Mirror energy differences of $2s_{1/2}$, $1d_{5/2}$, and $1f_{7/2}$ states

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I have examined mirror energy differences between $2s_{1/2}$, $1d_{5/2}$, and $1f_{7/2}$ single-particle states in neutron-excess light nuclei and their proton-excess mirrors. I expand on the earlier $2s_{1/2}$ treatment. For $1d_{5/2}$, I find that 11 such cases can be fitted by a simple expression, which I then use to compute the energies of other $1d_{5/2}$ states. Agreement with experimental values is good. Agreement is found to be even better for $1f_{7/2}$ states.

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I. INTRODUCTION

For many decades, two features have been obvious for the energies of $2s_{1/2}$ and $1d_{5/2}$ (abbreviated s and d here) single-particle (sp) states in light nuclei:

(1) As A decreases from 17 toward lighter nuclei, the s energy decreases relative to the d energy.
(2) The effect is much more pronounced for protons than for neutrons (the so-called Thomas-Ehrman effect).

Workers in the field have long known that both features are extremely well reproduced by simple sp calculations in a diffuse potential well—usually taken to be of Woods-Saxon shape. Two recent papers by Hoffman *et al.* have summarized the evidence for neutrons [1] and for protons [2]. Over the years, many workers have exploited these features in order to compute (or predict) mirror energy differences (MED's) of states that are predominantly sp in character, and/or to extract configuration-mixing amplitudes from experimental MED's.

I define

$$\mathrm{MED} = S_n(\text{neutron-excess nucleus}) - S_p(\text{proton-excess mirror}).$$

Recently, I discovered a simple parametrization for MED's of s states in several nuclei [3]. That analysis considered states whose dominant configuration was an s nucleon plus a core that contained no $2s$ nucleons. As mentioned above, in many of these cases, the energies of the proton-rich nuclei have been previously calculated, with reasonable success, in a simple potential model, under the assumption of mirror symmetry. In Ref. [3], I sought a global representation of the mirror energy differences without introducing a potential or a spectroscopic factor. The aim was to treat the states as single-particle states as long as the spectroscopic factor was known to be large, without the need for a specific numerical value. Such a simple parametrization should prove extremely useful, because in many cases the mass excess of the neutron-excess member of a mirror pair is known, and one wishes to estimate the mass excess of the proton-excess member.

I emphasize that MED's can be defined not only for ground states (g.s.), but also for mirror excited states, making use of the relationship

$$S_{n,p}(E_x) = S_{n,p}(\text{g.s.}) - E_x(+E_x(\text{core})).$$

The term in parentheses is present whenever the state in question is a sp coupled to an excited state of the core. Unless indicated otherwise, separation energies are taken from the most recent mass evaluation [4] and excitation energies from the compilations [5–9].

Within the area of calculating mirror energy differences, workers have long known that for a given mirror pair and a given ℓ value, computed MED's are well described with a quadratic function of the neutron separation energy S_n, for a wide range of S_n. Of course, a computed Coulomb energy should contain a factor Z/R (or $Z/A^{1/3}$). Apparently, no one had noticed that, if a factor of $Z/A^{1/3}$ is removed, the quadratic function is the same for different nuclei. Thus, with the definition

$$\mathrm{MED} = (Z/A^{1/3})\mathrm{MED}',$$

values of MED′ can be compared for different nuclei. This was the basis of my paper [3] on $2s_{1/2}$ sp states. The fact that these MED′ did not exhibit any A dependence is a clear indication that they do not depend on radius—other than in the factor relating MED and MED′.

One product of the $2s_{1/2}$ analysis was a prediction of the resonance energy for ^{10}N(g.s.) $= {}^{9}$C $+ p$, as indicated in Table I. This energy was recently measured [10], and agreement is acceptable. Here, I explore some additional s states, and I then investigate whether a similar procedure can be applied to d and f sp states.

