

Nuclear spin dependence of time reversal invariance violating effects in neutron scattering

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The spin structure of parity violating and time reversal invariance violating effects in neutron scattering is discussed. The explicit relations between these effects are presented in terms of functions nuclear spins and neutron partial widths of p -wave resonances.

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I. INTRODUCTION

Time reversal invariance violating (TRIV) effects in nuclear reactions which can be measured in the transmission of polarized neutrons through a polarized target [1,2] attract interest since they can be enhanced [3,4] by a factor as large as 10^6 . Similar enhancement [5,6] was already observed for parity violating (PV) effects related in neutron transmission through nuclear targets. For example, the PV asymmetry in the 0.734-eV p -wave resonance in ^{139}La has been measured to be $(9.56 \pm 0.35) \times 10^{-2}$ (see, for example, Ref. [7] and references therein). The recent proposals for searches TRIV in neutron-nucleus scattering (see, for example, Ref. [8] and references therein) demonstrated the existence of a class of experiments that are free from false asymmetries, which can have a discovery potential that is 10^2 to 10^4 more sensitive than current limits due to the enhancement of TRIV effects for neutron energies corresponding to p -wave resonances in the compound nuclear system. Also, a search for TRIV in neutron transmission gives the opportunity to use many different nuclear targets. This provides assurance that possible “accidental” cancellation of T-violating effects due to unknown structural factors related to the strong interactions in the particular nucleus can be avoided. Taking into account that different models of the CP violation may contribute differently to a particular T/CP-odd observable, which may have unknown theoretical uncertainties, TRIV nuclear effects could be considered complementary to electric dipole moment (EDM) measurements, whose status as null tests of T invariance is more widely known. It has been shown [4,9] that the ratio of TRIV effects to PV effects measured at the same p -wave resonance has almost complete cancellation of nuclear reaction effects resulting in a ratio of TRIV to PV matrix elements taken between the same states with opposite parities. This eliminates

a bulk of nuclear model uncertainties in TRIV effects.¹ The coefficient of proportionality between the observables and the corresponding matrix elements involves neutron partial widths with different channel spins, which are unknown and must be determined from independent measurements of angular correlations in neutron induced reactions (see, for example, Ref. [14]). Since these coefficients have a “natural” value of an order of 1, they were mostly ignored in previous studies of TRIV effects in neutron scattering. However, for development of experimental proposals, it is very important to know the exact spin structure in the relation of PV and TRIV effects to be able to choose the optimal target for TRIV experiments and for the further analysis of the experimental data.

II. GENERAL FORMALISM

TRIV and parity violating (PV) effects, related to the T-odd correlation, $\vec{\sigma} \cdot (\hat{k} \times \hat{I})$, can be observed in the transmission of polarized neutrons through the polarized target, where $\vec{\sigma}$, \hat{I} , and \hat{k} are unit vectors parallel to the neutron spin, the target nuclear spin, and the incident neutron momentum. This correlation leads to a P-odd and T-odd difference between the total neutron cross sections [2] $\Delta\sigma_{\text{PT}}$ for $\vec{\sigma}$ parallel and antiparallel to $\hat{k} \times \hat{I}$

$$\Delta\sigma_{\text{PT}} = \frac{4\pi}{k} \text{Im}(f_{\uparrow} - f_{\downarrow}). \quad (1)$$

Here, $f_{\uparrow, \downarrow}$ are the zero-angle scattering amplitudes for neutrons polarized parallel and antiparallel to the $\vec{k} \times \vec{I}$ axis, respectively.

¹It should be noted that we do not discuss here the relations between the ratio of TRIV to PV matrix elements and the corresponding ratio of nucleon coupling constants. It was shown [9] that these two ratios have the same order of magnitude with a possible suppression for the nucleon coupling constants by about a factor of 3, which is consistent with the detailed statistical analysis [10–13] of the correlator of TRIV and PV matrix elements leading to the suppression factor by about of $\sim A^{1/3}$ (A is the atomic number).

