α decay and cluster radioactivity of nuclei of interest to the synthesis of $Z = 119, 120$ isotopes

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Super-heavy nuclei of interest for the forthcoming synthesis of the isotopes with $Z = 119, 120$ are investigated. One of the very interesting latest experiments was performed at the velocity filter SHIP (GSI Darmstadt) trying to produce $^{299}120$ in a fusion reaction $^{248}\mathrm{Cm}(^{54}\mathrm{Cr},3n)^{299}120$. We report calculations of α-decay half-lives using four models: AKRA (Akrawy), ASAF (analytical superasymmetric fission), UNIV (universal formula), and semFIS (semi-empirical formula based on fission theory). The released energy, Q, is calculated using the theoretical model of atomic masses, WS4. For $^{92,94}\mathrm{Sr}$ cluster radioactivity of $^{300,302}120$ we predict a branching ratio relative to α decay of -0.10 and 0.49, respectively, meaning that it is worth trying to detect such kinds of decay modes in competition with α decay.

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I. INTRODUCTION

The interest in α decay (αD) is strongly stimulated by the search for heavier and heavier superheavies (SHs)—nuclides with $Z > 103$, produced by fusion reactions, which may be identified easily if a chain of αD leading to a known nucleus may be measured. Superheavy nuclei (SHN) [1–3], with atomic number Z up to 118, have been produced by two kinds of fusion reactions: (1) almost cold fusion (with one evaporated neutron) at GSI Germany [4,5] and RIKEN Japan [6] based on the doubly magic target $^{208}\mathrm{Pb}$ or its neighbor $^{209}\mathrm{Bi}$, and (2) hot fusion (with three or four evaporated neutrons) at JINR Dubna Russia and Livermore National Laboratory, USA [7,8], with the $^{48}\mathrm{Ca}$ projectile. One of the very interesting latest experiments was performed at the velocity filter SHIP (GSI Darmstadt) trying to produce $^{299}120$ in a fusion reaction $^{248}\mathrm{Cm}(^{54}\mathrm{Cr},3n)^{299}120$ [9].

Wang *et al.* [10] compared 20 models of atomic masses and 18 relationships used to calculate αD half-lives. They found that the "SemFIS2 (semi-empirical based on fission theory) formula is the best one to predict the alpha-decay half-lives .... In addition, the UNIV2 (universal formula) formula with fewest parameters ... work well in prediction on the SHN alpha-decay half-lives." Among these, an important role is played by Refs. [11,12]. Very interesting recent results are reported in Refs. [13–15]. For atomic mass models Wang *et al.* recommended W4 [16,17]. Nevertheless, for $^{297,299}119$ nuclei we could not get the Q values by using the model W4; hence, in these particular cases the model KTUY05 [18] was used.

From the attempts to synthesize $Z = 119,120$ isotopes we selected that from Ref. [9] dealing with $Z = 120$, without any positive result until now; we compare one of the chains starting with $^{299}120$ and ending with the fissioning nucleus $^{283}\mathrm{Rg}$.

In order to calculate αD half-lives, we use semFIS, UNIV, ASAF (analytical superasymmetric fission model) [19–25], and AKRA (Akrawy) models [26]. A FORTRAN77 computer program [27] gives us the possibility to improve the parameters of the ASAF model in agreement with a given set of experimental data. The UNIV (universal curve) model was updated in 2011 [28]. In the decay modes we are studying, a parent nucleus, ^{A}Z, disintegrates with emission of a light particle, $^{A_e}Z_e$, and a heavy daughter $^{A_d}Z_d$:

$$^{A}Z \to {}^{A_e}Z_e + {}^{A_d}Z_d. \tag{1}$$

The kinetic energy of the α particle is related to the Q value by the relationship $E_k = QA_d/A$ and the Q value is calculated from the atomic masses.

In the region of SHN the majority of researchers prefer to use the Viola-Seaborg formula [29]. Recently for nuclei with $Z = 84-110$ and $N = 128-160$, for which both Q_α^{expt} and T_{expt} values are available, new optimum parameter values [11] have been determined. A new semiempirical formula for the α-decay half-lives [19] was developed. The analytical and numerical superasymmetric fission models (ASAF and NUSAF) [20] were used together with fragmentation theory developed by the Frankfurt School, and with penetrability calculations, to predict cluster (or heavy-particle) radioactivity [30,31]. The extended calculations, e.g., Ref. [32], were used to guide the experiments and as a reference for many theoretical developments. For some isotopes of SHs, with $Z > 121$, cluster decay modes may compete with αD and spontaneous fission [33,34].

