

Light clusters in warm stellar matter: Explicit mass shifts and universal cluster-meson couplingsHelena Pais,¹ Francesca Gulminelli,² Constança Providência,¹ and Gerd Röpke^{3,4}¹*CFisUC, Department of Physics, University of Coimbra, 3004-516 Coimbra, Portugal*²*LPC (CNRS/ENSICAEN/Université de Caen Normandie), UMR6534, 14050 Caen Cédex, France*³*Institut für Physik, Universität Rostock, D-18051 Rostock, Germany*⁴*National Research Nuclear University (MEPhI), 115409 Moscow, Russia*

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In-medium modifications of light cluster properties in warm stellar matter are studied within the relativistic mean-field approximation. In-medium effects are included by introducing an explicit binding energy shift analytically calculated in the Thomas-Fermi approximation, supplemented with a phenomenological modification of the cluster couplings to the σ meson. A linear dependence on the σ meson is assumed for the cluster mass, and the associated coupling constant is fixed by imposing that the virial limit at low density is recovered. The resulting cluster abundances come out to be in reasonable agreement with constraints at higher density coming from heavy-ion collision data. Some comparisons with microscopic calculations are also shown.

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Neutron stars are compact objects where a wide range of densities, pressures, and temperatures can be achieved. In the outer part of the star, where the baryonic density is below the central density of atomic nuclei, matter is inhomogeneous and clustered into nuclei. If matter is catalyzed as in equilibrium neutron stars, the crust is a Wigner solid, and the nuclear components are sufficiently heavy to be treatable within the density functional theory [1]. However, above the crystallization temperature, the crust melts and light clusters with only a few nucleons contribute to the equilibrium [2]. At sufficiently low density and high temperature, corresponding to a large fraction of the post-bounce supernova dynamics, they constitute the main baryonic contribution [3], and we can expect that these light particles will play an important role in neutron star cooling, accreting systems, and binary mergers.

The light clusters will eventually melt when high enough temperatures are achieved, but they can appear in either warm neutron stars where $T \lesssim 2$ MeV, core-collapse supernova environments where $T \lesssim 20$ MeV, or binary star mergers with $T \lesssim 10$ MeV, as these environments have the perfect conditions for their formation. Moreover, all these clusters, light and heavy, may have a non-negligible effect in the core-collapse supernova mechanism [3], as they affect the neutrino mean free path and, consequently, the cooling of the star.

Light clusters in nuclear matter have been included within different approaches: in the single nucleus approximation (SNA)—such as in the Lattimer and Swesty (LS) [4] equation of state (EoS) based on the compressible liquid droplet model or the Shen [5] EoS using a relativistic mean-field (RMF) model—nonhomogeneous matter is described in the Wigner-Seitz approximation, with a single nucleus in equilibrium with a gas of neutrons, protons, electrons, and α clusters. The limit of these approaches is that they only consider α particles, while many other nuclear species are expected to contribute to the equilibrium.

The nuclear statistical equilibrium (NSE) models [6,7] go beyond SNA, because they consider all possible nuclear species in statistical equilibrium. However, in the original formulation of the approach [8], the system is considered as an ideal gas of clusters, and nuclear interactions of the clusters among themselves, as well as with the surrounding gas of free nucleons, are neglected. As a result, the expected cluster melting at high density is not observed [2], showing that in-medium effects must be introduced. In the Hempel and Schaffner-Bielich [7] model, such effects are included within a geometric excluded-volume mechanism. A more complex isospin dependence is proposed in the Raduta and Gulminelli model [9], where an excluded volume-like correction is derived as a mass shift from the extended Thomas-Fermi energy density functional [10]. Still, all these approaches are semiclassical and do not account for temperature effects, which might explain why they only qualitatively agree with more microscopic treatments [11]. In particular, the excluded volume mechanism appears to provide a realistic treatment only at high densities close to the cluster dissolution density [12]. A better way to describe light clusters is the quantum statistical (QS) approach [13] that can describe quantum correlations with the medium, and takes into account the excited states and temperature effect. However, the mass shifts calculated within the QS approach are available only for a few nuclear species and a limited density domain, therefore they can be implemented in a complete NSE description of stellar matter only within some approximations [14].

A different approach was developed within the relativistic mean-field framework and uses mean-field concepts, such as the ones used in recent works [2,15–17], where light clusters are considered as new degrees of freedom. As such, they are characterized by a density- and possibly temperature-dependent effective mass, and they interact with the medium via meson couplings. In-medium effects can thus be incorporated via the meson couplings, the effective mass shift, or

both. In Ref. [2] the description of light clusters was achieved with a modification of the effective mass, which introduces a density and temperature dependence of the binding energy of the clusters. These quantities have been fitted to quantum statistical outputs for light clusters in warm matter. The meson-cluster couplings were taken to be proportional to the atomic number of the cluster, taking as basis the meson-nucleon couplings. However, it is not mandatory that the nucleons within the cluster feel the same mean field as free nucleons. In Ref. [17], the main idea was to obtain adequate phenomenological parametrizations for the clusters-mesons couplings. In particular, the authors looked for parametrizations that better describe both experimentally obtained chemical equilibrium constants for the formation of light clusters in heavy-ion collisions [18] and microscopic results obtained from quantum statistical calculations. It was one goal of the work [17] to discuss the combination of light cluster approaches with pasta structure concepts, which are important if going to high densities. As pointed out there, the coupling to the isoscalar-vector field was renormalized by a global parameter η to keep the parameter space restricted, but one should consider different couplings for the different clusters in future work to optimize the description of measured data, such as chemical equilibrium constants. In particular, whereas the α particles are well described by a suitable fit of η , the chemical equilibrium constants of the other light elements are not well reproduced.

To proceed toward a satisfactory description of light cluster degrees of freedom within the RMF framework, in this article we explore the possibility of both in-medium mass shifts and in-medium modification of the cluster couplings. We aim to obtain a universal though phenomenological set for the cluster-meson couplings, with the purpose of having a formalism where different cluster species of arbitrary mass and charge can be described. The inclusion of heavier clusters, i.e., pasta phases, will be left for a future work.

At very low densities, a model-independent constraint can be considered: this is the one set by the virial EoS (VEoS) [19,20], which only depends on the experimentally determined binding energies and scattering phase shifts, and provides the correct zero density limit for the equation of state at finite temperature. We therefore fix the cluster-meson couplings so that, at very low densities, the VEoS particle fractions obtained in Ref. [20] are well reproduced. The deuteron, which is weakly bound, needs special treatment and will be considered later on. We know that the VEoS breaks down when the interactions between particles become stronger as the density increases. In this regime, we use the fact that the cluster dissolution mechanism is reasonably well described by the geometrical excluded volume mechanism [11,12], and employ the Thomas-Fermi formulation of Ref. [9] in order to evaluate the associated cluster mass shift.

The final result is a simple analytical formula for the effective mass shift. To reproduce empirical data, an in-medium modified coupling of cluster j with the scalar meson σ of the form $g_{sj} = x_s A_j g_s$ is proposed, where g_s is the coupling constant with the nucleons (n, p), A_j the cluster mass number, and x_s is a universal cluster coupling fraction, with an associated uncertainty.

II. FORMALISM

In this section, we present the model used in the rest of the paper and discuss how light clusters, which are considered as point-like particles, are included within our approach.

A. Lagrangian

Our system includes light clusters: both bosons [deuterons (d , ${}^2\text{H}$) and α particles (${}^4\text{He}$)] and fermions [tritons (t , ${}^3\text{H}$) and helions (h , ${}^3\text{He}$)]. They are immersed in a gas of neutrons (n) and protons (p), neutralized by electrons. The Lagrangian density of our system reads [2,15–17]

$$\mathcal{L} = \sum_{j=n,p,d,t,h,\alpha} \mathcal{L}_j + \mathcal{L}_\sigma + \mathcal{L}_\omega + \mathcal{L}_\rho + \mathcal{L}_{\omega\rho}. \quad (1)$$

In the following, the couplings of the clusters to the mesons are defined in terms of the couplings g_s, g_v, g_ρ of the nucleons to, respectively, the σ, ω , and ρ mesons. We will take for the vacuum proton and neutron mass an average value, $m = 939$ MeV. For the fermionic clusters, $j = t, h$, we have

$$\mathcal{L}_j = \bar{\psi} [\gamma_\mu i D_j^\mu - M_j^*] \psi, \quad (2)$$

with

$$i D_j^\mu = i \partial^\mu - g_{vj} \omega^\mu - \frac{g_\rho}{2} \boldsymbol{\tau}_j \cdot \mathbf{b}^\mu, \quad (3)$$

where $\boldsymbol{\tau}_j$ is the isospin operator and g_{vj} is the coupling of cluster j to the vector meson ω and, in the present work, it is defined as $g_{vj} = A_j g_v$ for all clusters. The effective mass M_j^* will be defined in the next section.

