# Universal odd-even staggering in isotopic fragmentation and spallation cross sections of neutron-rich fragments

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An evident odd-even staggering (OES) in fragment cross sections has been experimentally observed in many fragmentation and spallation reactions. However, quantitative comparisons of this OES effect in different reaction systems are still scarce for neutron-rich nuclei near the neutron drip line. By employing a third-order difference formula, the magnitudes of this OES in extensive experimental cross sections are systematically investigated for many neutron-rich nuclei with $(N - Z)$ from 1 to 23 over a broad range of atomic numbers ($Z \approx 3$–50). A comparison of these magnitude values extracted from fragment cross sections measured in different fragmentation and spallation reactions with a large variety of projectile-target combinations over a wide energy range reveals that the OES magnitude is almost independent of the projectile-target combinations and the projectile energy. The weighted average of these OES magnitudes derived from cross sections accurately measured in different reaction systems is adopted as the evaluation value of the OES magnitude. These evaluated OES magnitudes are recommended to be used in fragmentation and spallation models to improve their predictions for fragment cross sections.

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## I. INTRODUCTION

Fragmentation and spallation reactions are two of the most important techniques utilized to produce exotic nuclei near the drip lines at many present [1–11] and future [12–14] radioactive beam facilities around the world. Spallation reactions are also widely applied in spallation neutron sources [15–18] and accelerator-driven subcritical reactors systems [19]. In addition, investigations of spallation reactions are essential for interpreting the propagation of cosmic-ray nuclei in the galaxy and the determination of the composition of the galactic cosmic-ray source [20,21]. Finally, a good knowledge of fragmentation and spallation reactions is required for applications in cancer therapy using heavy nuclei [22]. An accurate estimation or measurement of fragment cross sections is of significant importance for the above applications.

A pronounced odd-even staggering (OES) in fragment cross sections (yields), namely, an enhanced production of even-$Z$ nuclides compared to the neighboring odd-$Z$ ones, has been experimentally observed in many fragmentation and spallation reactions with different projectile-target combinations over a wide energy range (see, e.g., Refs. [9,10,23–45]). However, full $A$ and $Z$ identification was not achieved in many of these experiments (see, e.g., [24,25,28,33,34]). Moreover, fragment cross sections measured in most of the above experiments have very large uncertainties, especially for nuclei far from the valley of $\beta$ stability. In such cases, a quantitative and accurate investigation of the OES can hardly be performed. In recent works [9,10], production yields of some neutron-deficient nuclei from fragmentation reactions have been accurately measured by using a time-of-flight detector in a storage ring at IMP [46]. For these neutron-deficient nuclei, quantitative studies of the OES in different fragmentation reactions reveal that this OES seems to be almost independent of the projectile-target combinations and the projectile energy [9,10]. However, this (projectile, target, and energy) independence of the OES in cross sections is still not clear for neutron-rich nuclei, where quantitative studies of this OES in different reaction systems are scarce. Thus, more quantitative investigations of the OES are needed for neutron-rich nuclei produced by various fragmentation and spallation reactions to understand mechanisms of these reactions and accurately predict fragment cross sections. Additionally, it is essential to check whether this OES is a universal quantity for different fragmentation and spallation reactions.

The OES in fragment cross sections is believed to originate in excited nuclei during the late evaporation process and seems to be dominated by the OES of the particle-emission threshold energies, where the nuclear structure effects, e.g., pairing [9,10,32,47] and shell [9,10], exist. On the basis of the above results in Refs. [9,10,32,47], the OES in fragment cross sections has been taken into account in recent empirical analytical models, i.e., FRACS [48] and SPACS [49], for fragmentation and spallation reactions, respectively. However, the OES correction factors included in these empirical models are based on limited experimental data, and thus more quantitative studies are required to validate the OES factors

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used in these models. On the other hand, many different Monte Carlo models, e.g., the abrasion-ablation model [47] and the improved statistical multifragmentation model with secondary decay [44], have been tried to study this OES, but they can hardly reproduce the measured OES in cross sections of fragments over a large range of $Z$ (see, e.g., Refs. [9,10,32,44] for details). Thus, further investigations are needed to explore the origin of this OES, to reproduce the measured OES, and to improve theoretical predictions for cross sections of exotic nuclei produced in fragmentation as well as spallation reactions.