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These TRIV effects can be enhanced [3] by a factor as large as 10^6 . Similar enhancement was already observed for PV effects related to $(\vec{\sigma} \cdot \hat{k})$ correlation in neutron transmission through unpolarized nuclear targets. This PV and TRI-conserving difference of total cross sections $\Delta\sigma_P$ can be written in terms of differences of zero-angle elastic scattering amplitudes with negative and positive neutron helicities as

$$\Delta\sigma_P = \frac{4\pi}{k} \text{Im}(f_- - f_+). \quad (2)$$

To obtain the explicit spin-dependent coefficients for these two observables, we consider the reaction matrix \hat{T} , which is related to the scattering matrix \hat{S} and the matrix \hat{R} as

$$2\pi i \hat{T} = \hat{1} - \hat{S} = \hat{R}. \quad (3)$$

It is convenient to relate this matrix to the matrix \hat{R} in the integral of motion representation of the S matrix [15]

$$\langle S'l'\alpha' | S^J | Sl\alpha \rangle \delta_{JJ'} \delta_{MM'} \delta(E' - E), \quad (4)$$

where J and M are the total spin and its projection, S is the channel spin, l is the orbital momentum, and α represents the other internal quantum numbers. Taking into account that the spin channel is a sum of the neutron spin \vec{s} and nucleus spin \vec{I}

$$\vec{S} = \vec{s} + \vec{I}, \quad (5)$$

the total angular momentum is

$$\vec{J} = \vec{l} + \vec{S}. \quad (6)$$

Then by choosing quantization axis along axis z , one can write T -matrix elements for elastic forward scattering as

$$\begin{aligned} 2\pi i \langle \vec{k}\mu | T | \vec{k}\mu \rangle = & \sum_{JM_J l m l' m' S m_s S' m'_s} Y_{l'm'}(\theta, \phi) \langle s\mu I M | S' m'_s \rangle \langle l' m' S' m'_s | J M_J \rangle \\ & \times \langle S'l'\alpha' | R^J | Sl\alpha \rangle \langle J M_J | l m S m_s \rangle \langle S m_s | s\mu I M \rangle Y_{lm}^*(\theta, \phi), \end{aligned} \quad (7)$$

where angles (θ, ϕ) describe the direction of the neutron momentum \vec{k} , and μ is the projection of the neutron spin along the axis of quantization.

Using the relation for a reaction amplitude \hat{f} : $\hat{f} = -\pi(k_i k_f)^{-1/2} \hat{T}$, where $k_{i,f}$ are values of initial and final momenta, respectively, we can write the elastic scattering amplitude as

$$\begin{aligned} f = & \frac{i}{2k} \sum_{JM_J l' S m'_s S' m'_s} Y_{Lm_L}(\theta, \phi) \langle s\mu I M | S' m'_s \rangle \langle S m_s | s\mu I M \rangle \langle S'l'\alpha' | R^J | Sl\alpha \rangle (-1)^{J+S'+l'+l} (2J+1) \\ & \times \sqrt{\frac{(2l+1)(2l'+1)}{4\pi(2S+1)}} \langle l0l'0 | L0 \rangle \langle Lm_L S' m'_s | S m_s \rangle \begin{Bmatrix} l' & l & L \\ S & S' & J \end{Bmatrix}. \end{aligned} \quad (8)$$