The Lagrangian density for the bosonic clusters, $j = d, \alpha$, is given by

$$\mathcal{L}_\alpha = \frac{1}{2} (i D_\alpha^\mu \phi_\alpha)^* (i D_{\mu\alpha} \phi_\alpha) - \frac{1}{2} \phi_\alpha^* (M_\alpha^*)^2 \phi_\alpha, \quad (4)$$

$$\mathcal{L}_d = \frac{1}{4} (i D_d^\mu \phi_d^v - i D_d^v \phi_d^\mu)^* (i D_{d\mu} \phi_{dv} - i D_{dv} \phi_{d\mu}) - \frac{1}{2} \phi_d^{\mu*} (M_d^*)^2 \phi_{d\mu}, \quad (5)$$

with

$$i D_j^\mu = i \partial^\mu - g_{vj} \omega^\mu \quad (6)$$

For the nucleonic gas, $j = n, p$, we have

$$\mathcal{L}_j = \bar{\psi} [\gamma_\mu i D^\mu - m^*] \psi \quad (7)$$

with

$$i D^\mu = i \partial^\mu - g_v \omega^\mu - \frac{g_\rho}{2} \boldsymbol{\tau}_j \cdot \mathbf{b}^\mu, \quad (8)$$

$$m^* = m - g_s \phi_0. \quad (9)$$

For the fields, we have the standard RMF expressions:

$$\begin{aligned} \mathcal{L}_\sigma &= +\frac{1}{2} (\partial_\mu \phi \partial^\mu \phi - m_s^2 \phi^2 - \frac{1}{3} \kappa \phi^3 - \frac{1}{12} \lambda \phi^4), \\ \mathcal{L}_\omega &= -\frac{1}{4} \Omega_{\mu\nu} \Omega^{\mu\nu} + \frac{1}{2} m_v^2 V_\mu V^\mu, \\ \mathcal{L}_\rho &= -\frac{1}{4} \mathbf{B}_{\mu\nu} \cdot \mathbf{B}^{\mu\nu} + \frac{1}{2} m_\rho^2 \mathbf{b}_\mu \cdot \mathbf{b}^\mu, \\ \mathcal{L}_{\omega\rho} &= g_{\omega\rho} g_\rho^2 g_v^2 V_\mu V^\mu \mathbf{b}_\nu \cdot \mathbf{b}^\nu, \end{aligned} \quad (10)$$

where $\Omega_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$ and $\mathbf{B}_{\mu\nu} = \partial_\mu \mathbf{b}_\nu - \partial_\nu \mathbf{b}_\mu - g_\rho(\mathbf{b}_\mu \times \mathbf{b}_\nu)$.

B. Mass shift in the clusters

The total binding energy of a light cluster j is given by

$$B_j = A_j m^* - M_j^*, \quad j = d, t, h, \alpha, \quad (11)$$

with M_j^* the effective mass of cluster j , which is determined by the meson coupling as well as by a binding energy shift:

$$M_j^* = A_j m - g_{sj} \phi_0 - (B_j^0 + \delta B_j). \quad (12)$$

Within the RMF approach, the nucleons are considered as independent moving particles, neglecting any correlations. The account of correlations via the introduction of bound states (clusters) will modify the coupling to the mesonic fields within the effective Lagrangian as denoted by the coupling constants g_{sj} and g_{vj} . There is no reason to consider them as the sum of the couplings of the individual constituents of the cluster, but they have to be introduced as new empirical parameters which are fitted to results from microscopic theories or to measured data. We discuss the choice of the coupling constants g_{sj} in Sec. III; see also Eqs. (21) and (22) below. In expression (12), B_j^0 is the binding energy of the cluster in the vacuum, and these constants are fixed to experimental values. Following the formalism of Ref. [9,10], we write for the binding energy shift δB_j

$$\delta B_j = \frac{Z_j}{\rho_0} (\epsilon_p^* - m\rho_p^*) + \frac{N_j}{\rho_0} (\epsilon_n^* - m\rho_n^*), \quad (13)$$

which is the energetic counterpart of the excluded volume mechanism in the Thomas-Fermi approximation. Here, ρ_0 is the nuclear saturation density. The energy states already occupied by the gas are excluded in the calculation of the cluster binding energy, thus avoiding double counting of the particles of the gas and the ones of the clusters. The energy density ϵ_j^* and the density ρ_j^* are given by

$$\epsilon_j^* = \frac{1}{\pi^2} \int_0^{p_{F_j}(\text{gas})} p^2 e_j(p) [f_{j+}(p) + f_{j-}(p)] dp, \quad (14)$$

$$\rho_j^* = \frac{1}{\pi^2} \int_0^{p_{F_j}(\text{gas})} p^2 [f_{j+}(p) + f_{j-}(p)] dp \quad (15)$$

for $j = p, n$, and correspond to the gas energy density and the gas nucleonic density associated with the gas lowest energy levels. In the preceding expressions, $f_{j\pm}$ are the usual Fermi distribution functions for the nucleons and respective antiparticles, p_{F_j} is the Fermi momentum of nucleon j , given by $p_{F_j} = (3\pi^2 \rho_j)^{1/3}$, and $e_j(p) = \sqrt{p_j^2 + m^{*2}}$ is the corresponding single-particle energy of the nucleon j .

We treat the binding energy shifts δB_j as in Ref. [2]: we replace the density dependence of these quantities by a vector meson dependence. This is equivalent, in our present study, to consider in the shifts δB_j the neutron and proton densities replaced by

$$\rho_n = \frac{m_v^2}{2g_v} V_0 - \frac{m_\rho^2}{2g_\rho} b_0, \quad \rho_p = \frac{m_v^2}{2g_v} V_0 + \frac{m_\rho^2}{2g_\rho} b_0.$$

With the inclusion of this extra term, the equations for the fields read

$$\begin{aligned} m_{\rho,\text{eff}}^2 b_0 &= \frac{g_\rho}{2} (\rho_p - \rho_n + \rho_h - \rho_t) \\ &\quad - \frac{m_\rho^2}{g_\rho \rho_0} \left(-\frac{\partial \epsilon^*}{\partial \rho_n} + \frac{\partial \epsilon^*}{\partial \rho_p} + \frac{m \partial \rho^*}{\partial \rho_n} - \frac{m \partial \rho^*}{\partial \rho_p} \right) \\ &\quad \times \sum_j A_j \rho_s^j, \end{aligned} \quad (16)$$

$$\begin{aligned} m_{v,\text{eff}}^2 V_0 &= g_v (\rho_p + \rho_n) + \sum_j g_{vj} \rho_j \\ &\quad - \frac{m_v^2}{2g_v^2 \rho_0} \left(-\frac{\partial \epsilon^*}{\partial \rho_n} - \frac{\partial \epsilon^*}{\partial \rho_p} + \frac{m \partial \rho^*}{\partial \rho_n} + \frac{m \partial \rho^*}{\partial \rho_p} \right) \\ &\quad \times \sum_j A_j \rho_s^j, \end{aligned} \quad (17)$$

$$\begin{aligned} m_s^2 \phi_0 &+ \frac{k}{2} \phi_0^2 + \frac{\lambda}{6} \phi_0^3 \\ &= g_s (\rho_s^p + \rho_s^n) + \sum_j g_{sj} \rho_s^j, \end{aligned} \quad (18)$$

with $\epsilon^* = \epsilon_p^* + \epsilon_n^*$, $\rho^* = \rho_p^* + \rho_n^*$, and

$$m_{\rho,\text{eff}}^2 = m_\rho^2 + 2g_{\omega\rho} g_\rho^2 g_v^2 V_0^2, \quad (19)$$

$$m_{v,\text{eff}}^2 = m_v^2 + 2g_{\omega\rho} g_\rho^2 g_v^2 b_0^2 + \frac{1}{6} \xi g_v^4 V_0^2. \quad (20)$$

For a given baryonic density, proton fraction and temperature, Eqs. (16)–(18) have to be solved self-consistently.