II. ANALYSIS AND RESULTS

A. Additional s states

Results for additional s states are listed in Table II. In ^{19}O, the lowest $1/2^+$ state is primarily of the structure $(d_0^2)s$, so it meets the criterion of having no $2s$ nucleons in the core. The predicted separation energy of its mirror in ^{19}Na misses the experimental value [12] by only 24 keV. The 0^- state in ^{18}N is predominantly a $p_{1/2}$ proton hole in the $1/2^+$ state of ^{19}O, and it therefore also meets the criterion of the model. The predicted

TABLE I. Resonance energy of ^{10}N(g.s.) = ^{9}C + p.

Source	E_p (MeV)	Ref.
Quadratic fit	1.79	[3]
Linear fit	1.86	[3]
Potential model	1.8	[9]
Potential model	1.81–1.94	[11]
Experiment	1.9(2)[a]	[10]

[a]Reference [10], assuming $J^\pi = 1^-$.

energy of its mirror in ^{18}Na agrees reasonably well with the experimental value [13].

The next two states are in the mirror pair ^{15}N/^{15}O, considered as n and p sp states, respectively, plus a ^{14}N(1$^+$) core. Here the agreement is not as good, but both states are known to also contain a small d component [14,15] whose presence decreases the predicted separation energy [15]. The 1^- and 0^- states of ^{14}C are well described as an s neutron coupled to ^{13}C(g.s.). Predictions of their mirror energies in ^{14}O are very good, missing by only 12 keV in both cases. I note that in all these examples, the predicted separation energy is larger than the experimental value.

I have also included the 1^- state of ^{12}Be and the $1/2^+$ state of ^{9}Be. The first is well described as an s nucleon plus the $1/2^-$ first-excited state of ^{11}Be. Its mirror is unknown in ^{12}O, but my prediction is $E_p = 1.49$ MeV. The predicted $1/2^+$ energy in ^{9}B is $E_p = 1.31$ MeV.

I turn now to s states near $N = Z = 14$. The supposition of a closed subshell here is not very good, but we see what ensues. Results for seven such ground states are listed in Table III. The magnitude of the miss for the pair ^{29}Si/^{29}P may indicate that those $1/2^+$ states are not very good sp states. In fact, in the reaction ^{29}Si(p,d), the spectroscopic factor for the first 2^+ state is quite large—0.58 or 0.86 [16]—indicating a significant $2 \times d$ component in the $1/2^+$ g.s. However, the next two predictions are surprisingly accurate. Perhaps the analysis for $^{23-26}$P will spur experiments to measure their masses. The last column of the table is discussed later, in Sec. III.

B. The 1$d_{5/2}$ fit

Here, I investigate the situation for d states. The procedure treats the neutron-excess state as a core plus a d neutron and its proton excess mirror as a core′ plus a d proton. If the states being considered have isospin $T = 1/2$, then core = core′. If $T > 1/2$, core and core′ are mirrors. Other members of an isospin multiplet do not have unique neutron and proton parentage. That is why I restrict attention to the $T_z = \pm T$ extremes.

Results for 11 such states are listed in Table IV. Values of MED′ are plotted in Fig. 1 vs S_n. Unless indicated otherwise, excitation energies and J values have been taken from the compilations [5–9]. These states are of two types: (1) $5/2^+$ states whose dominant structure is a 1$d_{5/2}$ nucleon coupled to a 0^+ core and (2) states of dominant d sp character coupled to $J^\pi \neq 0^+$ cores, whose J^π's are such as to not allow admixture of a core + s configuration. For reasons mentioned above, I have investigated both linear and quadratic fits. The aim is to use the simplest function that works. It can be seen that a quadratic fit is slightly better than a linear fit. The mirror pair that most influences the preference for a quadratic over linear fit is ^{13}C/^{13}N. As mentioned above, for any given Z, A, and ℓ, experience has demonstrated that the MED is well fitted by a quadratic function of S_n. In the following, I concentrate on the quadratic fit (QF), for which, in units of MeV, the fitting function is MED′ = $0.9783 + 0.064 S_n - 0.0056 S_n^2$.

