Interacting hadron resonance gas model in the K-matrix formalism

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An extension of hadron resonance gas (HRG) model is constructed to include interactions using relativistic virial expansion of partition function. The noninteracting part of the expansion contains all the stable baryons and mesons and the interacting part contains all the higher mass resonances which decay into two stable hadrons. The virial coefficients are related to the phase shifts which are calculated using K-matrix formalism in the present work. We have calculated various thermodynamics quantities like pressure, energy density, and entropy density of the system. A comparison of thermodynamic quantities with noninteracting HRG model, calculated using the same number of hadrons, shows that the results of the above formalism are larger. A good agreement between equation of state calculated in K-matrix formalism and lattice QCD simulations is observed. Specifically, the lattice QCD calculated interaction measure is well described in our formalism. We have also calculated second-order fluctuations and correlations of conserved charges in K-matrix formalism. We observe a good agreement of second-order fluctuations and baryon-strangeness correlation with lattice data below the crossover temperature.

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I. INTRODUCTION

Relativistic heavy-ion collisions have contributed immensely to our understanding of strongly interacting matter at finite temperature (T) and baryon chemical potential (μ_B). Lattice quantum chromodynamics (LQCD) [1–8] calculation provides a first-principles approach to study strongly interacting matter at zero baryon chemical potential (μ_B) and finite temperature (T), which indicates a smooth crossover transition [1] from hadronic to a quark-gluon plasma (QGP) phase [8]. On the other hand, at high baryon chemical potential, the nuclear matter is expected to have a first-order phase transition [9] which ends at a critical point. Several experimental programs have been devoted to study strongly interacting matter in a wide range of temperature and baryon chemical potential. At present, the properties of matter at high temperature and small baryon chemical potential are being investigated using ultrarelativistic heavy-ion collisions at the Large Hadron Collider (LHC), Conseil Européen pour la Recherche Nucléaire (CERN), and the Relativistic Heavy Ion Collider (RHIC), Brookhaven National Laboratory (BNL). The beam energy scan (BES) program of RHIC [10] is currently investigating the matter at large baryon chemical potential and the location of the critical point [11]. The HADES experiment at Gesellschaft für Schwerionenforschung mbH (GSI), Darmstadt, is also investigating a medium with very large baryon chemical potential [12]. In the future, the compressed baryonic matter (CBM) experiment [13] at the Facility for Antiproton and Ion Research (FAIR) at GSI and the Nuclotron-Based Ion Collider Facility (NICA) [14] at Joint Institute for Nuclear Research, Dubna, will study nuclear matter at large baryon chemical potential.

Hadron resonance gas [15–70] is a popular model to study the QCD matter formed in heavy-ion collisions at finite temperature and chemical potential. Varieties of HRG models exist in the literature, some of which consider interaction between hadrons and some of which do not. The ideal hadron resonance gas (HRG) model is successful in reproducing the zero chemical potential LQCD data of bulk properties of the QCD matter at moderate temperatures $T \approx 150$ MeV [2,3,5–7]. This model is also successful in describing the hadron yields, at chemical freezeout, created in central heavy-ion collisions from Schwerionensynchrotron up to RHIC energies [18,19,22,28]. The ideal HRG model assumes that the microscopic thermal system consists of noninteracting pointlike hadrons and resonances, and hence the width of the resonances are ignored. There are several approaches to include interaction in the HRG model. One such model is the excluded volume HRG (EVHRG) model where van der Waals–type repulsive interaction [15–17,20,21,30–35,44–48,50,68] is introduced by considering the geometrical sizes of the hadrons. However, the long-distance repulsive interactions are ignored in this model. Another major issue of the model is fixing the radii of various hadrons. In Refs. [31,35], it was shown that the LQCD data of different thermodynamic quantities can be described in the EVHRG model with the fixed radius parameter between 0.2 and 0.3 fm. The mass-dependent hadronic radius is also considered in the EVHRG model to study the hadronic multiplicities at LHC energy and a reasonable agreement between model and experimental data is found [68]. Repulsive interaction can also be introduced via repulsive mean field approach [71,72]. The van der Waals (VDW)–type interaction with both attractive and repulsive parts has been introduced recently in the HRG model [55–57,60,61,69]. Such a model introduces more parameters and fixing them using existing

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information has its own drawbacks. The two van der Waals parameters can be fixed either by reproducing the properties of the nuclear matter at zero temperature [56] or by fitting the LQCD data at zero chemical potential [69]. In addition to that, the VDWHRG model does not even include the interactions of mesons since the number densities diverge when the chemical potential becomes comparable to the meson mass [55]. Compared to the ideal HRG model, both EVHRG and VDWHRG describe better the lattice QCD data in the crossover region. As discussed above, in lieu of introducing interactions, both the interacting HRG models bring in additional parameters compared to the ideal HRG model. The assumptions involved in fixing the additional parameters in the interacting HRG models are debatable.

