Systematic study of α decay of nuclei around the $Z = 82$, $N = 126$ shell closures within the cluster-formation model and proximity potential 1977 formalism

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In the present work, we systematically study the α decay preformation factors P_α within the cluster-formation model and α decay half-lives by the proximity potential 1977 formalism for nuclei around $Z = 82$, $N = 126$ closed shells. The calculations show that the realistic P_α is linearly dependent on the product of valance protons (holes) and valance neutrons (holes) $N_p N_n$. It is consistent with our previous works [Sun *et al.*, Phys. Rev. C **94**, 024338 (2016); Deng *et al.*, *ibid.* **96**, 024318 (2017)], in which P_α are model dependent and extracted from the ratios of calculated α half-lives to experimental data. Combining with our previous works, we confirm that the valance proton-neutron interaction plays a key role in the α preformation for nuclei around $Z = 82$, $N = 126$ shell closures whether the P_α is model dependent or microcosmic. In addition, our calculated α decay half-lives by using the proximity potential 1977 formalism taking P_α evaluated by the cluster-formation model can well reproduce the experimental data and significantly reduce the errors.

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I. INTRODUCTION

In 1928, the phenomenon of α decay for nuclei was independently explained by Gurney and Condon [1] and Gamow [2] using the quantum tunnel theory. Since then, α decay has long been perceived as one of the most powerful tools to investigate unstable nuclei, neutron-deficient nuclei, and superheavy nuclei, and has been an active area of research of nuclear physics [3–22].

Within Gamow's theory, the α decay process is described as a preformed α particle penetrating the Coulomb barrier. Thus an α preformation factor should be introduced into α decay theories, which denotes the probability of an α cluster preformation. There are a lot of models devoted to determining α preformation factors. Microscopically, α preformation factors can be calculated by the overlap between initial wave function and α decaying wave function [23]. In the R-matrix method, the α preformation can be obtained from the initial tailored wave function of the parent nucleus [24–28]. Röpke *et al.* [29] and Xu *et al.* [30] calculated α preformation factors using an approach of the Tohsaki-Horiuchi-Schuck-Röpke wave function, which was also successfully used to describe the cluster structure of light nuclei. In the cluster model, the α preformation factor is tread as a constant less than 1 for a certain type of nuclei and the value of even-even nuclei > odd$-A$ nuclei > doubly odd nuclei [31–36]. Xu and Ren systematically studied the α decay of medium mass nuclei using the density-dependent cluster model (DDCM) [37]. Their results indicated that the α preformation factors are 0.43 for even-even nuclei, 0.35 for odd-A nuclei, and 0.18 for doubly odd nuclei. Because of the complicated structure of quantum many-body systems, phenomenologically, the α preformation factors are extracted from the ratios of calculations to experimental α decay half-lives [38–40]. Nevertheless, the obtained preformation factors are strongly model dependent.

Recently, Ahmed *et al.* presented a new quantum-mechanical theory named cluster-formation model (CFM) to calculate the α preformation factors P_α of even-even nuclei [11,12], which suggests that the initial state of the parent nucleus should be a linear combination of different possible clusterization states. They successfully determined the $P_\alpha = 0.22$ for ^{212}Po using CFM, which could well reproduce the calculations of Varga *et al.* [24,28] and the values of Ni and Ren [41] in different microscopic ways. Very recently, Ahmed *et al.* and Deng *et al.* extended CFM to odd-A and doubly odd nuclei through modifying the formation energy of the interior α cluster for various types of nuclei (i.e., even-Z–odd-N, odd-Z–even-N, and doubly odd nuclei) and considered the effects of unpaired nucleon [13–15,42]. In 2011, Seif *et al.* put forward that the α preformation factor is linearly dependent on $N_p N_n$ for even-even nuclei around proton $Z = 82$, neutron $N = 126$ closed shells, where N_p and N_n denote valance protons (holes) and valance neutrons (holes) [7]. In our previous works, the extracted α preformation factors from ratios of calculated α decay half-life to experimental data for cases of odd-A and doubly odd nuclei α decay also

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satisfy this relationship [43,44]. It is interesting to validate whether the realistic α preformation factor within CFM is also proportional to N_pN_n. In addition, many researchers adopted the Coulomb and proximity potential model (CPPM) to investigate α decay leaving P_α out of consideration or assuming as $P_\alpha = 1$, thus the deviations between calculated α decay half-lives and experimental data were considerable [45–47]. For confirming CFM and diminishing the difference between theoretical and experimental data, we also calculate α decay half-lives within the proximity potential 1977 formalism (Prox.1977) [48] taking $P_\alpha = 1$ and the realistic P_α evaluated by CFM, respectively. Our calculated α decay half-lives within Prox.1977 taking P_α evaluated by CFM can significantly reduce the deviations between calculations and experimental data.

This article is organized as follows. In next section, the theoretical framework of the CFM, α decay half-life and Prox.1977 are briefly presented. The detailed calculations and discussion are given in Sec. III. In this section, we investigate the α preformation factors from the viewpoint of the valence proton-neutron interaction, and calculate α decay half-lives by Prox.1977 with $P_\alpha = 1$ and P_α calculated by CFM, respectively. Section IV is a brief summary.

II. THEORETICAL FRAMEWORK

A. Cluster-formation model

Within the cluster-formation model (CFM) [11–15], the total clusterization state Ψ of parent nuclei is assumed as a linear combination of all its n possible clusterization states Ψ_i. It can be represented as

$$\Psi = \sum_{i=1}^{n} a_i \Psi_i, \tag{1}$$

$$a_i = \int \Psi_i^* \Psi d\tau, \tag{2}$$

where a_i denotes the superposition coefficient of Ψ_i, on the basis of orthogonality condition,

$$\sum_{i=1}^{n} |a_i|^2 = 1. \tag{3}$$

The total wave function is an eigenfunction of the total Hamiltonian H. Similarly, H can be expressed as

$$H = \sum_{i=1}^{n} H_i, \tag{4}$$

where H_i is the Hamiltonian for the ith clusterization state Ψ_i. On account of all the clusterizations describing the same nucleus, they are assumed as sharing the same total energy E of the total wave function. Thus the total energy E can be expressed as

$$E = \sum_{i=1}^{n} |a_i|^2 E = \sum_{i=1}^{n} E_{fi}, \tag{5}$$

where E_{fi} denotes the formation energy of cluster in the ith clusterization state Ψ_i. Therefore, the α preformation factor

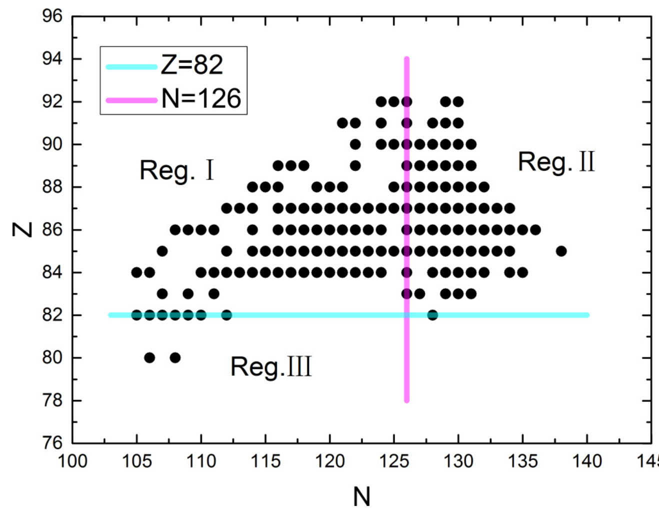

FIG. 1. Nuclide chart is divided into three regions. The cyan and magenta lines denote the $Z = 82$, $N = 126$ nuclear shell closures, respectively.

can be defined as

$$P_\alpha = |a_\alpha|^2 = \frac{E_{f\alpha}}{E}, \tag{6}$$

where a_α denotes the coefficient of the α clusterization state. $E_{f\alpha}$ is the formation energy of the α cluster. E is composed of the $E_{f\alpha}$ and the interaction energy between α cluster and daughter nuclei. In the framework of CFM [11–15], the α cluster-formation energy $E_{f\alpha}$ and total energy E of a considered system can be expressed as four different cases, as follows:

case I for even-even nuclei:

$$E_{f\alpha} = 3B(A,Z) + B(A-4,Z-2) \\ - 2B(A-1,Z-1) - 2B(A-1,Z), \tag{7a}$$

$$E = B(A,Z) - B(A-4,Z-2); \tag{7b}$$

case II for even Z-odd N, i.e., even-odd nuclei,

$$E_{f\alpha} = 3B(A-1,Z) + B(A-5,Z-2) \\ - 2B(A-2,Z-1) - 2B(A-2,Z), \tag{7c}$$

$$E = B(A,Z) - B(A-5,Z-2); \tag{7d}$$

case III for odd Z-even N, i.e., odd-even nuclei,

$$E_{f\alpha} = 3B(A-1,Z-1) + B(A-5,Z-3) \\ - 2B(A-2,Z-2) - 2B(A-2,Z-1), \tag{7e}$$

$$E = B(A,Z) - B(A-5,Z-3); \tag{7f}$$

case IV for doubly odd nuclei,

$$E_{f\alpha} = 3B(A-2,Z-1) + B(A-6,Z-3) \\ - 2B(A-3,Z-2) - 2B(A-3,Z-1), \tag{7g}$$

$$E = B(A,Z) - B(A-6,Z-3), \tag{7h}$$

where $B(A,Z)$ denotes the binding energy of nucleus with the mass number A and proton number Z.

