

Origin of a maximum of the astrophysical S factor in heavy-ion fusion reactions at deep subbarrier energies

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The hindrance phenomenon of heavy-ion fusion cross sections at deep subbarrier energies often accompanies a maximum of an astrophysical S factor at a threshold energy for fusion hindrance. We argue that this phenomenon can naturally be explained when the fusion excitation function is fitted with two potentials, with a larger (smaller) logarithmic slope at energies lower (higher) than the threshold energy. This analysis clearly suggests that the astrophysical S factor provides a convenient tool to analyze the deep subbarrier hindrance phenomenon, even though the S factor may have a strong energy dependence for heavy-ion systems unlike that for astrophysical reactions.

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I. INTRODUCTION

Coupled-channels calculations [1], taking into account low-lying collective excitations of colliding nuclei as well as several transfer channels, have enjoyed a great success in reproducing experimental fusion excitation functions for many heavy-ion systems at energies around the Coulomb barrier [2–6]. The effect of channel coupling has now been well understood in terms of fusion barrier distributions [3,7,8], that is, fusion cross sections are given as a weighted sum of those for a few eigenbarriers.

In 2002, Jiang *et al.* measured fusion excitation function for the $^{60}\text{Ni} + ^{89}\text{Y}$ system down to the 100 nb and discovered for the first time that fusion cross sections fall off much steeper at deep subbarrier energies as compared to a theoretical extrapolation based on the coupled-channels calculations [9]. Subsequently, a similar deep subbarrier fusion hindrance has been found also in many other systems, see Ref. [5] and references therein. Two theoretical models have been proposed in order to interpret this phenomenon, based either on the sudden approximation [10,11] or on the adiabatic approximation [12–14]. Even though the origin of the hindrance is different in these two models, both of them expose the importance of dynamical effects after two colliding nuclei touch with each other [15].

The deep subbarrier fusion hindrance phenomenon has often been analyzed in terms of the astrophysical S factor [5,16], even though the S factor itself may not provide a useful tool for heavy-ion reactions—unlike light systems in which penetration of the Coulomb repulsive potential makes a dominant contribution to reaction dynamics. A somewhat surprising observation was that the experimental data often show a maximum in astrophysical S factor as a function of incident energy [5]. Jiang *et al.* argued that deep subbarrier

hindrance sets in at the peak energy of the astrophysical S factor [5]. See also Ref. [17], which briefly discusses, in terms of the action integral, the relation between the logarithmic slope of fusion excitation functions and the astrophysical S factor.

Even though the threshold energy so determined well follows the value of several global internucleus potentials at the touching configuration [15], the exact cause of the S factor maximum has not yet been clarified. One could question if the S factor can be used as a representation of fusion cross sections, provided that the physical meaning of the S factor maximum is clarified. The aim of this paper is to address this question. To this end, we introduce a two-potential fit to fusion cross sections at deep subbarrier energies [18], and show that the S factor maximum can be naturally accounted for with this method. An important fact here is that the energy derivative of the astrophysical S factor is determined by a cancellation of two terms, that is, the nuclear and the Coulomb contributions, and the relative importance between them changes precisely around the threshold energy for fusion hindrance.

II. TWO-POTENTIAL FIT AND THE ASTROPHYSICAL S FACTOR

In Ref. [18], we have fitted an experimental fusion excitation function for several systems using a single-channel potential model. To this end, we used two different Woods-Saxon potentials for the subbarrier and the deep subbarrier energy regions, which we define as the regions in which a fusion cross section is between 10^{-2} and 10^0 mb, and below 10^{-3} mb, respectively. Examples of the fit are shown in Figs. 1(a) and 2(a) for the $^{64}\text{Ni} + ^{64}\text{Ni}$ and $^{28}\text{Si} + ^{64}\text{Ni}$ systems, respectively. The values for the Woods-Saxon potentials are listed in Table I (note that we have used a slightly different parameter set for