Another approach to include interaction in a system consisting of hadronic gas is the S-matrix approach [73]. This approach expands the partition function using relativistic virial expansion. The virial coefficients are related to phase shifts, which needs to calculated either theoretically [74,75] or obtained from experiments. The type of the interaction depends on the sign of the derivative of phase shift. A positive sign corresponds to the attractive interaction and a negative sign corresponds to the repulsive interaction. For an example, the authors of Ref. [76] had found that π-π channel has the attractive δ_0^0, δ_1^1 phase shifts (phase shift is defined as δ_l^I where l is the orbital angular momentum and I is the isospin of the channel) and also the repulsive δ_0^2 phase shift, while Refs. [76–79] used experimental phase shifts in their study. A theoretical way of calculating phase shifts is to use the K-matrix formalism [80,81]. The resonances, contributing to the interaction, appear as a sum of poles in the K matrix. This approach preserves the unitarity of the S matrix and neatly handles multiple resonances, unlike the popular Breit-Wigner parametrization of the resonance spectral function. We would like to mention here that Refs. [18,28,82–84] used Breit-Wigner parametrization with an *ad hoc* profile function [84]. All these issues motivate us to use the K-matrix formalism consistently to calculate phase shifts in the virial expansion approach. The K-matrix formalism has been applied previously to calculate shear viscosity and interaction measure for interacting hadronic gas in Ref. [80]. However, our result on interacting measure agrees better with the lattice QCD result on including additional resonances. Further, we calculate susceptibilities of the conserved charges within the K-matrix formalism.

The paper is organized as follows. In Secs. II and III, we discuss K-martix formalism and the Breit-Wigner parametrization of the resonance spectral function respectively. A comparison between the above approaches is given in Sec. IV. Section V discusses relativistic virial expansion. We discuss numerical results and comparison of our calculations with ideal HRG and LQCD in Sec VI. We conclude our findings in Sec. VII.

II. THE K-MATRIX FORMALISM

The K-matrix formalism elegantly expresses the unitarity of the S matrix for the processes of type $ab \to cd$, where a, b and c, d are hadrons. We provide only a brief summary of the formalism in this section; interested readers are referred to Refs. [80,85,86].

In general, the amplitude that an initial state $|i\rangle$ to be scattered to the final state $|f\rangle$ is

$$S_{fi} = \langle f|S|i\rangle, \tag{1}$$

where S is called the scattering operator. Splitting the probability of noninteraction I and interaction by defining the transition operator T, we have

$$S = I + 2iT, \tag{2}$$

where I is the identity operator. Conservation of probability implies that scattering matrix S should be unitary, i.e.,

$$SS^\dagger = S^\dagger S = I. \tag{3}$$

From the unitarity of S, one gets

$$T - T^\dagger = 2iT^\dagger T = 2iTT^\dagger. \tag{4}$$

One may further rearrange this expression into

$$(T^{-1} + iI)^\dagger = T^{-1} + iI. \tag{5}$$

Let us introduce the Hermitian operator K via

$$K^{-1} = T^{-1} + iI. \tag{6}$$

Since the operator K is Hermitian, the matrix is symmetric and the the eigenvalues are real. One can rewrite the components of T matrix in terms of K matrix as

$$\begin{aligned} \mathrm{Re}\, T &= (I + K^2)^{-1} K = K(I + K^2)^{-1}, \\ \mathrm{Im}\, T &= (I + K^2)^{-1} K^2 = K^2(I + K^2)^{-1}. \end{aligned} \tag{7}$$

Resonances appear as sum of poles in the K matrix as

$$K_{ab\to cd} = \sum_R \frac{g_{R\to ab}(\sqrt{s}) g_{R\to cd}(\sqrt{s})}{m_R^2 - s}, \tag{8}$$

where the sum on R runs over the number of resonances with mass m_R, and the residue functions are given by

$$g_{R\to ab}(\sqrt{s}) = m_R \Gamma_{R\to ab}(\sqrt{s}), \tag{9}$$

where $\sqrt{s}$ is the center-of-mass energy. The energy-dependent partial decay widths [85] are given by

$$\Gamma_{R\to ab}(\sqrt{s}) = \Gamma^0_{R\to ab}(\sqrt{s}) \frac{m_R}{\sqrt{s}} \frac{q_{ab}}{q_{ab0}} [B^l(q_{ab}, q_{ab0})]^2. \tag{10}$$

The momentum q_{ab} is given as

$$q_{ab}(\sqrt{s}) = \frac{1}{2\sqrt{s}} \sqrt{[s - (m_a + m_b)^2][s - (m_a - m_b)^2]}, \tag{11}$$

where m_a and m_b are the masses of decaying hadrons a and b.

In Eq. (10), $q_{ab0} = q_{ab}(m_R)$ is the resonance momentum at $\sqrt{s} = m_R$ and Γ^0_R is the width of the pole at half maximum. The $B^l(q_{ab}, q_{ab0})$ are the Blatt-Weisskopf barrier factors which can be expressed in terms of momentum q_{ab} and resonance momentum q_{ab0} for the orbital angular momentum l as

$$B^l_{R\to ab}(q_{ab}, q_{ab0}) = \frac{F_l(q_{ab})}{F_l(q_{ab0})}. \tag{12}$$

The barrier factors $F_l(q)$ can be obtained using the following definition:

$$F_l(z) = \frac{|h_l^{(1)}(1)|}{|z h_l^{(1)}(z)|}, \tag{13}$$

where $h_l^{(1)}(z)$ are spherical Hankel functions of the first kind and $z = (q/q_R)^2$, with $q_R = 0.1973$ GeV corresponding to 1 fm.