TABLE I. Calculations of α decay half-lives and the α preformation factors of even-even nuclei in region I—III around $Z = 82$, $N = 126$ closed shells. The experimental α decay half-lives, spin, and parity are taken from the latest evaluated nuclear properties table NUBASE2016 [54], and the α decay energies are taken from the latest evaluated atomic mass table AME2016 [55,56]. The α preformation factors P_α are calculated within the CFM [11–15].

α transition	Q_α (MeV)	$j_p^\pi \to j_d^\pi$	l_{min}	P_α	$T_{1/2}^{expt}$ (s)	$T_{1/2}^{calc1}$ (s)	$T_{1/2}^{calc2}$ (s)	$T_{1/2}^{calc3}$ (s)
				Nuclei in region I				
$^{190}Po \to ^{186}Pb$	7.693	$0^+ \to 0^+$	0	0.262	2.46×10^{-3}	5.97×10^{-4}	2.28×10^{-3}	2.75×10^{-3}
$^{194}Po \to ^{190}Pb$	6.987	$0^+ \to 0^+$	0	0.235	3.92×10^{-1}	1.31×10^{-1}	5.56×10^{-1}	6.45×10^{-1}
$^{196}Po \to ^{192}Pb$	6.658	$0^+ \to 0^+$	0	0.222	5.67×10^{0}	2.19×10^{0}	9.87×10^{0}	1.12×10^{1}
$^{198}Po \to ^{194}Pb$	6.310	$0^+ \to 0^+$	0	0.206	1.85×10^{2}	5.61×10^{1}	2.72×10^{2}	2.97×10^{2}
$^{200}Po \to ^{196}Pb$	5.981	$0^+ \to 0^+$	0	0.187	6.20×10^{3}	1.57×10^{3}	8.44×10^{3}	8.66×10^{3}
$^{202}Po \to ^{198}Pb$	5.700	$0^+ \to 0^+$	0	0.178	1.39×10^{5}	3.42×10^{4}	1.92×10^{5}	1.95×10^{5}
$^{204}Po \to ^{200}Pb$	5.485	$0^+ \to 0^+$	0	0.158	1.88×10^{6}	4.18×10^{5}	2.64×10^{6}	2.49×10^{6}
$^{206}Po \to ^{202}Pb$	5.327	$0^+ \to 0^+$	0	0.145	1.39×10^{7}	2.85×10^{6}	1.96×10^{7}	1.77×10^{7}
$^{208}Po \to ^{204}Pb$	5.216	$0^+ \to 0^+$	0	0.135	9.15×10^{7}	1.15×10^{7}	8.51×10^{7}	7.47×10^{7}
$^{194}Rn \to ^{190}Po$	7.862	$0^+ \to 0^+$	0	0.262	7.80×10^{-4}	1.04×10^{-3}	3.99×10^{-3}	3.83×10^{-3}
$^{196}Rn \to ^{192}Po$	7.617	$0^+ \to 0^+$	0	0.257	4.70×10^{-3}	5.89×10^{-3}	2.29×10^{-2}	2.28×10^{-2}
$^{200}Rn \to ^{196}Po$	7.043	$0^+ \to 0^+$	0	0.228	1.17×10^{0}	5.19×10^{-1}	2.28×10^{0}	2.25×10^{0}
$^{202}Rn \to ^{198}Po$	6.773	$0^+ \to 0^+$	0	0.213	1.23×10^{1}	5.26×10^{0}	2.47×10^{1}	2.43×10^{1}
$^{204}Rn \to ^{200}Po$	6.547	$0^+ \to 0^+$	0	0.194	1.03×10^{2}	4.05×10^{1}	2.09×10^{2}	2.00×10^{2}
$^{206}Rn \to ^{202}Po$	6.384	$0^+ \to 0^+$	0	0.181	5.46×10^{2}	1.86×10^{2}	1.02×10^{3}	9.84×10^{2}
$^{208}Rn \to ^{204}Po$	6.260	$0^+ \to 0^+$	0	0.163	2.33×10^{3}	6.07×10^{2}	3.73×10^{3}	3.47×10^{3}
$^{210}Rn \to ^{206}Po$	6.159	$0^+ \to 0^+$	0	0.152	8.99×10^{3}	1.62×10^{3}	1.07×10^{4}	1.00×10^{4}
$^{212}Rn \to ^{208}Po$	6.385	$0^+ \to 0^+$	0	0.121	1.43×10^{3}	1.44×10^{2}	1.19×10^{3}	9.79×10^{2}
$^{202}Ra \to ^{198}Rn$	7.880	$0^+ \to 0^+$	0	0.248	4.10×10^{-3}	4.50×10^{-3}	1.82×10^{-2}	1.65×10^{-2}
$^{204}Ra \to ^{200}Rn$	7.637	$0^+ \to 0^+$	0	0.237	6.00×10^{-2}	2.62×10^{-2}	1.10×10^{-1}	1.04×10^{-1}
$^{208}Ra \to ^{204}Rn$	7.273	$0^+ \to 0^+$	0	0.199	1.27×10^{0}	4.20×10^{-1}	2.11×10^{0}	2.00×10^{0}
$^{214}Ra \to ^{210}Rn$	7.273	$0^+ \to 0^+$	0	0.139	2.44×10^{0}	3.25×10^{-1}	2.34×10^{0}	2.21×10^{0}
$^{212}Th \to ^{208}Ra$	7.958	$0^+ \to 0^+$	0	0.205	3.17×10^{-2}	1.14×10^{-2}	5.59×10^{-2}	5.64×10^{-2}
$^{214}Th \to ^{210}Ra$	7.827	$0^+ \to 0^+$	0	0.196	8.70×10^{-2}	2.84×10^{-2}	1.45×10^{-1}	1.63×10^{-1}
$^{216}U \to ^{212}Th$	8.530	$0^+ \to 0^+$	0	0.215	6.90×10^{3}	1.06×10^{-3}	4.93×10^{-3}	5.83×10^{-3}
				Nuclei in regions II and III				
$^{186}Hg \to ^{182}Pt$	5.204	$0^+ \to 0^+$	0	0.247	5.02×10^{5}	1.86×10^{5}	7.53×10^{5}	7.67×10^{5}
$^{188}Hg \to ^{184}Pt$	4.707	$0^+ \to 0^+$	0	0.239	3.33×10^{9}	1.56×10^{8}	6.53×10^{8}	6.53×10^{8}
$^{188}Pb \to ^{184}Hg$	6.109	$0^+ \to 0^+$	0	0.222	2.68×10^{2}	6.70×10^{1}	3.01×10^{2}	3.16×10^{2}
$^{190}Pb \to ^{186}Hg$	5.697	$0^+ \to 0^+$	0	0.215	1.76×10^{4}	5.17×10^{3}	2.40×10^{4}	2.44×10^{4}
$^{192}Pb \to ^{188}Hg$	5.221	$0^+ \to 0^+$	0	0.210	3.52×10^{6}	1.54×10^{6}	7.35×10^{6}	7.29×10^{6}
$^{194}Pb \to ^{190}Hg$	4.738	$0^+ \to 0^+$	0	0.198	1.71×10^{10}	1.23×10^{9}	6.19×10^{9}	5.79×10^{9}
$^{210}Pb \to ^{206}Hg$	3.793	$0^+ \to 0^+$	0	0.107	9.26×10^{16}	1.20×10^{16}	1.13×10^{17}	5.67×10^{16}
$^{210}Po \to ^{206}Pb$	5.408	$0^+ \to 0^+$	0	0.105	1.20×10^{7}	8.70×10^{5}	8.31×10^{6}	4.11×10^{6}
$^{212}Po \to ^{208}Pb$	8.954	$0^+ \to 0^+$	0	0.221	2.95×10^{-7}	5.78×10^{-8}	2.62×10^{-7}	2.69×10^{-7}
$^{214}Po \to ^{210}Pb$	7.834	$0^+ \to 0^+$	0	0.213	1.64×10^{-4}	7.03×10^{-5}	3.30×10^{-4}	3.23×10^{-4}
$^{216}Po \to ^{212}Pb$	6.907	$0^+ \to 0^+$	0	0.205	1.45×10^{-1}	9.63×10^{-2}	4.71×10^{-1}	4.36×10^{-1}
$^{218}Po \to ^{214}Pb$	6.115	$0^+ \to 0^+$	0	0.196	1.86×10^{2}	1.72×10^{2}	8.77×10^{2}	7.66×10^{2}
$^{214}Rn \to ^{210}Po$	9.208	$0^+ \to 0^+$	0	0.228	2.70×10^{-7}	6.95×10^{-8}	3.04×10^{-7}	3.19×10^{-7}
$^{216}Rn \to ^{212}Po$	8.198	$0^+ \to 0^+$	0	0.237	4.50×10^{-5}	3.42×10^{-5}	1.44×10^{-4}	1.53×10^{-4}
$^{218}Rn \to ^{214}Po$	7.263	$0^+ \to 0^+$	0	0.234	3.38×10^{-2}	3.52×10^{-2}	1.50×10^{-1}	1.53×10^{-1}
$^{220}Rn \to ^{216}Po$	6.405	$0^+ \to 0^+$	0	0.221	5.56×10^{1}	7.97×10^{1}	3.61×10^{2}	3.37×10^{2}
$^{222}Rn \to ^{218}Po$	5.591	$0^+ \to 0^+$	0	0.222	3.30×10^{5}	6.49×10^{5}	2.93×10^{6}	2.68×10^{6}
$^{216}Ra \to ^{212}Rn$	9.526	$0^+ \to 0^+$	0	0.239	1.82×10^{-7}	5.88×10^{-8}	2.46×10^{-7}	2.66×10^{-7}
$^{218}Ra \to ^{214}Rn$	8.546	$0^+ \to 0^+$	0	0.242	2.52×10^{-5}	1.96×10^{-5}	8.09×10^{-5}	8.50×10^{-5}
$^{220}Ra \to ^{216}Rn$	7.592	$0^+ \to 0^+$	0	0.240	1.79×10^{-2}	1.73×10^{-2}	7.21×10^{-2}	7.22×10^{-2}
$^{216}Th \to ^{212}Ra$	8.072	$0^+ \to 0^+$	0	0.159	2.60×10^{-2}	4.11×10^{-3}	2.59×10^{-2}	1.94×10^{-2}
$^{218}Th \to ^{214}Ra$	9.849	$0^+ \to 0^+$	0	0.251	1.17×10^{-7}	4.92×10^{-8}	1.96×10^{-7}	2.20×10^{-7}
$^{220}Th \to ^{216}Ra$	8.953	$0^+ \to 0^+$	0	0.247	9.70×10^{-6}	8.11×10^{-6}	3.28×10^{-5}	3.43×10^{-5}
$^{218}U \to ^{214}Th$	8.775	$0^+ \to 0^+$	0	0.189	5.50×10^{-4}	1.85×10^{-4}	9.75×10^{-4}	8.72×10^{-4}
$^{222}U \to ^{218}Th$	9.478	$0^+ \to 0^+$	0	0.246	4.70×10^{-6}	1.79×10^{-6}	7.30×10^{-6}	7.40×10^{-6}