For the further calculations, we assume that the direction of the target polarization \vec{I} is along axis z , and the direction of the momentum \vec{k} is along the axis y . Then, for low-energy neutrons ($l, l' \leq 1$) we can obtain from Eq. (8) the differences of the amplitudes related to TRIV correlation $(\vec{\sigma} \cdot [\hat{k} \times \hat{I}])$ and P -odd correlation $(\vec{\sigma} \cdot \hat{k})$ as

$$\begin{aligned} \Delta f_{PT} = f_{\uparrow} - f_{\downarrow} = & M \frac{\sqrt{3}I}{8\pi k \sqrt{2I+1}} \left[\frac{\langle (I-1/2), 0 | R^{I-1/2} | (I+1/2), 1 \rangle - \langle (I+1/2), 1 | R^{I-1/2} | (I-1/2), 0 \rangle}{\sqrt{I+1}} \right. \\ & \left. + \frac{\langle (I+1/2), 0 | R^{I+1/2} | (I-1/2), 1 \rangle - \langle (I-1/2), 1 | R^{I+1/2} | (I+1/2), 0 \rangle}{\sqrt{I}} \right], \end{aligned} \quad (9)$$

and

$$\begin{aligned} \Delta f_P = f_+ - f_- = & -\frac{i}{4\pi k \sqrt{6I+3}} \{ I \sqrt{2I-1} [\langle (I-1/2), 0 | R^{I-1/2} | (I-1/2), 1 \rangle + \langle (I-1/2), 1 | R^{I-1/2} | (I-1/2), 0 \rangle] \\ & + 2\sqrt{I}(I+1) [\langle (I+1/2), 0 | R^{I+1/2} | (I-1/2), 1 \rangle + \langle (I-1/2), 1 | R^{I+1/2} | (I+1/2), 0 \rangle] \\ & - 2I\sqrt{I+1} [\langle (I-1/2), 0 | R^{I-1/2} | (I+1/2), 1 \rangle + \langle (I+1/2), 1 | R^{I-1/2} | (I-1/2), 0 \rangle] \\ & - (I+1)\sqrt{2I+3} [\langle (I+1/2), 0 | R^{I+1/2} | (I+1/2), 1 \rangle + \langle (I+1/2), 1 | R^{I+1/2} | (I+1/2), 0 \rangle] \}. \end{aligned} \quad (10)$$

Both TRIV and PV amplitudes can be calculated using the distorted wave Born approximation to first order in the parity and time reversal violating interactions (see, for example, Ref. [3]). Then for slow neutrons the PV and TRIV matrix elements in the above expressions can be written in the Breit-Wigner resonance approximation with one s resonance and

one p resonance as [3]

$$\begin{aligned} \langle S'l' | R^J | Sl \rangle = & \frac{\sqrt{\Gamma_l^n(S')}(-iv+w)\sqrt{\Gamma_l^n(S)}}{(E-E_l+i\Gamma_l/2)(E-E_{l'}+i\Gamma_{l'}/2)} \\ & \times e^{i(\delta_{l'}(S')+\delta_l(S))} \end{aligned} \quad (11)$$

where E_l , Γ_l , and $\Gamma_l^n(S)$ are the energy, the total width, and the partial neutron width of the l th nuclear compound resonance, E is the neutron energy, and δ_l is the potential scattering phase shift, $l \neq l'$, and v and w are PV and TRIV nuclear matrix elements between s -wave and p -wave compound states, correspondingly:

$$\langle \varphi_s | V_P + W_{PT} | \varphi_p \rangle = \langle \varphi_s | V_P | \varphi_p \rangle + \langle \varphi_s | W_{PT} | \varphi_p \rangle = -v - iw.$$

Here φ_s and φ_p are compound states' wave functions, and V_P and W_{PT} are PV and TRIV interactions' nuclear operators. Therefore (see, for example, Refs. [3,4,8,9]), at each resonance

the ration of TRIV and PV effects is related to the ration of TRIV and PV nuclear matrix elements multiplied by a spin-dependent factor κ

$$\frac{\Delta\sigma_{PT}}{\Delta\sigma_P} = \kappa \frac{w}{v}. \quad (12)$$

The important point is that the factor κ includes amplitudes of the partial neutron widths which depend on spin channels. To see this dependence, let us consider the ratio of TRIV and PV effects at resonances with a total spin $J = I + 1/2$ and $J = I - 1/2$ separately, assuming 100% polarization of the target for TRIV effect ($M = I$) and unpolarized target for PV effect. Then, using Eqs. (9), (10), and (11), we obtain

