Absorption effects in nuclear particle correlations

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We calculate the contribution of absorption effects to the correlation function of two identical particles emitted by a fixed source. These effects result from the successive collisions undergone by the emitted particles before they leave the source of emission.

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I. INTRODUCTION

Much effort, both experimental and theoretical, has gone into nuclear particle interferometry studies since the pioneering paper of Goldhaber [1]. It became known that the correlation functions defined as the convolution of the modulus squared of the pair of particles wave functions with the emitting source distribution are not only sensitive to the spatial extension of the emitting source but also to the time dependence of the emission processes and to the final-state interactions [2,3]. The influence of the mean field of the emitter source including the Coulomb field on the particles correlation function is mentioned and analyzed by a few authors [4,5]. However, there is another effect, which is generally ignored, which comes from the very fact that the emitted particles should undergo a succession of collisions with the nucleons inside the source before leaving the source's volume. The aim of the present paper is to give an approximated way to evaluate the contribution of such specific collision mechanism on the two-particle correlation function.

II. GENERAL THEORETICAL EXPRESSIONS

Let us consider two incoherent identical particles emitted from pointlike sources located at $\bar{r}_i$ $(i = 1,2)$ inside a nuclear extended source of spatial density $\rho(\bar{r}_i)$.

The probability of observing simultaneously both particles (bosons or fermions) with wave vector $\bar{k}_a$ and $\bar{k}_b$ at detectors located at $\boldsymbol{a}$ and $\boldsymbol{b}$, respectively, is

$$P(\bar{k}_a,\bar{k}_b) = \int d\bar{r}_1 \int d\bar{r}_2 |\Psi(\bar{r}_1,\bar{k}_a;\bar{r}_2,\bar{k}_b)|^2 \rho(\bar{r}_1)\rho(\bar{r}_2). \quad (1)$$

In (1) the wave function $\Psi(\bar{r}_1,\bar{k}_a;\bar{r}_2,\bar{k}_b)$ for two identical bosons or fermions is replaced by:

$$\Psi(\bar{r}_1,\bar{k}_a;\bar{r}_2,\bar{k}_b) = \frac{1}{\sqrt{2}}[\phi_{1,a}(\bar{k}_a,\bar{r}_1)\phi_{2,b}(\bar{k}_b,\bar{r}_2) \pm \phi_{1,b}(\bar{k}_b,\bar{r}_1)\phi_{2,a}(\bar{k}_a,\bar{r}_2)], \quad (2)$$

where $\phi_{i,a}$ defines the single-particle wave function for the particle emitted at $\bar{r}_i$ with a wave vector $\bar{k}_a$.

The general expression of the correlation function is

$$C(\bar{k}_a,\bar{k}_b) = \frac{P(\bar{k}_a,\bar{k}_b)}{P(\bar{k}_a)P(\bar{k}_b)} \quad (3)$$

or

$$C(\bar{k}_a,\bar{k}_b) = 1 + \alpha R(\bar{k}_a,\bar{k}_b), \quad (4)$$

where $P(\bar{k}_a,\bar{k}_b)$ is replaced by (1) and α is a number dependent of the type of particles and of the statistics to be applied.

To take into account the succession of collisions of the emitted particles with the nucleons inside the source, we refer to the Glauber [6] approach in pion or nucleon scattering at high energy by semitransparent nuclei. We assume that the wavelength of the emitted particles is very small compared to the spatial expansion of the source so as to take for the single-particle wave functions in (2) the high-energy approximation:

$$\phi_{i,j}(\bar{k}_j,\bar{r}_i) = \exp i\bar{k}_j\bar{r}_i \exp i\chi_i^G \quad (5)$$

with $j = a,b$ and $i = 1,2$.

The phase χ_i^G is the Glauber phase shift given by

$$\chi_i^G = -\frac{1}{\hbar v}\int V_{\text{opt}}(\bar{r}_i)d\bar{r}_i. \quad (6)$$

For the scattering by a many-particle system, the Glauber phase shift χ_i^G corresponds to an optical potential $V_{\text{opt}}(\bar{r})$ having the same functional form as the density $\rho(\bar{r})$ of the system. We shall adopt the parametrization:

$$\rho(\bar{r}) = \rho_0 v(\bar{r}) \quad V_{\text{opt}}(\bar{r}) = (V_0 + iW_0)v(\bar{r}). \quad (7)$$

The phase shift (6) in the single-particle wave functions (5) of the emitted particles should be calculated on the straight-line trajectories between the point sources $\bar{r}_i$ $(i = 1,2)$ and the detectors located at $\boldsymbol{a}$ and $\boldsymbol{b}$.