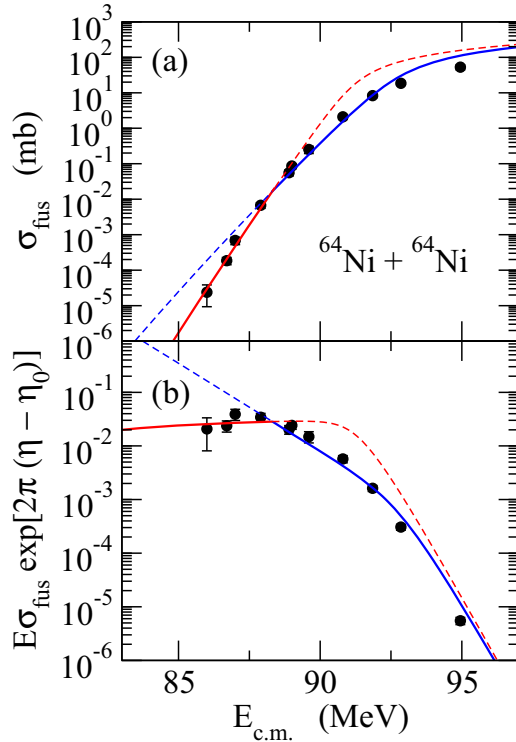


FIG. 1. The fusion cross sections (top) and the astrophysical S factor (bottom) for the $^{64}\text{Ni} + ^{64}\text{Ni}$ system. The astrophysical S factor is scaled with $\eta_0 = 75.23$ (see text), and is given in units of (mb MeV). The solid curves denote the result of the two-potential fit, whereas the dashed curves show an extrapolation of the calculations to the region outside the fitting areas. The experimental data are taken from Ref. [19].

the $^{64}\text{Ni} + ^{64}\text{Ni}$ system from that shown in Ref. [18] in order to get a better fit for the astrophysical S factor). In general, the surface diffuseness parameter a in the Woods-Saxon potential is around 0.65 fm in the subbarrier region, however, it increases to a much larger value in the deep subbarrier region [17]. In Ref. [18], we defined the threshold energy for the deep subbarrier hindrance, E_{thr} , as the energy at which the fusion excitation functions obtained with the two potentials cross with each other.

The astrophysical S factor,

$$\tilde{S}(E) = E\sigma_{\text{fus}}(E)e^{2\pi(\eta-\eta_0)}, \quad (1)$$

is plotted in the bottom panel of Figs. 1 and 2. Here, $\sigma_{\text{fus}}(E)$ is the fusion cross section at energy E , and $\eta = Z_P Z_T e^2 / \hbar v$ is the Sommerfeld parameter, Z_P and Z_T being the atomic number of the projectile and the target, respectively, and v being the velocity for the relative motion in the center of mass frame. For the purpose of a clear presentation, we scale the S factor by introducing a constant η_0 in the exponent. As one can see in the figures, the energy dependence of the S factor changes at the threshold energy, E_{thr} . At energies below the threshold energy, the S factor has a positive slope, whereas the slope becomes negative at energies above E_{thr} . As a consequence, the astrophysical S factor takes a maximum at $E = E_{\text{thr}}$.

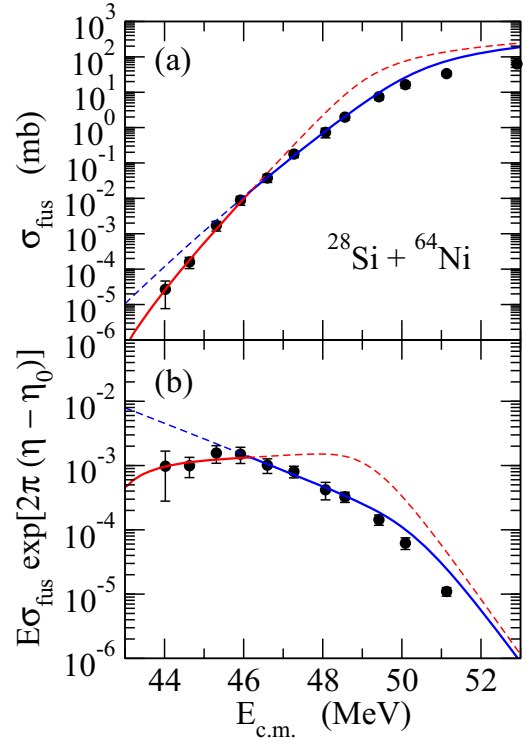


FIG. 2. Same as Fig. 1, but for the $^{28}\text{Si} + ^{64}\text{Ni}$ system. The astrophysical S factor is scaled with $\eta_0 = 41.25$. The experimental data are taken from Ref. [20].