$$\kappa(J = I + 1/2) = \left(\frac{\Delta\sigma_{PT}}{\Delta\sigma_P} \right)_{J=I+1/2} = \left[\frac{3\sqrt{I}(2I+1)}{2(I+1)} \right] \frac{\sqrt{\Gamma_p^n(I-1/2)}}{[\sqrt{2I+3}\sqrt{\Gamma_p^n(I+1/2)} - 2\sqrt{I}\sqrt{\Gamma_p^n(I-1/2)}]}, \quad (13)$$

and

$$\kappa(J = I - 1/2) = \left(\frac{\Delta\sigma_{PT}}{\Delta\sigma_P} \right)_{J=I-1/2} = \left[\frac{3(2I+1)}{2\sqrt{I+1}} \right] \frac{\sqrt{\Gamma_p^n(I+1/2)}}{[2\sqrt{I+1}\sqrt{\Gamma_p^n(I+1/2)} - \sqrt{2I-1}\sqrt{\Gamma_p^n(I-1/2)}]}. \quad (14)$$

For the further analysis, it is convenient to use a ratio of $[\Gamma_p^n(I+1/2)/\Gamma_p^n(I-1/2)]^{1/2} \equiv \gamma$ and relative fractions of the amplitudes of the neutron decay widths

$$x_S = [\Gamma_p^n(I-1/2)/\Gamma_p^n]^{1/2}, \quad (15)$$

$$y_S = [\Gamma_p^n(I+1/2)/\Gamma_p^n]^{1/2}, \quad (16)$$

where $\Gamma_p^n = \Gamma_p^n(I+1/2) + \Gamma_p^n(I-1/2)$ is a total p -wave neutron width. Then, Eqs. (13) and (14) can be written as²

$$\begin{aligned} \kappa(I+1/2) &= \left[\frac{3\sqrt{I}(2I+1)}{2(I+1)} \right] \frac{1}{[\gamma\sqrt{2I+3} - 2\sqrt{I}]}, \\ \kappa(I-1/2) &= \left[\frac{3(2I+1)}{2\sqrt{I+1}} \right] \frac{1}{[2\sqrt{I+1} - \sqrt{2I-1}/\gamma]}. \end{aligned} \quad (17)$$

Unfortunately, the parameters γ , x_S , and y_S are unknown and must be obtain from additional experiments, for example, from the measurements of different angular correlations in (n, γ) reactions. The complete analysis of angular correlations in (n, γ) reactions for low-energy neutrons has been done in Ref. [14], where a different spin coupling scheme was

used: $\vec{J} = (\vec{s} + \vec{l}) + \vec{I}$. The relations between these two spin coupling schemes are given in the appendix. In the coupling scheme used in Ref. [14], one can define the corresponding fractions of the amplitudes of neutron widths as

$$x = (\Gamma_{p1/2}^n / \Gamma_p^n)^{1/2}, \quad (18)$$

$$y = (\Gamma_{p3/2}^n / \Gamma_p^n)^{1/2}, \quad (19)$$

where Γ_{pj}^n correspond to $j = 1/2$ and $j = 3/2$ with $\vec{j} = \vec{s} + \vec{l}$, and $\Gamma_p^n = \Gamma_{p1/2}^n + \Gamma_{p3/2}^n$.

Then, taking into account that for $J = I + 1/2$,

$$\begin{aligned} [\Gamma_{p1/2}^n]^{1/2} &= \frac{1}{\sqrt{3(2I+1)}} \{ -2\sqrt{I}[\Gamma_p^n(I-1/2)]^{1/2} \\ &\quad + \sqrt{2I+3}[\Gamma_p^n(I+1/2)]^{1/2} \}, \end{aligned} \quad (20)$$