III. RESULTS

In the following, we look for a possible universal parametrization for all clusters which only accounts for the differences through the atomic number and isospin projection. In the last section, we test the proposed parametrizations by comparing the predicted chemical equilibrium constants with the recent experimental results published in Ref. [18]. All the calculations are performed for the FSU [21] model, at finite fixed temperatures and for fixed proton fractions y_p which describe the ratio of the total proton density to the baryon density. For this model, the values of the nucleon coupling constants are $g_s^2 = 112.1996$, $g_v^2 = 204.5469$, and $g_\rho^2 = 138.4701$ and the nuclear saturation density is $\rho_0 = 0.148 \text{ fm}^{-3}$. Further constants (the meson masses and the couplings of the nonlinear meson terms) are found in Ref. [21]. This model has been chosen because it describes adequately the properties of nuclear matter at saturation and subsaturation densities. It has the drawback of not predicting a two-solar-mass neutron star (NS). However, it is possible to include excluded volume-like effects above saturation density, making the EoS hard enough at high density [22,23]. We have tested the formalism with two other models that have good properties at saturation density and below, and, additionally, describe two-solar-mass NSs: the NL3 $\omega\rho$ [24] and the TM1 $\omega\rho$ models [25,26] with the symmetry energy slope $L \sim 55 \text{ MeV}$. The results obtained were within the uncertainty bands of our approach and, therefore, we do not include them in the present

TABLE I. Virial cluster fraction X_j for the light clusters triton, helion, and α , at different densities ρ , for $T = 4$ and 10 MeV used in the present work and taken from Ref. [20]. The densities are in units of 10^{-6} fm^{-3} .

ρ	1.1	5.3	12.0	52.5	91.2
	X_j				
cluster	$T = 4 \text{ MeV}$				
$t(^3\text{H})$	1.3×10^{-6}	3.0×10^{-5}	1.5×10^{-4}	2.6×10^{-3}	6.8×10^{-3}
$h(^3\text{He})$	1.1×10^{-6}	2.5×10^{-5}	1.3×10^{-4}	2.1×10^{-3}	5.7×10^{-3}
$\alpha(^4\text{He})$	2.7×10^{-8}	2.9×10^{-6}	3.2×10^{-5}	2.4×10^{-3}	1.1×10^{-1}
cluster	$T = 10 \text{ MeV}$				
$t(^3\text{H})$	2.3×10^{-8}	5.2×10^{-7}	2.7×10^{-6}	5.1×10^{-5}	1.5×10^{-4}
$h(^3\text{He})$	2.1×10^{-8}	4.8×10^{-7}	2.5×10^{-6}	24.7×10^{-5}	1.4×10^{-4}
$\alpha(^4\text{He})$	6.0×10^{-12}	6.7×10^{-10}	7.9×10^{-9}	6.4×10^{-8}	2.3×10^{-6}

study. A more complete thermodynamical study will be left for a future work.

A. Low-density limit and cluster-meson couplings

We will first take as reference the virial EoS (VEoS) [20]. There, the account of continuum correlations (scattering phase shifts), which is necessary to obtain the correct second virial coefficient, was performed by introducing a temperature-dependent effective resonance energy $E_{ij}(T)$ in each ij channel. The cluster-meson couplings are obtained from the best fit of the RMF cluster fractions, defined as $X_j = A_j n_j / n$, to these data, using the FSU parametrization. The fit is done with the choice of a sufficiently low density, close to the cluster onset, where the virial EoS is still valid and at the same time the interaction already has non-negligible effects; see Table I. We have considered densities between 10^{-6} and 10^{-4} fm^{-3} , although, for small temperatures, 10^{-4} fm^{-3} is close to the limit of validity of the VEoS. Still, we expect that at these densities the VEoS is a good approximation. In this low-density

domain, the binding energy shift δB_j of Eq. (13) is completely negligible and does not affect the particle fractions (see also Fig. 2 below), therefore it was put to zero for this calculation. However, already at $5 \times 10^{-6} \text{ fm}^{-3}$ the cluster fractions are sensitive to the meson couplings.

In principle, both scalar and vector couplings could be considered for this fit. However, for the present constraints, it was shown in Ref. [15] that the $\{g_{sj}, g_{vj}\}$ parameter space is somewhat redundant and very similar results can be obtained either by modifying the scalar coupling (i.e., decreasing the nuclear attraction) or the vector one (i.e., increasing the nuclear repulsion). In contrast to Ref. [15] where g_v was scaled, in this work we only optimize the g_{sj} parameters,

$$g_{sj} = x_{sj} A_j g_s, \quad (21)$$

while the vector couplings are set to

$$g_{vj} = A_j g_v. \quad (22)$$

We have performed calculations for $T = 4$ and 10 MeV, keeping the proton fraction at 0.5 .

It is clear that we are not able to reproduce the deuteron fractions predicted by the VEoS. This is somewhat expected, due to the specificity of the deuteron. Indeed such a loosely bound structure which is known to correspond to a highly delocalized wave function can be hardly described in a mean-field approximation. As detailed in Ref. [13], if the binding energy per nucleon is small compared with T , the contributions of the continuum as given by the scattering phase shifts are essential. For the other clusters our coupling parametrizations are reasonable within the range of temperatures between 4 and 10 MeV.

Reasonable values for g_{sj} are $(0.85 \pm 0.05) A_j g_s$; see Fig. 1, where the colored bands show the range of particle fractions covered by this interval at low densities, for $T = 4$ and 10 MeV. The solid vertical black lines represent the upper limit of the region of validity of the VEoS. This region can be estimated by imposing $\rho_j \lambda_j^3 \ll 1$, where $\lambda_j = \sqrt{2\pi/(m_j T)}$ is the thermal

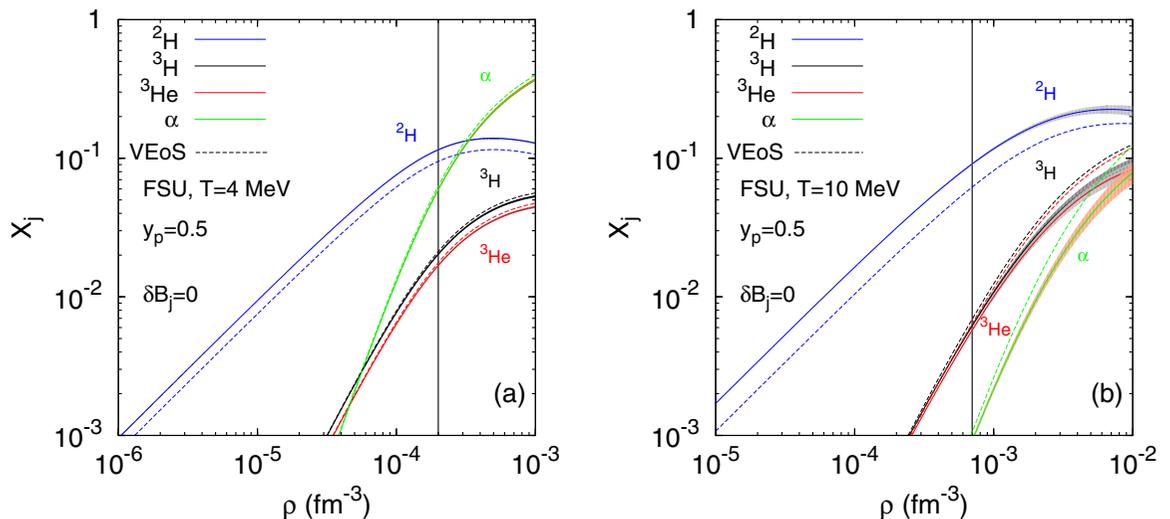


FIG. 1. Fraction of deuteron, X_d , triton, X_t , helion, X_h , and α , X_α , as a function of the density for FSU, $T = 4$ MeV (a) and 10 MeV (b), with proton fraction $y_p = 0.5$, taking $\delta B = 0$, $x_{sj} = 0.85 \pm 0.05$ (variation indicated by the spreading of the bands), and comparing with results of the virial EoS from [20]. Solid vertical black lines are given by $\rho \lambda_n^3 = 1/10$. For more details, see the text.

