Role of hexadecapole deformation of projectile ^{28}Si in heavy-ion fusion reactions near the Coulomb barrier

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The vast knowledge regarding the strong influence of quadrupole deformation β_2 of colliding nuclei in heavy-ion sub-barrier fusion reactions inspires a desire to quest the sensitivity of fusion dynamics to higher order deformations, such as β_4 and β_6 deformations. However, such studies have rarely been carried out, especially for deformation of projectile nuclei. In this article, we investigated the role of β_4 of the projectile nucleus in the fusion of the ^{28}Si + ^{92}Zr system. We demonstrated that the fusion barrier distribution is sensitive to the sign and value of the β_4 parameter of the projectile, ^{28}Si, and confirmed that the ^{28}Si nucleus has a large positive β_4. This study opens an indirect way to estimate deformation parameters of radioactive nuclei using fusion reactions, which is otherwise difficult because of experimental constraints.

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I. INTRODUCTION

Gaining insight into the role of nuclear intrinsic degrees of freedom in heavy-ion fusion reactions has motivated many experimental and theoretical studies in nuclear research [1–7]. During the fusion process, the nuclear intrinsic degrees of freedom, such as inelastic excitations, neutron transfers, and static or dynamical deformation, are coupled to the relative motion of the interacting nuclei and significantly affect the fusion dynamics. Experimental signatures of these couplings have been observed via a sub-barrier fusion enhancement of fusion cross sections and a deviation of fusion barrier distributions from a simple one-peaked function [1–6]. Comparisons of these experimental data with coupled-channel calculations have established the role of various couplings in the heavy-ion fusion mechanism [2,4].

An important question to be addressed concerns the relevant degrees of freedom one needs to consider in a description of the fusion dynamics. For deformed nuclei, the role of quadrupole deformation β_2 of the colliding nuclei in fusion is significant and has been well established [3]. With increasing experimental knowledge about the role of quadrupole collectivity in fusion, the sensitivity to the hexadecapole deformation β_4 is the next topic to explore. A measurement by Lemmon *et al.* [8] for ^{16}O + ^{154}Sm and ^{16}O + ^{186}W fusion reactions has clearly shown the sensitivity of fusion barrier distributions to the sign of β_4 of the target nuclei (see also Refs. [9,10]). The effect of the β_6 (hexacontatetrapole) deformation has also been investigated in Refs. [11,12].

A study of β_4 is important in connection with the synthesis of superheavy elements (SHEs). That is, the hexadecapole deformation may significantly affect the height of fusion barrier, which in turn influences the fusion probability and thus the formation probability of SHEs [13]. It has theoretically been argued that a β_4 deformation may help fusion (both hot and cold fusion reactions) leading to SHEs, depending on the choice of the reaction partners [14].

In this respect, an interesting observation has appeared recently while investigating the experimental fusion barrier distribution for the ^{28}Si + ^{154}Sm system [15]. In this experiment, the barrier distribution was extracted using quasielastic back-scattering [10,16]. Despite the well-established rotational nature of ^{28}Si (having both quadrupole and hexadecapole deformations), it was found that a coupled-channel calculation with a vibrational coupling to its first 2^+ state reproduces the structure of the barrier distribution rather well. Subsequently, it was observed that the resolution of this anomaly lies in the large hexadecapole deformation parameter of ^{28}Si, which has the opposite sign of the quadrupole deformation parameter. That is, the contribution to the reorientation coupling ($2_1{}^+ \rightarrow 2_1{}^+$) from the quadrupole deformation is largely canceled out by that from the hexadecapole deformation, making the rotational coupling scheme look like the vibrational coupling scheme for this system. This leads to almost identical results for the two coupling schemes. Since the quasielastic backward scattering is a process complementary to fusion, it shows a sensitivity of fusion mechanism to the hexadecapole deformation of ^{28}Si.

