Further studies of the multiplicity derivative in models of heavy ion collision at intermediate energies as a probe for phase transitions

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(Received 30 January 2018; revised manuscript received 15 February 2018; published 12 April 2018)

In conjunction with models, the experimental observable of total multiplicity can be used to check if the data contain the signature of phase transition and if it is first order. Two of the models reach similar conclusions. The third one is quite different.

DOI: 10.1103/PhysRevC.97.044605

I. INTRODUCTION

This paper deals with identifying the order of phase transition from experimental data in intermediate-energy heavy ion collisions. We focus here on the total multiplicity M resulting from central collisions of two heavy ions; M is a function of the beam energy. The derivative of M with energy as a function of energy may go through a maximum. In a previous paper [1] we claimed that the appearance of this maximum is a signature of a first-order phase transition in the collision. The absence of a maximum would imply there is no first-order phase transition. We used a canonical thermodynamic model (CTM) [2] to reach this conclusion. As is usual in canonical model calculations the M derivative is easiest to obtain with respect to temperature and can then be mapped in terms of energy.

The model is based on the ansatz that in heavy ion collisions a heated conglomeration of nucleons in an expanded volume is formed. Nucleons get grouped into various composites and the total number of composites plus monomers is the total multiplicity M. This system of particles can go through a phase transition [3–7]. The system is characterized by a temperature T and has an average energy E. At the phase transition temperature, C_v, the derivative of energy with respect to temperature goes through a maximum. The quintessential problem is how to recognize this maximum experimentally. Using CTM we found that the maximum of dE/dT and the maximum of dM/dT coincide. Because dM/dE is experimentally accessible, the signal for a first-order transition can be recognized.

Although the calculations in Ref. [1] were done with CTM only, we expect similar results with microcanonical models [8,9]. The basic physics assumptions are the same. In examples where microcanonical and canonical calculations were compared [10] they were found to be very close. We note in passing that both canonical and microcanonical models are found to give in general very good fits to experimental data.

Here we examine features of M derivatives for models different from standard thermodynamic models. Of particular interest is the percolation model [11,12], which has been extensively used in the past to establish a link between experimental data and phase transition. In the context of the present work percolation results will be very interesting because percolation is a model of continuous phase transition. We next examine the M derivative in the lattice gas model which uses geometry similar to that of percolation but is much more elaborate with the insertion of a Hamiltonian. First-order transition is possible here [7,13,14].

II. TOTAL MULTIPLICITY AND ITS DERIVATIVE IN THE PERCOLATION MODEL

We consider a system of 6^3 nucleons in the bond percolation model. The model does not distinguish between neutrons and protons. There are 6^3 boxes and each box contains one nucleon. Nearest neighbors (these have a common wall) can bind together with a probability p_s. If p_s is 1 there is just one nucleus with 6^3 nucleons and $M = 1$. If p_s is 0 there are 6^3 monomers and $M = 6^3$. For intermediate values of p_s, nucleons can group into several composites. For an "event" this is obtained by Monte Carlo sampling. Let the average number of clusters of a nucleons be n_a. Then $M = \sum_a n_a$. In the bond percolation model there is just one parameter, p_s. Thus we can plot M against p_s and examine the M derivative. Instead of plotting M against p_s we plot M against $p_b = 1 - p_s$, which is the bond-breaking probability. If p_b is 0, then 6^3 nucleons appear as one cluster and $M = 1$. If p_b is 1 then we have 6^3 monomers and $M = 6^3$. Figure 1 plots M and dM/dp_b in the range of p_b 0 to 1. For reference in Fig. 2 we have plotted M and dM/dT as was obtained in the CTM [1]. Both M and M derivatives are very different in the two models. The percolation model has no first-order phase transition and as conjectured before [1] there is no maximum in the M derivative. Also note that the CTM calculations are quite realistic. The inputs were liquid-drop-model energies for composites. Coulomb interactions between composites are included approximately. If one omits the Coulomb interactions between the different composites the maximum in the M derivative becomes sharper.

The well-known function of p_b that is normally used is not $M(p_b)$ but a second moment function, $m_2(p_b)$. That function has a maximum at about $p_b = 0.8$ (equivalently $p_s = 0.2$). We use that function in Sec. IV.