TABLE II. Same as Table I, but for favored α decay of odd-A nuclei. "()" means uncertain spin and/or parity, and "#" means values estimated from trends in neighboring nuclides with the same Z and N parities, which are taken from NUBASE2016 [54].

α transition	Q_α (MeV)	$j_p^\pi \rightarrow j_d^\pi$	$l_{\min}$	P_α	$T_{1/2}^{\text{expt}}$ (s)	$T_{1/2}^{\text{calc1}}$ (s)	$T_{1/2}^{\text{calc2}}$ (s)	$T_{1/2}^{\text{calc3}}$ (s)
Nuclei in region I								
^{195}Po $\rightarrow$ ^{191}Pb	6.745	$(3/2^-) \rightarrow (3/2^-)$	0	0.170	4.92×10^{0}	1.04×10^{0}	6.11×10^{0}	6.26×10^{0}
^{197}Po $\rightarrow$ ^{193}Pb	6.405	$(3/2^-) \rightarrow (3/2^-)$	0	0.162	1.20×10^{2}	2.30×10^{1}	1.42×10^{2}	1.44×10^{2}
^{199}Po $\rightarrow$ ^{195}Pb	6.075	$(3/2^-) \rightarrow 3/2^-$	0	0.152	4.36×10^{3}	6.01×10^{2}	3.95×10^{3}	3.92×10^{3}
^{201}Po $\rightarrow$ ^{197}Pb	5.799	$3/2^- \rightarrow 3/2^-$	0	0.139	8.26×10^{4}	1.14×10^{4}	8.20×10^{4}	7.77×10^{4}
^{205}Po $\rightarrow$ ^{201}Pb	5.325	$5/2^- \rightarrow 5/2^-$	0	0.120	1.53×10^{7}	3.05×10^{6}	2.55×10^{7}	2.28×10^{7}
^{207}Po $\rightarrow$ ^{203}Pb	5.216	$5/2^- \rightarrow 5/2^-$	0	0.111	9.85×10^{7}	1.18×10^{7}	1.07×10^{8}	9.33×10^{7}
^{197}At $\rightarrow$ ^{193}Bi	7.105	$(9/2^-) \rightarrow (9/2^-)$	0	0.220	4.04×10^{-1}	1.21×10^{-1}	5.50×10^{-1}	6.51×10^{-1}
^{199}At $\rightarrow$ ^{195}Bi	6.778	$9/2^{(-)} \rightarrow 9/2^{(-)}$	0	0.200	7.83×10^{0}	1.92×10^{0}	9.58×10^{0}	1.09×10^{1}
^{201}At $\rightarrow$ ^{197}Bi	6.473	$(9/2^-) \rightarrow (9/2^-)$	0	0.177	1.19×10^{2}	3.07×10^{1}	1.73×10^{2}	1.85×10^{2}
^{203}At $\rightarrow$ ^{199}Bi	6.210	$9/2^- \rightarrow 9/2^-$	0	0.167	1.42×10^{3}	3.95×10^{2}	2.37×10^{3}	2.53×10^{3}
^{205}At $\rightarrow$ ^{201}Bi	6.019	$9/2^- \rightarrow 9/2^-$	0	0.146	1.99×10^{4}	2.76×10^{3}	1.90×10^{4}	1.88×10^{4}
^{207}At $\rightarrow$ ^{203}Bi	5.873	$9/2^- \rightarrow 9/2^-$	0	0.132	6.52×10^{4}	1.28×10^{4}	9.73×10^{4}	9.38×10^{4}
^{209}At $\rightarrow$ ^{205}Bi	5.757	$9/2^- \rightarrow 9/2^-$	0	0.121	4.70×10^{5}	4.49×10^{4}	3.71×10^{5}	3.53×10^{5}
^{211}At $\rightarrow$ ^{207}Bi	5.983	$9/2^- \rightarrow 9/2^-$	0	0.093	6.21×10^{4}	3.23×10^{3}	3.49×10^{4}	2.76×10^{4}
^{195}Rn $\rightarrow$ ^{191}Po	7.694	$3/2^- \rightarrow (3/2^-)$	0	0.183	7.00×10^{-3}	3.45×10^{-3}	1.88×10^{-2}	1.51×10^{-2}
^{197}Rn $\rightarrow$ ^{193}Po	7.410	$(3/2^-) \rightarrow (3/2^-)$	0	0.182	5.40×10^{-2}	2.83×10^{-2}	1.56×10^{-1}	1.31×10^{-1}
^{203}Rn $\rightarrow$ ^{199}Po	6.629	$3/2^-\# \rightarrow (3/2^-)$	0	0.155	6.58×10^{1}	1.93×10^{1}	1.24×10^{2}	1.10×10^{2}
^{207}Rn $\rightarrow$ ^{203}Po	6.251	$5/2^- \rightarrow 5/2^-$	0	0.135	2.61×10^{3}	6.95×10^{2}	5.15×10^{3}	4.64×10^{3}
^{209}Rn $\rightarrow$ ^{205}Po	6.155	$5/2^- \rightarrow 5/2^-$	0	0.122	1.00×10^{4}	1.76×10^{3}	1.44×10^{4}	1.28×10^{4}
^{199}Fr $\rightarrow$ ^{195}At	7.816	$1/2^+\# \rightarrow 1/2^+$	0	0.247	6.60×10^{-3}	3.09×10^{-3}	1.25×10^{-2}	1.33×10^{-2}
^{201}Fr $\rightarrow$ ^{197}At	7.519	$(9/2^-) \rightarrow (9/2^-)$	0	0.231	6.28×10^{-2}	2.75×10^{-2}	1.19×10^{-1}	1.28×10^{-1}
^{203}Fr $\rightarrow$ ^{199}At	7.274	$9/2^- \rightarrow 9/2^{(-)}$	0	0.211	5.50×10^{-1}	1.83×10^{-1}	8.67×10^{-1}	9.20×10^{-1}
^{205}Fr $\rightarrow$ ^{201}At	7.054	$9/2^- \rightarrow (9/2^-)$	0	0.188	3.82×10^{0}	1.09×10^{0}	5.80×10^{0}	5.99×10^{0}
^{207}Fr $\rightarrow$ ^{203}At	6.894	$9/2^- \rightarrow 9/2^-$	0	0.174	1.55×10^{1}	4.16×10^{0}	2.39×10^{1}	2.50×10^{1}
^{209}Fr $\rightarrow$ ^{205}At	6.777	$9/2^- \rightarrow 9/2^-$	0	0.153	5.66×10^{1}	1.12×10^{1}	7.29×10^{1}	7.44×10^{1}
^{211}Fr $\rightarrow$ ^{207}At	6.662	$9/2^- \rightarrow 9/2^-$	0	0.140	2.13×10^{2}	3.03×10^{1}	2.16×10^{2}	2.27×10^{2}