For a constant density $\rho(\bar{r})$ the phase shift due to the real part of the optical potential (7) can be evaluated for these two trajectories and it consists only of replacing the value $q = 2k\sin(\frac{\vartheta}{2})$ of the transfer momentum $\bar{q} = \bar{k}_a - \bar{k}_b$ with $k = |\bar{k}_a| = |\bar{k}_b|$ by the value $q_n = q + 2\frac{|V_0|}{\hbar v}\sin(\frac{\vartheta}{2})$.

For small angles ϑ, the two trajectories $\bar{r}_{i,a}$ and $\bar{r}_{i,b}$ are very close to each other inside the volume source so that one can replaced these ones by mean trajectories along the z_i axis $(i = 1,2)$ (see Fig. 1). With this approximation there is no phase

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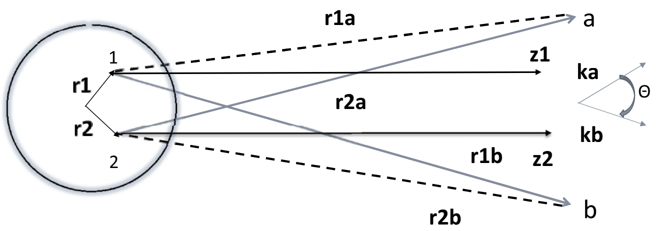

FIG. 1. Schematic illustration of the two-particle correlation measurement from incoherent pointlike sources located at $\bar{r}_1$ and $\bar{r}_2$ inside a nuclear extended source of density $\rho(\bar{r}_i)$. The emitted particles of momentum $\bar{k}_a$ and $\bar{k}_b$ are counted by detectors $\boldsymbol{a}$ and $\boldsymbol{b}$.

shift due to the real part V_0 of the optical potential and the phase shift due to the absorptive part is given by

$$\chi_i^G = i\frac{|W_0|}{\hbar v}\int_{z_i}^{\infty} v(\bar{r})dz \tag{8}$$

Using this expression (8) in the single-particle wave function (5), the correlation function $R(\bar{k_a},\bar{k_b})$ is

$$R(\bar{k_a},\bar{k_b}) = \frac{\left|\int d\bar{r}\,S(\bar{r})\exp i\bar{q}\bar{r}\right|^2}{\left|\int d\bar{r}\,S(\bar{r})\right|^2} \tag{9}$$

with

$$\begin{aligned} S(\bar{r}) &= \rho(\bar{r})\exp i\chi^G(\bar{r}) \\ &= \rho(\bar{r})\exp\left(-\frac{|W_0|}{\hbar v}\int_z^{\infty} v(\bar{r}')dz'\right). \end{aligned} \tag{10}$$

The function $S(\bar{r})$ defined as the product of the density in the source and of the probability that the emitted particle survives the absorption when traveling inside the source can be interpreted as the *source function*. Using cylindrical coordinates $\bar{r} = (b,\phi,z)$ and operating the integrations over ϕ and z successively, the function $R(q)$ of (9) is

$$R(q) = \frac{\left|\int_0^{\infty} bdbJ_0(qb)\int dzS(b,z)\right|^2}{\left|\int_0^{\infty} bdb\int dzS(b,z)\right|^2} \tag{11}$$

or, more explicitly,

$$R(q) = \frac{\left|\int_0^{\infty} bdbJ_0(qb)\left[1-\exp\left(-\frac{|W_0|}{\hbar v}\int_{-\infty}^{+\infty} v(b,z)dz\right)\right]\right|^2}{\left|\int_0^{\infty} bdb\left[1-\exp\left(-\frac{|W_0|}{\hbar v}\int_{-\infty}^{+\infty} v(b,z)dz\right)\right]\right|^2}. \tag{12}$$

This expression tells us that the q dependence of the correlation function (11) is the same as the one obtained for the differential cross section in the elastic scattering of high-energy incident particles (nucleons or pions) by absorbing nuclei. However, the measured angular distributions of these two phenomena are, in fact, quite different. When the pion or nucleon scattering exhibits an Airy-like oscillatory pattern, the correlation function for such particles emitted in high-energy heavy-ion collisions shows a curve peaked forward and slowly decreases [7]. As we shall see, this can be interpreted as due to the difference between the absorption effects in these two situations. In particles with high-energy scattering, the absorption effects are produced by the target nuclei viewed as spheres of constant density. In the interference process, the pattern generally observed corresponds to absorption effects produced by a diffuse Gaussian-like density of the excited emitting source.