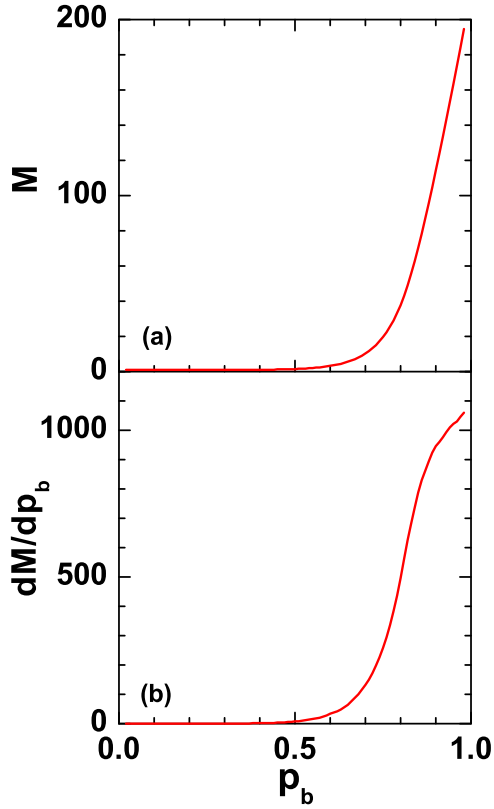

FIG. 1. Variation of M (a) and dM/dp_b (b) with p_b obtained from the bond percolation model for a system of 6^3 nucleons.

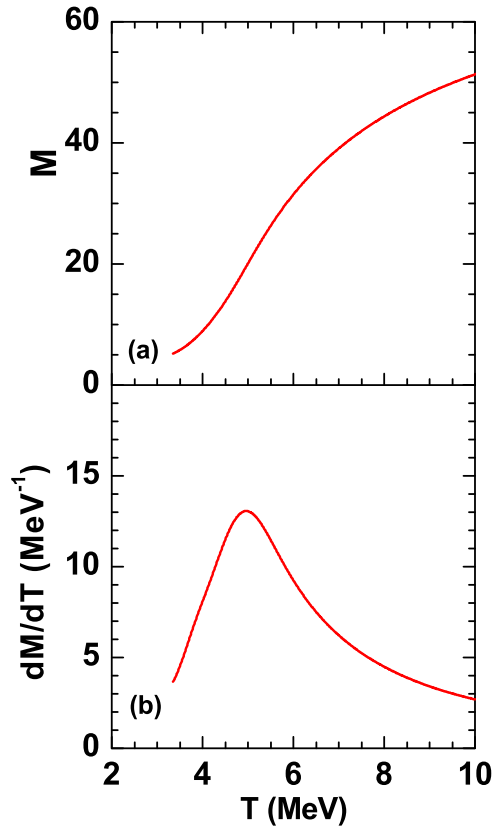

FIG. 2. Variation of M (a) and dM/dT (b), with T obtained from the CTM for a fragmenting system having $Z = 82$ and $N = 126$.

III. TOTAL MULTIPLICITY AND ITS DERIVATIVE IN THE LATTICE GAS MODEL

The next topic we deal with is the M derivative in the lattice gas model. This is shown in Fig. 3. Here M and its derivative are plotted against the temperature T. The lattice gas model is considerably more complicated than the percolation model but expositions of the model exist [7,13,14] and we refer to Ref. [14] for details. Let $A = N + Z$ be the number of nucleons in the system that dissociate. We consider D^3 cubic boxes where each cubic box has a volume of (1.0/0.16) fm^3. D^3 is larger than A (they have the same value in the bond percolation model). Here $D^3/A = V_f/V_0$, where V_0 is the normal volume of a nucleus with A nucleons and V_f is the freeze-out volume where the partitioning of nucleons into clusters is computed. For nuclear forces one adopts nearest-neighbor interactions. Following normal practice, we use the neutron-proton interaction $v_{np} = -5.33$ MeV and set $v_{nn} = v_{pp} = 0.0$. The Coulomb interaction between protons is included. Each cube can contain 1 or 0 nucleon. There is a very large number of configurations that are possible (a configuration designates which cubes are occupied by neutrons, which are occupied by protons, and which are empty; we sometimes call a configuration an event). Each configuration has an energy. If a temperature is specified, the occupation probability of each configuration is proportional to its energy: $P \propto \exp(-E/T)$. This is achieved by Monte Carlo sampling using the Metropolis algorithm.

The calculation of clusters need further work. Once an event is chosen we ascribe to each nucleon a momentum. The momentum of each nucleon is picked by Monte-Carlo sampling of a Maxwell-Boltzmann distribution for the prescribed temperature T. Two neighboring nucleons are part of the same cluster if $\vec{P}_r^2/2\mu + \epsilon < 0$, where ϵ is v_{np}, v_{nn}, or v_{pp}. Here $\vec{P}_r$ is the relative momentum of the two nucleons and μ is the reduced mass. If nucleon i is bound with nucleon j and nucleon j is bound with nucleon k, then i, j, and k are part of the same cluster. At each temperature we calculate 50 000 events to obtain the average energy $\langle E \rangle$ and the average multiplicity n_a (where a is the mass number of the cluster) of all clusters. A cluster with 1 nucleon is a monomer, one with 2 nucleons is a dimer and so on. The total multiplicity is $M = \sum n_a$ and $\sum a n_a = A$ where $A = N + Z$ is the mass number of the dissociating system. Plots of dM/dT and $d\langle E \rangle/dT$ are shown in Fig. 4. Note that c_v goes through a maximum at some temperature which is a hallmark of a first-order phase transition and this occurs at the same temperature where dM/dT maximizes. This is remarkably different from the percolation model results but very similar to the CTM results of Ref. [1], corroborating the evidence that the appearance of a maximum in dM/dT is indicative of a first-order phase transition.