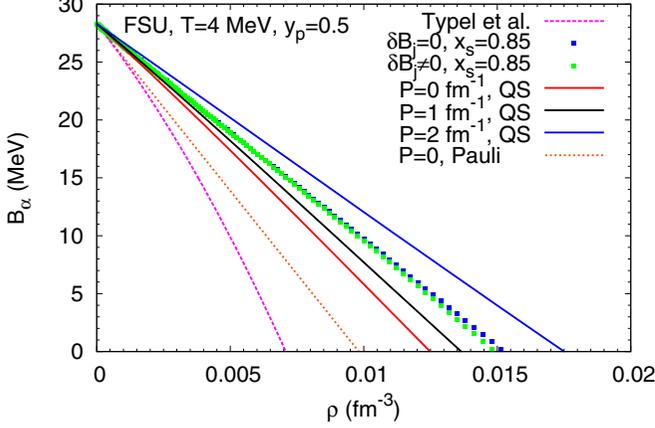


FIG. 2. Binding energy of α for the RMF-FSU calculation (this work), $T = 4$ MeV, and $y_p = 0.5$ obtained with Eq. (12). For comparison, results neglecting the binding energy shift (13) ($\delta B_j = 0$), as well as QS calculations of a perturbative approach [27] (Pauli), Eq. (24), the empirical form Eq. (25) from Typel *et al.* [2], and results obtained from a recent QS approach [13] for different center-of-mass momenta P are also shown.

wavelength of particle j , ρ_j its density, and m_j its mass [20]. The vertical line was defined by $\rho\lambda_n^3 = 1/10$, ρ being the baryon density and λ_n the nucleon thermal wavelength. In Table II, we compare explicitly the RMF abundances of clusters obtained under these conditions with the ones coming from the VEOs. Different values for the clusters' σ -meson couplings g_{sj} , Eq. (21), were considered for five different values for the density. In particular, we have taken for the fraction x_{sj} three values which best describe the VEOs. For the lowest density in Table II, the cluster fractions are almost independent of x_{sj} because for this low density the clusters behave like free particles. At $T = 10$ ($T = 4$) MeV the largest deviations obtained are below 2% (5%) of relative difference for the largest density considered, the largest deviations occurring for the α clusters. Choosing the best couplings, these relative differences can be reduced to $\sim 1\%$ ($\sim 2\%$).

Larger values of x_{sj} were considered but we found the problem already discussed in Ref. [15]: taking $g_{vj} = A_j g_v$, the light clusters will not dissolve if $x_{sj} \geq 1$. We have confirmed that even including the contribution δB_j , the clusters would not dissolve with this value of x_{sj} .

Next we will discuss the effect of introducing a nonzero binding energy shift δB_j , Eq. (13). In Fig. 2, we compare the binding energy of the α clusters obtained taking δB_α defined by Eq. (13) with the binding energy

$$B_j = B_j^0 + \delta B_j^{\text{QS}} \quad (23)$$

obtained from QS calculations. In particular, a perturbation theory was given in Ref. [27], where the following result for the Pauli blocking shift of α particles with center-of-mass momentum (wave number) $P = 0$ was obtained in the lowest order of density ρ :

$$\delta B_\alpha^{\text{Pauli}}(P = 0; \rho_n, \rho_p, T) = -\frac{164371 \rho}{(T + 10.67)^{3/2}} \quad (24)$$

TABLE II. Relative difference in percentage of the cluster fractions between the VEOs and the RMF EoS [$\Delta_{\text{rel}} = 100 \times (X_j^{\text{RMF}} - X_j^{\text{VEoS}})/X_j^{\text{VEoS}}$] for different couplings $g_{sj} = x_{sj} A_j g_s$, and $g_{vj} = A_j g_v$, and baryon number densities ρ for the light clusters triton (t), helion (h), and α , with $T = 4$ and 10 MeV. The densities are in units of 10^{-6} fm^{-3} .

ρ	1.1	5.3	12.0	52.5	91.2
	$\Delta_{\text{rel}}(\%)$				
$T = 4$ MeV					
triton (t)					
0.80	0.24	0.01	-0.37	-2.33	-3.76
0.85	0.26	0.07	-0.22	-1.74	-2.85
0.90	0.27	0.14	-0.06	-1.13	-1.92
helion (h)					
0.80	0.11	-0.13	-0.36	-2.48	-3.91
0.85	0.12	-0.06	-0.36	-1.88	-2.99
0.90	0.14	0.01	-0.20	-1.28	-2.06
α					
0.80	0.07	-0.24	-0.74	-3.35	-5.22
0.85	0.09	-0.15	-0.53	-2.55	-4.02
0.90	0.11	-0.06	-0.33	-1.76	-2.78
$T = 10$ MeV					
triton (t)					
0.80	0.62	0.57	0.39	-0.67	-1.64
0.85	0.62	0.59	0.45	-0.40	-1.18
0.90	0.63	0.62	0.51	-0.13	-0.73
helion (h)					
0.80	0.48	0.43	0.25	-0.81	-1.78
0.85	0.49	0.46	0.31	-0.54	-1.32
0.90	0.49	0.49	0.38	-0.27	-0.87
α					
0.80	0.34	0.27	0.03	-1.36	-2.64
0.85	0.34	0.31	0.12	-1.01	-2.03
0.90	0.35	0.35	0.20	-0.65	-1.43

(in units of MeV, with T in units of MeV and ρ in units of fm^{-3}). Typel *et al.* [2] performed RMF calculations for a wide density region. To suppress cluster formation at higher densities, an empirical quadratic form was introduced,

$$\delta B_\alpha^{\text{Typel}}(T) = \delta B_\alpha^{\text{Pauli}}(P = 0; \rho_n, \rho_p, T) \times \left[1 - \frac{\delta B_\alpha^{\text{Pauli}}(P = 0; \rho_n, \rho_p, T)}{2B_\alpha^0} \right]. \quad (25)$$

Both results are shown in Fig. 2, together with more recent calculations for $\delta B_j^{\text{QS}}(P; \rho_n, \rho_p, T)$ according to Ref. [13]. In contrast to the Pauli blocking, assuming an ideal Fermi distribution in the nuclear medium, correlations in the medium have been taken into account, and the distribution function of the nucleons in the medium is parametrized there by a Fermi distribution with effective chemical potentials and temperature. Also shown in Fig. 2 are QS calculations for different center-of-mass momenta $P = 0, 1, 2 \text{ fm}^{-1}$.

The QS calculations show two effects:

- (i) Due to the in-medium correlations the perturbation theory result for the Mott density, according to [27], is shifted to higher densities.
- (ii) The Pauli blocking is strongly dependent on the center-of-mass momentum P of the cluster. For an α cluster at temperatures and densities considered here, typical values of P for the four-nucleon contribution to the EoS are of the order 1 fm^{-1} .

We can also see that the shifts given by the empirical reduction of the coupling to the σ meson field proposed in this work to reproduce the VEOs, and the microscopically calculated shifts of the binding energies of the QS calculations, are of the same order. This is a prerequisite for the description of the composition and the chemical equilibrium constants, as discussed in the following sections.

The additional binding energy shift δB_j given by Eq. (13) is completely negligible in the domain of validity of the VEOs, which means that the cluster couplings extracted in Table II do not depend on this term. Even at higher density, this extra correction is small in the range of densities where the binding energies of the clusters are still positive, but rises fast for larger densities; see Fig. 3. It gives an important contribution to the self-consistent calculation of matter in thermodynamic equilibrium at higher densities, as will be shown in Figs. 4, 5, and 6.

It is also interesting to discuss the effect of the coupling x_{sj} and temperature T on the binding energy shift. From Figs. 3 we conclude that, for larger x_{sj} , $-\delta B_j$ increases more slowly, and also that a larger temperature determines a softer behavior, with $-\delta B_j$ taking larger values at the lower densities and smaller ones close to the dissolution density.

The change of slope for $\rho > 0.02 \text{ fm}^{-3}$ occurs at the maximum of the cluster fraction, corresponding to the onset of decreasing cluster fraction.

However, we should stress that the representations in Figs. 2 and 3 do not give a complete picture of the in-medium effects and cluster dissolution mechanism. As we can see from Eqs. (16)–(18), the mass shift deeply modifies the equations of motion for the meson fields. The particle fractions are thus affected in a highly complex way because of the self-consistency of the approach, which additionally induces temperature effects.

B. Global cluster distributions

We are interested in extending the calculation of thermodynamic properties from the low-density region where perturbation theory can be applied to the entire subsaturation region $\rho \leq \rho_0$. It is expected that the light clusters will be dissolved below ρ_0 , and the RMF approach is applicable there. Correlations which are always present in nuclear matter are included in this density-functional approach, and the fit to data at saturation density presumes that no further correlations are considered.