In order to understand the energy dependence of the S factor, let us take its first energy derivative [5,16]. From Eq. (1), one obtains

$$\frac{1}{\tilde{S}} \frac{d\tilde{S}}{dE} = L(E) - \frac{\pi\eta}{E}, \quad (2)$$

where

$$L(E) = \frac{1}{E\sigma_{\text{fus}}} \frac{d}{dE}(E\sigma_{\text{fus}}) = \frac{d}{dE} \ln(E\sigma_{\text{fus}}) \quad (3)$$

is the logarithmic slope of a fusion excitation function [5,16]. One can see that the energy derivative of the astrophysical S factor consists of two terms. The first term, $L(E)$, originates from the nuclear potential, while the second term, $\pi\eta/E$,

TABLE I. Parameters for the Woods-Saxon potential defined by $V(r) = -V_0 / \{1 + \exp[(r - R_0)/a]\}$, with $R_0 = r_0(A_P^{1/3} + A_T^{1/3})$, where A_P and A_T are the mass number of the projectile and the target nuclei, respectively. The subbarrier region is defined as the energy region in which a fusion cross section is between 10^{-2} and 10^0 mb, while the deep subbarrier region is the region in which a fusion cross section is below 10^{-3} mb.

Systems	Regions	V_0 (MeV)	r_0 (fm)	a (fm)
$^{64}\text{Ni} + ^{64}\text{Ni}$	subbarrier	180	1.15	0.676
	deep subbarrier	98.0	1.1	1.1
$^{28}\text{Si} + ^{64}\text{Ni}$	subbarrier	70.5	1.2	0.71
	deep subbarrier	46.5	1.19	0.99

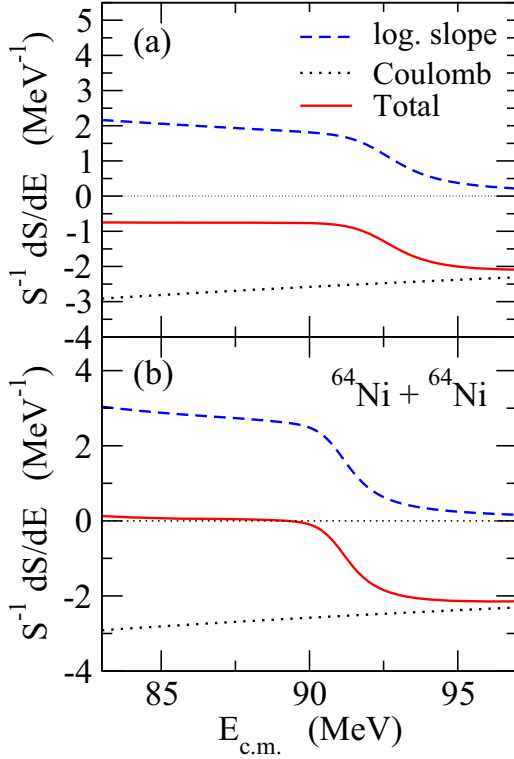


FIG. 3. The first derivative of the astrophysical S factor ($1/S$) dS/dE for the $^{64}\text{Ni} + ^{64}\text{Ni}$ system. The dashed and the dotted curves show the first and the second terms in Eq. (2), respectively, while the solid curves show the sum of these two contributions. The top panel is obtained with the potential for the subbarrier region, while the bottom panel with the potential for the deep subbarrier region.

originates from the pure Coulomb interaction. These two terms have opposite signs, and a strong cancellation may occur. Figures 3 and 4 show those contributions separately for the $^{64}\text{Ni} + ^{64}\text{Ni}$ and $^{28}\text{Si} + ^{64}\text{Ni}$ reactions, respectively. The top and the bottom panels of these figures are obtained with the potentials for the subbarrier and the deep subbarrier regions, respectively (see Table I). For the potentials for the subbarrier region, the logarithmic slope (the dashed lines) is relatively small, and the second term in Eq. (2) (the dotted lines) gives a larger contribution. The energy derivative of the S factor is then negative at subbarrier energies. That is, the astrophysical S factor is a decreasing function of energy in this region. On the other hand, for the potentials for the deep subbarrier region, the logarithmic slope is considerably larger than that in the subbarrier region, and the first term in Eq. (2) is comparable to the second term. Consequently, the energy derivative of the S factor is slightly positive in the deep subbarrier region (see the solid lines), and thus the S factor becomes an increasing function of energy. This observation is consistent with the astrophysical S factors shown in Figs. 1 and 2.