IV. THE SECOND MOMENT m_2 IN THE MODELS

Although the percolation model curves that we have shown above are even qualitatively different from those emerging

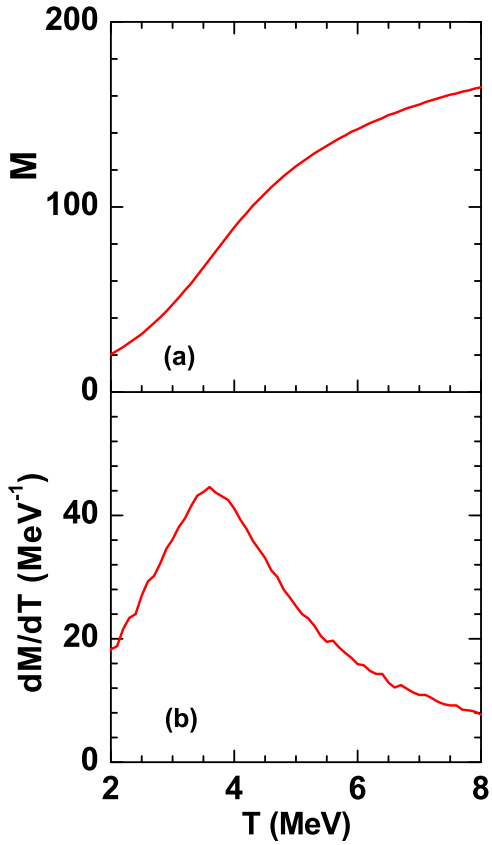

FIG. 3. Variation of M (a) and dM/dT (b), with T obtained from the lattice gas model for a fragmenting system having $Z = 82$ and $N = 126$.

from the lattice gas model and the CTM, there is one curve that is similar and was used a great deal when percolation was the only available microscopic model to link experimental multifragmentation data to phase transition. We call this the second moment curve m_2. Consider the percolation curve of Fig. 1 where we chose the dissociating system to consist of $A = 216$ nucleons. Define reduced multiplicity as $n = M/A$, where M is the total multiplicity and A is the mass of the dissociating system; n varies from $1/A \approx 0$ to 1 as p_b goes from 0 to 1. We expect M to increase if more energy is pumped into the system. For example, in counter experiments one can gate on central collisions and vary the beam energy. In emulsion experiments [11,15] there is no selection on the impact parameter and in collisions at different impact parameters different amounts of energies are pumped in for multifragmentation. For our illustration purposes we consider central collisions for two models, the percolation model and the lattice gas model in a range of energies. For these we plot m_2 as a function of n. Define m_2 as

$$m_2 = \frac{\left[\sum a^2 n_a - a_{\max}^2\right]}{A}. \quad (1)$$

We denote the largest cluster in an event by $a_{\max}$. For percolation we pick a p_b and get n, n_a, and $a_{\max}^2$ by averaging over 50 000 events. Thus we can plot m_2 against n. For the lattice gas model we take 50 000 events at each temperature and follow the same procedure. The m_2 curves are given in Fig. 5. Note that m_2

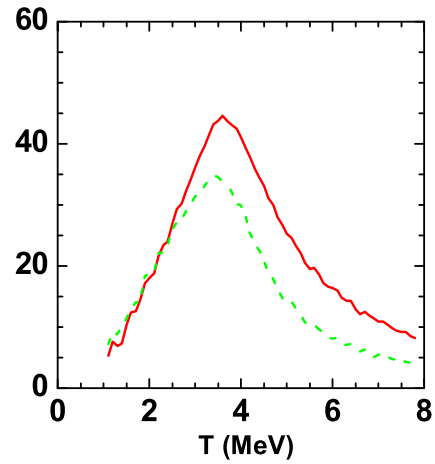

FIG. 4. Variation of dM/dT (red solid lines) and C_v (green dashed lines) with temperature from the lattice gas model at $D = 8$ (see text) for a fragmenting system having $Z = 82$ and $N = 126$. To draw dM/dT and C_v in the same scale, C_v is normalized by a factor of 1/10; dM/dT is given in units of MeV^{-1}.

curves for the lattice gas model and the percolation models are quite similar, and from experimental data (which can be fitted only approximately) one could choose either a percolation model or a lattice gas model. However the models have in fact even different orders of transition. If we define $m_1 = \sum a n_a/A$ the answer is identical in both the models with value 1; just a straight line with a value of 1 for all n's. One can build a little bit of structure if we define $m_1 = [\sum a n_a - a_{\max}]/A$, but the m_2 is the first interesting quantity, though not a confirmatory signal.