In Ref. [17], Newton *et al.* studied the experimental fusion barrier distribution for the ^{28}Si + ^{92}Zr system and reached the same conclusion as in Ref. [15] for the ^{28}Si + ^{154}Sm system. That is, the authors of Ref. [17] have reported that treating the 2^+ state in ^{28}Si as a phonon state rather than a rotational state with oblate deformation gives a somewhat better fit to the experimental fusion barrier distribution. Moreover, treating the ^{28}Si nucleus as a prolate rotor leads to a poor representation of the data. They have argued that there is not strong evidence from the fusion data to distinguish between ^{28}Si being a vibrational nucleus or an oblate deformed nucleus.

The aim of this paper is to reanalyze the fusion barrier distribution for the $^{28}\mathrm{Si} + ^{92}\mathrm{Zr}$ system which Newton *et al.* have studied and to clarify the role of hexadecapole deformation of the ^{28}Si nucleus. We shall show that a large positive value for β_4 leads to fusion barrier distributions calculated with the rotational coupling scheme which look similar to those with the vibrational scheme. This result cannot be regarded as a direct measurement of β_4, but it strongly suggests that ^{28}Si is a deformed nucleus with a large positive hexadecapole parameter, β_4.

II. COUPLED-CHANNEL CALCULATIONS FOR $^{28}\mathrm{Si} + ^{92}\mathrm{Zr}$ SYSTEM

To clarify the influence of hexadecapole deformation of ^{28}Si on the fusion of the $^{28}\mathrm{Si} + ^{92}\mathrm{Zr}$ system, we have performed the coupled-channel calculations using the computer code CCFULL [18]. To this end, we have used a Woods-Saxon potential whose diffuseness parameter was fixed to be $a_0 = 1.03$ fm. Notice that a large value of diffuseness parameter has been found to reproduce high-precision fusion cross sections in many systems [17]. The exact origin of this phenomenon has not been clarified, and the phenomenon has been referred to as the surface diffuseness anomaly. Here, we follow Ref. [17] and take $a_0 = 1.03$ fm. We have checked that the agreement of the calculation with the experimental data becomes worse if we use a smaller value of a_0, such as $a_0 = 0.7$ fm. Notice that results are almost independent of the precise values of V_0 and R_0 as long as the barrier height is reproduced. For excitations in the target nucleus, ^{92}Zr, we have included a coupling to the one-quadrupole-phonon state at 0.934 MeV with the deformation parameter of 0.13.

The dashed line in Fig. 1 shows the fusion barrier distribution for the $^{28}\mathrm{Si} + ^{92}\mathrm{Zr}$ system when the coupling to the 2^+ state in ^{28}Si is included assuming an oblate rotor with $\beta_2 = -0.407$ [19] and $\beta_4 = 0$. Here, the fusion barrier distribution is defined as [20]

$$D_{\mathrm{fus}}(E) = \frac{d^2(E\sigma_{\mathrm{fus}})}{dE^2}, \tag{1}$$

where E is the incident energy in the center-of-mass frame and σ_{fus} is a fusion cross section. The experimental fusion barrier distribution was extracted with a point difference formula with $\Delta E \sim 2$ MeV [17], and the same procedure was applied to the theoretical fusion barrier distribution as well. In the figure, one can find that this calculation captures the main structure of the barrier distribution, but the experimental data around $E_{\mathrm{c.m}} = 75$ MeV are not well accounted for. The calculation is somewhat improved by taking into account a finite value of β_4, e.g., $\beta_4 = +0.10$, the value which was employed in Ref. [21], as is shown by the dashed line with crosses. On the other hand, when the quadrupole deformation of ^{28}Si was taken to be positive, the shape of fusion barrier distribution becomes inconsistent with the experimental data (see the dashed line with triangles), supporting an oblate deformation of ^{28}Si [22]. The solid line in the figure shows the result with the vibrational excitation ^{28}Si, in which the first 2^+ state is treated as a one-phonon state in the harmonic oscillator approximation. One can clearly see that this calculation better reproduces the experimental fusion barrier distribution, compared to the rotational coupling with $\beta_4 = 0.10$, as has been pointed out in Ref. [17].