This analysis provides an interesting view of the astrophysical S factor for deep subbarrier fusion reactions. As has been argued in Refs. [5,16], the maximum of astrophysical S factor is well related to the deep subbarrier hindrance phenomenon. The hindrance of fusion cross sections leads to a steep falloff

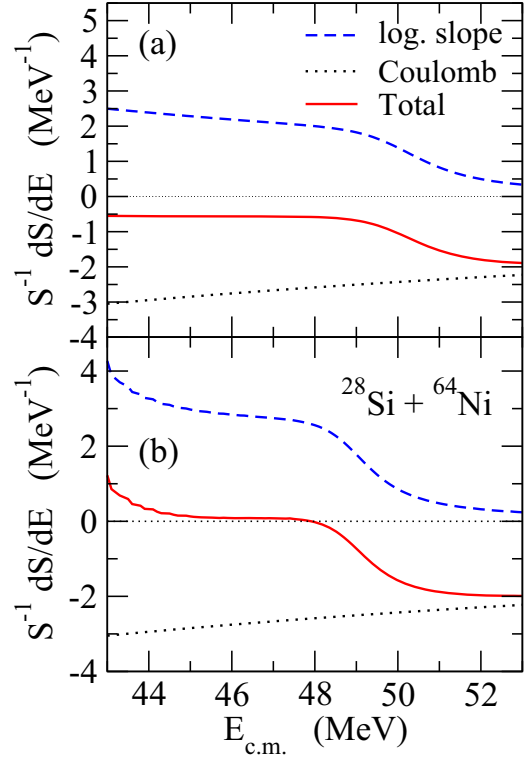


FIG. 4. Same as Fig. 3, but for the $^{28}\text{Si} + ^{64}\text{Ni}$ system.

of fusion cross sections, and thus a large logarithmic slope. When the logarithmic slope becomes larger than $\pi\eta/E$, the S factor has a positive slope as a function of energy. As the energy increases, the logarithmic slope of fusion excitation function then turns to a normal value at the threshold energy, which results in a negative slope of S factor. This leads to a maximum in astrophysical S factor at $E = E_{\text{thr}}$. We mention that Jiang *et al.* proposed to find the value of E_{thr} by looking for an intersect between the logarithmic slope, $L(E)$, and a function $\pi\eta/E$ [16], which has provided a convenient and efficient way to define the threshold energy for deep subbarrier fusion hindrance [5].

There remains the question concerning the cause of the change in the logarithmic slope of fusion excitation functions at the threshold energy, and the amount logarithmic slope changes for each system. In order to address the latter question, one would need microscopic calculations, such as those carried out in Ref. [14] for vibrational excitations in a two-body system. This is beyond the scope of this paper, and we defer it to a future study. On the other hand, it is likely that dynamical effects after the touching configuration play an important role [15] for the deep subbarrier fusion hindrance. A static effect, such as the reaction Q value, has also been conjectured [5], for which the argument is that a fusion cross section must drop to zero at the reaction threshold for a system with a negative Q value. Even though this effect should certainly appear at some energy as long as the Q value is negative, it would be less important than the dynamical effect, since the deep subbarrier fusion hindrance has been observed not only in systems with a negative Q value but also in systems with a positive Q value [5].

III. SUMMARY

We discussed the relation between a maximum of astrophysical S factor and the hindrance phenomenon in heavy-ion fusion reactions at deep subbarrier energies. To this end, we applied the method of two-potential fit to fusion cross sections. We showed that the logarithmic slope increases at deep subbarrier energies, which results in a positive energy slope in astrophysical S factor, whereas the energy slope is negative at subbarrier energies. This leads to a maximum in astrophysical S factor, which has been observed in many systems. This analysis provides a clear interpretation of the S factor maximum, which occurs as a consequence of the change in the logarithmic slope of fusion excitation function at the threshold energy for the hindrance.