^{213}Fr $\rightarrow$ ^{209}At	6.905	$9/2^- \rightarrow 9/2^-$	0	0.110	3.43×10^{1}	2.92×10^{0}	2.66×10^{1}	2.49×10^{1}
^{203}Ra $\rightarrow$ ^{199}Rn	7.735	$(3/2^-) \rightarrow (3/2^-)$	0	0.175	3.60×10^{-2}	1.28×10^{-2}	7.34×10^{-2}	5.70×10^{-2}
^{209}Ra $\rightarrow$ ^{205}Rn	7.143	$5/2^- \rightarrow 5/2^-$	0	0.144	4.71×10^{0}	1.22×10^{0}	8.49×10^{0}	7.36×10^{0}
^{205}Ac $\rightarrow$ ^{201}Fr	8.096	$9/2^-\# \rightarrow (9/2^-)$	0	0.247	8.00×10^{-2}	2.14×10^{-3}	8.67×10^{-3}	9.22×10^{-3}
^{207}Ac $\rightarrow$ ^{203}Fr	7.849	$9/2^-\# \rightarrow 9/2^-$	0	0.225	3.10×10^{-2}	1.21×10^{-2}	5.39×10^{-2}	5.81×10^{-2}
^{211}Ac $\rightarrow$ ^{207}Fr	7.619	$9/2^- \rightarrow 9/2^-$	0	0.184	2.13×10^{-1}	6.05×10^{-2}	3.30×10^{-1}	3.71×10^{-1}
^{213}Pa $\rightarrow$ ^{209}Ac	8.395	$9/2^-\# \rightarrow (9/2^-)$	0	0.207	7.00×10^{-3}	1.21×10^{-3}	5.82×10^{-3}	6.84×10^{-3}
^{215}Pa $\rightarrow$ ^{211}Ac	8.235	$9/2^-\# \rightarrow 9/2^-$	0	0.195	1.40×10^{-2}	3.44×10^{-3}	1.76×10^{-2}	2.35×10^{-2}
Nuclei in regions II and III								
^{191}Pb $\rightarrow$ ^{187}Hg	5.463	$(3/2^-) \rightarrow 3/2^{(-)}$	0	0.160	1.55×10^{4}	7.72×10^{4}	4.83×10^{5}	4.73×10^{5}
^{213}Po $\rightarrow$ ^{209}Pb	8.536	$9/2^+ \rightarrow 9/2^+$	0	0.180	3.71×10^{-6}	6.82×10^{-7}	3.78×10^{-6}	3.97×10^{-6}
^{215}Po $\rightarrow$ ^{211}Pb	7.527	$9/2^+ \rightarrow 9/2^+$	0	0.177	1.78×10^{-3}	6.50×10^{-4}	3.66×10^{-3}	3.66×10^{-3}
^{219}Po $\rightarrow$ ^{215}Pb	5.916	$9/2^+\# \rightarrow 9/2^+\#$	0	0.167	2.19×10^{3}	1.41×10^{3}	8.43×10^{3}	7.47×10^{3}
^{213}At $\rightarrow$ ^{209}Bi	9.254	$9/2^- \rightarrow 9/2^-$	0	0.187	1.25×10^{-7}	2.40×10^{-8}	1.28×10^{-7}	1.40×10^{-7}
^{215}At $\rightarrow$ ^{211}Bi	8.178	$9/2^- \rightarrow 9/2^-$	0	0.178	1.00×10^{-4}	1.60×10^{-5}	8.98×10^{-5}	8.85×10^{-5}
^{217}At $\rightarrow$ ^{213}Bi	7.202	$9/2^- \rightarrow 9/2^-$	0	0.168	3.26×10^{-2}	2.15×10^{-2}	1.28×10^{-1}	1.14×10^{-1}
^{219}At $\rightarrow$ ^{215}Bi	6.342	$(9/2^-) \rightarrow (9/2^-)$	0	0.158	5.98×10^{1}	5.02×10^{1}	3.17×10^{2}	2.54×10^{2}
^{215}Rn $\rightarrow$ ^{211}Po	8.839	$9/2^+ \rightarrow 9/2^+$	0	0.182	2.30×10^{-6}	5.80×10^{-7}	3.20×10^{-6}	3.22×10^{-6}
^{217}Rn $\rightarrow$ ^{213}Po	7.888	$9/2^+ \rightarrow 9/2^+$	0	0.192	5.40×10^{-4}	2.90×10^{-4}	1.51×10^{-3}	1.51×10^{-3}
^{215}Fr $\rightarrow$ ^{211}At	9.541	$9/2^- \rightarrow 9/2^-$	0	0.201	8.60×10^{-8}	2.46×10^{-8}	1.23×10^{-7}	1.38×10^{-7}
^{217}Fr $\rightarrow$ ^{213}At	8.470	$9/2^- \rightarrow 9/2^-$	0	0.204	1.68×10^{-5}	1.34×10^{-5}	6.58×10^{-5}	7.00×10^{-5}
^{219}Fr $\rightarrow$ ^{215}At	7.449	$9/2^- \rightarrow 9/2^-$	0	0.198	2.00×10^{-2}	2.05×10^{-2}	1.04×10^{-1}	9.96×10^{-2}
^{217}Ra $\rightarrow$ ^{213}Rn	9.161	$(9/2^+) \rightarrow 9/2^+\#$	0	0.185	1.63×10^{-6}	4.50×10^{-7}	2.43×10^{-6}	2.38×10^{-6}
^{215}Ac $\rightarrow$ ^{211}Fr	7.746	$9/2^- \rightarrow 9/2^-$	0	0.130	1.70×10^{-1}	1.88×10^{-2}	1.44×10^{-1}	1.15×10^{-1}
^{217}Ac $\rightarrow$ ^{213}Fr	9.832	$9/2^- \rightarrow 9/2^-$	0	0.218	6.90×10^{-8}	2.47×10^{-8}	1.14×10^{-7}	1.35×10^{-7}
^{219}Ac $\rightarrow$ ^{215}Fr	8.827	$9/2^- \rightarrow 9/2^-$	0	0.216	1.18×10^{-5}	7.63×10^{-6}	3.53×10^{-5}	3.75×10^{-5}
^{219}Th $\rightarrow$ ^{215}Ra	9.511	$9/2^+\# \rightarrow 9/2^+\#$	0	0.189	1.02×10^{-6}	3.05×10^{-7}	1.61×10^{-6}	1.54×10^{-6}
^{217}Pa $\rightarrow$ ^{213}Ac	8.488	$9/2^-\# \rightarrow 9/2^-\#$	0	0.155	3.48×10^{-3}	5.29×10^{-4}	3.42×10^{-3}	3.24×10^{-3}
^{219}Pa $\rightarrow$ ^{215}Ac	10.084	$9/2^- \rightarrow 9/2^-$	0	0.236	5.30×10^{-8}	3.08×10^{-8}	1.30×10^{-7}	1.63×10^{-7}