The description of the fractions of different components is difficult not only because of the problems with the many-body theory at high densities, but also the conceptual definitions

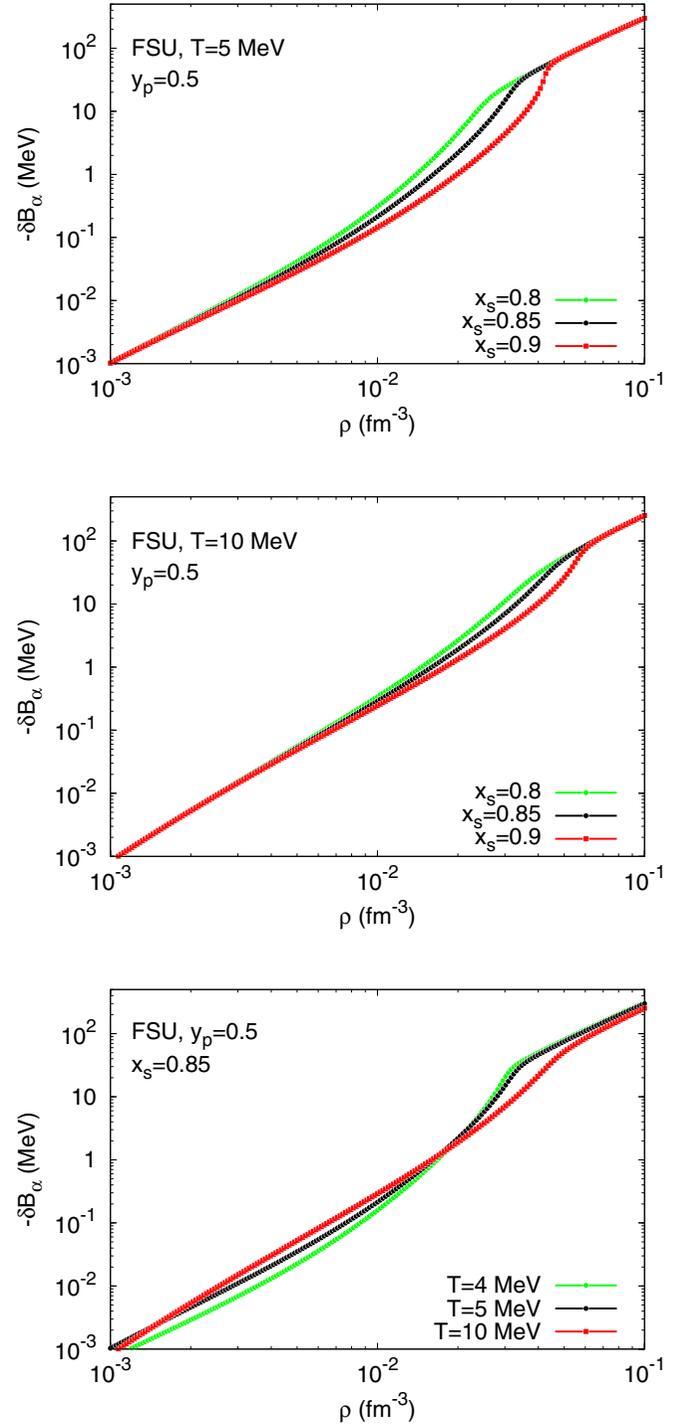


FIG. 3. Binding energy shift of α , δB_α , as given by Eq. (13) for the RMF-FSU calculation, $y_p = 0.5$, $T = 5 \text{ MeV}$ (top), $T = 10 \text{ MeV}$ (middle), for $x_s = 0.8, 0.85, 0.9$. The bottom panel shows the same shift for all the temperatures and keeping $x_s = 0.85$.

of bound states near the Mott density is problematic. The QS theory has been worked out for the two-particle case and extrapolated for the other light clusters; see Ref. [13]. A further problem is that at higher densities also other structures such as pasta structures are of relevance, so that one cannot discuss

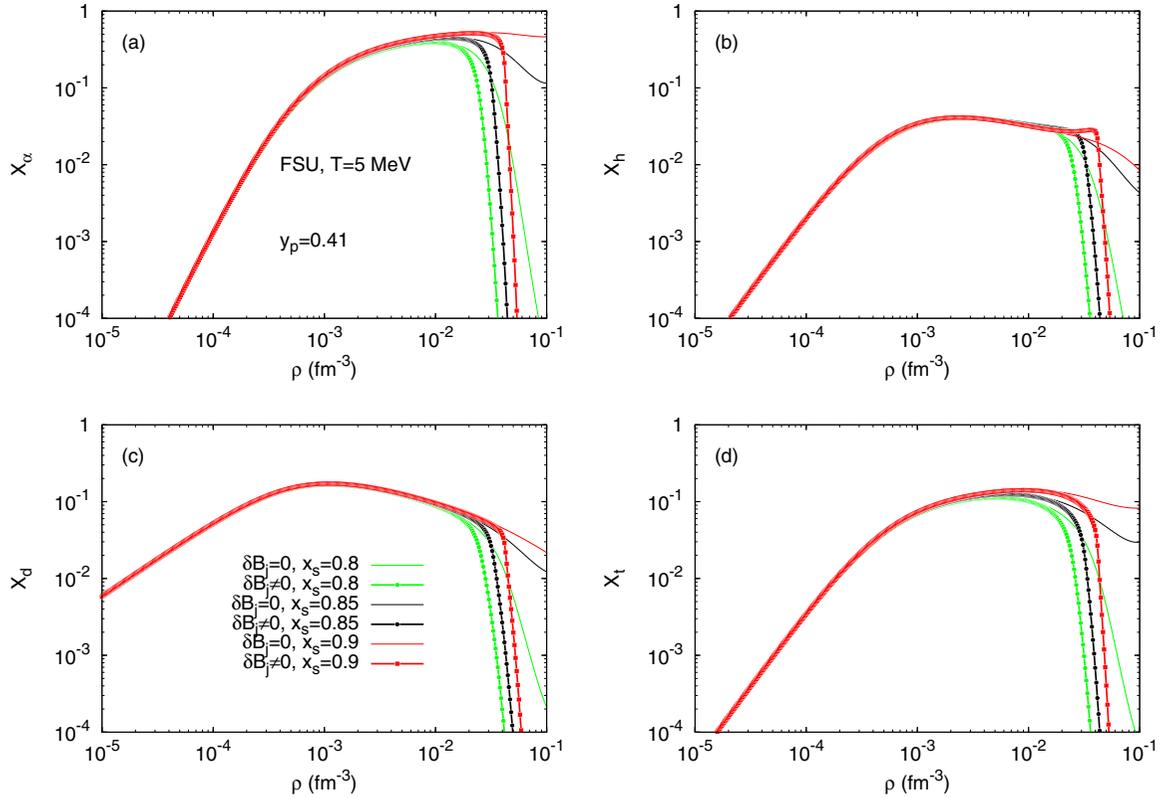


FIG. 4. Fraction of α , X_α (a), helion, X_h (b), deuteron, X_d (c), and triton, X_t (d), as a function of the density for FSU, $T = 5$ MeV, and $y_p = 0.41$, with and without δB_j , for $x_{sj} = 0.8, 0.85, 0.9$, keeping $g_{vj} = A_j g_v$.

the thermodynamics at higher densities without accounting for droplets and other structures.

We show in the present subsection that the clusters are dissolved below ρ_0 . This has been achieved, for instance, by Typel *et al.* [2], who introduced an empirical quadratic form (25). A more microscopic approach to the suppression of the cluster fraction was given in Ref. [13], where the dissolution of the bound states and the virial contributions for the partial partition functions are considered. In this work we show that the account of the binding energy shift δB_j , Eq. (13), gives similar results.

The global effect of the modified meson couplings and binding energy shifts is presented in Figs. 4, 5, and 6.

In Fig. 4, we show the cluster fractions for matter with a fixed proton fraction of $y_p = 0.41$ and $T = 5$ MeV, keeping $g_{vj} = A_j g_v$ and using different values for $g_{sj} = x_{sj} A_j g_s$: $x_{sj} = 0.8, 0.85, 0.9$. We extend the cluster fractions to larger densities in order to analyze the δB_j contribution. Neglecting this term, the clusters do not dissolve. Taking $x_{sj} = 0.9$, the clusters seem to dissolve but there is just a local reduction of clusters followed by a reappearance of similar fractions. The role of the extra term in the binding energy is precisely to dissolve the clusters at large densities, and the larger the value of x_{sj} , the larger the dissolution density. Typical values for the dissolution ($X_j < 10^{-4}$) of light clusters at conditions considered here are densities $0.04 < \rho < 0.06 \text{ fm}^{-3}$.

