Reply to "Comment on 'Stability of the wobbling motion in an odd-mass nucleus and the analysis of ^{135}Pr'"

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We reply to the preceding Comment.

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Wobbling motion is defined as a precessional motion of angular momentum $\vec{I}$ around the principal axis either with the maximum or minimum moment of inertia (MoI) of a triaxially deformed body. Its quantal nature appears as the incremental alignment of angular momentum along the wobbling axis with the maximum or the minimum MoI in a quantized unit -1 [1,2], and also the incremental alignment of $\vec{R}$ $(= \vec{I} - \vec{j}$ with $\vec{j}$ the single-particle angular momentum) along the same axis is -1 in the odd-A nucleus (see Figs. 9 and 15 in Ref. [2]). These kinematical characteristics are well expressed by Holstein-Primakoff boson realization of angular momentum algebra [1,2]. We have adopted the rigid MoI for the β_2 and γ dependences of MoI, because the γ deformed Nilsson single-particle orbit should compose the rotor core. In other words, the cranking formula for MoI based on the triaxial harmonic oscillator gives the rigid MoI but not the hydrodynamical MoI [1]. As a result, the calculated electromagnetic transition rates well reproduce the experimental data (see Fig. 8 in Ref. [3] where the I dependence of MoI is not yet included). Compared with Fig. 19 in Ref. [4], the rigid MoI for the rotor core is superior to the hydrodynamical MoI in $B(M1)_{\text{out}}/B(E2)_{\text{in}}$.

In the Comment, the statement in the middle of the second paragraph, "Quantal rotation about a symmetry axis is not possible" is correct, but the subsequent part "i.e., the moment of inertia of a symmetry axis is zero" is meaningless, because quantum mechanically there is no way to observe the moment of inertia about the symmetry axis of the system.

In the low-spin region where pairing correlation fully works, the hydrodynamical MoI may work as shown by Ref. [5]. However, we have to notice the mismatch caused by the different periodicities in γ space between a set of the hydrodynamical MoI's and the one of the oscillator strength of Nilsson potential together with the core radius.

The rotor core is constructed from nucleons in the γ-deformed Nilsson levels correlated through the residual pairing interaction. The Coriolis-antipairing (CAP) effect [6] plays an important role, i.e., the Coriolis force coming from the nuclear rotation starts to dissolve the pairs in the high spin single-particle orbital, and the cranking formula for the MoI becomes the rigid MoI in the limit where all the pair are dissociated. We have obtained the analytic formulas for I dependence of MoI both for odd- and even-mass nuclei by the second-order perturbation approximation applied to the self-consistent Hartree-Fock-Bogoliubov (HFB) equation under the number and I constraints [7]. The $I - I_0$ dependence of MoI (I_0 is the bandhead value of I) is shown in Fig. 9 in Ref. [7] both for even- and odd-mass nucleus. To simulate the I dependence of MoI as in Fig. 9, we have expressed the MoI as $\mathcal{J}_k^{\text{rig}} g(I)$ and studied two cases [2,8]. The first case is highly excited high-spin states such as the triaxial, strongly deformed (TSD) bands in ^{163}Lu [8] with $j = 13/2$, where the gap Δ is small and the wobbling mode is found in the highly excited levels. The second case is low-excitation low-spin states in ^{135}Pr with $j = 11/2$, where Δ is large enough not to be negligible, and the wobbling mode is observed in the low-excitation and low-spin region before the first backbending. For ^{135}Pr, I-dependent MoI well reproduces the experimental energy levels (Fig. 17 in Ref. [2]), and $B(E2)_{\text{out}}/B(E2)_{\text{in}}$, $B(M1)_{\text{out}}/B(E2)_{\text{in}}$, and the mixing ratio δ (Table III in Ref. [2]).