TABLE II. (*Continued.*)

α transition	Q_α (MeV)	$j_p^\pi \to j_d^\pi$	l_{min}	P_α	$T_{1/2}^{expt}$ (s)	$T_{1/2}^{calc1}$ (s)	$T_{1/2}^{calc2}$ (s)	$T_{1/2}^{calc3}$ (s)
$^{221}Pa \to {}^{217}Ac$	9.251	$9/2^- \to 9/2^-$	0	0.229	5.90×10^{-6}	3.02×10^{-6}	1.32×10^{-5}	1.41×10^{-5}
$^{221}U \to {}^{217}Th$	9.889	$(9/2^+) \to 9/2^+$#	0	0.186	6.60×10^{-7}	1.83×10^{-7}	9.82×10^{-7}	8.87×10^{-7}

B. α decay half-life and proximity potential 1977 formalism

The α decay half-life can be calculated by decay width Γ or decay constant λ and expressed as

$$T_{1/2} = \frac{\hbar \ln 2}{\Gamma} = \frac{\ln 2}{\lambda}, \tag{8}$$

where $\hbar$ is the Planck constant. In the framework of the Proximity potential 1977 formalism (Prox.1977) [48], the α decay constant λ is calculated by

$$\lambda = P_\alpha \nu P, \tag{9}$$

where P_α denotes α preformation factors. In CPPM, the P_α is left out of consideration or assumed as $P_\alpha = 1$. The assault frequency ν can be obtained by the oscillation frequency ω [21], and expressed as

$$\nu = \frac{\omega}{2\pi} = \frac{(2n_r + l + \frac{3}{2})\hbar}{2\pi\mu R_n^2} = \frac{(G + \frac{3}{2})\hbar}{1.2\pi\mu R_0^2}, \tag{10}$$

where $\mu = \frac{m_d m_\alpha}{m_d + m_\alpha}$ denotes the reduced mass between daughter nucleus and preformed α particle with the mass of daughter nucleus m_d and α particle m_α. The nucleus root-mean-square (rms) radius $R_n = \sqrt{\frac{3}{5}} R_0$ with $R_0 = 1.240 A^{1/3}(1 + \frac{1.646}{A} - 0.191\frac{A-2Z}{A})$ [49], where A and Z are mass number and proton number of parent nucleus. $G = 2n_r + l$ denotes the principal quantum number with radial quantum number n_r and angular momentum quantum number l. For α decay [50], G can be obtained by

$$G = 2n_r + l = \begin{cases} 18, & N \leqslant 82, \\ 20, & 82 < N \leqslant 126, \\ 22, & N > 126. \end{cases} \tag{11}$$

P, the semiclassical Wentzel-Kramers-Brillouin (WKB) barrier penetrate probability, can be calculated by

$$P = \exp\left(-2\int_{r_{in}}^{r_{out}} k(r)dr\right), \tag{12}$$

where $k(r) = \sqrt{\frac{2\mu}{\hbar^2}|Q_\alpha - V(r)|}$ is the wave number of the α particle. r is the center of mass distance between the daughter nucleus and the preformed α particle. $V(r)$ and Q_α are the total α-core potential and α decay energy, respectively. r_{in} and r_{out} are the classical turning points, they satisfy the conditions $V(r_{in}) = V(r_{out}) = Q_\alpha$.

The total interaction potential $V(r)$ between α particle and daughter nucleus is composed of three parts: the nuclear potential $V_N(r)$, the Coulomb potential $V_C(r)$, and the centrifugal potential $V_l(r)$. It can be expressed as

$$V(r) = V_N(r) + V_C(r) + V_l(r). \tag{13}$$

TABLE III. Same as Tables I and II, but for unfavored α decay of odd-A nuclei.

α transition	Q_α (MeV)	$j_p^\pi \to j_d^\pi$	l_{min}	P_α	$T_{1/2}^{expt}$ (s)	$T_{1/2}^{calc1}$ (s)	$T_{1/2}^{calc2}$ (s)	$T_{1/2}^{calc3}$ (s)
				Nuclei in region I				
$^{209}Bi \to {}^{205}Tl$	3.138	$9/2^- \to 1/2^+$	5	0.094	6.34×10^{26}			
$^{189}Po \to {}^{185}Pb$	7.694	$(5/2^-) \to 3/2^-$	2	0.191	3.80×10^{-3}	1.25×10^{-3}	6.53×10^{-3}	7.14×10^{-3}
$^{203}Po \to {}^{199}Pb$	5.496	$5/2^- \to 3/2^-$	2	0.134	1.97×10^{6}	8.15×10^{5}	6.09×10^{6}	5.99×10^{6}
$^{205}Rn \to {}^{201}Po$	6.386	$5/2^- \to 3/2^-$	2	0.143	6.88×10^{2}	3.95×10^{2}	2.77×10^{3}	2.54×10^{3}
$^{207}Ra \to {}^{203}Rn$	7.269	$5/2^-$# $\to 3/2^-$#	2	0.160	1.60×10^{0}	9.17×10^{-1}	5.73×10^{0}	5.24×10^{0}
$^{213}Ra \to {}^{209}Rn$	6.862	$1/2^- \to 5/2^-$	2	0.126	2.03×10^{2}	2.61×10^{1}	2.06×10^{2}	2.08×10^{2}
$^{215}Th \to {}^{211}Ra$	7.665	$(1/2^-) \to 5/2(^-)$	2	0.142	1.20×10^{0}	1.93×10^{-1}	1.36×10^{0}	1.51×10^{0}
$^{217}U \to {}^{213}Th$	8.425	$1/2^-$# $\to 5/2^-$#	2	0.152	8.00×10^{-4}	4.11×10^{-3}	2.70×10^{-2}	3.14×10^{-2}
				Nuclei in regions II and III				
$^{187}Pb \to {}^{183}Hg$	6.393	$3/2^- \to 1/2^-$	2	0.166	1.60×10^{2}	8.89×10^{0}	5.35×10^{1}	9.39×10^{1}
$^{189}Pb \to {}^{185}Hg$	5.915	$3/2^- \to 1/2^-$	2	0.163	9.75×10^{3}	1.03×10^{3}	6.33×10^{3}	1.09×10^{4}
$^{213}Bi \to {}^{209}Tl$	5.988	$9/2^- \to 1/2^+$	5	0.092	1.31×10^{5}			
$^{223}At \to {}^{219}Bi$	4.723	$3/2^-$# $\to 9/2^-$#	4	0.161	6.25×10^{5}			
$^{213}Rn \to {}^{209}Po$	8.245	$9/2^+$# $\to 1/2^-$	5	0.098	1.95×10^{-2}	8.47×10^{-4}	8.65×10^{-3}	8.09×10^{-3}
$^{219}Rn \to {}^{215}Po$	6.946	$5/2^+ \to 9/2^+$	2	0.193	3.96×10^{0}	1.02×10^{0}	5.26×10^{0}	6.19×10^{0}
$^{221}Rn \to {}^{217}Po$	6.162	$7/2^+ \to (9/2^+)$	2	0.185	6.98×10^{3}	1.97×10^{3}	1.07×10^{4}	1.07×10^{4}
$^{221}Fr \to {}^{217}At$	6.457	$5/2^- \to 9/2^-$	2	0.182	2.88×10^{2}	2.89×10^{2}	1.59×10^{3}	1.49×10^{3}
$^{215}Ra \to {}^{211}Rn$	8.864	$9/2^+$# $\to 1/2^-$	5	0.109	1.67×10^{-3}	8.07×10^{-5}	7.39×10^{-4}	7.36×10^{-4}
$^{219}Ra \to {}^{215}Rn$	8.138	$(7/2)^+ \to 9/2^+$	2	0.190	1.00×10^{-2}	5.88×10^{-4}	3.08×10^{-3}	3.47×10^{-3}
$^{217}Th \to {}^{213}Ra$	9.435	$9/2^+$# $\to 1/2^-$	5	0.122	2.47×10^{-4}	1.31×10^{-5}	1.08×10^{-4}	1.14×10^{-4}
$^{221}Th \to {}^{217}Ra$	8.625	$7/2^+$# $\to (9/2^+)$	2	0.190	1.78×10^{-3}	1.23×10^{-4}	6.47×10^{-4}	6.35×10^{-4}

TABLE IV. Same as Tables I and II, but for favored α decay of doubly odd nuclei.