The scattering amplitude $f(\theta)$ can be expressed as

$$f(\sqrt{s},\theta) = \frac{1}{q_{ab}} \sum_l (2l+1) T^l P_l(\cos\theta), \tag{14}$$

in terms of the interaction matrix $T^l(s)$. Here $P_l(\cos\theta)$ are the Legendre polynomials for the angular momentum l and θ is the center-of-mass scattering angle. The cross section for the process $ab \to cd$ can be given in terms of terms of scattering amplitude

$$\sigma(\sqrt{s},\theta) = |f(\sqrt{s},\theta)|^2. \tag{15}$$

If we use partial decomposition of the T matrix,

$$T^l = e^{i\delta_l} \sin\delta_l, \tag{16}$$

one can relate the phase shift in a single resonance of mass m_1 to the K matrix using the relations in Eq. (7),

$$K = \frac{m_1 \Gamma_1(\sqrt{s})}{m_1^2 - s} = \tan\delta_l. \tag{17}$$

A. Three-body decay

Let the resonance R with mass m_R decay into three other particles a, b, and c of masses m_a, m_b, and m_c. The residue function is given by

$$g_{R\to abc}(\sqrt{s}) = \frac{1}{2m_\pi^2} \int d\phi_3 |\Gamma(\sqrt{s})|^2, \tag{18}$$

where ϕ_3 is the three-body Lorentz-invariant phase space and we have scaled it by pion mass (m_π) to make it dimensionless. The three-body phase space can be expressed as

$$\begin{aligned}\phi_3 &= \int \frac{d^3p_1}{(2\pi)^3}\frac{1}{2E_1}\frac{d^3p_2}{(2\pi)^3}\frac{1}{2E_2} \\ &\quad \times \frac{d^3p_3}{(2\pi)^3}\frac{1}{2E_3}(2\pi)^4\delta^4\left(p - \sum_{i=1}^{3} p_i\right) \\ &= \frac{R_3(\sqrt{s})}{(2\pi)^5},\end{aligned} \tag{19}$$

where E_i's and p_i's are energies and the momenta of the decaying particles in the resonance rest frame. The function $R_3(\sqrt{s})$ is expressed as

$$R_3(\sqrt{s}) = \frac{\pi^2}{4s} \int_{s_2^{\min}}^{s_2^{\max}} \frac{ds_2}{s_2} \lambda^{\frac{1}{2}}(s_2,s,m_a^2)\lambda^{\frac{1}{2}}(s_2,m_b^2,m_c^2), \tag{20}$$

where $s_2^{\min} = (m_b + m_c)^2$ and $s_2^{\max} = (\sqrt{s} - m_a)^2$ and the λ's are the Kallen functions [87,88]. They can be defined as

$$\lambda(x,y,z) = (x - y - z)^2 - 4yz. \tag{21}$$

If we assume that the width $\Gamma(\sqrt{s})$ is a slowly varying function of energy, it can be pulled out of the integration sign and then finally we have

$$g_{R\to abc}(\sqrt{s}) = \frac{1}{(2\pi)^5} \frac{R_3(\sqrt{s})|\Gamma(\sqrt{s})|^2}{2m_\pi^2}. \tag{22}$$

III. THE BREIT-WIGNER PARAMETRIZATION

The interaction matrix or the T matrix that was defined in Eq. (2) for the relativistic single-particle resonance can be parametrized in the Breit-Wigner form as [89]

$$T = \frac{m_R \Gamma_{R\to ab}(\sqrt{s})}{(m_R^2 - s) - i m_R \Gamma_R^{\rm tot}(\sqrt{s})}, \tag{23}$$

where $\Gamma_R^{\rm tot} = \sum_{i,j} \Gamma_{R\to ij}$ is the total width and $\Gamma_{R\to ij}$ is the partial width for a given channel $R \to ij$ of the resonance R respectively.

The cross section for an elastic scattering reaction $a + b \to R \to a + b$ is then given as

$$\sigma(\sqrt{s},\theta) = \frac{g_{I,l}}{q_{ab}^2} \frac{m_R^2 \Gamma_{R\to ab}^2}{(m_R^2 - s)^2 + m_R^2 \Gamma_R^{\rm tot\,2}} P_l(\cos\theta), \tag{24}$$

where $g_{I,l}$ are the symmetry factors containing the isospin and spin multiplicities of the corresponding resonance R. The center-of-mass momentum q_{ab} is the same as given in Eq. (11), $P_l(\cos\theta)$ are the Legendre polynomials for the angular momentum l, and θ is the center-of-mass scattering angle. The partial decay widths $\Gamma_{R\to ab}(\sqrt{s})$ are same as given in Eq. (10),