Recently, extensive fragment cross sections have been accurately measured with a small relative uncertainty for many different fragmentation and spallation reactions with a large variety of projectile-target combinations at energies between a few tens of MeV/nucleon and a few GeV/nucleon (see, e.g., Refs. [2,3,5,6,9,50–55]). In this work, the OES in these accurate experimental cross sections is quantitatively investigated for many neutron-rich nuclei with $(N-Z)$ from 1 to 23 over a broad range of atomic numbers ($Z \approx 3$–50). A systematical comparison of this OES extracted from different reaction systems at different energies allows one to explore the possible dependence of the OES on the projectile, target, and beam energy. Furthermore, the OES magnitudes for different neutron-rich fragments are evaluated by using many experimental data from various reaction systems. The evaluated OES value can be used in fragmentation and spallation reaction models to improve their predictions for fragment cross sections; see, e.g., Refs. [48,49].

## II. ODD-EVEN STAGGERING IN EXPERIMENTAL CROSS SECTIONS

As already demonstrated in Refs. [9,10,48], it is very suitable to quantitatively investigate the OES in fragment cross sections along a constant isospin $T_z$ chain, where the staggering effect and the impact of nuclear structure effects become very evident. In this work, the magnitude of this OES for four neighboring fragments (centered at $Z+3/2$) along a constant $T_z$ chain is calculated by using the following third-order difference formula [9,10,48,56]:

$$D_{CS}(Z,N) = \tfrac{1}{8}(-1)^{Z+1}\{\ln Y(Z+3,N+3) - \ln Y(Z,N) - 3[\ln Y(Z+2,N+2) - \ln Y(Z+1,N+1)]\}, \quad (1)$$

where $Y(Z,N)$ is the value of the production cross section (yield) for a particular nucleus with an atomic number $Z$ and thus a neutron number $N = Z + 2T_z$. A positive (negative) value of $D_{CS}$ indicates an enhanced production of even-$Z$