This formulation of the source function and of the associated correlation function as defined in (10) and (12), respectively, will be used with different forms, Gaussian and constant, of the source density $\rho(\bar{r})$. In the analysis performed by different authors [8–10] of the experimental data in such interference process in nuclear or particle physics, it appears that the source function (as defined by these authors) displays significant non-Gaussian forms.

In some papers [1,11], it is also mentioned that similar correlation functions are obtained indifferently with a Gaussian or a constant source density $\rho(\bar{r})$ according to a specific relation between the radii used in both forms. We shall analyze some special cases in the next paragraph.

III. THE MODEL SOURCE FUNCTIONS AND THE CORRESPONDING CORRELATION $R(q)$

We shall consider two model source functions as follows:

(1) A source function associated to a Gaussian density $\rho(\bar{r}) = \rho_0 exp - \frac{r^2}{r_0^2}$ modified by the factor describing the absorption effect as defined in (10):

$$\begin{aligned} S(b,z) &= \rho_0\exp-\frac{b^2+z^2}{r_0^2} \\ &\quad\times\exp\left[-\frac{|W_0|}{\hbar v}\exp\left(-\frac{b^2}{r_0^2}\right)\int_z^{\infty}\exp-\frac{z'^2}{r_0^2}dz'\right]. \end{aligned} \tag{13}$$

In this case the correlation function $R(q)$ (12) is given by:

$$R(q) = \frac{\left|\frac{R^2}{qR}J_1(qR) - \int_0^{\infty} bdbJ_0(qb)\exp\left(-\frac{|W_0|}{\hbar v}\sqrt{\pi}r_0\exp-\frac{b^2}{r_0^2}\right)\right|^2}{\left|\frac{R^2}{2} - \int_0^{\infty} bdb\exp\left(-\frac{|W_0|}{\hbar v}\sqrt{\pi}r_0\exp-\frac{b^2}{r_0^2}\right)\right|^2} \tag{14}$$

with a radius $R \to \infty$.

(2) A source function associated to a constant density for the nuclear emitting sphere of radius R; $\rho(\bar{r}) = \rho_0$ for $0 \leqslant r \leqslant R$. In this case, the source function (10) is

$$S(b,z) = \rho_0\exp\left(-\frac{|W_0|}{\hbar v}\int_z^{\sqrt{R^2-b^2}} dz'\right) \tag{15}$$

and the expression of the correlation function $R(q)$ is

$$R(q) = \frac{\left|\frac{R^2}{qR}J_1(qR) - \int_0^R bdbJ_0(qb)\exp\left(-2\frac{|W_0|}{\hbar v}\sqrt{R^2-b^2}\right)\right|^2}{\left|\frac{R^2}{2} - \int_0^R bdb\exp\left(-2\frac{|W_0|}{\hbar v}\sqrt{R^2-b^2}\right)\right|^2}. \tag{16}$$

In these expressions we have only two parameters $\frac{|W_0|}{\hbar v}$ and r_0 or R for the extension of the density $\rho(\bar{r})$. Note that the parameter $\frac{|W_0|}{\hbar v}$ defines how the emitted particles of velocity v are absorbed along their trajectories inside the nuclear emitting source. When neglecting the particles final-state interaction, the correlation function appears generally as the Fourier transform of