V. BACK TO PERCOLATION

In the previous section we compared m_2 obtained from the lattice gas model and the percolation model. For that purpose it was convenient to plot m_2 as a function of n. Now we concentrate on the percolation model only and it is more convenient to draw m_2 as a function of p_b. In Fig. 5 we drew

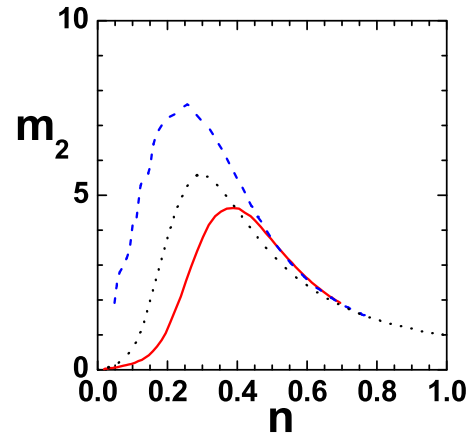

FIG. 5. Variation of m_2 with n calculated with the lattice gas model at $D = 7$ (red solid line) and $D = 8$ (blue dashed line) and with the percolation model (black dotted line) for a fragmenting system having $Z = 82$ and $N = 126$.

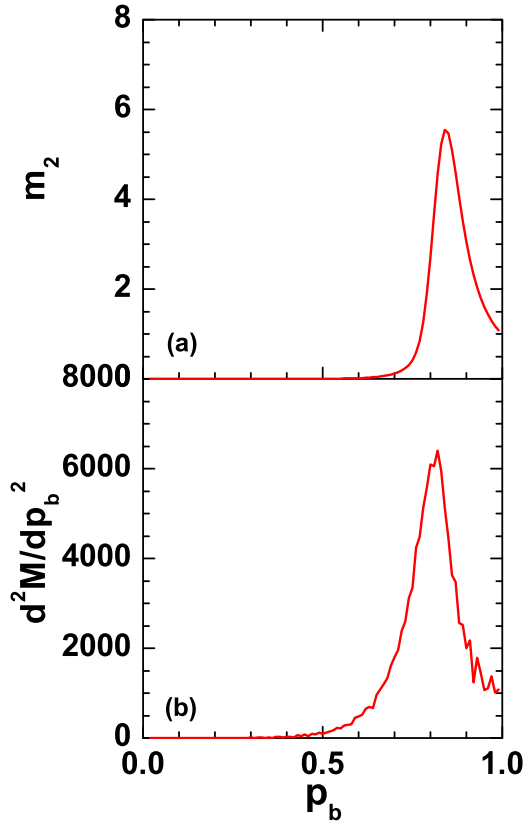

FIG. 6. Variation of m_2 (a) and d^2M/dp_b^2 (b), with p_b obtained from the bond percolation model for a system of 6^3 nucleons.

a curve of m_2 as a function of n. In this section it is more convenient to draw m_2 as a function of p_b. In Fig. 1 of Sec. II we drew a curve of both M and dM/dp_b. We now draw a curve of d^2M/dp_b^2 and compare it with $m_2(p_b)$ in Fig. 6. The similarity of the two is remarkable. The mathematical procedures used in computing m_2 and the second derivative are very different. One is tempted to conclude that the second derivative of M having a maximum is an indication that this is a case of second-order phase transition.

VI. DISCUSSION

Recognition of phase transition in intermediate-energy collisions has been an interesting and intriguing problem of long standing. A popular approach has been to try to best fit individual multiplicity n_a to a form suggestive of critical phenomenon: $n_a = a^{-\tau} f[a^{\sigma}(T - T_c)]$ [11,12,16–18]. It is impossible to get a very good fit as the masses a need to be big for the model to work and in heavy ion collisions in the laboratory fragment sizes are limited. As fits are only approximate very different models can give similar quality fits. Thus the conclusions are ambiguous. Here we have specialized to an observable that is very feasible to scan and will give an unambiguous answer. In addition we have identified an interesting feature of M in the percolation model that was not recognized before.

ACKNOWLEDGMENT

We thank the anonymous referee for useful comments and advice.

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