IV. COMPARISONS BETWEEN K-MATRIX AND BREIT-WIGNER APPROACHES

Consider a $\pi\pi$ scattering at center-of-mass energy $\sqrt{s}$, which has two resonance with mass m_1 and m_2 coupling to a certain channel l. From Eq. (8), we have

$$K = \frac{m_1 \Gamma_1(\sqrt{s})}{m_1^2 - s} + \frac{m_2 \Gamma_2(\sqrt{s})}{m_2^2 - s}; \tag{25}$$

i.e., the resonances are summed in the K matrix. We can use Eq. (6) to get the T matrix as

$$\begin{aligned}T &= \frac{m_1 \Gamma_1(\sqrt{s})}{(m_1^2 - s) - i m_1 \Gamma_1(\sqrt{s}) - i\frac{m_1^2 - s}{m_2^2 - s} m_2 \Gamma_2(\sqrt{s})} \\ &\quad + \frac{m_2 \Gamma_2(\sqrt{s})}{(m_2^2 - s) - i m_2 \Gamma_2(\sqrt{s}) - i\frac{m_2^2 - s}{m_1^2 - s} m_1 \Gamma_1(\sqrt{s})}.\end{aligned} \tag{26}$$

If m_1 and m_2 are far apart relative to their widths, then K is dominated either by m_1 or m_2 depending on whether $\sqrt{s}$ is near m_1 or m_2. The transition amplitude is then given using Eq. (26) approximately as the sum

$$T \approx \frac{m_1 \Gamma_1(\sqrt{s})}{(m_1^2 - s) - i m_1 \Gamma_1(\sqrt{s})} + \frac{m_2 \Gamma_2(\sqrt{s})}{(m_2^2 - s) - i m_2 \Gamma_2(\sqrt{s})}, \tag{27}$$

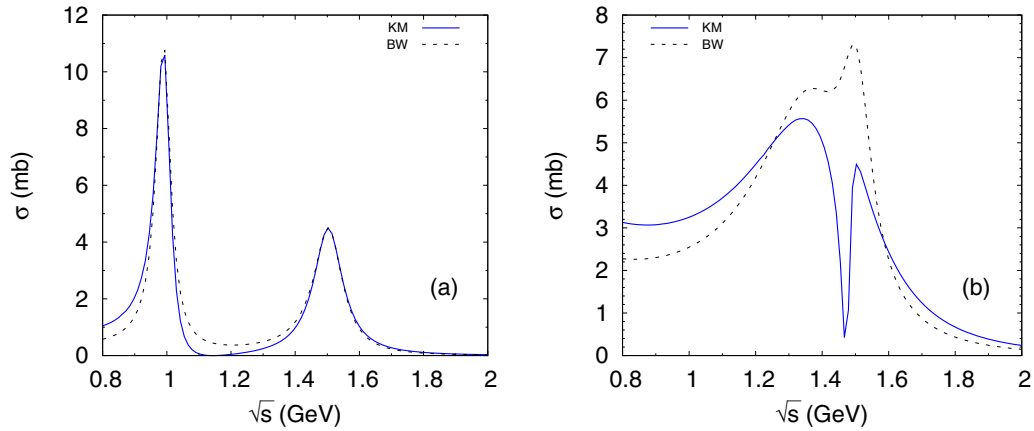

FIG. 1. The variation of total cross section as a function of center-of-mass energy. Left panel shows total cross section of separated resonances $f_0(980)$ and $f_0(1500)$; right panel shows total cross section of overlapping resonances $f_0(1370)$ and $f_0(1500)$. The calculations using the K-matrix formalism are shown using a solid blue line (KM). Calculations using Breit-Wigner parametrization are shown using a dashed black line (BW).

which shows that the result is the same as adding two Breit-Wigner forms Eq. (23) with masses m_1, m_2 and widths Γ_1, Γ_2. The left panel of Fig. 1 compares the results of total cross section in the K matrix and Breit-Wigner formalism for two separated resonances $f_0(980)$ and $f_0(1500)$ of mass $m_1 = 990$ MeV, $\Gamma_1 = 55$ MeV and $m_2 = 1505$ MeV, $\Gamma_2 = 109$ MeV. The results are almost identical except that the peak in Breit-Wigner formalism is slightly larger than that in the K-matrix formalism.

In the limit in which the two states have the same masses, i.e., $m_c = m_1 = m_2$, then the transition amplitude becomes

$$T = \frac{m_c[\Gamma_1(\sqrt{s}) + \Gamma_2(\sqrt{s})]}{\left(m_c^2 - s\right) - i m_c[\Gamma_1(\sqrt{s}) + \Gamma_2(\sqrt{s})]}, \tag{28}$$

which shows that the result is a single Breit-Wigner form but its total width is now the sum of the two individual widths. The right panel of Fig. 1 compares the results of the total cross section in the K matrix and Breit-Wigner formalism for two overlapping resonances $f_0(1370)$ and $f_0(1500)$ of mass $m_1 = 1370$ MeV, $\Gamma_1 = 350$ MeV and $m_2 = 1505$ MeV, $\Gamma_2 = 109$ MeV. The results shows that the Breit-Wigner parametrization overestimates the cross section both at the peak and in the middle of the overlapping resonances. In such cases of two nearby resonances the Breit-Wigner form, Eq. (23) is not strictly valid and the correct equation Eq. (25) must be used.