To show the quantal nature of the typical wobbling, we display the calculated root-mean-square values of the alignment of $\vec{I}$ in the left panel and $\vec{R}$ in the right panel for ^{163}Lu in Fig. 1. The parameter set is the same as in Ref. [8], which successfully reproduces the experimental energy levels as shown in Figs. 5–8 in Ref. [8] and $B(E2)_{\text{in}}$, $B(E2)_{\text{out}}$, and $B(M1)_{\text{out}}$ as quoted in Table 1 in Ref. [9]. In Fig. 1, TSD1 is the band with $I - j$ being even, TSD2 with $I - j$ being odd (the yrast wobbling band), and TSD3 with $I - j$ being even (the yrare wobbling band). As for $\langle I_x^2 \rangle^{1/2}$ in the left panel, we see that the vertical difference between the line connecting closed circles (TSD1) and the one connecting open circles (TSD2) is one, while the difference between the line connecting closed circles (TSD1) and the one connecting closed triangles (TSD3) is two. Such a regularity among TSD1, TSD2, and TSD3 is not seen in $\langle I_y^2 \rangle^{1/2}$. As for $\langle j_x^2 \rangle^{1/2}$ and $\langle j_y^2 \rangle^{1/2}$, corresponding curves for TSD1, TSD2, and TSD3 are almost degenerate, indicating there is no j precession. The left panel in Fig. 1 shows that $\vec{I}$ wobbles around the x axis with the maximum MoI in ^{163}Lu. For $\langle R_x^2 \rangle^{1/2}$ in the right panel, the closed circles (TSD1), open circles (TSD2), and the closed triangles (TSD3) show almost the same alignments, i.e., $\langle R_x^2 \rangle_I^{1/2} \sim \langle R_x^2 \rangle_{I+1}^{1/2} \sim \langle R_x^2 \rangle_{I+2}^{1/2}$ for

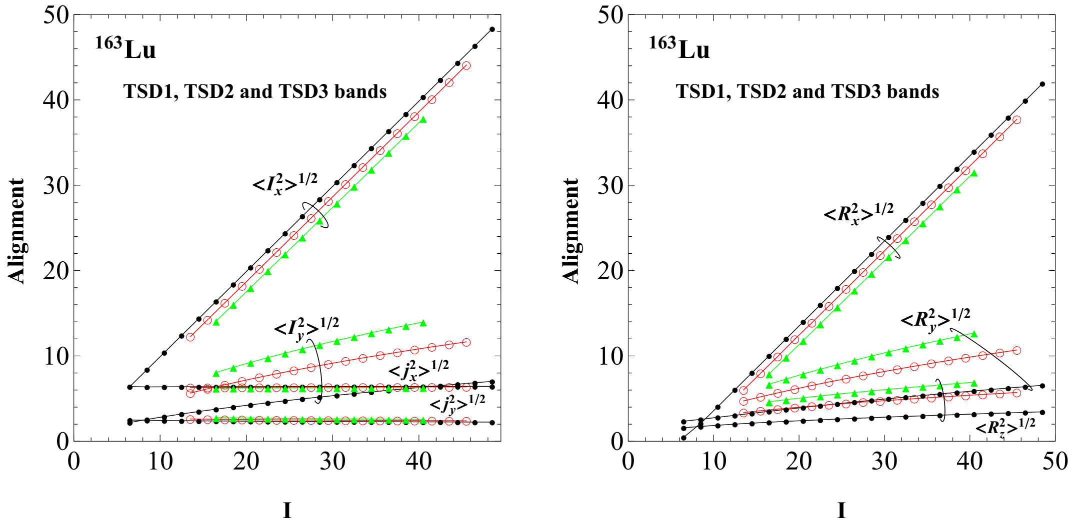

FIG. 1. The alignments of $\langle I_x^2\rangle^{1/2}$, $\langle I_y^2\rangle^{1/2}$, $\langle j_x^2\rangle^{1/2}$, and $\langle j_y^2\rangle^{1/2}$ in the left panel and $\langle R_x^2\rangle^{1/2}$, $\langle R_y^2\rangle^{1/2}$, and $\langle R_z^2\rangle^{1/2}$ in the right panel as functions of I for ^{163}Lu. In both panels, the closed circles correspond to TSD1 band, open circles correspond to TSD2 band, and closed triangles correspond to TSD3 band. In the left panel, the symbols corresponding to TSD1, TSD2, and TSD3 bands for $\langle j_x^2\rangle^{1/2}$ or $\langle j_y^2\rangle^{1/2}$ overlap each other.