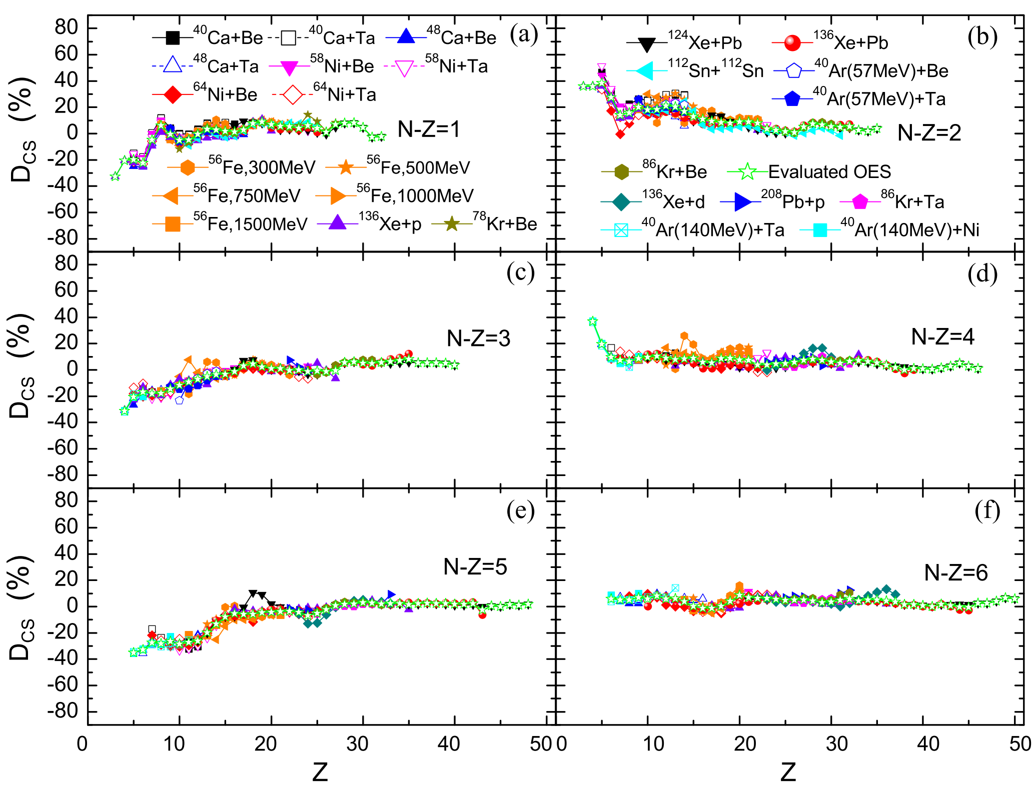


FIG. 1. Magnitudes of the OES calculated by Eq. (1) using experimental data of 26 different reactions, i.e., 140 MeV/nucleon $^{40,48}$Ca+Be/Ta [6], 140 MeV/nucleon $^{58,64}$Ni+Be/Ta [6], $^{56}$Fe + $p$ at 300, 500, 750, 1000, and 1500 MeV/nucleon [50], 1000 MeV/nucleon $^{136}$Xe + $p$ [51], 483 MeV/nucleon $^{78}$Kr+Be [9], 1000 MeV/nucleon $^{124,136}$Xe+Pb [2], 1000 MeV/nucleon $^{112}$Sn + $^{112}$Sn [3], 57 MeV/nucleon $^{40}$Ar+Be/Ta [52], 64 MeV/nucleon $^{86}$Kr+Be/Ta [53], 500 MeV/nucleon $^{136}$Xe + $d$ [5], 1000 MeV/nucleon $^{208}$Pb + $p$ [54], and 140 MeV/nucleon $^{40}$Ar+Ni/Ta [55]. The evaluated magnitudes (green open stars) are obtained from the weighted average of the above measured OES values by using Eq. (2). For clarity, experimental error bars (around 8% in most cases) are not shown. The data are shown from (a) $N-Z=1$ to (f) $N-Z=6$.