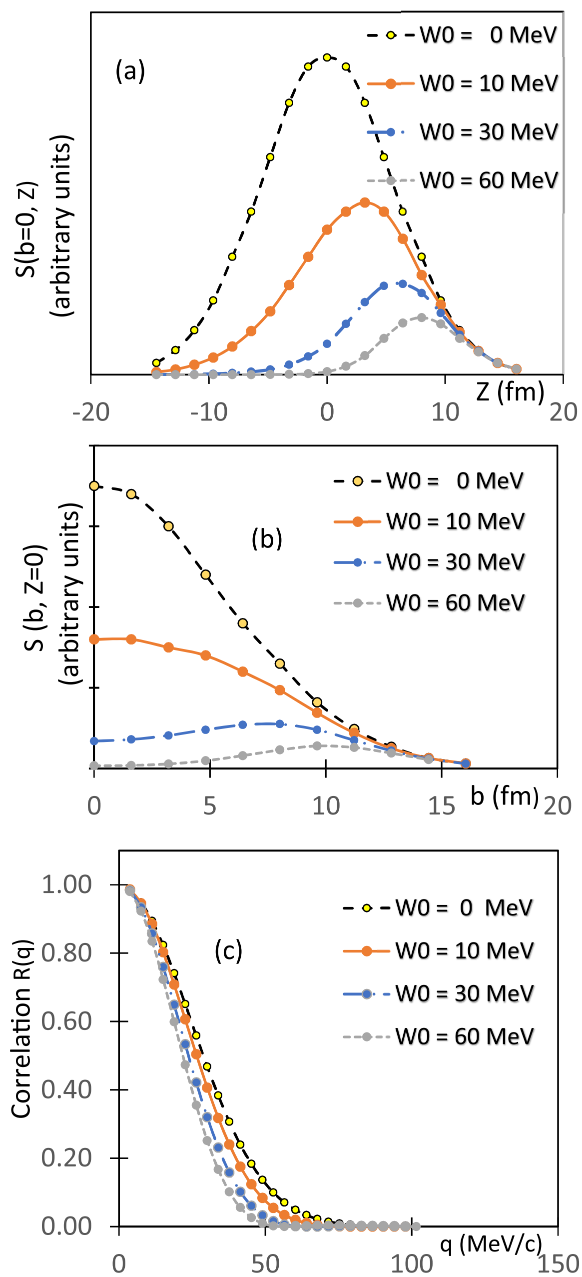

FIG. 2. Emission of two nucleons at $E/p = 100$ MeV. The source functions $S(b,z)$ for a Gaussian density ($r_0 = 8$ fm) and several values of the absorption parameter W_0 are drawn in (a) and (b). On (c) we have reported the correlation functions $R(q)$ as function of the variable q (MeV/c) associated to the different source functions reported in (a) and (b).

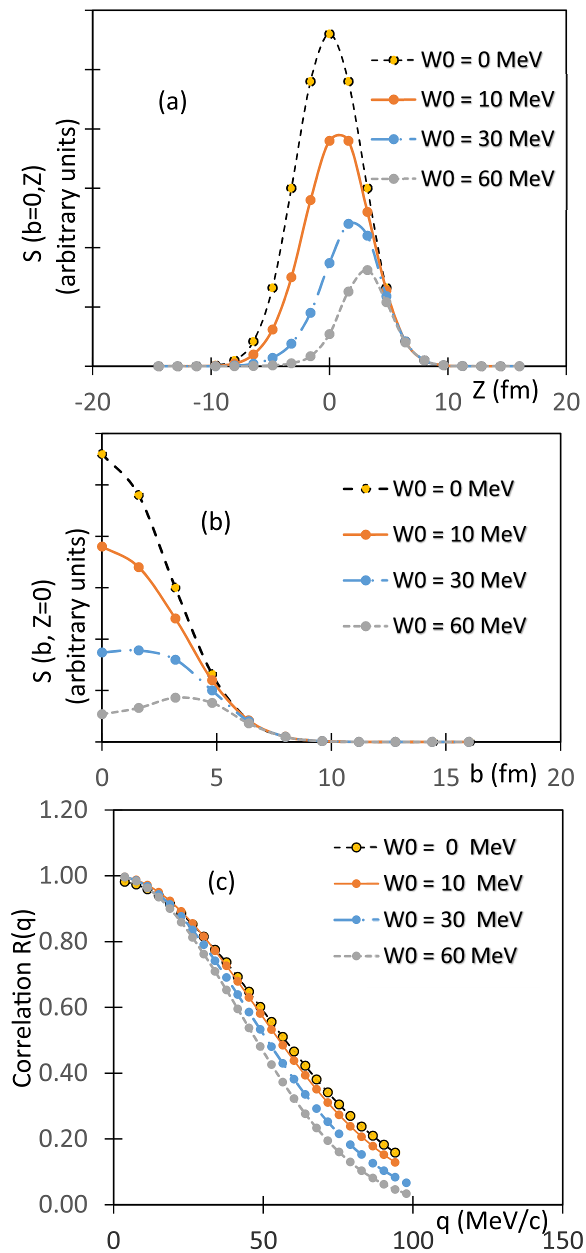

FIG. 3. The same as for Fig. 2 for a Gaussian density with $r_0 = 4$ fm.

