-0.05}$	4.33(5)	0.45(8)
$3/2^{(+)}$	3.41	4.77	1.718[a]	No candidate			
$5/2^-$	3.89	5.93	1.520	$6.030^{+0.042}_{-0.034}$	$1.45^{+0.13}_{-0.12}$	5.98(10)	0.10(6)
$3/2^-$	3.96	4.79	0.074	No candidate			
$5/2^-$	5.24	6.55	~ 0.070	$6.81^{+0.20}_{-0.41}$[b]	$0.01^{+0.34}_{-0.01}$	6.54(10)	0.10(6)

[a]Calculated g.s. width is only 98 keV.
[b]Authors assumed J^π is $3/2^-$.

structure ^{10}Be$(2^+) \otimes s$. Its experimental width for decay to ^{10}Be(g.s.) is 113(13) keV, which corresponds to a $d_{3/2}$ spectroscopic factor of 0.10(1). Decay to the 2^+ is energetically forbidden in ^{11}Be, but allowed in ^{11}N. If I assume that spectroscopic factor is about 0.9, the predicted width in ^{11}N is about 1.6 MeV to the 2^+, and only about 100 keV to the g.s.

The $5/2^-$ state at 3.89 MeV in ^{11}Be preferentially decays to the 2^+ [24,25], despite the fact that that decay energy is only 20 keV. That result is not surprising, because of the very small $f_{5/2}$ strength expected. This 2^+ decay corresponds to a spectroscopic factor of 0.59(24) [26], to be compared with a shell-model value of 0.66 [22] for this p-shell state. In ^{11}N, I have used the shell-model value, because of the large uncertainty on the experimental value in ^{11}Be. The expected width in ^{11}N is thus about 1.5 MeV.

The $3/2^-$ state at 3.96 MeV in ^{11}Be has been identified as the lowest $(sd)^2$ state [20], with the configuration ^{9}Be(g.s.) $\otimes$ $(sd)^2_0$. It preferentially decays to the 2^+ of ^{10}Be [24,25] despite a factor of about 80 favoring g.s. decay from the single-particle widths. Spectroscopic factors are about 0.7×10^{-3} for g.s. decay and 0.21(2) for 2^+ decay [26]. Of course, both of these would be zero in the absence of configuration mixing. The 2^+ value seems a bit large to have arisen from such mixing, but that is what the data show. In ^{11}N, the S's correspond to about 2 and 72 keV for g.s. and 2^+ decays, respectively. Both are seen to be extremely small for a state at such a high unbound energy.

The $5/2^-$ state at 5.24 MeV is thought to have the major configuration ^{9}Be$\otimes(sd)^2_2$ [20]. The predicted resonance energy for its mirror in ^{11}N is 6.55 MeV. In ^{11}Be, its width for decay to the 2^+ state is 37(7) keV, resulting in a spectroscopic factor of 0.026(5), a value small enough to indicate that this is not a p-shell state. In ^{11}N, this S produces an expected width of about 70 keV.

Insufficient information is available for higher states in ^{11}Be. So, now I turn to a comparison of experimental ^{11}N energies and widths with my predictions. Guimaraes *et al.* observed four resonances that are good candidates for predicted ones (Table III), even though their width for their 4.6-MeV resonance is too small, and they assumed $J^\pi = 3/2^-$ for their 6.81-MeV resonance, compared to a probable $J^\pi = 5/2^-$ assignment in ^{11}Be. They do not observe any candidates for the other resonances, but that is understandable because those resonances have one or two protons in the sd shell, and they would be expected to be extremely weak in the (^{3}He, ^{6}He) reaction. Their 6.03-MeV resonance is in excellent agreement with predictions, in both energy and width. Oliveira's.4.33-MeV resonance has a better width, but the energy is too low. Because of its extremely small width, it is difficult to know what their 5.98-MeV resonance could be. However, their 6.54-MeV resonance agrees well in both energy and width for the mirror of the 5.24-MeV state in ^{11}Be. The resonances that were not observed in these two experiments would also likely not have been populated in ^{10}C + p elastic scattering, because of their very small g.s. widths.

Lepine-Szily, *et al.*, in the reaction ^{12}C(^{14}N, ^{15}C), observed a narrow ($\Gamma < 0.2(1)$ MeV) resonance at 4.39(5) MeV, which they suggested could be the mirror of ^{11}Be$(3/2^-)$ at 3.96 MeV. It is narrow enough to correspond to this mirror, but the energy is significantly low. Other resonances observed in this reaction in the relevant region are compared with my predictions in Table IV. I have discussed these results elsewhere [27], and I suggested the two narrow peaks in this experiment might not correspond to resonances, but rather to two peaks that arise from interference of two resonances of the same J^π: one wide, one narrow.

The three reactions ^{14}N(^{3}He, ^{6}He), ^{12}C(^{14}N, ^{15}C), and ^{10}B(^{14}N, ^{13}B) might be expected to have different selectivity for different types of states. All three have ^{14}N in the initial state and ^{11}N in the final state, but the mechanisms of the three are quite different. The ^{14}N(^{3}He, ^{6}He) reaction will preferentially populate states that can be reached in three-neutron removal from the g.s. of ^{14}N. Because ^{14}N(g.s.) contains a very small percentage of sd-shell occupancy, states of ^{11}N with one or two protons in the sd shell would be expected to be weak. One possible mechanism of the ^{12}C(^{14}N, ^{15}C) reaction is a two-step process of ^{12}C(^{14}N, ^{13}C,) followed by ^{13}N(^{13}C, ^{15}C).

TABLE IV. Comparison of present predictions with results of the reaction ^{12}C(^{14}N, ^{15}C). (Energies and widths are both in MeV.)