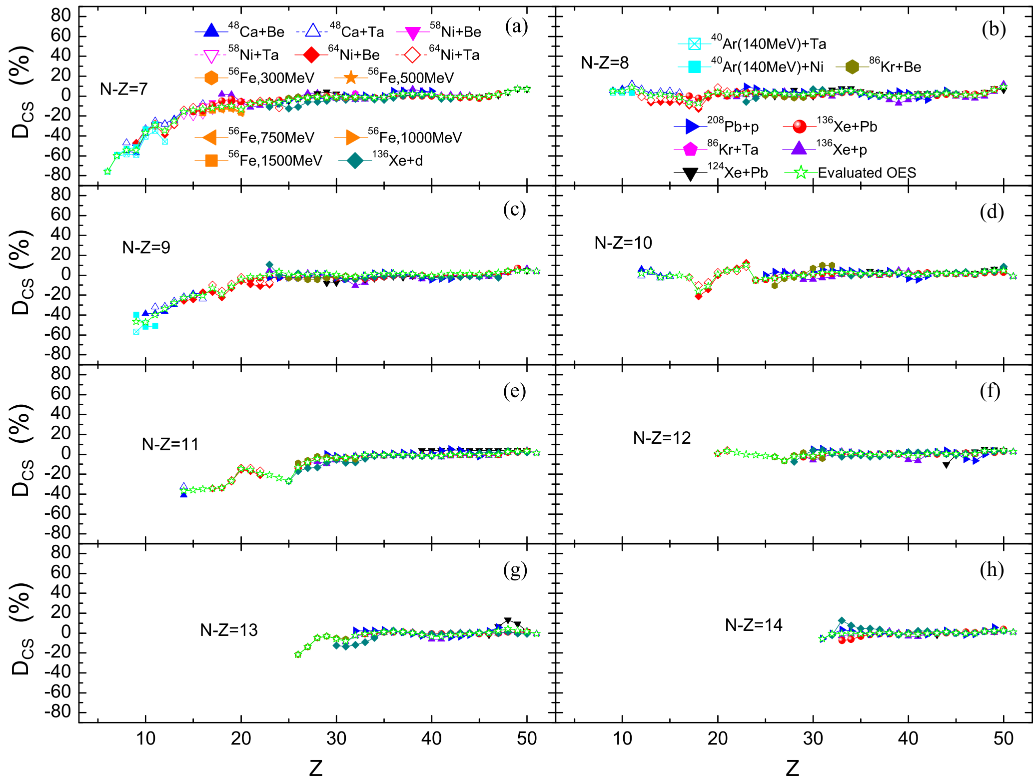


FIG. 2. Magnitudes of the OES extracted from experimental data of different reactions, i.e., 140 MeV/nucleon $^{48}$Ca+Be/Ta [6], 140 MeV/nucleon $^{58,64}$Ni+Be/Ta [6], $^{56}$Fe + $p$ at 300, 500, 750, 1000, and 1500 MeV/nucleon [50], 1000 MeV/nucleon $^{136}$Xe + $p$ [51], 1000 MeV/nucleon $^{124,136}$Xe+Pb [2], 64 MeV/nucleon $^{86}$Kr+Be/Ta [53], 500 MeV/nucleon $^{136}$Xe + $d$ [5], 1000 MeV/nucleon $^{208}$Pb + $p$ [54], and 140 MeV/nucleon $^{40}$Ar+Ni/Ta [55]. The evaluated magnitudes (green open stars) are derived from the weighted average of the above measured OES values. For the $N-Z=10$ chain with $Z=16$, the $N-Z=11$ chain with $Z=15$, 16, 23, and 24, and the $N-Z=12$ chain with $Z$ from 22 to 25, the evaluated OES magnitudes are obtained by interpolating neighboring ones since there are no experimental data. For clarity, experimental error bars (below 8% in most cases) are not shown. The data are shown from (a) $N-Z=7$ to (h) $N-Z=14$.

(odd-$Z$) nuclei, while its absolute value represents the strength of this OES.

To accurately investigate the magnitude of the OES and avoid the staggering structures caused by the large errors in experimental data, only experimental cross sections with a small relative uncertainty less than about 15% are applied in the calculations of the OES magnitude. Recently, extensive cross sections have been accurately measured with a small relative uncertainty in the following fragmentation or spallation reactions: 1000 MeV/nucleon $^{124,136}$Xe + Pb [2], 1000 MeV/nucleon $^{112}$Sn + $^{112}$Sn [3], 500 MeV/nucleon $^{136}$Xe + $d$ [5], 140 MeV/nucleon $^{40,48}$Ca+Be/Ta [6], 140 MeV/nucleon $^{58,64}$Ni+Be/Ta [6], 483 MeV/nucleon $^{78}$Kr+Be [9], $^{56}$Fe + $p$ at 300, 500, 750, 1000, and 1500 MeV/nucleon [50], 1000 MeV/nucleon $^{136}$Xe + $p$ [51], 57 MeV/nucleon $^{40}$Ar+Be/Ta [52], 64 MeV/nucleon $^{86}$Kr+Be/Ta [53], 1000 MeV/nucleon $^{208}$Pb + $p$ [54], and 140 MeV/nucleon $^{40}$Ar+Ni/Ta [55]. The magnitudes of the OES in the above accurate experimental cross sections are calculated by using Eq. (1) for many neutron-rich fragments with $(N-Z)$ from 1 to 23 over a very wide range of atomic numbers ($Z \approx 3$–50).

Figure 1 presents the magnitudes of the OES for neutron-rich fragments with $(N-Z)$ from 1 to 6, which are derived from fragment cross sections measured in the above-mentioned fragmentation or spallation reactions with a large variety of projectile-target combinations over a wide energy range [2,3,5,6,9,50–55]. For odd-mass fragments ($N-Z=1$, 3, and 5), a large reversed OES with an enhanced production of odd-$Z$ nuclei is observed for very light ones and there is a transition from a large negative $D_{CS}$ (reversed OES) to a very small positive value around 0 (OES) with increasing $Z$. The reversed OES occurs because these very light odd-mass nuclei predominantly emit neutrons and the neutron separation energy, which determines the production yield of the final fragment in the evaporation phase, is large for odd-$Z$ (even-$N$) fragments but small for even-$Z$ (odd-$N$) ones; see Ref. [9]. For even-mass nuclei ($N-Z=2$, 4, and 6), $D_{CS}$ decreases rapidly from a large positive value to about 0 as $Z$ increases. The large positive OES for very light even-mass nuclei is caused by the large neutron separation energy for even-$Z$ (even-$N$) fragments but a small one for odd-$Z$ (odd-$N$) fragments. Additionally, the OES magnitudes from different reactions are in very good agreement within their uncertainties (around 8%) and they show the same evolution tendency along a constant isospin chain, as displayed in Fig. 1.