the source function. Thus a Gaussian pattern will correspond to a Gaussian source function. But there is no reason to expect the source function to be a Gaussian. By applying imagine techniques, different authors have investigated the possibility of having non-Gaussian sources [8–10]. To have agreement with the experimental data at low q values, some authors introduce a factor λ replacing $R(q)$ in the correlation function $C(q)$ by $\lambda R(q)$, considering that there is only a fraction of pairs of particles which will interfere, λ being a simple parameter of absorption [5] or a parameter connected to a mean lifetime τ [2]. This also means that the true source function has a non-Gaussian tail. More recently, exponential tails have been tested and compared with the image analysis of the experimental data in $C(q)$ [10].

IV. QUALITATIVE INSIGHT OF THE ABSORPTION EFFECTS: APPLICATIONS

In the present calculations, we give a qualitative insight of the effect due to the absorption as defined in (13)–(16) on the source function $S(b,z)$ and on the correlation function $R(q)$ independently of all effects other than the statistic requirement for identical emitted particles. Within the Glauber approach,

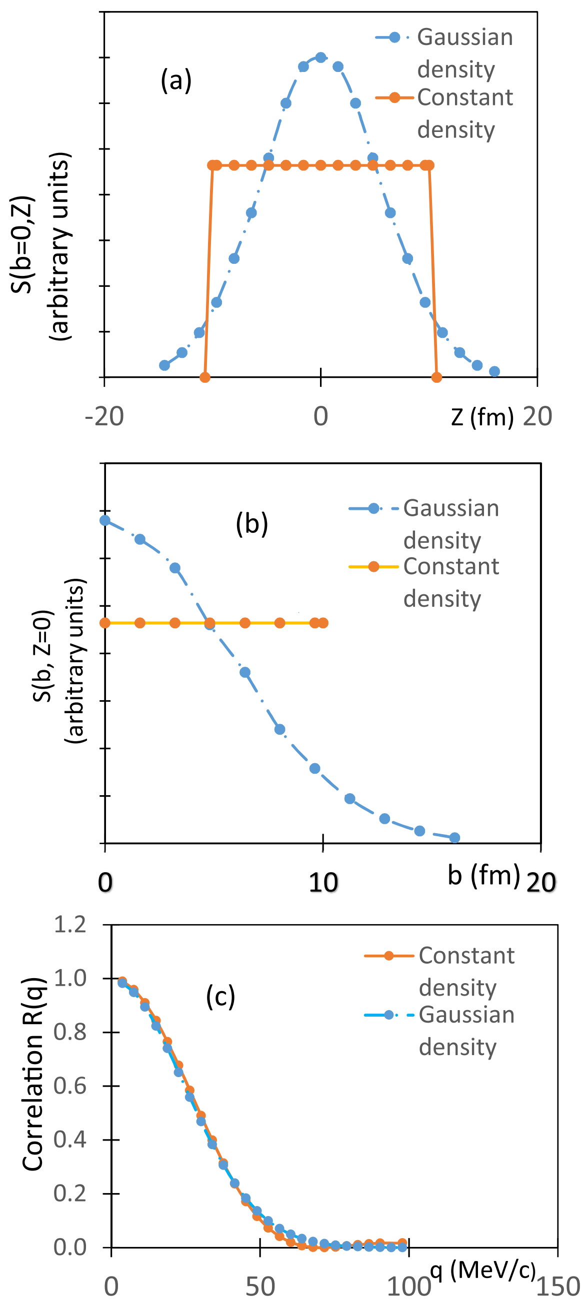

FIG. 4. The source functions $S(b,z)$ [(a) and (b)] for a Gaussian density $r_0 = 8$ fm and for a constant density $R = 1.35 r_0$ fm without any absorption ($W_0 = 0$ MeV) and the corresponding correlation functions $R(q)$ (c).