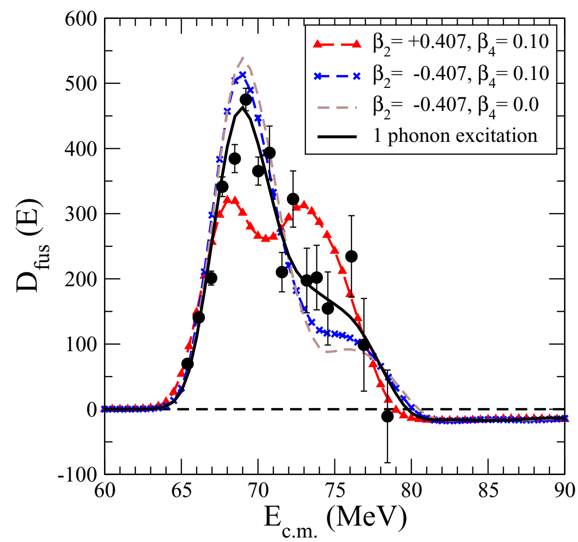

FIG. 1. A comparison of fusion barrier distributions for the $^{28}\mathrm{Si} + ^{92}\mathrm{Zr}$ system obtained with several coupling schemes for the coupled-channel calculations. The solid line shows the result with the vibrational coupling to the first 2^+ state of ^{28}Si, along with the vibrational excitation of ^{92}Zr. The dashed line shows the result of the rotational couplings to the 2^+ state of ^{28}Si with deformation parameters of $\beta_2 = -0.407$ and $\beta_4 = 0.0$. On the other hand, the dashed lines with triangles and crosses are obtained with $\beta_2 = +0.407$ and $\beta_2 = -0.407$, respectively, together with $\beta_4 = 0.1$. The experimental data, taken from Ref. [17], are shown with filled circles.

In order to see the sensitivity of the results to β_4 in the rotational coupling scheme, the dashed line with triangles in Fig. 2 shows the barrier distribution obtained with a larger value of β_4, that is, $\beta_4 = 0.25$. This is the value obtained by Möller and Nix [23] by using the finite-range droplet model with spherical-harmonic expansions. This value is also consistent with the one obtained with proton scattering experiments, i.e., $+0.25 \pm 0.08$ [24]. Earlier experiments for electron scattering [25], neutron scattering [26,27], and α-particle scattering [28] indicate that the value of β_4 in ^{28}Si is $+0.10$, $+0.18 \pm 0.02$/ $+0.20 \pm 0.05$, and $+0.08 \pm 0.01$, respectively. Although these values are somewhat different from each other, all of these values point to a large value of β_4. Interestingly, the rotational calculation with $\beta_4 = 0.25$ yields an almost identical result to the result of the vibrational coupling scheme shown by the solid line in the figure. This is in the same situation as in the $^{28}\mathrm{Si} + ^{154}\mathrm{Sm}$ system discussed in Ref. [15].

Within the space of the ground state (0^+) and the first 2^+ state, the difference between the rotational and the harmonic vibrational coupling schemes is found only in the reorientation term. That is, there is no coupling from the 2^+ state to the same state, 2^+, in the vibrational coupling, while this coupling is finite in the rotational coupling (compare between Eqs. (3.41) and (3.49) in Ref. [4]). It is important to notice here that the