Interesting calculations have been performed in Refs. [35,36].

II. THE MODELS

In the following we give some information concerning the AKRA, ASAF, UNIV, and semFIS models [26,37]. More details can be found in Ref. [38]. We express the half-lives in

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TABLE I. Cluster radioactivities of even-even emitters. Q values were obtained using the W4 model, and half-lives with the ASAF model.

A	Z	A_e	Z_e	Q_c (MeV)	$\log_{10} T_c$(s)	$B_a = T_\alpha - T_c$
300	120	92	38[Sr]	321.36	−5.73	−0.10
302	120	94	38[Sr]	320.04	−5.26	0.49

decimal logarithm of the values in seconds, $T = \log_{10} T_{1/2}$(s). The half-life of a parent nucleus AZ against the split into a cluster A_eZ_e and a daughter A_dZ_d,

$$T = [(h \ln 2)/(2E_v)] \exp(K_{ov} + K_s), \quad (2)$$

is calculated by using the Wentzel-Kramers-Brillouin (WKB) quasiclassical approximation, according to which the action integral is expressed as [39]

$$K = \frac{2}{\hbar} \int_{R_a}^{R_b} \sqrt{2B(R)[E(R) - Q]} dR \quad (3)$$

with $B = \mu$ the reduced mass, $K = K_{ov} + K_s$ (overlapping and separated fragments), and $E(R)$ the total deformation energy. R_a and R_b are the turning points, defined by $E(R_a) - Q = E(R_b) - Q = 0$.

The AKRA model [26] was derived by adding a few parameters to the one developed by Royer [40]; this formula is defined as

$$T_{1/2} = a + bA^{1/6}\sqrt{Z} + \frac{cZ}{\sqrt{Q_\alpha}} \quad (4)$$

with initial parameters $a = -27.657$, -28.408, -27.408, and -24.763; $b = -0.966$, -0.920, -1.038, and -0.907; and $c = 1.522$, 1.519, 1.581, and 1.410 for even-even (e-e), even-odd (e-o), odd-even (o-e), and odd-odd (o-o) nuclei, respectively.

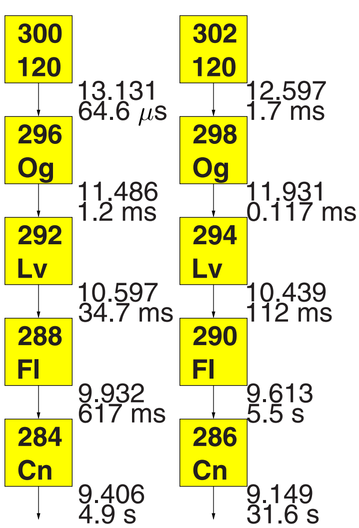

FIG. 1. A few α-decay chains of even-even SH emitters. We give the two emitters for which the branching ratios of cluster decay with respect to α decay is close to unity. We give the kinetic energy (in MeV) and the half-life of the parent nucleus. Kinetic energies are calculated with the W4 model, and half-lives with the ASAF model.

TABLE II. Cluster radioactivities of even-odd emitters. Q values obtained using the W4 model, and half-lives with the ASAF model.

A	Z	A_e	Z_e	Q_c (MeV)	$\log_{10} T_c$(s)	$B_a = T_\alpha - T_c$
299	120	91	38[Sr]	321.48	−2.70	−1.49
301	120	93	38[Sr]	320.58	−3.86	−1.17

The new relationship is obtained by introducing $I = (N - Z)/A$ and the new parameters d and e:

$$T_{1/2} = a + bA^{1/6}\sqrt{Z} + \frac{cZ}{\sqrt{Q_\alpha}} + dI + eI^2, \quad (5)$$

where after a fit with a comprehensive set of experimental data [26] we got $a = -27.989$, $b = -0.940$, $c = 1.532$, $d = -5.747$, and $e = 11.336$.