and for $J = I - 1/2$,

$$\begin{aligned} [\Gamma_{p1/2}^n]^{1/2} &= \frac{1}{\sqrt{3(2I+1)}} \{ -\sqrt{2I-1}[\Gamma_p^n(I-1/2)]^{1/2} \\ &\quad + 2\sqrt{I+1}[\Gamma_p^n(I+1/2)]^{1/2} \}, \end{aligned} \quad (21)$$

we can rewrite Eqs. (13) and (14) as

$$\begin{aligned} \kappa(I+1/2) &= \left[\frac{\sqrt{3I}(2I+1)}{2(I+1)} \right] \sqrt{\frac{\Gamma_p^n(I-1/2)}{\Gamma_{p1/2}^n}}, \\ \kappa(I-1/2) &= \left[\frac{\sqrt{3(2I+1)}}{2\sqrt{I+1}} \right] \sqrt{\frac{\Gamma_p^n(I+1/2)}{\Gamma_{p1/2}^n}}, \end{aligned} \quad (22)$$

²These expressions are different from the presented ones in previous papers [4,8,9], where a misprint from Ref. [9] was propagated without been noticed.

or as

$$\begin{aligned}\kappa(I + 1/2) &= \left[\frac{\sqrt{I}}{2(I + 1)} \right] \frac{(-2\sqrt{I}x + \sqrt{2I + 3}y)}{x}, \\ \kappa(I - 1/2) &= \left[\frac{1}{2\sqrt{I+1}} \right] \frac{(2\sqrt{I+1}x + \sqrt{2I-1}y)}{x}.\end{aligned}\quad (23)$$

Since the parameter κ is a coefficient of proportionality between the observed value of PV effect and the unknown value of TRIV effect, it is very important to know κ for the estimation of the accuracy of TRIV effects in the proposed experiments. The larger value of the κ leads to better statistics in the measurements of TRIV effects. The obtained expressions for the κ parameter gives us the opportunity to calculate explicitly the ratio of TRIV and PV effects at each p -wave nuclear compound resonance in terms of the amplitudes of neutron widths, or in terms of two sets of parameters: x and y or x_s and y_s , which satisfy the constraints $x^2 + y^2 = 1$ and $x_s^2 + y_s^2 = 1$, correspondingly. The values of amplitudes of neutron widths are randomly fluctuated on different p -wave compound resonances for the same nuclei, which may lead to different sensitivities of TRIV effects for different resonances at the same nuclear target. However, there is one exceptional case for $J = 0$ p -wave resonances for nuclei with spin $1/2$ where the parameter κ does not depend on neutron partial widths at all and, as a consequence, can be calculated exactly as $\kappa = 1$ [see Eq. (23)]. Another interesting observation from Eq. (23) is related to the fact that PV effects are proportional to the parameter x (or to $\Gamma_{p1/2}^n$), while TRIV effects to the linear superposition of x and y parameters [or to $\Gamma_p^n(I \pm 1/2)$]. Therefore, TRIV effects can be observed even on p -wave resonances which do not show visible PV effects.

In spite of the fact that total p -wave neutron widths are measured for many p -wave resonances with a good accuracy [7,16], the parameters x and y (or x_s and y_s) currently are practically unknown. They cannot be calculated due to a complex structure of compound resonances; therefore, they can be obtained only from experiments which are sensitive to the values and signs of amplitudes of neutron widths. For example, as was mentioned above, they can be obtained from angular distributions of γ rays in resonance neutron radiative capture reactions or by measuring of pseudomagnetic neutron spin rotation in the vicinity of p -wave resonances [17].

III. DISCUSSION AND CONCLUSIONS

The obtained expressions of spin-dependent factors for TRIV and PV effects give us the opportunity to clarify a number of issues for the proposed search for TRIV effects in transmission of polarized neutron through polarized targets. We can see that the parameter κ has a “natural” value of an order of unity with the possible fluctuations on different resonances due to unknown spin-dependent partial neutron width amplitudes of p -wave resonances. In some cases, this can lead to a quite significant enhancement or suppression of TRIV effects at different p -wave resonances. Therefore, it is

very important to measure these neutron partial widths for the choice of the optimal target. Also, one can see that, depending on the values of the neutron partial widths, resonances with the largest PV effects may not have the largest TRIV effects, and otherwise one can observe reasonably large TRIV effects at p -wave resonances with a “regular” or small values of PV effects. This suggests that all p -wave resonances with observed PV effects should be tested for the possible TRIV experiments by measuring their partial neutron widths.