To compare with QS calculations [13], we show in Fig. 5 the mass fractions of light clusters for the parameter region $0.8 \leq$

$x_{sj} \leq 0.9$ including the binding energy shift δB_j , Eq. (13), represented by a band, together with the QS calculation taking into account the Pauli blocking term in the residual virial coefficient $v_i(\mathbf{P}; T, \rho, y_p)$ as a function of the center-of mass momentum \mathbf{P} ; see Sec. V D of Ref. [13]. Good agreement between the two approaches is seen up to density $\rho \approx 0.01 \text{ fm}^{-3}$.

The disappearance of the clusters at higher densities in the QS approach is not so sharp as expected from the assumption that, near the saturation density, nuclear matter is fully described by the RMF approach in an empirical way, similar to a density functional approach. In principle, one has to analyze which microscopic correlations are always contained in this effective mean-field approach. These correlations have to be removed from the contribution of light clusters to the thermodynamic properties. Whereas this problem has been solved in the low-density limit (see [13]), a rigorous solution analyzing continuum correlations near the saturation density is not in reach yet.

Two approaches to suppress the contribution of clusters at high densities are shown in Fig. 5: (i) As already discussed above, according to Typel *et al.* [2] a quadratic term is introduced in an empirical way to calculate the shift of the binding energy of clusters, Eq. (23). The result shown in Fig. 5 gives a suppression which is too strong compared with the other approaches; see also Fig. 2. (ii) A stronger suppression of clusters at increasing density is also obtained if the residual virial coefficient $v_i(0; T, \rho, y_p)$ is used, neglecting the \mathbf{P} dependence, as shown in [13]. This result for $\mathbf{P} = 0$

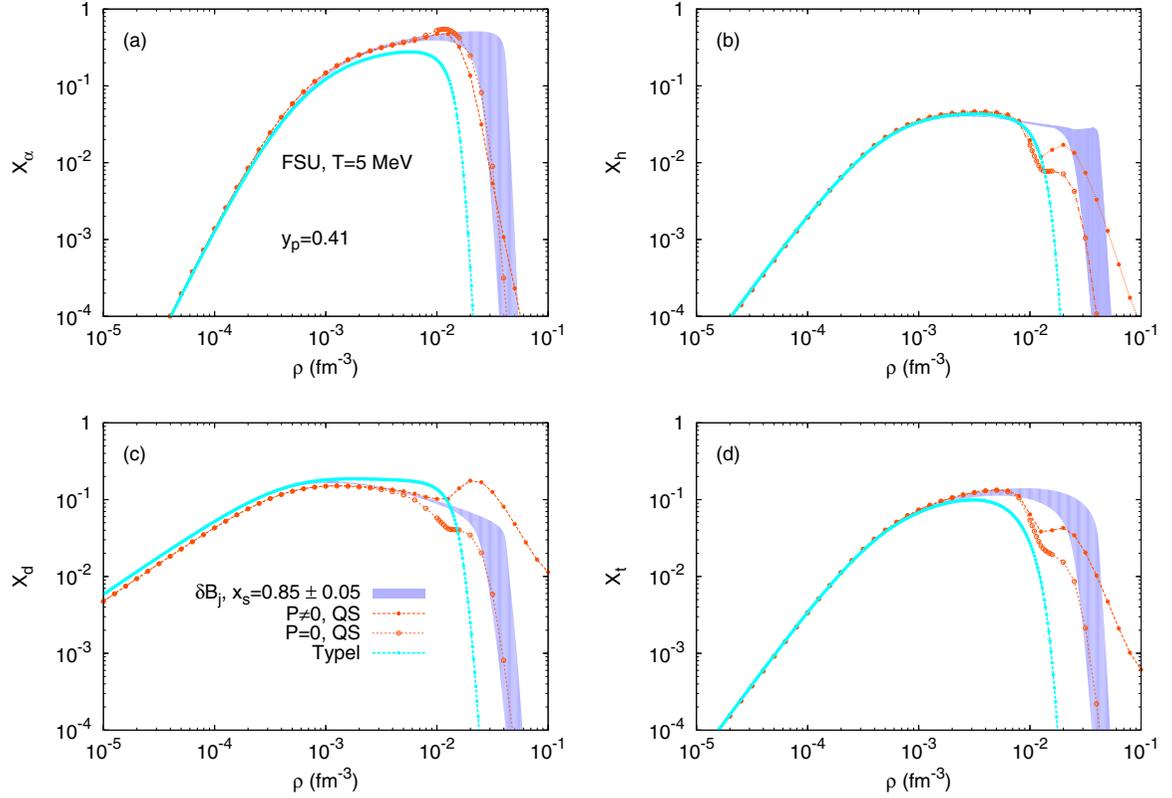


FIG. 5. Fraction of α , X_α (a), helium, X_h (b), deuteron, X_d (c), and triton, X_t (d), as a function of the density for FSU, $T = 5$ MeV, and $y_p = 0.41$, with δB_j , for $0.8 < x_s < 0.9$, keeping $g_{vj} = A_j g_v$, compared with the QS EoS from [13] with full \mathbf{P} dependence of the residual virial coefficient (red line with full dots) and neglecting the dependence ($\mathbf{P} = 0$) (red line with empty dots), and with the EoS given by Typel *et al.* in [2] (cyan full lines).

is also shown in Fig. 5. Good agreement with the RMF approach is obtained for the dissolution density, but stronger deviations (“bumps”) occur for densities $\rho > 0.01 \text{ fm}^{-3}$. This is a consequence of the fact that the composition is strongly interdependent: an overproduction of α particles is connected with an underproduction of other clusters.

In conclusion, the RMF approach considered here seems to be an appropriate description of the composition of nuclear matter also at high densities.

In Fig. 6, the same analysis is done at $T = 10$ MeV including the δB_j contribution. As in Fig. 5, our results are compared to the fractions obtained with the binding energy shift of Ref. [2] fitted on QS results, and the fully microscopic QS EoS of Ref. [13]. We can see that the different models agree very well at low density. The only sizable difference is a reduced deuteron fraction for the QS calculation, which is the only one to reproduce the VEoS for deuterons, as expected. At high density, the phenomenological models correctly obtain the cluster dissolution but the dissolution density is model dependent. This model dependence cannot be reduced using the QS microscopic results as a constraint, because these latter lack high order correlations at high density, where the different phenomenological prescriptions more strongly differ.

Concerning the temperature dependence of the cluster dissolution mechanism, we consider the fraction of each light cluster to be $X_j = 10^{-4}$ to get the dissolution density $\rho_{\text{diss}}(T)$. Results for the different clusters are shown in Table III. We can

quantify the temperature dependence of the cluster dissolution density by introducing a variable, also shown in Table III,

$$\Gamma_{\rho_{\text{diss}}} = \rho_{\text{diss}}(T = 10) / \rho_{\text{diss}}(T = 5), \quad (26)$$

defined by the ratio, for each particle species j , of its dissolution density at the higher temperature to the dissolution density at the lower temperature. We can see that the effect of temperature strongly depends on the chosen coupling, the biggest effect being obtained with the smallest value for the coupling x_s . The binding energy shift of Eq. (13) leads to $\Gamma_{\rho_{\text{diss}}}(j)$ of the order of 1.4 (1.6) for the α (deuteron) for $x_s = 0.85$; see Table III. Identical values are obtained choosing $x_s = 0.8$, while a slightly smaller temperature effect is seen with $x_s = 0.9$, $\Gamma_{\rho_{\text{diss}}}(j) \approx 1.4$ –1.5. Similar results are determined from the QS calculations with $\mathbf{P} = 0$ of Ref. [13]. We also compare with the dissolution density ρ_{diss} of Typel *et al.* [2], which shows a significantly larger temperature dependence of the clusters’ dissolution density expressed by $\Gamma_{\rho_{\text{diss}}}$.

Again, if the qualitative effect of a dissolution density increasing with increasing temperature is physically reasonable and well understood [2, 13], a quantitative determination is less obvious, and within the present constraints it is not easy to discriminate between the different predictions.