α transition	Q_α (MeV)	$j_p^\pi \to j_d^\pi$	$l_{\min}$	P_α	$T_{1/2}^{\text{expt}}$ (s)	$T_{1/2}^{\text{calc1}}$ (s)	$T_{1/2}^{\text{calc2}}$ (s)	$T_{1/2}^{\text{calc3}}$ (s)
				Nuclei in region I				
^{192}At $\to$ ^{188}Bi	7.696	$3^+\# \to 3^+\#$	0	0.190	1.15×10^{-2}	1.44×10^{-3}	7.60×10^{-3}	8.41×10^{-3}
^{200}At $\to$ ^{196}Bi	6.596	$(3^+) \to (3^+)$	0	0.147	8.26×10^{1}	9.88×10^{0}	6.73×10^{1}	7.10×10^{1}
^{202}At $\to$ ^{198}Bi	6.353	$3(^+) \to 3(^+)$	0	0.133	1.45×10^{3}	9.61×10^{1}	7.23×10^{2}	7.33×10^{2}
^{204}At $\to$ ^{200}Bi	6.071	$7^+ \to 7^+$	0	0.126	1.43×10^{4}	1.64×10^{3}	1.30×10^{4}	1.33×10^{4}
^{206}At $\to$ ^{202}Bi	5.886	$(5)^+ \to 5(^+\#)$	0	0.112	2.02×10^{5}	1.16×10^{4}	1.03×10^{5}	1.00×10^{5}
^{208}At $\to$ ^{204}Bi	5.751	$6^+ \to 6^+$	0	0.102	1.06×10^{6}	5.01×10^{4}	4.90×10^{5}	4.68×10^{5}
^{200}Fr $\to$ ^{196}At	7.615	$(3^+) \to (3^+)$	0	0.173	4.75×10^{-2}	1.36×10^{-2}	7.86×10^{-2}	7.47×10^{-2}
^{204}Fr $\to$ ^{200}At	7.170	$3^+ \to (3^+)$	0	0.153	1.82×10^{0}	4.20×10^{-1}	2.75×10^{0}	2.71×10^{0}
^{206}Fr $\to$ ^{202}At	6.924	$3^+ \to 3(^+)$	0	0.139	1.81×10^{1}	3.31×10^{0}	2.39×10^{1}	2.33×10^{1}
^{208}Fr $\to$ ^{204}At	6.784	$7^+ \to 7^+$	0	0.129	6.62×10^{1}	1.09×10^{1}	8.45×10^{1}	8.49×10^{1}
^{206}Ac $\to$ ^{202}Fr	7.959	$(3^+) \to 3^+$	0	0.172	2.50×10^{-2}	5.57×10^{-3}	3.25×10^{-2}	3.10×10^{-2}
				Nuclei in regions II and III				
^{214}At $\to$ ^{210}Bi	8.987	$1^- \to 1^-$	0	0.154	5.58×10^{-7}	1.05×10^{-7}	6.80×10^{-7}	7.02×10^{-7}
^{216}At $\to$ ^{212}Bi	7.950	$1(^-) \to 1(^-)$	0	0.150	3.00×10^{-4}	7.42×10^{-5}	4.95×10^{-4}	4.76×10^{-4}
^{218}At $\to$ ^{214}Bi	6.874	$1^-\# \to 1^-$	0	0.144	1.50×10^{0}	3.43×10^{-1}	2.38×10^{0}	2.11×10^{0}
^{216}Fr $\to$ ^{212}At	9.175	$(1^-) \to (1^-)$	0	0.161	7.00×10^{-7}	1.83×10^{-7}	1.14×10^{-6}	1.18×10^{-6}
^{218}Fr $\to$ ^{214}At	8.014	$1^- \to 1^-$	0	0.166	1.00×10^{-3}	2.94×10^{-4}	1.77×10^{-3}	1.76×10^{-3}
^{218}Ac $\to$ ^{214}Fr	9.374	$1^-\# \to (1^-)$	0	0.169	1.00×10^{-6}	2.96×10^{-7}	1.75×10^{-6}	1.82×10^{-6}
^{220}Pa $\to$ ^{216}Ac	9.651	$1^-\# \to (1^-)$	0	0.178	7.80×10^{-7}	3.09×10^{-7}	1.73×10^{-6}	1.83×10^{-6}

The Coulomb potential $V_C(r)$ is hypothesized as the potential of a uniformly charged sphere with sharp radius R and is expressed as

$$V_C(r) = \begin{cases} \frac{Z_d Z_\alpha e^2}{2R}\left[3 - \left(\frac{r}{R}\right)^2\right], & r < R, \\ \frac{Z_d Z_\alpha e^2}{r}, & r > R, \end{cases} \tag{14}$$

where $R = R_1 + R_2$ with $R_i = 1.28A_i^{1/3} - 0.76 + 0.8A_i^{-1/3}(i = 1,2)$. R_1 and R_2 denote the radius of the daughter nucleus and α particle, respectively. Z_d and Z_α are the proton number of daughter nucleus and α particle, respectively.

Because $l(l+1)\to(l+1/2)^2$ is a necessary correction for one-dimensional problems [51], we adopt the Langer modified centrifugal barrier $V_l(r)$, which can be expressed as

$$V_l(r) = \frac{\hbar^2(l+1/2)^2}{2\mu r^2}, \tag{15}$$

where l is the angular momentum taken away by an α particle. On the basis of the conservation laws of angular momentum and parity [52], the minimum angular momentum $l_{\min}$ taken

TABLE V. Same as Tables I and II, but for unfavored α decay of doubly odd nuclei.

α transition	Q_α (MeV)	$j_p^\pi \to j_d^\pi$	$l_{\min}$	P_α	$T_{1/2}^{\text{expt}}$ (s)	$T_{1/2}^{\text{calc1}}$ (s)	$T_{1/2}^{\text{calc2}}$ (s)	$T_{1/2}^{\text{calc3}}$ (s)
				Nuclei in region I				
^{190}Bi $\to$ ^{186}Tl	6.862	$(3^+) \to (2^-)$	1	0.163	8.16×10^{0}	2.01×10^{-1}	1.23×10^{0}	1.44×10^{0}
^{192}Bi $\to$ ^{188}Tl	6.381	$(3^+) \to (2^-)$	1	0.157	2.77×10^{2}	1.50×10^{1}	9.54×10^{1}	1.10×10^{2}
^{194}Bi $\to$ ^{190}Tl	5.918	$(3^+) \to 2(^-)$	1	0.152	2.05×10^{4}	1.59×10^{3}	1.04×10^{4}	1.19×10^{4}
^{210}At $\to$ ^{206}Bi	5.631	$(5)^+ \to 6(^+)$	2	0.095	1.66×10^{7}	4.11×10^{5}	4.34×10^{6}	3.61×10^{6}
^{210}Fr $\to$ ^{206}At	6.672	$6^+ \to (5)^+$	2	0.115	2.67×10^{2}	5.90×10^{1}	5.11×10^{2}	4.43×10^{2}
^{212}Fr $\to$ ^{208}At	6.529	$5^+ \to 6^+$	2	0.107	2.78×10^{3}	2.18×10^{2}	2.04×10^{3}	1.87×10^{3}
^{212}Pa $\to$ ^{208}Ac	8.415	$7^+\# \to (3^+)$	4	0.167	7.50×10^{-3}	1.07×10^{-2}	6.43×10^{-2}	5.89×10^{-2}
				Nuclei in regions II and III				
^{210}Bi $\to$ ^{206}Tl	5.037	$1^- \to 0^-$	2	0.082	4.13×10^{11}	6.60×10^{7}	8.05×10^{8}	8.54×10^{8}
^{212}Bi $\to$ ^{208}Tl	6.207	$1(^-) \to 5^+$	5	0.079	1.01×10^{4}			
^{214}Bi $\to$ ^{210}Tl	5.621	$1^- \to 5^+\#$	5	0.081	5.66×10^{6}			
^{212}At $\to$ ^{208}Bi	7.817	$(1^-) \to 5^+$	5	0.077	3.14×10^{-1}	7.35×10^{-3}	9.49×10^{-2}	8.91×10^{-2}
^{214}Fr $\to$ ^{210}At	8.588	$(1^-) \to (5)^+$	5	0.089	5.18×10^{-3}	2.02×10^{-4}	2.26×10^{-3}	2.30×10^{-3}
^{220}Fr $\to$ ^{216}At	6.800	$1^+ \to 1(^-)$	1	0.163	2.74×10^{1}	6.73×10^{0}	4.12×10^{1}	4.02×10^{1}
^{216}Ac $\to$ ^{212}Fr	9.235	$(1^-) \to 5^+$	5	0.103	4.40×10^{-4}	1.89×10^{-5}	1.83×10^{-4}	2.02×10^{-4}
^{220}Ac $\to$ ^{216}Fr	8.348	$(3^-) \to (1^-)$	2	0.171	2.64×10^{-2}	3.33×10^{-4}	1.95×10^{-3}	1.99×10^{-3}