For ASAF we replace in Eq. (3) $E(R) - Q$ by $[E(R) - E_{\text{corr}}] - Q$. E_{corr} is a correction energy similar to the Strutinsky shell correction [41]. The turning points of the WKB integral are $R_a = R_i + (R_t - R_i)[(E_v + E^*)/E_b^0]^{1/2}$ and $R_b = R_t E_c\{1/2 + [1/4 + (Q + E_v + E^*)E_l/E_c^2]^{1/2}\}/(Q + E_v + E^*)$, where E^* is the excitation energy concentrated in the separation degree of freedom, $R_i = R_0 - R_e$ is the initial separation distance, $R_t = R_e + R_d$ is the touching point separation distance, $R_j = r_0 A_j^{1/3}$ ($j = 0,e,d$; $r_0 = 1.2249$ fm) are the radii of parent, emitted, and daughter nuclei, and $E_b^0 = E_i - Q$ is the barrier height before correction. The two terms of the action integral K, corresponding to the overlapping (K_{ov}) and separated (K_s) fragments, are calculated by analytical formulas (approximated for K_{ov} and exact for K_s in the case of separated spherical shapes within the liquid drop model (LDM) [42]). Since 1984, the ASAF model results have been used to guide the experiments and to stimulate other theoretical works.

The UNIV (universal formula) was obtained starting with the decay constant $\lambda = \ln 2/T$, expressed as a product of three (model-dependent) quantities $\lambda = \nu S P_s$, where ν is the frequency of assaults on the barrier per second, S is the preformation probability of the cluster at the nuclear surface, and P_s is the quantum penetrability of the external potential barrier. By assuming that the frequency ν remains practically constant and the preformation depends only on the mass number of the emitted particle, A_e has a single straight line on a double logarithmic scale,

$$\log T = -\log P_s - 22.169 + 0.598(A_e - 1), \quad (6)$$

where $-\log P_s = c_{AZ}[\arccos\sqrt{r} - \sqrt{r(1-r)}]$ with $c_{AZ} = 0.22873(\mu_A Z_d Z_e R_b)^{1/2}$, $r = R_t/R_b$, $R_t = 1.2249(A_d^{1/3} + A_e^{1/3})$, $R_b = 1.43998 Z_d Z_e/Q$, and $\mu_A = A_d A_e/A$.

For α decay of even-even nuclei, $A_e = 4$, one has

$$\log T = -\log P_s + c_{ee}, \quad (7)$$

where $c_{ee} = \log S_\alpha - \log \nu + \log(\ln 2) = -20.375$. We can find new values for c_{ee} and we also can extend the relationship to even-odd, odd-even, and odd-odd nuclei by fitting a given set of experimentally determined α-decay data.

The semFIS (semiempirical relationship based on fission theory of α decay) model was derived in order to improve the

TABLE III. Comparison of α-decay half-lives obtained with four different models.

A	Z	Q_α (MeV)	$\log_{10} T_\alpha$(s) ASAF	$\log_{10} T_\alpha$(s) AKRA	$\log_{10} T_\alpha$(s) UNIV	$\log_{10} T_\alpha$(s) SemFis
297	119	12.210	−3.695	−0.450	−4.124	−3.717
299	119	12.696	−4.730	−1.357	−5.142	−4.739
300	120	13.308	−5.833	−4.415	−6.113	−5.765
302	120	12.766	−4.769	−4.769	−5.117	−4.682

behavior in the vicinity of magic numbers

$$\log T = 0.43429 K_s \chi - 20.446, \qquad (8)$$

where $K_s = 2.52956 Z_{da}[A_{da}/(A Q_\alpha)]^{1/2}[\arccos\sqrt{x} - \sqrt{x(1-x)}]$, and the numerical coefficient χ, close to unity, is a second-order polynomial $\chi = B_1 + B_2 y + B_3 z + B_4 y^2 + B_5 yz + B_6 z^2$ in the reduced variables y and z, expressing the distance from the closest magic-plus-one neutron and proton numbers N_i and Z_i:

$$y \equiv (N - N_i)/(N_{i+1} - N_i), \quad N_i < N \leqslant N_{i+1},$$
$$z \equiv (Z - Z_i)/(Z_{i+1} - Z_i), \quad Z_i < Z \leqslant Z_{i+1},$$

with $N_i = \ldots, 51, 83, 127, 185, 229, \ldots,$ $Z_i = \ldots, 29, 51, 83, 115, \ldots,$ and $Z_{da} = Z - 2$, $A_{da} = A - 4$. The coefficients B_i are obtained by fit with experimental data, using a computer program [27] in which the parameters are determined to ensure a minimum standard rms deviation:

$$\sigma = \left\{ \sum_{i=1}^{n} [\log(T_i/T_{\rm expt})]^2/(n-1) \right\}^{1/2}. \qquad (9)$$