V. RELATIVISTIC VIRIAL EXPANSION

The most natural way to incorporate interaction among a gas of hadrons is to use relativistic virial expansion introduced by Dashen *et al.* [73]. The formalism allows one to compute the thermodynamic variables of a system in a grand canonical ensemble, once the S matrix is known. In general, the logarithm of the partition function can be written as

$$\ln Z = \ln Z_0 + \sum_{i_1,i_2} z_1^{i_1} z_2^{i_2} b(i_1,i_2), \tag{29}$$

where z_1 and z_2 are fugacities of two species and $z = e^{\beta\mu}$. The chemical potential of jth particle is defined as $\mu_j = B_j\mu_B + S_j\mu_S + Q_j\mu_Q$, where B_j, S_j, and Q_j are baryon number, strangeness, and electric charge and μ's are the respective chemical potentials. The virial coefficients $b(i_1,i_2)$ are written as

$$b(i_1,i_2) = \frac{V}{4\pi i}\int \frac{d^3p}{(2\pi)^3}\int d\varepsilon \exp[-\beta(p^2+\varepsilon^2)^{1/2}] \times \left[A\left\{S^{-1}\frac{\partial S}{\partial \varepsilon} - \frac{\partial S^{-1}}{\partial \varepsilon}S\right\}\right]_c. \tag{30}$$

In the above, the inverse temperature is denoted by β while V, p, and ε stand for the volume, total center-of-mass momentum, and energy, respectively. The labels i_1 and i_2 refer to the channel of the S matrix which has an initial state containing $i_1 + i_2$ particles. The symbol A denotes the symmetrization (antisymmetrization) operator for a system of bosons (fermions) while the subscript c refers to trace over all linked diagrams. The lowest virial coefficient $b_2 = b(i_1,i_2)/V$ as $V \to \infty$ corresponds to the case where $i_1 = i_2 = 1$ and in which the present study is mostly interested.

The S matrix can be expressed in terms of phase shifts δ_l^I as [90]

$$S(\varepsilon) = \sum_{l,I}(2l+1)(2I+1)\exp\left(2i\delta_l^I\right), \tag{31}$$

where l and I denote angular momentum and isospin, respectively. On integrating Eq. (30) over the total momentum, we have

$$b_2 = \frac{1}{2\pi^3\beta}\int_M^\infty d\varepsilon\varepsilon^2 K_2(\beta\varepsilon)\sum_{l,I}{}' g_{I,l}\frac{\partial\delta_l^I(\varepsilon)}{\partial\varepsilon}. \tag{32}$$

The factor $g_{I,l} = (2I+1)(2l+1)$ is the degeneracy factor, M is the invariant mass of the interacting pair at threshold, and the factor $K_2(\beta\varepsilon)$ is the modified Bessel function of the second kind. The prime over the summation sign denotes that for given l the sum over I is restricted to values consistent with statistics.

Equation (32) shows that the contribution arising from interaction to thermodynamic variable are in terms of phase shifts weighted by thermal factors. This factors give positive

(attractive) or negative (repulsive) contribution depending on whether the derivative of phase shifts are positive or negative. The b_2 or alternatively phase shifts are obtained from experiments or from theoretical calculations. In the present work, we determine the phase shifts from two different parametrization of the T matrix, (i) K-matrix parametrization and (ii) Breit-Wigner parametrization, which were discussed in Secs. II and III. Since in this work we are interested only in the part corresponding to the second virial coefficient $b_2(\varepsilon)$ in the partition function Eq. (29), by inserting Eq. (32) into Eq. (29) one can immediately compute all the thermodynamic variables. We adopt the following relations from Ref. [76]:

$$\mathrm{P_{int}} = \frac{1}{\beta}\frac{\partial \ln Z_{\mathrm{int}}}{\partial V} = \frac{z_1 z_2}{2\pi^3\beta^2}\int_M^\infty d\varepsilon\varepsilon^2 K_2(\beta\varepsilon)\sum_{I,l}{}' g_{I,l}\frac{\partial\delta_l^I(\varepsilon)}{\partial\varepsilon}, \tag{33}$$

$$\begin{aligned}\varepsilon_{\mathrm{int}} &= -\frac{1}{V}\left(\frac{\partial \ln Z_{\mathrm{int}}}{\partial\beta}\right)_z \\ &= \frac{z_1 z_2}{8\pi^3\beta}\int_M^\infty d\varepsilon\varepsilon^3[K_1(\beta\varepsilon)+3K_3(\beta\varepsilon)]\sum_{I,l}{}' g_{I,l}\frac{\partial\delta_l^I(\varepsilon)}{\partial\varepsilon},\end{aligned} \tag{34}$$

$$\begin{aligned}\mathrm{s_{int}} &= -\frac{\beta^2}{V}\left[\frac{\partial(T\ln Z_{\mathrm{int}})}{\partial\beta}\right]_{V,\mu} \\ &= \frac{z_1 z_2}{2\pi^3}\int_M^\infty d\varepsilon\varepsilon^3 K_3(\beta\varepsilon)\sum_{I,l}{}' g_{I,l}\frac{\partial\delta_l^I(\varepsilon)}{\partial\varepsilon} \\ &\quad -(\mu_1+\mu_2)\beta^2\,\mathrm{P_{int}}\,,\end{aligned} \tag{35}$$

$$\mathrm{n_{int}} = \frac{T}{V}\left(\frac{\partial\ln Z_{\mathrm{int}}}{\partial\mu}\right)_{V,T} \tag{36}$$