From the model dependence point of view, we have seen that choosing different coupling fractions for the different nuclear species, within the constraint of the VEoS at low density,

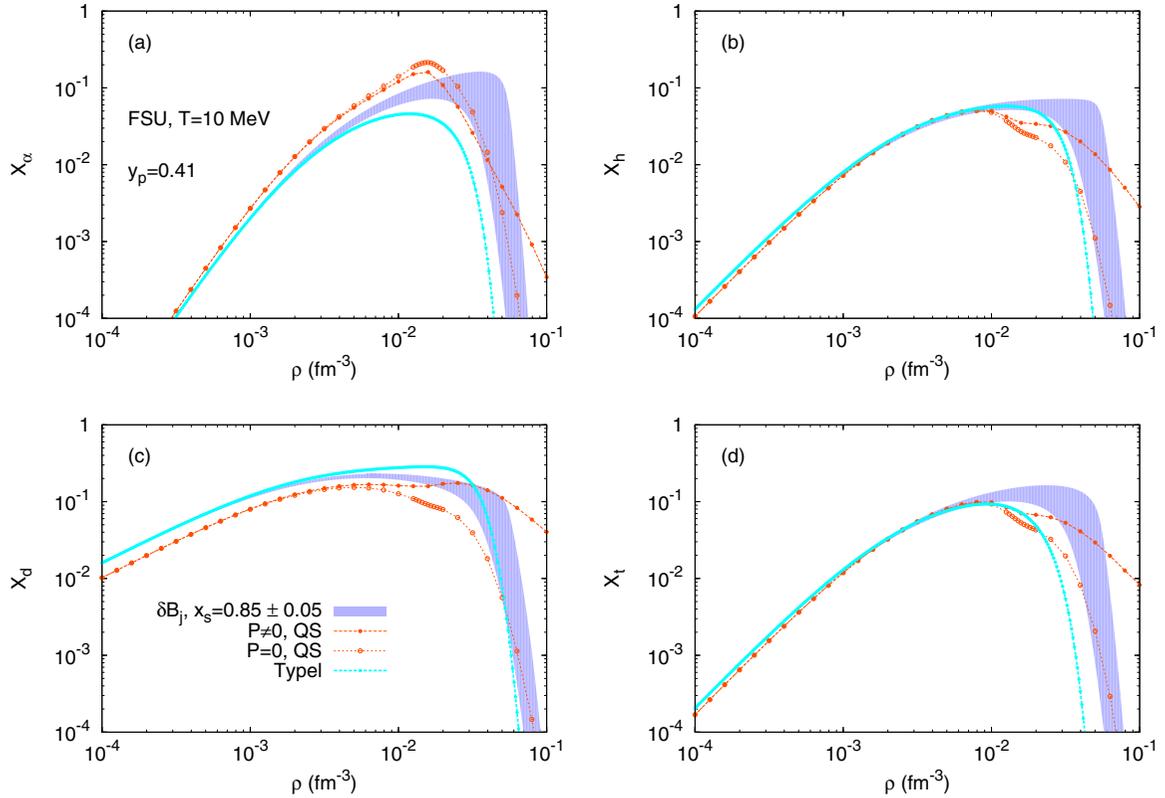


FIG. 6. Fraction of α , X_α (a), helium, X_h (b), deuteron, X_d (c), and triton, X_t (d), as a function of the density for FSU, $T = 10$ MeV, and $y_p = 0.41$, with δB_j , for $0.8 < x_{s,j} < 0.9$, keeping $g_{vj} = A_j g_v$, compared with the QS EoS from [13] with full \mathbf{P} dependence of the residual virial coefficient (red line with full dots) and neglecting the dependence ($\mathbf{P} = 0$) (red line with empty dots), and with the EoS given by Typel *et al.* in [2] (cyan full lines).

still produces abundances which are within the uncertainty determined in Figs. 5 and 6 considering a universal coupling.

This is true concerning both the temperature and the density dependence. In the absence of more constraining observations/calculations, we can propose $x_s = 0.85 \pm 0.05$ as a reasonable universal value for the cluster couplings.

Note that we are only considering light clusters in the present study. Indeed, in-medium effects on heavy clusters can be reasonably well described within the excluded-volume or Thomas-Fermi approximation [9,11] and do not require a modification of the meson couplings. However, the presence of heavy clusters will have an indirect effect on the light cluster abundances. In Refs. [16,17], the authors have considered light clusters coexisting with a heavy cluster and a proton-neutron background gas and showed that the presence of heavy clusters shifts the light cluster Mott densities to larger values. Moreover, it was also shown that above a density $\sim 10^{-3} \text{ fm}^{-3}$ for $T = 5$ MeV and $\sim 10^{-2} \text{ fm}^{-3}$ for $T = 10$ MeV the presence of heavy clusters reduces the light cluster mass fractions. Introducing heavy clusters will, therefore, have an important effect precisely in the region where changing the coupling g_{si} has the largest effect, indicating that a more complete study which includes heavy clusters must be carried out.

Notice, however, that we do not have experimental results on the Mott densities that could put constraints on the model. On the other hand, we do have experimental results for the chemical equilibrium constants (EC) obtained by measuring

cluster formation in heavy-ion collisions. In the next section we will compare these quantities extracted from experimental data with the predictions of our models. In the experimental sample, particles are emitted in the mid-rapidity region of a collision between relatively light ions under a strong radial flow, implying that no heavy clusters are present. Therefore, it makes sense that we calculate these quantities only considering the light clusters, as previously done also in Ref. [28].

C. Equilibrium constants

A very interesting constraint at high density and temperature was recently proposed from heavy-ion collision experiments in Ref. [18].

This constraint should be taken with some caution, because the systematics of such measurement are very hard to estimate. First, the freeze-out concept was used to describe the expanding fireball, which is a strongly nonequilibrium process. In addition, the heavy-ion reaction used involves small nuclei, and might be sensitive to important finite-size and finite-particle-number effects. Moreover the detection was performed in a very limited angular range, and it is far from clear that the transient system formed during the collision and subject to a strong radial field is compatible with the laws of thermodynamical equilibrium. Finally, proton fraction (y_p), density, and temperature are not directly observable, and a

TABLE III. Dissolution density, ρ_{diss} , for each cluster, considering $x_s = 0.8, 0.85$, and 0.9 , for $T = 5$ and 10 with $y_p = 0.41$. We considered $X_j = 10^{-4}$ to get ρ_{diss} . The last part of the table shows $\Gamma_{\rho_{\text{diss}}} = \rho_{\text{diss}}(T = 10)/\rho_{\text{diss}}(T = 5)$. We also compare with the dissolution density of Typel *et al.* [2], and with the QS EoS from [13], neglecting the \mathbf{P} dependence ($\mathbf{P} = 0$).

$T = 5 \text{ MeV}$					
ρ_{diss}	$x_s = 0.8$	$x_s = 0.85$	$x_s = 0.9$	[2]	QS
deuteron	0.04164	0.04954	0.05903	0.02389	0.04793
triton	0.03615	0.04374	0.05295	0.01772	0.04244
helion	0.03577	0.04394	0.05404	0.01880	0.04004
α	0.03629	0.04429	0.05406	0.02113	0.04244
$T = 10 \text{ MeV}$					
ρ_{diss}	$x_s = 0.8$	$x_s = 0.85$	$x_s = 0.9$	[2]	QS
deuteron	0.06984	0.07983	0.09106	0.06490	0.08221
triton	0.05743	0.06706	0.07801	0.04259	0.06909
helion	0.05794	0.06860	0.08084	0.04856	0.06515
α	0.05298	0.06328	0.07511	0.04404	0.06623
$\Gamma_{\rho_{\text{diss}}}$	$x_s = 0.8$	$x_s = 0.85$	$x_s = 0.9$	[2]	QS
deuteron	1.6772	1.6114	1.5426	2.7166	1.7153
triton	1.5887	1.5331	1.4733	2.4035	1.6279
helion	1.6198	1.5612	1.4904	2.5830	1.6268
α	1.4599	1.4288	1.3894	2.0842	1.5604

strong model dependence is associated with the determination of these variables.

Still, these data are currently the unique existing constraint on in-medium modifications of light particle yields at high temperature, and in the following we will, therefore, examine how well our parametrizations can reproduce the equilibrium constants (EC) reported in Ref. [18].

With the same set of couplings determined in the last section, we calculate the chemical equilibrium constants

$$K_c[j] = \frac{\rho_j}{\rho_n^{N_j} \rho_p^{Z_j}} \quad (27)$$

where ρ_j is the number density of cluster j , with neutron number N_j and proton number Z_j , and ρ_p and ρ_n are, respectively, the number densities of free protons and neutrons. We will calculate the EC for a proton fraction equal to 0.41 , as was assumed in [18,28]. It has, however, been shown that dependence of the EC on the proton fraction is very small; see Ref. [17] and Ref. [28] for a discussion of this point.