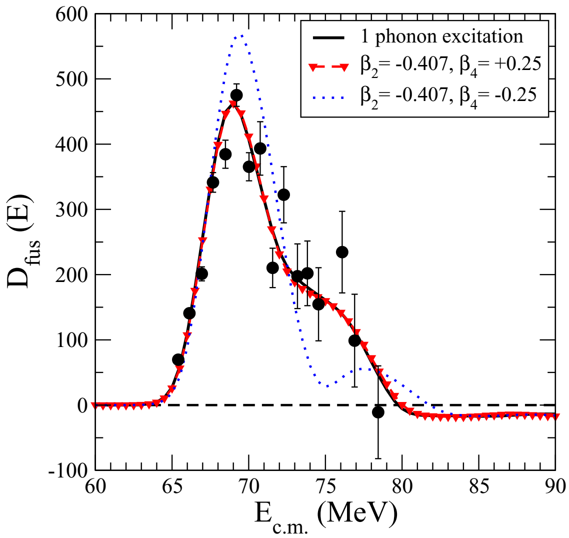

FIG. 2. The fusion barrier distribution for the ^{28}Si + ^{92}Zr system obtained with the rotational coupling scheme for ^{28}Si with $\beta_2 = -0.407$ and $\beta_4 = 0.25$ (the dashed line with triangles) and with $\beta_2 = -0.407$ and $\beta_4 = -0.25$ (the dotted line). The meaning of the solid line is the same as in Fig. 1.

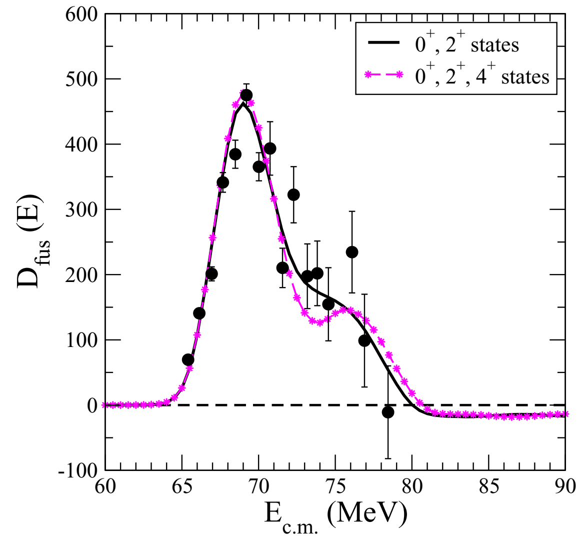

FIG. 3. A comparison of fusion barrier distributions for the ^{28}Si + ^{92}Zr system obtained with the rotational coupling scheme with a truncation of the ground-state rotational band of ^{28}Si at the 2^+ state (the solid line) and at the 4^+ state (the dashed line with stars) in the coupled-channel calculations.

2^+ state is coupled to itself by both the quadrupole and the hexadecapole deformations. In fact, the reorientation term is given by (see Eq. (3.58) in Ref. [4]),

$$O_{22} = \langle Y_{20}|\beta_2 R_P Y_{20}(\theta) + \beta_4 R_P Y_{40}(\theta)|Y_{20}\rangle, \tag{2}$$

$$= \frac{5\sqrt{5}}{\sqrt{4\pi}}\beta_2 R_P \begin{pmatrix} 2 & 2 & 2 \\ 0 & 0 & 0 \end{pmatrix}^2 + \frac{15}{\sqrt{4\pi}}\beta_4 R_P \begin{pmatrix} 2 & 4 & 2 \\ 0 & 0 & 0 \end{pmatrix}^2, \tag{3}$$

where R_P is the radius of the projectile nucleus and the $3j$ symbols read

$$\begin{pmatrix} 2 & 2 & 2 \\ 0 & 0 & 0 \end{pmatrix} = -\frac{2}{\sqrt{70}}, \quad \begin{pmatrix} 2 & 4 & 2 \\ 0 & 0 & 0 \end{pmatrix} = +\frac{2}{\sqrt{70}}. \tag{4}$$

With $\beta_2 = -0.407$ and $\beta_4 = +0.25$, the first and the second terms in Eq. (3) read $-0.073R_P$ and $+0.060R_P$, respectively, which are largely canceled with each other, leading to the situation which is close to the vibrational coupling scheme (the perfect cancellation is achieved for $\beta_4/\beta_2 = -\sqrt{5}/3 = -0.745$). As a matter of fact, the similarity disappears when we take $\beta_4 = -0.25$, as shown by the dotted line in Fig. 2. Therefore, even though the result with the vibrational coupling scheme may lead to a good reproduction of the experimental data, this does not imply that ^{28}Si is a vibrational spherical nucleus. A nice reproduction is simply due to an accidental cancellation of the reorientation term originated from the large value of β_4, and the rotational excitation of the ^{28}Si projectile still plays an important role in the fusion of this nucleus.

