Update on matter radii of $^{17-24}$O

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The appearance of new theoretical papers concerning matter radii of neutron-rich oxygen nuclei has prompted a return to this problem. New results provide no better agreement with experimental values than did previous calculations with a simple model. I maintain that there is no reason to adjust the ^{22}O core in the ^{24}O nucleus, and the case of ^{24}O should be reexamined experimentally.

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I. INTRODUCTION

The situation regarding matter radii of heavy oxygen nuclei ($^{22-24}$O) is somewhat confusing. Ozawa *et al.* [1] measured interaction cross sections for beams of about 950 A MeV $^{16-19}$O and $^{21-24}$O incident on a carbon target. They used a Glauber-model analysis and deduced effective matter radii of the nuclei. They found a large increase from ^{21}O to ^{22}O and an even larger increase from ^{22}O to ^{23}O, with small uncertainties in both cases. Standard calculations were unable to reproduce these results. For ^{23}O, the discrepancy was about an 8 σ effect. One explanation for the large change in matter radius was the possibility that ^{21,22}O cores in ^{22,23}O might have larger radii than the free nuclei. A later experiment for ^{22,23}O and new calculations appeared to have resolved the problem [2]. For ^{22}O, the new radius was considerably smaller than the earlier one, and with similar uncertainty—2.88(6) vs 2.75(7) fm. For ^{23}O, the difference was even larger, but the new value had a larger uncertainty—3.20(4) vs 2.97(11) fm. The possibility exists that the uncertainty for ^{23}O was underestimated [1].

II. ANALYSIS AND RESULTS

Using a simple model [3–8], Sherr and I [9] computed matter radii for $^{18-24}$O. If the ground state (g.s.) configuration is known, the procedure is exact, with only one parameter—the radius of ^{17}O. For all other nuclei, the computed radius for nucleus A is used as the core radius for nucleus $A+1$. The relevant equation is

$$A{R_m}^2 = (A-1){R_c}^2 + (A-1){R_v}^2/A, \qquad (1)$$

where R_m is the matter radius of nucleus A, R_c is the matter radius of the $A-1$ core, and R_v^2 is the expectation value of r^2 for the valence neutron, computed with the aid of a Woods-Saxon potential. The potential model is used to compute the wave function of a single particle of the appropriate ℓ value and separation energy, and then the expectation value of r^2 is computed by direct integration. The Woods-Saxon potential has geometric parameters $r_0, a = 1.25, 0.65$ fm.

The model is exact providing two conditions hold: (1) The matter density can be described as the density of a core plus that of one or two valence neutrons. (2) The configuration of the valence neutron(s) is known. If more than one configuration is important, the squares of the matter radii are weighted by the intensities of the various configurations. Of course, the model is useful only if the number of important configurations is small. For a chain of isotopes, the model contains only one parameter—namely the radius of one member of the chain, usually taken as the smallest A, but that is not required.

Our results were in excellent agreement with experimental values [1,10,11] for $A = 17-21$. For $A = 22$ and 23, our R_m's agreed with recent work of Ref. [2], but not with the earlier very large values [1]. We suggested that R_m for ^{24}O should be re-measured. Because of the overall agreement with all the nuclei, and especially with ^{22,23}O, we concluded that there is no longer any evidence to advocate change in the core to explain the observations.

Recently, theoretical groups have returned to this problem. Cipollone, Barbieri, and Navrátil [12] used three-nucleon forces and investigated the effect of correlations for several O nuclei, including ^{22,24}O. They concluded that their calculations could not reproduce the large radii for these nuclei. They suggested the reason was a lack of repulsion in their model of $NN+3N$ interactions.

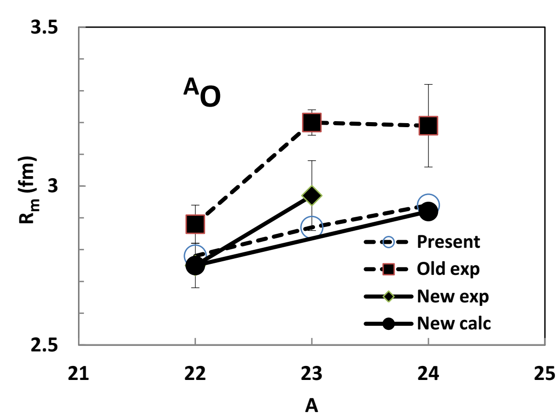

FIG. 1. Matter radii (fm) for $^{22-24}$O: Experimental, Ref. [1] (squares) and Ref. [2] (diamonds); calculated, Ref. [9] and present (open circles) and Ref. [13] (closed circles).