$$= \frac{z_1 z_2}{\pi^3\beta}\int_M^\infty d\varepsilon\varepsilon^2 K_2(\beta\varepsilon)\sum_{I,l}{}' g_{I,l}\frac{\partial\delta_l^I(\varepsilon)}{\partial\varepsilon}, \tag{37}$$

and the ideal gas counterpart can be obtained from the first term of Eq. (29) as follows:

$$\mathrm{P_{id}} = \sum_h \frac{g_h}{2\pi^2}m_h^2T^2\sum_{j=1}^\infty(\pm1)^{j-1}(z^j/j^2)K_2(j\beta m_h), \tag{38}$$

$$\varepsilon_{\mathrm{id}} = \sum_h \frac{g_h}{16\pi^2}m_h^4\sum_{j=1}^\infty(\pm1)^{j-1}z^j[K_4(j\beta m_h)-K_0(j\beta m_h)], \tag{39}$$

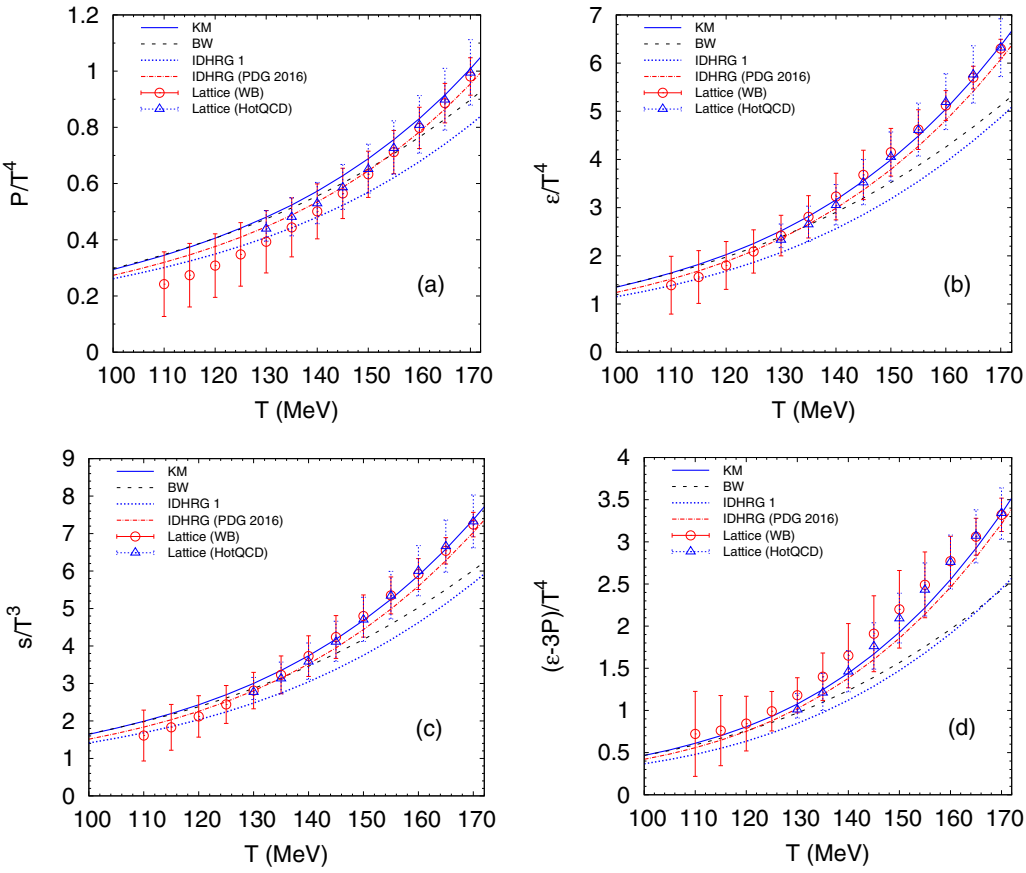

FIG. 2. Temperature dependence of various thermodynamic quantities at zero chemical potential. The calculations using K-matrix formalism are shown using a solid blue line (KM). Calculations using the Breit-Wigner parametrization are shown using a dashed black line (BW). IDHRG 1 corresponds to results of ideal HRG, with the same number of particles as used in KM/BW parametrization, whereas IDHRG (PDG 2016) includes all the hadrons and resonances listed in PDG 2016 [89]. Results are compared with lattice QCD data of Refs. [4] (WB) and [6] (HotQCD).

$$\mathrm{n_{id}} = \sum_h \frac{g_h}{2\pi^2} m_h^2 T \sum_{j=1}^{\infty} (\pm 1)^{j-1} (z^j/j) K_2(j\beta m_h), \quad (40)$$

$$\mathrm{s_{id}} = \beta(\varepsilon_{\mathrm{id}} + \mathrm{P_{id}} - \mu\, \mathrm{n_{id}}), \quad (41)$$

where h denotes the stable hadron index. The total pressure of the system is the sum of ideal and interacting parts, i.e.,

$$P = \mathrm{P_{id}} + \mathrm{P_{int}}\,, \quad (42)$$

and subsequent relationships hold for other quantities.