A large hexadecapole deformation of ^{28}Si should accompany a strong direct coupling from the ground state to the 4^+ state. The 4^+ state also couples to the 2^+ state with both the quadrupole and the hexadecapole terms (notice that there is no hexadecapole coupling between the 0^+ state and the 2^+ state). In order to check the influence of the 4^+ state, the dashed line with stars in Fig. 3 shows the result obtained by including the ground-state rotational band of ^{28}Si up to the 4^+ state with the deformation parameters of $\beta_2 = -0.407$ and $\beta_4 = +0.25$. The inclusion of the 4^+ state somewhat perturbs the shape of fusion barrier distribution, and the agreement with the experimental data is slightly worsened. However, the calculated fusion barrier distribution is still within the error bars of the experimental distribution and there remains a similarity to the barrier distribution for the vibrational coupling scheme. We have confirmed that the agreement is not significantly improved even with a larger value of β_4, that is, $\beta_4 = 0.30$. We have also checked the influence of the octupole excitation to the 3^- state at 6.878 MeV in ^{28}Si and have confirmed that the inclusion of this state simply shifts the barrier distribution in energy by ≈1.5 MeV without significantly changing its shape. As has been pointed out, e.g., in Ref. [4], excitation to a state with large excitation energy, such as the 3^- state in ^{28}Si, simply leads to a renormalization of the fusion barrier and thus does not significantly influence the fusion dynamics. We have also found that the results converge rapidly upon adding the higher members in the rotational band, beyond 4^+, of ^{28}Si due to the finite excitation energy. This latter fact is another necessary condition to have a similarity between the rotational coupling and the vibrational coupling schemes. That is, when higher members in the ground-state rotational band contribute significantly to the fusion dynamics, which is typically the case for fusion of medium-heavy nuclei such as ^{16}O + ^{154}Sm, the resultant fusion barrier distribution differs considerably from fusion barrier distributions for vibrational nuclei [3,4,9].

III. SUMMARY AND DISCUSSION

In summary, we have carried out the coupled-channel calculations for the ^{28}Si + ^{92}Zr fusion reaction and have shown that the fusion process is sensitive to the hexadecapole deformation of the ^{28}Si nucleus. We have demonstrated that the reorientation term for the 2^+ state is largely canceled out, leading to similar results between the rotational and the vibrational coupling schemes, even though in reality the ^{28}Si nucleus is not a spherical nucleus. This nicely follows the earlier conclusion obtained for the ^{28}Si + ^{154}Sm reaction [15], making a strong evidence for that ^{28}Si possesses a large positive hexadecapole moment.

In order to have such similarity between results with the rotational coupling scheme and those with the vibrational coupling scheme, the following two conditions are necessary. The first condition is that the quadrupole and hexadecapole deformation parameters have opposite signs with their ratio close to $\beta_4/\beta_2 = -\sqrt{5}/3 = -0.745$. The second condition, which is usually satisfied for light deformed nuclei, is that the excitation energy of the first 2^+ state is large so that higher members of the ground-state rotational band do not significantly contribute. The ^{28}Si nucleus satisfies both conditions. In addition to other Si isotopes, another candidate which shows the same kind of similarity might be ^{38}Ne. Even though several aspects related to the weakly bound nature of this neutron-rich nucleus would have also to be taken into account, this nucleus satisfies the two conditions, as the deformation parameters for this nucleus are predicted to be $\beta_2 = -0.302$ and $\beta_4 = +0.163$ with the FRDM(2012) mass model [29] and the energy of the 2^+ state is predicted to be around 1.05 MeV with a shell model calculation [30].