For neutron-rich fragments with $(N-Z)$ from 7 to 14 produced in different fragmentation or spallation reactions [2,5,6,50,51,53–55], their OES magnitudes calculated by Eq. (1) are shown in Fig. 2. For $D_{CS}$ of odd-mass fragments, a similar transition from a very large negative value to a small

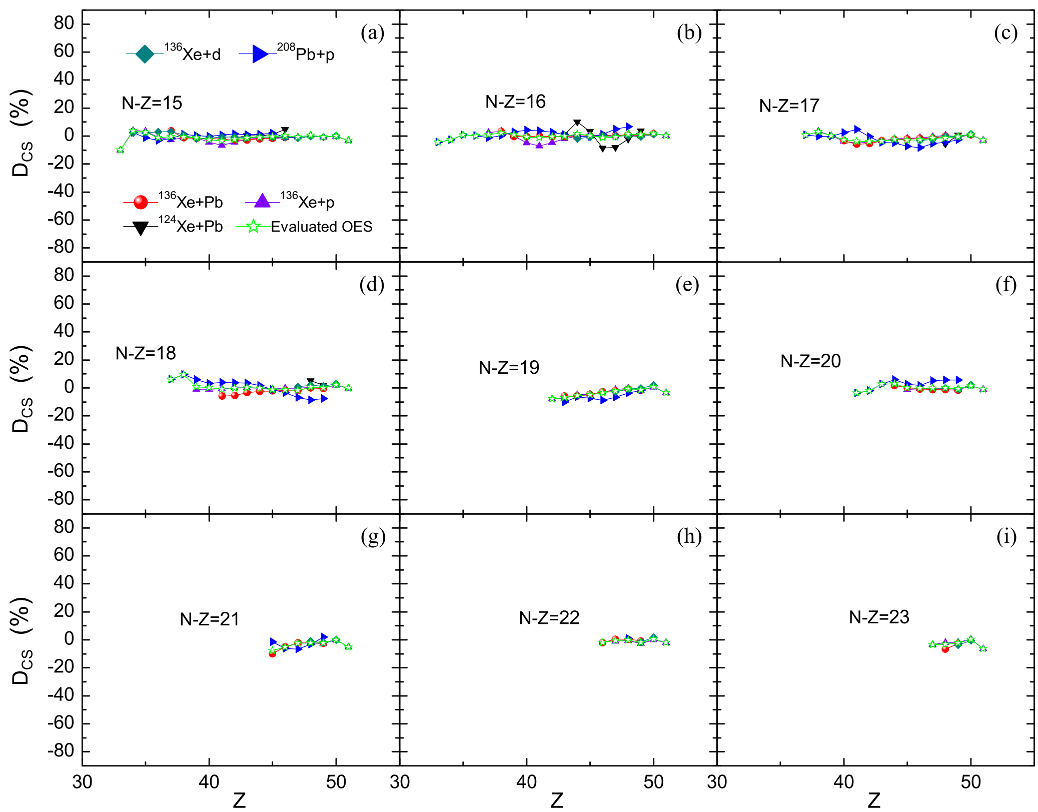


FIG. 3. Magnitudes of the OES obtained from experimental data of different reactions, i.e., 1000 MeV/nucleon $^{136}$Xe + $p$ [51], 1000 MeV/nucleon $^{124,136}$Xe+Pb [2], 500 MeV/nucleon $^{136}$Xe + $d$ [5], and 1000 MeV/nucleon $^{208}$Pb + $p$ [54]. The evaluated magnitudes (green open stars) are calculated from the weighted average of the above measured OES values by using Eq. (2). For clarity, experimental error bars (around 8% in most cases) are not shown. The data are shown from (a) $N - Z = 15$ to (i) $N - Z = 23$.

value around 0 is observed when $Z$ increases. A very large reversed OES of about −80%, which is the strongest reversed OES observed in experimental data, is reached for the light $N - Z = 7$ nuclei around $Z = 6$. This largest reversed OES value (around −80%) means that the measured cross section of the light odd-$Z$ (even-$Z$) fragment is larger (smaller) than the smooth distribution by a factor of about 2.2; see Ref. [48]. A very small OES (around 0%) is shown for measured even-mass nuclei ($N - Z = 8$, 10, 12, and 14), especially for those with $Z > 30$. In addition, a transition in the OES magnitude is shown for $N - Z = 10$ and 11 nuclei from $Z = 18$ to 23 and from $Z = 20$ to 25, respectively. This may be caused by the subshell closure at $N = 32$ and 34 [8]. Furthermore, a comparison of experimental data measured in various reaction systems at different energies demonstrates that the OES magnitude almost does not depend on the projectile-target combinations or the projectile energy.