$I - j$ being even, and they change by two unit regularly, i.e., $\langle R_x^2\rangle_{I+2}^{1/2} - \langle R_x^2\rangle_I^{1/2} \sim 2$ along each TSD band. These relations are the same as Eqs. (62a) and (62b) in our original publication, Ref. [2]. Such a regularity among TSD1, TSD2, and TSD3 is not seen in $\langle R_y^2\rangle^{1/2}$ and $\langle R_z^2\rangle^{1/2}$. The right panel in Fig. 1 shows the wobbling takes place surely around x axis with the maximum MoI.

The stability equations are based on the next-to-leading-order approximation and applied to investigate the plausibility of the boson approximation. We perform the test calculation by adopting $\mathcal{J}_x : \mathcal{J}_y : \mathcal{J}_z = 13 : 21 : 4$ for ^{135}Pr as given by Table I in Ref. [4]. Then we find that the stability domain with diagonal representation of I_x and j_x extends up to $I = 21/2$ at $\gamma = 26^\circ$ and $V = 1.6$ MeV. In the result of exact diagonalization of total H with these MoI, the behavior of alignments is almost similar to that in Figs. 10 and 14 in Ref. [2], although the crossing between the solid and the dashed lines for $\langle I_x^2\rangle^{1/2}$ moves from $I = 23/2$ (Fig. 10 in Ref. [2]) to $31/2$. However, there does not appear any quantized unit difference between the solid line and the dashed line for $\langle I_x^2\rangle^{1/2}$, indicating there is no evidence of the quantized unit in association with the wobbling around the x axis. As for $\langle R_x^2\rangle^{1/2}$, the crossing of the solid line and the dashed line moves from $I = 25/2$ (Fig. 14 in Ref. [2]) to $33/2$, and $\langle R_y^2\rangle^{1/2}$ is always larger than $\langle R_x^2\rangle^{1/2}$. Furthermore, the relations $\langle R_x^2\rangle_I^{1/2} \simeq \langle R_x^2\rangle_{I+1}^{1/2}$ for $I - j$ being even and $\langle R_x^2\rangle_{I+2}^{1/2} - \langle R_x^2\rangle_I^{1/2} \simeq 2$ do not occur, indicating there is no wobbling around the x axis.

In conclusion, the staggering behavior of $\langle I_x^2\rangle^{1/2}$ is described within a limited region of I by changing the ratios of three hydrodynamical MoI's. However, even with this ratio of three hydrodynamical MoI, the $\langle R_y^2\rangle^{1/2}$ keeps the largest, and the incremental alignment of $\langle R_x^2\rangle^{1/2}$ does not show the quantized unit through whole region of I. As illustrated in Fig. 16 in Ref. [2], the rotating core $\vec{R}$ does not move apart from the principal axis with the largest MoI. It is difficult to identify such a transient phenomenon as "wobbling."

[1] A. Bohr and B. R. Mottelson, *Nuclear Structure* (Benjamin, Reading, MA, 1975), Vol. II, Chap. 4.

[2] K. Tanabe and K. Sugawara-Tanabe, Phys. Rev. C **95**, 064315 (2017).

[3] K. Tanabe and K. Sugawara-Tanabe, Phys. Rev. C **73**, 034305 (2006); **75**, 059903(E) (2007).

[4] S. Frauendorf and F. Dönau, Phys. Rev. C **89**, 014322 (2014).

[5] J. M. Allmond and J. L. Wood, Phys. Lett. B **767**, 226 (2017).

[6] B. R. Mottelson and J. G. Valatin, Phys. Rev. Lett. **5**, 511 (1960).

[7] K. Tanabe and K. Sugawara-Tanabe, Phys. Rev. C **91**, 034328 (2015).

[8] K. Tanabe and K. Sugawara-Tanabe, Phys. Rev. C **77**, 064318 (2008).

[9] K. Sugawara-Tanabe, K. Tanabe, and N. Yoshinaga, Prog. Theor. Exp. Phys. **2014**, 063D01 (2014).