The coupled-channel calculations for the ^{28}Si + ^{92}Zr system presented in this paper suggest that the fusion mechanism is sensitive to projectile excitations. This is also relevant to the synthesis of superheavy elements. Very recently, barrier distributions were extracted using quasielastic scattering for reactions to form superheavy elements [31]. Quasielastic barrier distributions are complementary to fusion barrier distributions and they have smaller error bars on the high-energy side. It will be interesting in the future to study how the projectile excitations influence evaporation residue cross sections for fusion reactions of the ^{28}Si projectile to form superheavy elements. Note also that the method based on a quasielastic barrier distribution will be useful to discuss the shape of radioactive nuclei, for which the beam intensity is low [16].

In the past, α-particle scattering [32,33], electron scattering [34], and muonic x-ray methods [35] have been used in order to determine experimentally the shape of a deformed nucleus. However, β_4, especially its sign, is difficult to extract. All the available results for β_4 are model dependent and quite different from each other with large uncertainties. As we have discussed in this paper, fusion is sensitive not only to the target excitations but also to the projectile excitations, and the barrier distribution analysis will offer a powerful alternative method to extract the magnitude and sign of β_4 for deformed nuclei.

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[1] S. G. Steadman and M. J. Rhoades-Brown, Annu. Rev. Nucl. Part. Sci. **36**, 649 (1986).
[2] A. B. Balantekin and N. Takigawa, Rev. Mod. Phys. **70**, 77 (1998).
[3] M. Dasgupta, D. J. Hinde, N. Rowley, and A. M. Stefanini, Annu. Rev. Nucl. Part. Sci. **48**, 401 (1998).
[4] K. Hagino and N. Takigawa, Prog. Theor. Phys. **128**, 1061 (2012).
[5] B. B. Back, H. Esbensen, C. L. Jiang, and K. E. Rehm, Rev. Mod. Phys. **86**, 317 (2014).
[6] G. Montagnoli and A. M. Stefanini, Eur. Phys. J. A **53**, 169 (2017).
[7] M. V. Andres, N. Rowley, and M. A. Nagarajan, Phys. Lett. B **202**, 292 (1988).
[8] R. C. Lemmon, J. R. Leigh, J. X. Wei, C. R. Morton, D. J. Hinde, J. O. Newton, J. C. Mein, M. Dasgupta, and N. Rowley, Phys. Lett. B **316**, 32 (1993).
[9] J. R. Leigh, M. Dasgupta, D. J. Hinde, J. C. Mein, C. R. Morton, R. C. Lemmon, J. P. Lestone, J. O. Newton, H. Timmers, J. X. Wei, and N. Rowley, Phys. Rev. C **52**, 3151 (1995).
[10] H. Timmers, J. R. Leigh, M. Dasgupta, D. J. Hinde, R. C. Lemmon, J. C. Mein, C. R. Morton, J. O. Newton, and N. Rowley, Nucl. Phys. A **584**, 190 (1995).
[11] T. Rumin, K. Hagino, and N. Takigawa, Phys. Rev. C **61**, 014605 (1999).
[12] C. R. Morton, A. C. Berriman, R. D. Butt, M. Dasgupta, D. J. Hinde, A. Godley, J. O. Newton, and K. Hagino, Phys. Rev. C **64**, 034604 (2001).
[13] A. Iwamoto and P. Möller, Nucl. Phys. A **605**, 334 (1996).
[14] R. K. Gupta, M. Balasubramaniam, R. Kumar, N. Singh, M. Manhas, and W. Greiner, J. Phys. G: Nucl. Part. Phys. **31**, 631 (2005).
[15] G. Kaur, B. R. Behera, A. Jhingan, B. K. Nayak, R. Dubey, P. Sharma, M. Thakur, R. Mahajan, N. Saneesh, T. Banerjee *et al.*, Phys. Rev. C **94**, 034613 (2016).
[16] K. Hagino and N. Rowley, Phys. Rev. C **69**, 054610 (2004).
[17] J. O. Newton, C. R. Morton, M. Dasgupta, J. R. Leigh, J. C. Mein, D. J. Hinde, H. Timmers, and K. Hagino, Phys. Rev. C **64**, 064608 (2001).
[18] K. Hagino, N. Rowley, and A. T. Kruppa, Comp. Phys. Comm. **123**, 143 (1999).
[19] S. Raman, C. W. Nestor Jr., and P. Tikkanen, At. Data Nucl. Data Tables **78**, 1 (2001).
[20] N. Rowley, G. R. Satchler, and P. H. Stelson, Phys. Lett. B **254**, 25 (1991).
[21] L. S. Danu, B. K. Nayak, E. T. Mirgule, R. K. Choudhury, and U. Garg, Phys. Rev. C **89**, 044607 (2014).
[22] R. H. Spear and M. P. Fewell, Aust. J. Phys. **33**, 509 (1980).
[23] P. Möller and J. R. Nix, At. Data Nucl. Data Tables **59**, 185 (1995).
[24] R. de Swiniarski, C. Glashausser, D. L. Hendrie, J. Sherman, A. D. Bacher, and E. A. McClatchie, Phys. Rev. Lett. **23**, 317 (1969).
[25] Y. Horikawa, Y. Torizuka, A. Nakada, S. Mitsunobu, Y. Kojirna, and M. Kimura, Phys. Lett. B **36**, 9 (1971); Y. Horikawa, Prog. Theor. Phys. **47**, 867 (1972); Y. Horikawa, A. Nakada, and Y. Torizuka, *ibid.* **49**, 2005 (1973).
[26] C. R. Howell, R. S. Pedroni, G. M. Honoré, K. Murphy, R. C. Byrd, G. Tungate, and R. L. Walter, Phys. Rev. C **38**, 1552 (1988).