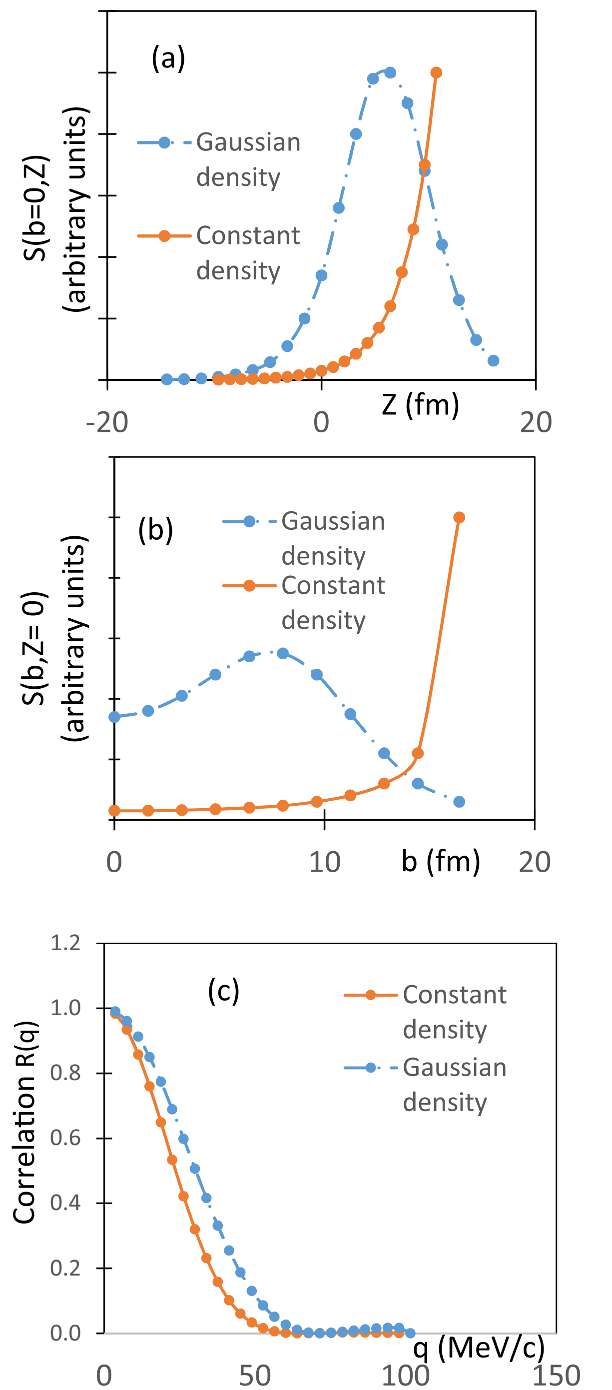

FIG. 5. The same as for Fig. 4 with an absorption parameter $W_0 = 30$ MeV.

we give an exact estimation of this effect of absorption. This parametrizes the assumption that the emitted particles should undergo a succession of collisions inside the source before to leave this. Such successive collisions have also been evaluated [12] assuming a cascade model and using the Cugnon's code (INC model).

As an example, we consider the emission of a pair of nucleons in an heavy-ion reaction, with a relative energy $E/$particle of about 25 and 100 MeV. The absorption coefficient W_0 is chosen so that we have the same value for the parameter $\frac{|W_0|}{\hbar v}$ at both relative energies $E/$particle. So we use $W_0 = 10(5), 30(15)$, and $60(30)$ MeV for $E/$particle $= 100(25)$ MeV. For a Gaussian density with $r_0 = 8$ fm and $r_0 = 4$ fm, the patterns of the source functions $S(b,z)$ as a function of z at $b = 0$ or of b at $z = 0$ are reported, respectively, in Figs. 2(a) and 2(b) and Figs. 3(a) and 3(b). The absorption effect inside the source volume reduces the source function $S(0,z)$ for negative and low positive z values, leaving unchanged the tail at large positive z. The effect is more or less the same for the source function $S(b,0)$ showing a reduced intensity at low positive b values and an unchanged tail at large b values. This reduces the values of the correlation function $R(q)$ at large q and modifies its qualitative behavior and tail compared to the pure Gaussian pattern ($W_0 = 0$), see Fig. 2(c) and Fig. 3(c).