away by the α particle can be obtained by

$$l_{\min} = \begin{cases} \Delta_j, & \text{for even } \Delta_j \text{ and } \pi_p = \pi_d, \\ \Delta_j + 1, & \text{for even } \Delta_j \text{ and } \pi_p \neq \pi_d, \\ \Delta_j, & \text{for odd } \Delta_j \text{ and } \pi_p \neq \pi_d, \\ \Delta_j + 1, & \text{for odd } \Delta_j \text{ and } \pi_p = \pi_d, \end{cases} \tag{16}$$

where $\Delta_j = |j_p - j_d|$, j_p, π_p, j_d, π_d denote the spin and parity values of the parent and daughter nuclei, respectively.

The nuclear potential $V_N(r)$ is obtained by

$$V_N(r) = 4\pi\gamma b\bar{R}\phi(\xi), \tag{17}$$

where γ, the surface energy coefficient, is obtained by the Myers and Świątecki formula [53] and expressed as

$$\gamma = \gamma_0(1 - k_s I^2), \tag{18}$$

where I denotes the isospin of the parent nucleus. The surface energy constant $\gamma_0 = 0.9517$ MeV/fm^2 and surface asymmetry constant $k_s = 1.7826$ [53]. The mean curvature radius $\bar{R}$ can be obtained by

$$\bar{R} = \frac{C_1 C_2}{C_1 + C_2}, \tag{19}$$

where $C_i = R_i[1 - (\frac{b}{R_i})^2](i = 1,2)$ with C_1 and C_2 representing the matter radius of daughter nucleus and α particle, respectively. b is the diffuseness of nuclear surface, which is taken as unity. The universal function $\phi(\xi)$ is expressed as

$$\phi(\xi) = \begin{cases} -\frac{1}{2}(\xi - 2.54)^2 - 0.0852(\xi - 2.54)^3, & \xi \leqslant 1.2511, \\ -3.437\exp\left(-\frac{\xi}{0.75}\right), & \xi \geqslant 1.2511, \end{cases} \tag{20}$$

where $\xi = (r - C_1 - C_2)/b$ denotes the minimum separation distance.

III. RESULTS AND DISCUSSION

The aims of this work are to study the α preformation factors and α decay half-lives of nuclei around $Z = 82$, $N = 126$ shell closures. Many researchers suggested that the smaller valance nucleons (holes) the nuclei have, the smaller the α preformation factors are [38–40]. In 2011, Seif *et al.* have put forward that the P_α of even-even nuclei around the $Z = 82$, $N = 126$ closed shells linearly depend on the product of the valance protons (holes) and neutrons (holes) N_pN_n [7]. Moreover, in our previous works, we systematically studied the P_α of the favored and unfavored α decay for odd-A and doubly odd nuclei, which was extracted from the ratio of calculated α decay half-life to the experimental data [43,44]. The results indicated that the P_α is linearly related to the N_pN_n although it is model dependent. Recently, the CFM [11–15] was proposed to calculate the P_α with the difference of binding energy. It is a simple, effective, and microscopic way. Once the binding energies of parent nuclei and neighboring nuclei are known, one can easily evaluate the P_α. Therefore, it is interesting to validate whether the realistic α preformation factor within CFM is also linearly dependent on N_pN_n. In addition, the Prox.1977 leaves P_α out of consideration or assumes as $P_\alpha = 1$, thus the deviation between calculated α decay half-life and experimental one is considerable [45–47]. For confirming CFM and diminishing the difference between theoretical calculation and experimental data, in this work, we also calculate α decay half-lives of 159 nuclei (including 50 even-even nuclei, 76 odd-A nuclei, and 33 doubly odd nuclei) around $Z = 82$, $N = 126$ shell closures within Prox.1977 taking $P_\alpha = 1$ and the realistic P_α evaluated by CFM, respectively.

For the purpose of a simple description, we plot a nuclide distribution map in Fig. 1, and the area is divided into three regions by magic numbers ($Z = 82$, $N = 126$). In region I, the proton numbers are above the $Z = 82$ shell closure and the neutron numbers are below the $N = 126$ closed shell, thus the N_pN_n are negative. By that analogy, in regions II and III the N_pN_n are positive. Therefore, both nuclei in regions II and III can be studied in a unified way.

First, we systematically calculate α preformation factors within the CFM [11–15]. The results are listed in the fifth column of Tables I–V. From these tables, we can find that the P_α sequence of nuclei from high to low is even-even nuclei, odd-A nuclei, and doubly odd nuclei, which satisfy the variation tendencies of P_α obtained by various models [31–36,57–60]. In order to have a deeper insight into P_α, we plot the relationship between P_α and $\frac{N_pN_n}{Z_0+N_0}$ of even-even nuclei, odd-A nuclei (including favored and unfavored α decay cases), and doubly odd nuclei (including favored and unfavored α decay cases) around $Z = 82$, $N = 126$ closed shells in Figs. 2–4, respectively. In these figures, the red circle and blue triangle represent the cases of favored and unfavored α decay, respectively. The red dash and blue solid lines represent the predictions of α preformation factors for corresponding cases, which are expressed as

$$P_\alpha = a\frac{N_pN_n}{Z_0 + N_0} + b, \tag{21}$$

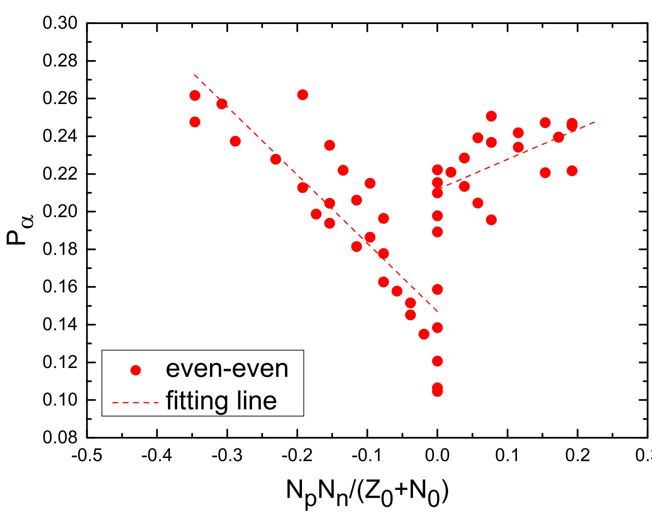

FIG. 2. The linear relationship between α preformation factors and $\frac{N_pN_n}{Z_0+N_0}$. N_p and N_n represent valence protons (holes) and neutrons (holes) of parent nucleus, respectively. Z_0 and N_0 mean the magic numbers of proton and neutron, respectively. The dash lines represent the fittings of α preformation factors.