Cross sections of some neutron-rich fragments with $(N - Z)$ from 15 to 23 have been accurately measured in Refs. [2,5,51,54] and the magnitudes of the OES in these experimental data are given in Fig. 3. The OES magnitude value is very small (around 0%) for all these measured fragments with $Z > 30$, which means that the measured cross section is almost smooth for these nuclei along a constant isospin chain.

According to all experimental data in Figs. 1–3, the magnitudes of the OES derived from extensive cross sections (about 3800) measured in 26 different fragmentation or spallation reactions over a broad energy range are in remarkable agreement within their uncertainties. All these experimental data support that the magnitude of this OES is almost independent of the projectile-target combinations as well as the projectile energy and thus it is a universal quantity for different (fragmentation and spallation) reaction systems.

## III. EVALUATION OF ODD-EVEN STAGGERING MAGNITUDE

A quantitative evaluation of the OES magnitude is very important for accurate predictions of fragmentation and spallation cross sections using theoretical models [48,49]. Since the OES magnitude is a universal quantity for fragmentation as well as spallation reactions, one can derive the evaluation value of this OES magnitude from the above experimental data measured in different reaction systems with a large variety of projectile-target combinations at different energies.

For a specific fragment with atomic number $Z$ and neutron number $N$, the weighted average of the OES magnitudes in different experimental data sets from various fragmentation or spallation reactions is adopted as the evaluated OES magnitude and it can be calculated by the following equation:

$$D_{\rm CS}^{\rm eval}(Z,N) = \frac{\sum_{i=1}^{n} \frac{D_{\rm CS}^{i}(Z,N)}{(\sigma^i)^2}}{\sum_{i=1}^{n} \frac{1}{(\sigma^i)^2}}, \quad (2)$$

where $D^i_{\mathrm{CS}}$ and $\sigma^i$ are the OES magnitude and its error, respectively, derived from one experimental data set, and $n$ is the number of experimental data sets. The evaluated OES magnitudes ($D^{\mathrm{eval}}_{\mathrm{CS}}$) are also given in Figs. 1–3 (see green open stars) and they are in excellent agreement with those magnitudes ($D^i_{\mathrm{CS}}$) in various experimental data sets. For the evaluated OES magnitude, an error of about 8% is estimated by a comparison of the evaluated magnitude and the magnitudes in various experimental data sets from different reaction systems. This error originates mainly from the uncertainty of the experimental data and the possible small dependence of the OES on the reaction system. These OES magnitudes evaluated from extensive experimental data sets are recommended to be implemented in the fragmentation and spallation models in order to accurately predict the fragment cross sections; see, e.g., Refs. [48,49]. In other cases where fragment cross sections have not been measured, the OES magnitudes in fragmentation and spallation models can be estimated from the nuclear separation energies according to Refs. [48,49], which can be calculated by theoretical models.

## IV. SUMMARY

In summary, the OES effect in extensive experimental cross sections is quantitatively studied for many neutron-rich fragments produced by various fragmentation and spallation reactions. For neutron-rich nuclei with $(N-Z)$ from 1 to 23 over a wide range of atomic numbers ($Z \approx 3$–50), the magnitudes of the OES in their production cross sections are systematically calculated by using a third-order difference formula. Quantitative comparisons of these magnitude values in various different fragmentation and spallation reaction systems over a wide energy range demonstrate that the OES magnitude almost does not depend on the projectile-target combinations as well as the projectile energy. Considering that the OES magnitude is a universal quantity for fragmentation as well as spallation reactions, the weighted average of the OES magnitudes from the above experimental data measured in different reactions is calculated and adopted as the evaluation value of the OES magnitude. These evaluated OES magnitudes can be incorporated into fragmentation and spallation models, e.g., FRACS [48] and SPACS [49], respectively, for improving their predictions of fragment cross sections. For instance, these OES magnitudes can be applied to a simplified FRACS model implemented in LISE++ v.10.1 [57], where the OES is not considered.

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