TABLE I. Matter radii (fm) of neutron-excess oxygen nuclei $^{22-24}$O.

A	ℓ	S_n (MeV)[a]	R_v[b]	R_m			
				Calc.[b]	Prev. exp.[c]	New exp.[d]	New calc.[e]
22	0	6.85(6)	3.77	2.79	2.88(6)	2.75(7)	2.75
	2		3.44	2.77			
23	0	2.73(11)	4.62	2.87	3.20(4)	2.97(11)	
24	0	4.19(14)	4.23	2.95	3.19(13)		2.92
		6.93/2	4.42	2.94			

[a]Reference [11].
[b]Reference [9] and present.
[c]Reference [1].
[d]Reference [2].
[e]Reference [13].

Quite recently, Itagaki and Tohsaki [13] published a paper entitled "Nontrivial origin for the large nuclear radii of dripline oxygen isotopes." They investigated ^{22,24}O in a model of $4\alpha + mn$, where $m = A - 16$ and n is a neutron. They attributed the increase of radius from ^{22}O to ^{24}O to a change in the size of the ^{22}O core. The size of the αclusters and the distance between them were taken as variational parameters. Their calculated root-mean-square (rms) matter radii for ^{22}O and ^{24}O were 2.75 and 2.92 fm, respectively. These are smaller than the experimental values, but a jump at ^{24}O from ^{22}O was reproduced.

Experimental and theoretical matter radii for these nuclei are summarized in Table I and Fig. 1. I note that the most recent calculated values for ^{22,24}O [13] are very close to (but slightly smaller than) our earlier results [9]. I emphasize that our calculated values used the free ^{22}O radius for the ^{22}O core in ^{23}O and ^{24}O. The increase in R_m from ^{22}O to ^{23}O is due to the two facts that the last neutron is in the $2s_{1/2}$ orbital, and the separation energy is somewhat smaller than that for nearby O nuclei. The matter radius of ^{24}O is then large because the radius of its ^{23}O core is large—even though the radius of a free ^{23}O nucleus was used in the computation for ^{24}O. As a check of the ^{24}O calculation, I also computed it using the $2n$ form [4,6,8,14–18] of Eq. (1), viz.

$$A R_m{}^2 = (A-2)\left(R_c{}^2 + 2R_v{}^2/A\right). \qquad (2)$$

The difference between the two procedures is only 0.01 fm. These comparisons reinforce the need to investigate ^{24}O again experimentally.

I turn now to lighter oxygen nuclei. Lapoux *et al.* [19] recently suggested that matter radii extracted from optical-model analysis of proton elastic-scattering angular distributions might be superior to those deduced from measurements of interaction cross sections (σ_I). They provided results for ^{18,20,22}O, with uncertainties of 0.1 fm. They stated, "we also conclude that uncertainties deduced from σ_I are underestimated." The two sets of values for these three nuclei are compared in Table II. It can be noted that the (p,p) radii are systematically larger than those from σ_I.

The simple model I am using requires as input the matter radius of one member of an isotopic chain. The radii of all the other nuclei then follow from the model. In Fig. 2, I have plotted the earlier comparison between our calculations and radii from σ_I, along with new calculations for radii extracted from (p,p). Both sets of computations were normalized at ^{18}O. The previous excellent agreement is easily seen. The general trend of the new (p,p) radii is a slightly faster increase with A than is present in the calculations. But, they all agree within the

TABLE II. Comparison of matter radii (fm) for ^{18,20,22}O extracted from interaction cross sections [1] and (p,p) elastic scattering [19].

A	18	20	22
R_m (σ_I)	2.61 (8)	2.69(3)	2.88(6) or 2.75(7)[a]
R_m (p,p)	2.77 (10)	2.9 (1)	3.0 (1)

[a]Reference [2].