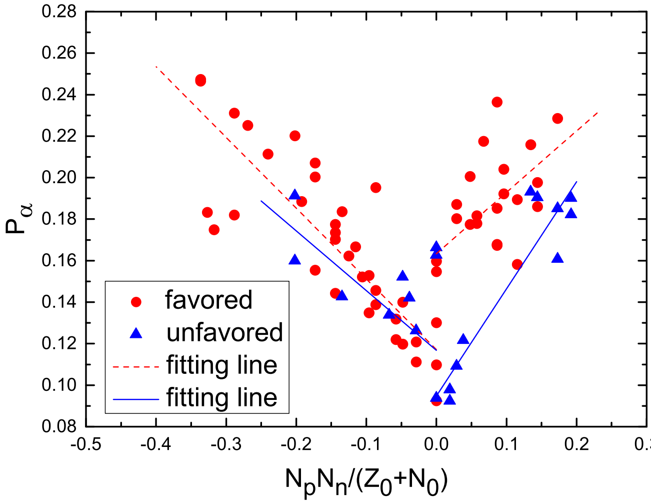

FIG. 3. Same as Fig. 2, but it depicts linear relationships between P_α and $\frac{N_pN_n}{Z_0+N_0}$ of odd-A nuclei. The red circle and blue triangle represent the cases of favored and unfavored α decay, respectively. The red dash and blue solid lines represent the fittings of α preformation factors for cases of favored and unfavored α decay, respectively.

where $Z_0 = 82$ and $N_0 = 126$ represent the magic number of proton and neutron. The a and b are adjustable parameters, which are extracted from fittings of Figs. 2–4 and listed in Table VI (the left hand side for favored α decays and the right hand side for unfavored ones). As shown in Figs. 2–4, we can clearly see that all the P_α are linearly dependent on N_pN_n for cases of even-even nuclei, odd-A nuclei, and doubly odd nuclei. It indicates that valance proton-neutron interaction plays a key role in the α preformation and the influence of proton-neutron pairs on the α cluster basically maintains invariably in the same region. In Fig. 3, we can distinctly find the linear relationship between P_α and N_pN_n for the cases of even-odd and odd-even nuclei without obvious difference. It manifests that in the N_pN_n scheme, the effect of the unpaired odd neutron or proton

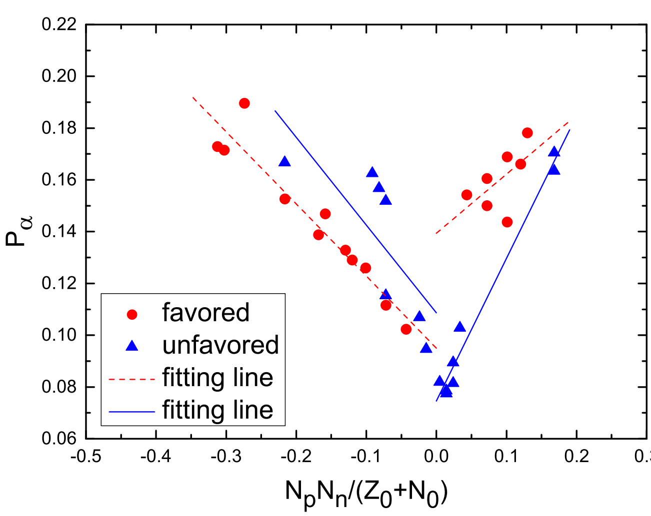

FIG. 4. Same as Figs. 2 and 3, but it depicts linear relationships between P_α and $\frac{N_pN_n}{Z_0+N_0}$ of doubly odd nuclei.

TABLE VI. The parameters of Eq. (21) that show α preformation factors are linearly related to N_pN_n.

Region	Favored decay		Unfavored decay	
	a	b	a	b
	Even-even nuclei			
I	− 0.36222	0.14703		
II, III	0.15948	0.21175		
	Odd-A nuclei			
I	− 0.34101	0.11712	− 0.28777	0.11684
II, III	0.29582	0.16333	0.51621	0.09475
	Doubly odd nuclei			
I	− 0.27858	0.09504	− 0.33891	0.10868
II, III	0.22820	0.13944	0.55115	0.07457

on P_α can be treated in an unified way. It also verifies that using different methods to calculate P_α of even-odd nuclei and odd-even nuclei in the CFM is appropriate. Combined with our previous works [43,44], we confirm that the P_α of nuclei around $Z = 82$, $N = 126$ closed shells is linearly dependent on N_pN_n whether the P_α is model dependent or microcosmic.

Second, we systematically calculate α decay half-lives of these nuclei within Prox.1977. The experimental α decay half-lives are taken from the latest evaluated nuclear properties table NUBASE2016 [54], and the α decay energies are taken from the latest evaluated atomic mass table AME2016 [55,56]. The detailed calculations are listed in Tables I–V. In these tables, the first four columns denote α decay, experimental decay energy, spin and parity transition, and the minimum angular momentum taken away by the α particle, respectively. The fifth column denotes α preformation factors calculated with CFM. The sixth one denotes experimental α decay half-life. The last three columns show calculated α decay half-life by Prox.1977 without considering P_α, with taking P_α by CFM, and with fitting P_α calculated by Eq. (21) and parameters listed in Table VI, which are denoted as $T_{1/2}^{\text{calc1}}$, $T_{1/2}^{\text{calc2}}$ and $T_{1/2}^{\text{calc3}}$, respectively. All tables are divided into two parts: the upper half shows the nuclei in region I and the lower one shows nuclei in regions II and III. From Tables I–V we find that although the $T_{1/2}^{\text{calc1}}$ can produce experimental data, the deviation is still considerable. So we calculate decay constant λ with P_α, which is evaluated by CFM. The new calculated α decay half-lives $T_{1/2}^{\text{calc2}}$ can better reproduce with $T_{1/2}^{\text{expt}}$ than $T_{1/2}^{\text{calc1}}$. In addition, we find that the $T_{1/2}^{\text{calc3}}$, which is calculated with fitting P_α, can well conform the $T_{1/2}^{\text{calc2}}$. It indicates that P_α

TABLE VII. The standard deviations between α decay half-lives of calculations and experimental data.

Nuclei	Favored decay			Unfavored decay		
	σ_1	σ_2	σ_3	σ_1	σ_2	σ_3
Even-even nuclei	0.583	0.380	0.383			
Odd-A nuclei	0.659	0.370	0.366	0.897	0.542	0.536
Doubly odd nuclei	0.813	0.215	0.213	1.631	0.940	0.926

is linearly well related to N_pN_n. In order to intuitively survey the deviations between α decay half-lives of calculations and experimental data, we calculate the standard deviation $\sigma = \sqrt{\sum(\log_{10} T_{1/2}^{\rm calc} - \log_{10} T_{1/2}^{\rm expt})^2/n}$. The results σ_1, σ_2, and σ_3 denote standard deviations between $T_{1/2}^{\rm calc1}$, $T_{1/2}^{\rm calc2}$, $T_{1/2}^{\rm calc3}$, and $T_{1/2}^{\rm expt}$, respectively, which are listed in Table VII. In this table, we can clearly see that the values of σ_2 significantly reduce compared to σ_1 and the σ_2 are basically equal to σ_3. It indicates that the calculations within Prox.1977 using P_α from CFM can better reproduce with experimental data than using $P_\alpha = 1$ and that the P_α have a linear relationship with N_pN_n. For nuclei ^{209}Bi, ^{213}Bi, and ^{223}At in Table III as well as nuclei ^{212}Bi and ^{214}Bi in Table V, we cannot obtain the classical turning points $r_{\rm in}$ through solving the equation $V(r_{\rm in}) = V(r_{\rm out}) = Q_\alpha$ due to the depths of the potential well above the Q_α. Therefore, we don't give the calculations of half-lives for these five nuclei. This phenomenon motivates our interest to further develop the theoretical model in the future.

IV. SUMMARY

In summary, we preformed the systematic study of α preformation factors within the cluster-formation model (CFM) and α decay half-lives within the proximity potential 1977 formalism (Prox.1977) for nuclei around $Z = 82$, $N = 126$ closed shells. Our results indicate that the realistic P_α calculated by CFM for nuclei around $Z = 82$, $N = 126$ shell closures are linear with N_pN_n. Combined with our previous works, it confirms that valance proton-neutron plays an important role in the α cluster formation. In addition, our calculated α decay half-lives, i.e., $T_{1/2}^{\rm calc2}$, using Prox.1977 taking P_α evaluated by CFM, can well reproduce the experimental data and significantly reduce the errors. It demonstrates that the CFM is credible. This work will be a reference for future experiments and theoretical researches.

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