The effects on the source function of changing the parameter r_0 of the Gaussian density $\rho(r)$ are shown by comparing Figs. 2 and 3. A shift of the maximum with increasing the absorption effect and the decrease of the density of pairs that will interfere are well observed; but the tail of the source functions remains almost the same. Moreover, this does not give a drastic modification of the associated correlation functions $R(q)$.

These patterns of $S(b,z)$ and of $R(q)$ are the same for the same values of r_0 and of the absorption parameter $\frac{|W_0|}{\hbar v}$ associated to different couples of values of W_0 and E/particle. For E/particle $= 25$ MeV with the same values of r_0 and $\frac{|W_0|}{\hbar v}$ (which means to have $W_0 = 5$, 15, and 30 MeV, respectively), the results are exactly the same. We have also compared the source functions associated to Gaussian and spherical constant densities $\rho(r)$. As mentioned in the literature, the correlation functions are very similar for $R_{\text{sphere}} \equiv 2.15 r_0$ as noted in [1] or for $R_{\text{sphere}} \equiv 1.52 r_0$ as noted in [11]. We note that for $W_0 \to 0$, the expression (16) of the correlation function is the same as that used by [2] for $q_0 = |E1 - E2| = 0$, it means

$$R(q) = \left| \frac{2 J_1(qR)}{qR} \right|^2 \tag{17}$$

and

$$\lim R(q)_{q \to 0} = \exp\left(- \frac{q^2 R^2}{4} \right). \tag{18}$$

And we also note that for $W_0 \to 0$, the expression (14) reduces to

$$R(q) = 1 - \frac{q^2 r_0^2}{4} \tag{19}$$

so that we have the approximated relation $R^2 = 2 r_0^2$ or $R = 1.44 r_0$ to be compared to the prescription of [1] or [11].

In Fig. 4 we compare the source functions $S(b,z)$ and the correlation $R(q)$ for Gaussian and constant densities without any absorption correction ($W_0 = 0$) for E/particle = 100 MeV and $r_0 = 8$ fm. The same functions are drawn on Fig. 5 but for $W_0 = 30$ MeV. The source functions are quite different; a shift to the surface with a Gaussian like shape at positive z values and $b = 0$ for the Gaussian density and a clear enhancement at the surface for the constant density. So one observes a small variation of the correlation $R(q)$ with an oscillation (see Fig. 6) in the very small intensities at large q values.

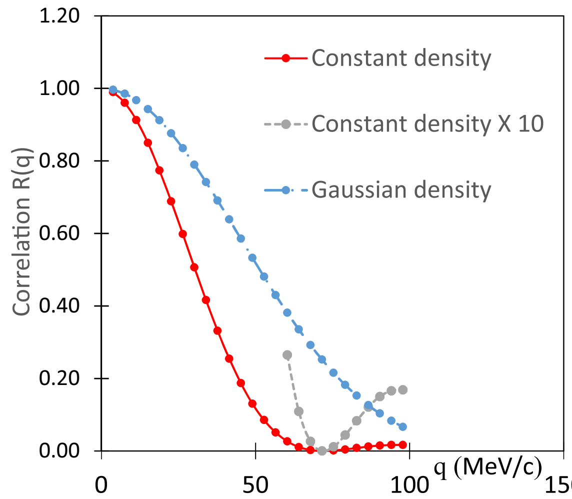

FIG. 6. The same as for Fig. 5 for a Gaussian and a Constant density with an absorption parameter $W_0 = 30$ MeV, $r_0 = 4$ fm, and $R = 2.7 r_0$.

In Fig. 6 the correlation functions are drawn for $W_0 = 30$ MeV with $r_0 = 4$ fm as parameter in the Gaussian density and $R = 2.7 r_0$. When zooming on the values at large transfer momentum q an oscillation appears clearly. This is a surface effect connected to the oscillatory behavior of the function $J_1(qR)$ (16) and (17). We have here the fingerprint of the first zero of this Bessel function.

V. CONCLUSION

With some simple academic examples, we have shown the qualitative and quantitative effects on the source functions and on the particle correlation of taking into account the absorption of the particles inside the source before emission. Such an effect is generally ignored in nuclear particles interferometry studies.

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