The susceptibilities of conserved charges can be calculated as [7]

$$\chi^{xyz}_{BSQ} = \frac{\partial^{x+y+z}(P/T^4)}{\partial(\mu_B/T)^x \partial(\mu_S/T)^y \partial(\mu_Q/T)^z}, \quad (43)$$

where x, y, and z are the order of derivatives of the quantities B, S, and Q.

For a very narrow resonance of mass m_R, the phase shift δ_l^I changes rapidly through π radians around $\varepsilon = m_R$ and can be approximated by a step function $\delta_l^I = \theta(\varepsilon - m_R)$. In such a limiting case $\partial\delta_l^I/\partial\varepsilon \approx \pi\delta(\varepsilon - m_R)$, and then from Eq. (32) we have

$$b_2 = \frac{g_{I,l}}{2\pi^2} m_R^2 T K_2(\beta m_R). \quad (44)$$

This is similar to a stable hadron in the Maxwell-Boltzmann statistics.

A few points to be noted here. First, notice that the K-matrix and Breit-Wigner forms can only be able to take resonance interaction based on known masses and widths of the resonances and do not carry any information about repulsive channels which are known to exist from perturbative calculation [91] and experimental phase shifts [76]. Recently, some works [78] started looking at these repulsive channels but an adequate treatment of such channels is still missing. Second, apart from the elastic interaction channel, many inelastic channels can exist for a gas of interacting hadrons, which the present study does not encompass [79]. Third, we have checked that the effects of three-body interaction in the equation of state is less than 5% in the range of temperature considered in this work. Therefore, we have not included the effect of the three-body interaction in Sec. VI.

VI. RESULTS

We have considered all the stable hadrons and resonances which have two-body decay channels listed in PDG (2016) [89]. In Fig. 2, we have shown temperature variation of different thermodynamic quantities such as scaled pressure, energy density, entropy density, and interaction measure $(\varepsilon - 3P)/T^4$. Results of the K-matrix parametrization are compared with the Breit-Wigner parametrization and two variants of HRG models are shown. The IDHRG 1 considers the same number of resonances (i.e., those decaying into two bodies) as the K-matrix–Breit-Wigner parametrization, and the IDHRG (PDG 2016) considers all the resonances listed in PDG (2016). Later, the results are compared with continuum extrapolated LQCD data of Wuppertal-Budapest (WB) [4] and the Hot QCD Collaboration [6]. We notice that all thermodynamic quantities calculated using the K-matrix–Breit-Wigner parametrization are larger than the results of IDHRG 1. This is because many resonances have finite width; for example, in the π-π channel resonances like $f_0(500)$, $\rho(770)$ or resonances like $N(1440)$, $N(1520)$ in π-N channel contribute substantially to the second virial coefficient. Differences in K-matrix and Breit-Wigner parametrizations persist because of the presence of many overlapping resonances, as mentioned in Sec. IV. In particular, we found that the interaction measure, which is a measure of interactions in a medium, is well described in the K-matrix formalism. Thermodynamic quantities in IDHRG (PDG 2016) are larger compared to previous IDHRG 1 because of increased number of degeneracies. It is important to mention that the all our results are meaningful below the hadronic to quark gluon crossover transition temperature T_c as predicted by LQCD [6], which is around 145–163 MeV. It must be noted that Ref. [80] also used the K-matrix formalism to calculate interaction measure in an interacting gas of π-K-N-η including the dominant resonances produced in the two-body elastic interaction. However, their result underestimates the lattice results close to T_c, which has been improved in the

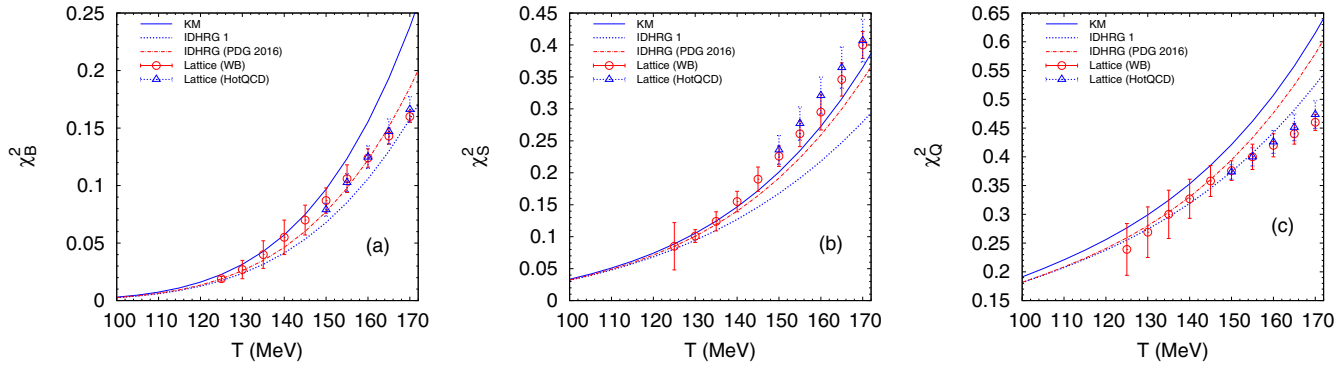

FIG. 3. Temperature dependence of second-order diagonal susceptibilities at zero chemical potential. The calculations using the K-matrix formalism are shown using a solid blue line (KM). IDHRG 1 corresponds to results of ideal HRG, with the same number of particles as used in the KM/BW parametrization, whereas IDHRG (PDG 2016) includes all the hadrons and resonances listed in PDG 2016 [89]. Results are compared with lattice QCD data of Refs. [2] (WB) and [3] (Hot QCD).