[27] G. Haouat, Ch. Lagrange, R. de Swiniarski, F. Dietrich, J. P. Delaroche, and Y. Patin, Phys. Rev. C **30**, 1795 (1984).
[28] H. Rebel, G. W. Schweimer, J. Specht, G. Schatz, R. Löhken, D. Habs, G. Hauser, and H. Klewe-Nebenius, Phys. Rev. Lett. **26**, 1190 (1971).
[29] P. Möller, A. J. Sierk, T. Ichikawa, and H. Sagawa, At. Data Nucl. Data Tables **109–110**, 1 (2016).
[30] E. Caurier, F. Nowacki, and A. Poves, Phys. Rev. C **90**, 014302 (2014).
[31] T. Tanaka, Y. Narikiyo, K. Morita, K. Fujita, D. Kaji, K. Morimoto, S. Yamaki, Y. Wakabayashi, K. Tanaka, M. Takeyama *et al.*, J. Phys. Soc. Jpn. **87**, 014201 (2018).
[32] I. Y. Lee, J. X. Saladin, J. Holden, J. O'Brien, C. Baktash, C. Bemis Jr., P. H. Stelson, F. K. McGowan, W. T. Milner, J. L. C. Ford Jr. *et al.*, Phys. Rev. C **12**, 1483 (1975).
[33] F. T. Baker, A. Scott, T. H. Kruse, W. Hartwig, E. Ventura, and W. Savin, Nucl. Phys. A **266**, 337 (1976).
[34] T. Cooper, W. Bertozzi, J. Heisemberg, S. Kowalski, W. Turchinetz, C. Williamson, L. Cardman, S. Fivozinsky, J. Lightbody Jr., and S. Penner, Phys. Rev. C **13**, 1083 (1976).
[35] R. J. Powers, F. Boehm, P. Vogel, A. Zehnder, T. King, A. R. Kunselman, P. Roberson, P. Martin, G. H. Miller, R. E. Welsh, and D. A. Jenkins, Phys. Rev. Lett. **34**, 492 (1975).