The general expressions in Eqs. (9) and (1) show that TRIV effects are proportional to the projection M of nuclear target spin on the axis of quantization. Therefore, for the case of a target with a superposition of different polarization modes (vector, tensor, etc.), only a vector part of the polarization leads to TRIV effects, and the value of TRIV effect is modified by a factor

$$P = \frac{1}{I} \sum_M w(M)M, \quad (24)$$

where $w(M)$ is a weight of a population for each spin projection quantum number M , or a weight in the density operator used for the description of the general polarization in terms of the density polarization matrix.

Finally, we would like to note again that the expressions for the parameter κ obtained in this paper are different from the presented in a number of publications (see, for example, Refs. [4,8,9]) due to misprints in Eq. (3) of Ref. [9] which were propagated to different publications without being noticed. Fortunately, the misprinted value of the κ does not change the results reported in these publications since for all estimates it was assumed that κ is unknown parameter of an order of unity. However, the knowledge of the exact value of κ is required for the choice of the target for the experiment and for the analysis of experimental data. In this relation, the first measurement of κ , x , and y parameters has been done recently [18] for 0.74-eV p -wave compound nuclear resonance of ^{139}La by measuring angular distribution of individual γ rays in neutron-induced radiative capture.

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APPENDIX: RELATIONS BETWEEN DIFFERENT SPIN-COUPLING SCHEMES

The relation between two spin coupling schemes $\vec{J} = \vec{l} + (\vec{I} + \vec{s})$ and $\vec{J} = (\vec{s} + \vec{l}) + \vec{I}$ is given by

$$\begin{aligned}&\left\langle \left[\left(l, \frac{1}{2} \right)_{j,I} \right] J \left[\left(l, \left(\frac{1}{2}, I \right) S \right) J \right] \right\rangle \\ &= (-1)^{l+1/2+I+J} \sqrt{(2j+1)(2S+1)} \begin{Bmatrix} l & \frac{1}{2} & j \\ I & J & S \end{Bmatrix}, \quad (A1)\end{aligned}$$

where $\vec{S} = \vec{I} + \vec{s}$ and $\vec{j} = \vec{s} + \vec{l}$.

Now, defining

$$\begin{aligned} x &= |j = \tfrac{1}{2}\rangle, \quad y = |j = \tfrac{3}{2}\rangle, \\ x_s &= |S = I - \tfrac{1}{2}\rangle, \quad y_s = |S = I + \tfrac{1}{2}\rangle, \end{aligned} \quad (\text{A2})$$

one can write for $l = 1$

$$\begin{aligned} x_s &= (-1)^{3/2+I+J} \sqrt{4I} \begin{Bmatrix} 1 & \frac{1}{2} & \frac{1}{2} \\ I & J & I - \frac{1}{2} \end{Bmatrix} x + (-1)^{3/2+I+J} \sqrt{8I} \begin{Bmatrix} 1 & \frac{1}{2} & \frac{3}{2} \\ I & J & I - \frac{1}{2} \end{Bmatrix} y, \\ y_s &= (-1)^{3/2+I+J} \sqrt{4(I+1)} \begin{Bmatrix} 1 & \frac{1}{2} & \frac{1}{2} \\ I & J & I + \frac{1}{2} \end{Bmatrix} x + (-1)^{3/2+I+J} \sqrt{8(I+1)} \begin{Bmatrix} 1 & \frac{1}{2} & \frac{3}{2} \\ I & J & I + \frac{1}{2} \end{Bmatrix} y. \end{aligned} \quad (\text{A3})$$

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