III. RESULTS

We present the results obtained using the four models. Generally speaking, a global indicator for a given model is the weighted mean value, e.g.,

$$\sigma_{\rm semFIS534} = \frac{173\sigma_{e-e} + 134\sigma_{e-o} + 123\sigma_{o-e} + 104\sigma_{o-o}}{534} = 0.40803 \qquad (10)$$

TABLE IV. α-decay chains of $^{299,300,302}120$ nuclei. Kinetic energy and the half-life of every decay mode are given. W4 and ASAF models are used.

A	Z	A_e	Z_e (symbol)	E_k (MeV)	T_α
300	120	296	118 (Og)	13.131	64.60 μs
299	120	295	118 (Og)	12.877	64.57 μs
296	118	292	116 (Lv)	11.486	1.20 ms
292	116	288	114 (Fl)	10.597	34.70 ms
288	114	284	112 (Cn)	9.932	617 ms
284	112	280	110 (Ds)	9.406	4.9 s
302	120	298	118 (Og)	12.597	1.70 ms
298	118	294	116 (Lv)	11.931	0.117 ms
294	116	290	114 (Fl)	9.613	5.5 s
290	114	286	112 (Cn)	9.149	31.6 s

to compare calculations within semFIS with experimental data for 534 emitters: 173 even-even, 134 even-odd, 123 odd-even, and 104 odd-odd.

We give in Table I the cluster emission with Q values calculated using the W4 model, and half-lives with the ASAF model. The most interesting results are those obtained for the heaviest nuclides: $^{300,302}120$ with branching ratios $B_\alpha = -0.10$ and 0.49, respectively; $^{299,301}120$ with $B_\alpha = -1.49$ and -1.17; $^{297,299}119$ with $B_\alpha = -1.99$ and -3.21; and $^{300}119$ with $B_\alpha = -3.75$. Similarly, in Table II there are results for even-odd emitters. A comparison of alpha decay half-lives obtained with four models is made in Table III.

Few possible α-decay chains of even-even and odd-mass SH emitters are given in Fig. 1 and Table IV and Fig. 2 and Table V, respectively.

In Table III we compare the half-lives calculated with four models: ASAF, AKRA, UNIV, and semFIS. The highest difference appears for AKRA in the case of odd Z nuclides, for which AKRA is too "optimistic" by about three orders of magnitude compared to ASAF and other models. From the results presented in this table we may give the following values of the error bars: 1.7 and 0.4 orders of magnitude for $^{300}120$ and $^{302}120$, respectively, and 3.7 and 3.8 orders of magnitude for $^{297}119$ and $^{299}119$, respectively.

A comparison between the data given in Ref. [43] and our calculations using the model W4 for E_k and ASAF for

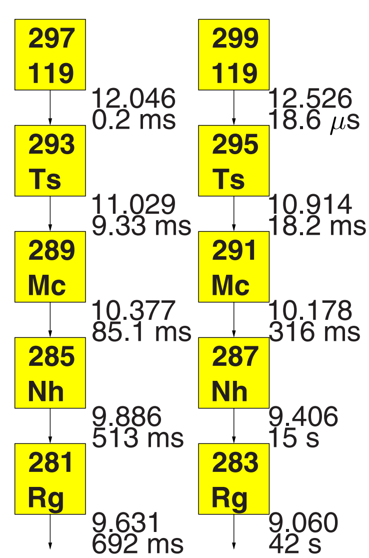

FIG. 2. A few α-decay chains of odd-mass SH emitters. In this case the branching ratio of cluster decay with respect to α decay is far from unity. Calculations are performed with the same models as for Fig. 1.