The astrophysical S factor has originally been introduced for light systems in order to remove the trivial energy dependence of the Coulomb penetration factor, so that an extrapolation of fusion cross sections down to astrophysically relevant energies can be done easily. Although this original purpose

of introducing an astrophysical S factor does not apply to heavy-ion systems, the analysis presented in this paper clearly shows that the S factor can still be used as a convenient tool to analyze the deep subbarrier hindrance phenomenon, especially to identify the threshold energy for the hindrance.

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- [1] K. Hagino, N. Rowley, and A. T. Kruppa, *Comput. Phys. Commun.* **123**, 143 (1999).
 - [2] A. B. Balantekin and N. Takigawa, *Rev. Mod. Phys.* **70**, 77 (1998).
 - [3] M. Dasgupta, D. J. Hinde, N. Rowley, and A. M. Stefanini, *Annu. Rev. Nucl. Part. Sci.* **48**, 401 (1998).
 - [4] K. Hagino and N. Takigawa, *Prog. Theor. Phys.* **128**, 1061 (2012).
 - [5] B. B. Back, H. Esbensen, C. L. Jiang, and K. E. Rehm, *Rev. Mod. Phys.* **86**, 317 (2014).
 - [6] G. Montagnoli and A. M. Stefanini, *Eur. Phys. J. A* **53**, 169 (2017).
 - [7] N. Rowley, G. R. Satchler, and P. H. Stelson, *Phys. Lett. B* **254**, 25 (1991).
 - [8] J. R. Leigh, M. Dasgupta, D. J. Hinde, J. C. Mein, C. R. Morton, R. C. Lemmon, J. P. Lestone, J. O. Newton, H. Timmers, J. X. Wei, and N. Rowley, *Phys. Rev. C* **52**, 3151 (1995).
 - [9] C. L. Jiang, H. Esbensen, K. E. Rehm, B. B. Back, R. V. F. Janssens, J. A. Caggiano, P. Collon, J. Greene, A. M. Heinz, D. J. Henderson, I. Nishinaka, T. O. Pennington, and D. Seweryniak, *Phys. Rev. Lett.* **89**, 052701 (2002).
 - [10] Ş. Mişicu and H. Esbensen, *Phys. Rev. Lett.* **96**, 112701 (2006); *Phys. Rev. C* **75**, 034606 (2007).
 - [11] C. Simenel, A. S. Umar, K. Godbey, M. Dasgupta, and D. J. Hinde, *Phys. Rev. C* **95**, 031601(R) (2017).
 - [12] T. Ichikawa, K. Hagino, and A. Iwamoto, *Phys. Rev. C* **75**, 057603 (2007); *Phys. Rev. Lett.* **103**, 202701 (2009).
 - [13] T. Ichikawa, *Phys. Rev. C* **92**, 064604 (2015).
 - [14] T. Ichikawa and K. Matsuyanagi, *Phys. Rev. C* **88**, 011602 (2013); **92**, 021602(R) (2015).
 - [15] T. Ichikawa, K. Hagino, and A. Iwamoto, *Phys. Rev. C* **75**, 064612 (2007).
 - [16] C. L. Jiang, H. Esbensen, B. B. Back, R. V. F. Janssens, and K. E. Rehm, *Phys. Rev. C* **69**, 014604 (2004).
 - [17] K. Hagino, N. Rowley, and M. Dasgupta, *Phys. Rev. C* **67**, 054603 (2003).
 - [18] E. S. Z. Thein, N. W. Lwin, and K. Hagino, *Phys. Rev. C* **85**, 057602 (2012).
 - [19] C. L. Jiang, K. E. Rehm, R. V. F. Janssens, H. Esbensen, I. Ahmad, B. B. Back, P. Collon, C. N. Davids, J. P. Greene, D. J. Henderson, G. Mukherjee, R. C. Pardo, M. Paul, T. O. Pennington, D. Seweryniak, S. Sinha, and Z. Zhou, *Phys. Rev. Lett.* **93**, 012701 (2004).
 - [20] C. L. Jiang, B. B. Back, and H. Esbensen, *Phys. Lett. B* **640**, 18 (2006).