In Fig. 7, we show the chemical equilibrium constants for all the light clusters considered, taking the range of the couplings to be $g_{sj} = (0.85 \pm 0.05)A_j g_s$. In this figure, we also show results for the parametrization obtained in Ref. [17] for $\eta = 0.70$ (black squares). This model describes quite well the experimental results for the α cluster, because the parametrization was fitted to the α -equilibrium constants. However, it completely fails to reproduce the EC of the deuteron and the triton.

Taking the coupling fractions $x_{sj} = 0.85 \pm 0.05$ essentially describes the experimental equilibrium constants. We have checked that $x_s = 0.95$ would be too large.

In Ref. [28], the authors have compared the EC calculated within different models with the experimental data of Qin

et al. [18], and, in particular, tested the cluster formation and the in-medium modification of the cluster properties. They have shown that the QS formalism gives an excellent description of the experimental EC. Also, the generalized relativistic-density functional (gRDF) discussed in [2,20] gives a very good description of the EC. The gRDF model is a meson-exchange based effective relativistic mean-field model which includes as degrees of freedom nucleons, light nuclei, and heavy nuclei, and considers medium dependent binding energy shifts of nuclei. In the gRDF model, the in-medium binding energy shifts were fitted to the QS results for the light nuclei and a Thomas-Fermi calculation for the heavy ones, and include a temperature dependence. Another model giving a good description of the EC is the Hempel and Schaffner-Bielich (HS) EoS [7] with the DD2 interaction of Typel *et al.* [2]. This model consists of a mixture of nuclei and unbound nucleons in nuclear statistical equilibrium, with the nucleons being described within a relativistic mean-field, in this case the DD2 interaction.

Our present approach gives results similar to the last two approaches and even to the QS prediction, except for the deuteron. Therefore, we consider that our proposal for the effective description of the in-medium effects on the light clusters, given by the temperature independent binding energy shifts defined in Eq. (13), is justified, and presents the extra advantage to be applicable also to heavier hydrogen and helium isotopes, which are predicted to be abundant in high temperature neutron rich matter [9].

Instead of the DD2 version used in other approaches, we used the FSU version of the RMF model. We suppose that the different RMF models will not show a large effect on the results in the low-density region ($n < 0.03 \text{ fm}^{-3}$) considered here. Larger deviations between the different versions of the RMF model are expected for nucleon densities above the saturation density.

This experimental data seem to set extra constraints that, together with VEoS, suggest that a good universal coupling for all clusters is $g_{sj} = (0.85 \pm 0.05)A_j g_s$. For the deuteron, the experimental data seem to be described by the upper limit $x_s = 0.9$. Possibly a more detailed approach would allow for a different coupling g_{sj} for each cluster. According to the experimental data, the deuteron seems to be more adequately described by taking $g_{sd} = (0.9 \pm 0.03)2g_s$. It should be stressed that the deuteron is only a weakly bound state consisting of two nucleons.

IV. CONCLUSIONS

In this paper we have proposed a simple parametrization of in-medium effects acting on light clusters, in the framework of the relativistic mean field approximation. The interactions of the clusters with the surrounding medium are described with a phenomenological modification of the coupling constant to the σ meson. A coupling proportional to the cluster size is proposed, with a correction factor which is obtained by imposing that the cluster fractions exhibit the correct virial behavior in the low-density limit. The phenomenon of cluster dissolution at high density is described by introducing a simple binding energy shift which can be analytically derived in the

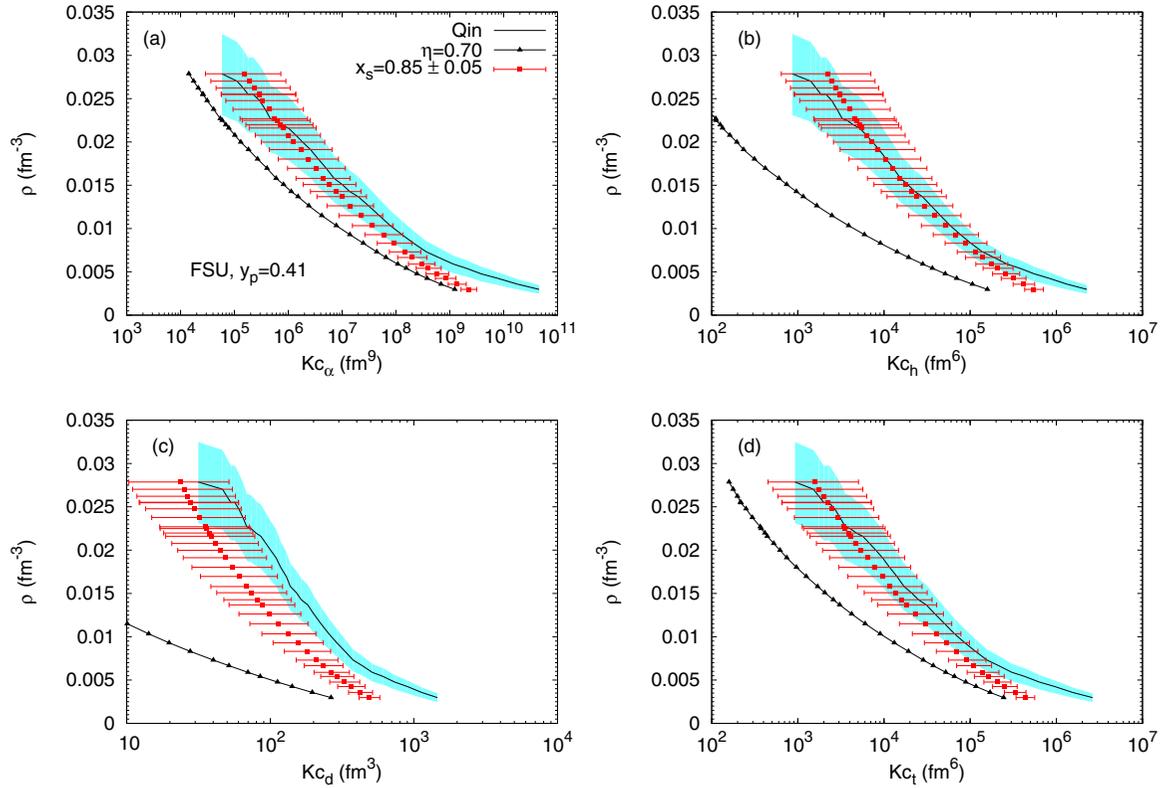


FIG. 7. Chemical equilibrium constants of α (a), helion (b), deuteron (c), and triton (d) for FSU, and $y_p = 0.41$, and for the $\eta = 0.70$ (black squares) fitting (check Ref. [17] for the complete parameter sets) and the universal g_{sj} fitting with $g_{sj} = (0.85 \pm 0.05)A_j g_s$, (red dotted lines). The experimental results of Qin *et al.* [18] (light blue region) are also shown.

Thomas-Fermi approximation as the energetic counterpart of the classical exclusion volume mechanism. With a universal cluster coupling fraction $x_s = 0.85 \pm 0.05$, we reproduce reasonably well both the virial limit and the equilibrium constants extracted from heavy-ion data. A correct description of the deuteron is probably out of scope within the mean-field approximation. Our results are qualitatively similar to the ones obtained with more microscopic approaches in Refs. [2,13], and have the advantage of being applicable also to other light clusters, which might have a non-negligible contribution in warm asymmetric stellar matter, as it is produced in proto-neutron stars, supernova environments, and neutron star mergers.

The uncertainty in the coupling has a negligible influence for densities below $\approx 10^{-2} \text{ fm}^{-3}$. At higher densities the dispersion becomes larger and the predictions of the different models show a considerable deviation. The dissolution density can thus vary of a factor 2–3 depending on the model, as well as on the choice of the coupling constants. The evolution of the dissolution density with temperature also varies approximately by a factor of 2 within the error bar of the couplings. Besides,

as discussed above, heavier clusters and the formation of pasta structures also become of relevance and should be explicitly included.

More sophisticated prescriptions allowing for different couplings for each cluster, a nonlinear mass dependence, or an explicit temperature dependence could be envisaged. However, to improve the present phenomenological description and fix these additional parameters, extra constraints from experimental data and/or microscopic calculations around the dissolution density will be needed.

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