	^{11}Be	^{11}N calc.		^{11}N exp. Lépine-Szily *et al.*,		
J^π	E_x	E_p	Γ_{tot}	E_p	Γ	J^π
$3/2^-$	2.69	4.56	0.615	5.12(8)	<0.22(10)	$(5/2^-)$
$3/2^{(+)}$	3.41	4.77	1.718	No candidate		
$5/2^-$	3.89	5.93	1.520	5.87(15)	0.7(2)	$(7/2^-)$
$3/2^-$	3.96	4.79	0.074	4.39(5)	<0.22(10)	$(3/2^-)$

The second step is two-neutron removal from ^{13}N(g.s.) or excited states. Because ^{13}N has two low-lying *sd*-shell states ($1/2^+$ and $5/2^+$) that would be strongly excited in the first step, this reaction should populate states of ^{11}N with one proton in the *sd* shell, but not two. For ^{10}B(^{14}N, ^{13}B), one mechanism is ^{10}B(^{14}N, ^{12}B) followed by ^{12}N(^{12}B, ^{13}B). This second step involves one-neutron removal from the ground or excited states of ^{12}N. The first step of adding two protons to ^{10}B would allow population of states in ^{12}N with two *sd*-shell protons leading in the second step to states of ^{11}N with two protons in the *sd* shell.

In an investigation of decays of ^{11}N formed in the ^{9}Be(^{12}N, ^{11}N) reaction, in addition to proton decays of various states to the g.s., evidence was reported for a peak at 1.45 MeV [4]. This peak was attributed to decay of an excited resonance to the 2^+ of ^{10}C. In fact, the suggestion was made that the decaying state was the *p*-shell $3/2^-$ state expected in the appropriate energy region. Because of the very different Coulomb effects, the *p*-shell $3/2^-$ state at 2.654 MeV and the $(sd)^2$ $3/2^-$ state at 3.96 MeV in ^{11}Be have very similar energies in ^{11}N. In ^{11}Be, the *p*-shell shell-model calculations [4] predict appreciable spectroscopic factors to both the g.s. and the 2^+ state. So, both ^{11}N states will probably decay to both ^{10}C states. However, the total widths and branching ratios will be considerably different. In Ref. [4], the total calculated width of the *p*-shell $3/2^-$ state was estimated to be about 500 keV (compared to 615 keV here) but the observed peak was considerably wider.

III. CONCLUSIONS

It thus appears that, despite all the research that has involved resonances in ^{11}N, it is likely that two or three resonances have never been observed. They all have one or two nucleons in the *sd* shell, small g.s. widths, and favorable decays to the 2^+ state of ^{10}C. Perhaps experiments to locate them would do well to include coincidence with the ^{10}C γ ray.

ACKNOWLEDGMENT

I acknowledge useful correspondence with A. Lépine-Szily.

[1] W. Benenson, E. Kashy, D. H. Kong-A-Siou, A. Moalem, and H. Nann, Phys. Rev. C **9**, 2130 (1974).
[2] L. Axelsson *et al.*, Phys. Rev. C **54**, R1511 (1996).
[3] A. Lépine-Szily *et al.*, Phys. Rev. Lett. **80**, 1601 (1998).
[4] A. Azhari *et al.*, Phys. Rev. C **57**, 628 (1998).
[5] J. M. Oliveira, Jr. *et al.*, Phys. Rev. Lett. **84**, 4056 (2000).
[6] K. Markenroth *et al.*, Phys. Rev. C **62**, 034308 (2000).
[7] V. Guimaraes *et al.*, Phys. Rev. C **67**, 064601 (2003).
[8] E. Casarejos *et al.*, Phys. Rev. C **73**, 014319 (2006).
[9] J. H. Kelley, E. Kwan, J. E. Purcell, C. G. Sheu, and H. R. Weller, Nucl. Phys. A **880**, 88 (2012).
[10] H. T. Fortune, D. Koltenuk, and C. K. Lau, Phys. Rev. C **51**, 3023 (1995).
[11] D. L. Auton, Nucl. Phys. A **157**, 305 (1970).
[12] B. Zwieglinski, W. Benenson, R. G. H. Robertson, and W. R. Coker, Nucl. Phys. A **315**, 124 (1979).
[13] R. Sherr and H. T. Fortune, Phys. Rev. C **64**, 064307 (2001).
[14] D. R. Goosman, E. G. Adelberger, and K. A. Snover, Phys. Rev. C **1**, 123 (1970).
[15] H. T. Fortune, Phys. Rev. C **74**, 034328 (2006).
[16] F. C. Barker, Phys. Rev. C **76**, 027602 (2007).
[17] H. T. Fortune, Phys. Rev. C **85**, 044304 (2012).
[18] M. MacCormick and G. Audi, Nucl. Phys. A **925**, 61 (2014).
[19] G. Audi *et al.*, Nucl. Phys. A **729**, 3 (2003).
[20] G.-B. Liu and H. T. Fortune, Phys. Rev. C **42**, 167 (1990).
[21] H. T. Fortune and R. Sherr, Phys. Rev. C **83**, 054314 (2011).
[22] D. J. Millener, Nucl. Phys. A **693**, 394 (2001).
[23] S. Cohen and D. Kurath, Nucl. Phys. A **101**, 1 (1967).
[24] Y. Hirayama *et al.*, Phys. Lett. B **611**, 239 (2005).
[25] P. J. Haigh, M. Freer, N. I. Ashwood, T. Bloxham, N. Curtis, P. McEwan, H. G. Bohlen, T. Dorsch, Tz. Kokalova, Ch. Schulz, and C. Wheldon, Phys. Rev. C **79**, 014302 (2009).
[26] H. T. Fortune, Phys. Rev. C **86**, 037302 (2012).
[27] H. T. Fortune, Nucl. Phys. A **968**, 342 (2017).