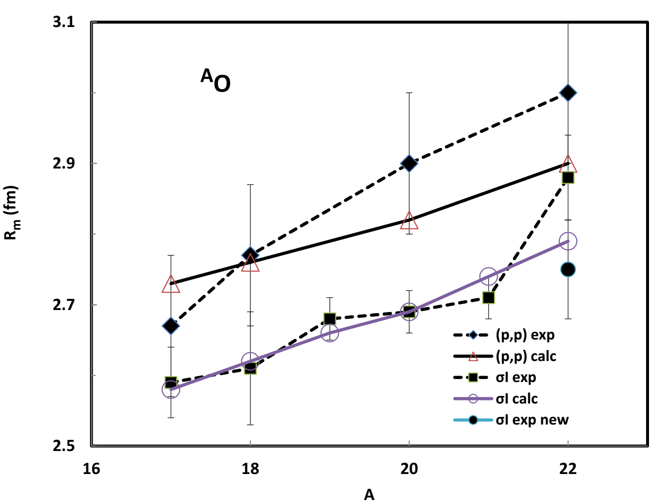

FIG. 2. Matter radii for $^{17-22}$O. Dashed lines connect values derived from experiment: diamonds from (p,p) elastic [19], squares from interaction cross sections (σ_I) [1]. Closed circle is a more recent experimental σ_I value [2]. Solid lines connect present computed values (normalized at ^{18}O); open triangles for results from (p,p), open circles from σ_I.

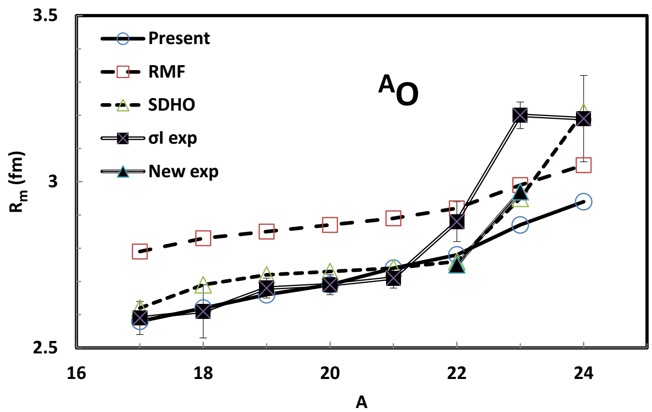

FIG. 3. Matter radii for $^{17-24}$O: Theoretical results from RMF [20], open squares connected by long-dash line; SDHO [20], open triangles connected by short-dash line; simple-model calculations ([9] and present), open circles connected by solid line; early experimental results, closed squares connected by double line; and more recent experimental results for ^{22,23}O, closed triangles connected by solid thin line.

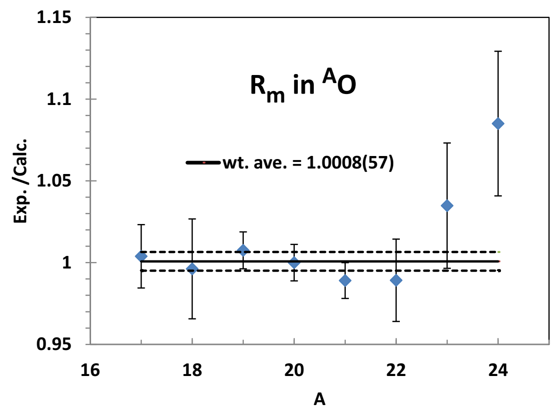

FIG. 4. The ratios of experimental matter radii to the present calculated ones are plotted vs A.

uncertainties. It would be interesting to pursue this idea further, in order to investigate whether the (p,p) method contains physics not present in the procedure that uses interaction cross sections.

Recently, Ahmad *et al.* [20] obtained densities of $^{16-26}$O using two separate approaches: the relativistic mean-field (RMF) method, and with Slater determinants consisting of harmonic oscillator single-particle wave functions (SDHOs). The RMF procedure appears to be a first-principles approach, whereas the SDHO technique contains two variable parameters for each nucleus (one for protons, one for neutrons) that can be adjusted in order to reproduce matter radii extracted from interaction cross sections. Their results are compared with present calculations and with experimental matter radii in Fig. 3. The RMF calculations are larger than the experimental values for most nuclei. For the SDHO approach, it would appear that the authors chose to fit the new radii for ^{22,23}O [2], but the earlier one (the only existing value) for ^{24}O [1].

A different form of comparison of experimental and calculated matter radii is presented in Fig. 4, where I have plotted the exp./calc. ratios vs A. The weighted average of these ratios is remarkably close to unity. This comparison emphasizes the point that ^{24}O should be remeasured.

III. SUMMARY

The recent appearance of several new theoretical papers concerning matter radii of neutron-rich oxygen nuclei has encouraged me to return to this problem. Contrary to a recent claim, I maintain that there is no reason to adjust the ^{22}O core in the ^{24}O nucleus. Furthermore, the case of ^{24}O should be revisited experimentally. For the entire range of $^{17-24}$O, new results provide no better agreement with experimental values than did previous calculations with a simple model.

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