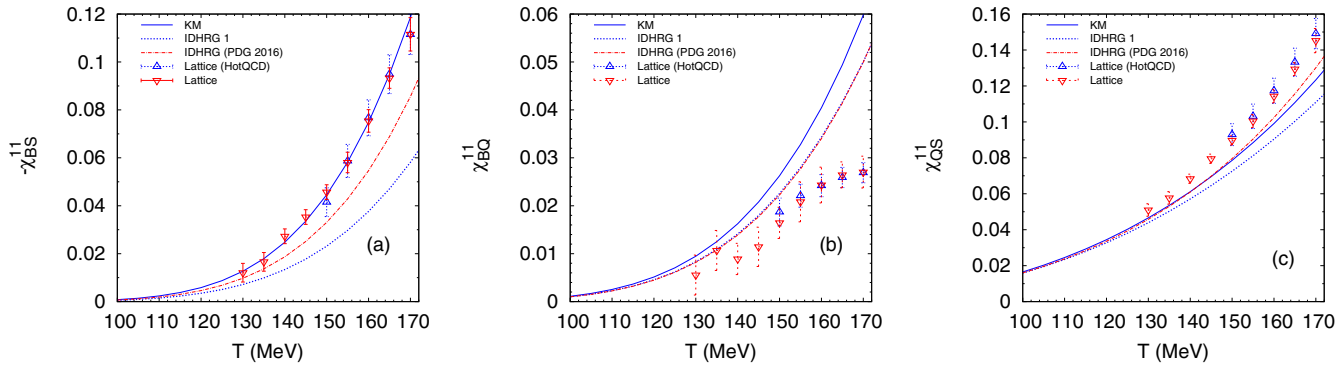

FIG. 4. Temperature dependence of second-order off-diagonal susceptibilities at zero chemical potential. The calculations using K-matrix formalism are shown using a solid blue line (KM). IDHRG 1 corresponds to results of the ideal HRG, with same number of particles as used in KM/BW parametrization, whereas IDHRG (PDG 2016) includes all the hadrons and resonances listed in PDG 2016 [89]. Results are compared with lattice QCD data of Refs. [3] (Hot QCD) and [7] (lattice).

present work by the inclusion of additional hadrons Ξ-Λ-Σ and the corresponding resonances. The above results show that our approach of using the K-matrix formalism is in good agreement with LQCD.

The temperature dependencies of diagonal susceptibilities χ_B^2, χ_S^2, and χ_Q^2 are shown in Fig. 3. We compare our results with continuum extrapolated LQCD data of Refs. [2] (WB) and [3] (Hot QCD). Results of all diagonal susceptibilities in the K-matrix formalism are in better agreement with LQCD data, especially χ_S^2, up to crossover temperature than IDHRG 1. However, it should be noted that χ_B^2 and χ_Q^2 in IDHRG (PDG 2016) also agree with LQCD data but this is due to the increase in the number of degeneracies, as mentioned earlier. Results of Breit-Wigner parametrization are not shown in the comparison because of its inherent inadequacy in treating in multiple resonances, which leads to violation of unitarity.

The temperature dependencies of off-diagonal susceptibilities χ_{BS}^{11}, χ_{BQ}^{11}, and χ_{QS}^{11} are shown in Fig. 4. We compare our results with the continuum extrapolated LQCD data of Refs. [3] (Hot QCD) and [7] (lattice). We have found that K-matrix formalism agrees with lattice data for χ_{BS}^{11} but not for χ_{BQ}^{11} and χ_{QS}^{11}. We think this might happen because [76] many channels, mostly in the N-N channels, have dominant repulsive channels which could negate the influence of positive phase shifts, thereby contributing to the correlations.

VII. CONCLUSION

To summarize, we have included the interaction properly in the hadron resonance gas model using the quantum virial expansion approach. The thermodynamic quantities were calculated by parametrizing the two-body phase shifts using the K-matrix formalism, which preserves the unitarity of the S matrix. Good agreement with lattice QCD calculations is found for the equation of state using the above formalism. Specifically, we found that the interaction measure $[(\varepsilon - 3P)/T^4]$ as a function of temperature is well described in the K-matrix formalism and has been improved compared to the previous studies in Ref. [80] by the inclusion of additional hadrons. We found that IDHRG 1 (considering those resonances that decay into two stable hadrons) underestimates the lattice data for all the thermodynamic variables. However, IDHRG (PDG 2016) matches lattice data because of the increased number of degeneracies. Additionally we have calculated the diagonal and off-diagonal susceptibilities of conserved charges in the K-matrix formalism. The results of susceptibilities calculated in the K-matrix formalism resembles lattice data quite well, especially in the strangeness sector below the crossover region. However, observables χ_{BQ}^{11} and χ_{QS}^{11} are not described satisfactorily in the present work. This could be improved by incorporating inelastic collisions and repulsive interactions and it would be interesting to introduce them in a future work.

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