TABLE V. α-decay chains of $^{297,299}119$ nuclei. Kinetic energy and the half-life of every decay mode are given. W4 and ASAF models are used.

A	Z	A_e	Z_e (symbol)	E_k (MeV)	T_α
297	119	293	117 (Ts)	12.046	0.20 ms
293	117	289	115 (Mc)	11.029	9.33 ms
289	115	285	113 (Nh)	10.377	85.10 ms
285	113	281	111 (Rg)	9.886	513 ms
281	111	277	109 (Mt)	9.630	692 ms
299	119	295	117 (Ts)	12.526	18.60 μs
295	117	291	115 (Mc)	10.914	18.20 ms
291	115	287	113 (Nh)	9.406	15 s
287	113	283	111 (Rg)	9.060	42 s

half-lives for α decay is made in Table VI. E_k^S and T_α^S are taken from Ref. [43]. We can see that the kinetic energies are comparable, but the half-lives are sometimes very different, e.g., 0.0645 s (our calculation) compared to 5.4 s given in Ref. [43] for the α emitter $^{299}120$, 29.7 s compared to 0.78 s for $^{287}114$, 9.42 h versus 0.0183 s for $^{267}104$, etc. Maybe some of these discrepancies are due to the fact that there are isomeric states which have not been taken into account before now. Among the best agreement, one has 0.0372 s compared to 0.037 s for $^{291}116$, 14.70 s versus 19.9 s for $^{287}113$, 0.85 s and 0.42 s for $^{279}110$, 0.0912 s and 0.0180 s for $^{286}114$, 0.154 s and 0.261 s for $^{293}116$, etc. A need to remeasure the half-lives with better accuracy is evident because the half-lives for different α emitters ($^{286}114$, $^{282}112$, $^{275}108$, $^{271}106$, $^{267}104$, $^{288}114$, $^{284}112$, $^{289}114$, $^{285}112$, $^{281}110$, and $^{277}108$) are almost identical.

In conclusion, we introduced a weighted mean value of the rms standard deviation, allowing one to compare the global properties of a given model. We made a few predictions concerning possible αD chains of future SHs. A comparison between the data reported in Ref. [43] and our calculations shows either a good agreement (e.g., $^{291,293}116$, $^{286}114$, $^{287}113$, and $^{293}116$) or a large disagreement (e.g., $^{299}120$, $^{297}114$, and $^{267}104$). In the future it would be useful to take into account a detailed structure of different states, some of them being isomeric states.

TABLE VI. Comparison between the data given in Ref. [43] and our calculations using the model W4 for E_k and ASAF for half-lives for α decay. E_k^S and T_α^S are taken from Ref. [43].

A	Z	E_k^S (MeV)	E_k (MeV)	T_α^S(s)	T_α(s)
299	120	13.14	12.88	5.4	0.0645
295	118	11.81	11.54	0.261	0.0059
291	116	10.70	10.74	0.037	0.0372
287	114	10.025	10.02	0.78	29.7
287	113	10.14	9.41	19.9	14.70
283	112	9.521	9.71	6.5	34.3
279	110	9.706	9.94	0.42	0.85
294	118	13.14	11.65	5.4	0.000485
290	116	11.81	10.84	0.261	0.00961
286	114	10.70	10.23	0.0180	0.0912
282	112	10.70	9.97	0.0184	0.112
275	108	10.70	9.30	0.0190	9.77
271	106	10.70	8.76	0.0185	74.13
267	104	10.70	7.77	0.0183	9.42 h
292	116	11.81	10.63	0.261	0.035
288	114	10.70	9.93	0.0182	0.617
284	112	10.70	9.41	0.0181	4.90
293	116	11.81	10.53	0.261	0.154
289	114	10.70	9.83	0.0179	191.0
285	112	10.70	9.87	0.0180	2818.0
281	110	10.70	9.19	0.0179	288.0
277	108	10.70	8.92	0.0181	257.0

For ^{92,94}Sr cluster radioactivity of $^{300,302}120$ we predict a branching ratio relative to α decay of -0.10 and 0.49, respectively, meaning that it is worth trying to detect such kinds of decay modes in competition with α decay. The error bars for the half-lives of even-even nuclei are lower (0.4–1.7 orders of magnitude) than for odd atomic number $^{297,299}119$.

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