The average rms deviation between experimental and fitted MED values is 33 keV. In many cases for which detailed wave functions are reliably known, potential-model calculations of mirror energy differences have agreed with experimental values to within 30–40 keV. Here a very simple description

TABLE III. Energies (MeV) and J^π of additional $s_{1/2}$ ground states discussed herein.

J^π	Nucl.	S_n(g.s.)	Mirror	S_p(expt.)[a]	S_p(fit)	ImKG[b]
$1/2^+$	^{29}Si	8.474	^{29}P	2.749	2.959	
3^+	^{28}Al	7.725	^{28}P	2.052	2.135	
$1/2^+$	^{27}Mg	6.443	^{27}P	0.870(26)	0.879	
3^+	^{26}Na	5.574(4)	^{26}P	0.14(20)[c]	0.085	−0.119(16)
$1/2^+$	^{25}Ne	4.18(4)	^{25}P	−1.71(40)[c]	−1.008(17)	−1.507(45)
$(1,2,3)^+$	^{24}F	3.82(9)	^{24}P	−2.33(71)[c]	−1.313(58)	−1.866(124)
$1/2^+$	^{23}O	2.73(11)	^{23}P		−2.020(71)	−2.955(107)

[a]Reference [4], unless otherwise noted.
[b]Discussed later, in Sec. III.
[c]Estimate from systematics.

TABLE II. Energies (MeV) and J^π of additional $s_{1/2}$ excited states discussed herein.

J^π[a]	Nucl.	$E_x(n)$[a]	S_n(g.s.)	$S_n(E_x)$	Mirror	$E_x(p)$[a]	S_p(g.s.)	S_p(expt.)	S_p(fit)[b]
$1/2^+$	^{19}O	1.471	3.956	2.485	^{19}Na	0.743	−0.323	−1.066	−1.042
0^-	^{18}N	1.768	2.828	1.07	^{18}Na	0.704	−2.25	−1.954	−1.914
$3/2^+$	^{15}N	7.301	10.833	3.532	^{15}O	6.793	7.297	0.504	0.594
$1/2^+$	^{15}N	8.313	10.833	2.520	^{15}O	7.556	7.297	−0.259	−0.161
1^-	^{14}C	6.094	8.176	2.082	^{14}O	5.173	4.627	−0.543	−0.531
0^-	^{14}C	6.903	8.176	1.273	^{14}O	5.71	4.627	−1.083	−1.071
1^-	^{12}Be	2.70	3.171	0.79	^{12}O				−1.49
$1/2^+$	^{9}Be	1.684	1.665	−0.019	^{9}B		−0.186		−1.31

[a]References [5–9].
[b]In units of MeV, the fitting function is MED′ = $0.6377 + 0.1447 S_n - 0.0091 S_n^2$.

TABLE IV. Energies (MeV) and J^π of predominantly $d_{5/2}$ states included in the fit.

J^{π} [a]	Nucl.	$E_x(n)$ [a]	S_n(g.s.)	$S_n(E_x)$	Mirror	$E_x(p)$ [a]	S_p(g.s.)	S_p(expt.)	S_p(fit) [b]
$5/2^+$	^{17}O	0	4.143	4.143	^{17}F	0	0.600	0.600	0.573
2^-	^{16}N	0	2.489	2.489	^{16}F	0.424	−0.536	−0.960	−1.012
3^-	^{16}N	0.298	2.489	2.191	^{16}F	0.721	−0.536	−1.257	−1.275
$5/2^+$	^{15}N	7.155	10.833	3.678	^{15}O	6.859	7.297	0.438	0.448
$7/2^+$	^{15}N	7.567	10.833	3.266	^{15}O	7.276	7.297	0.021	0.065
3^-	^{14}C	6.728	8.176	1.448	^{14}O	6.28	4.627	−1.653	−1.628
2^-	^{14}C	7.341	8.176	0.835	^{14}O	6.769	4.627	−2.142	−2.150
$5/2^+$	^{13}C	3.854	4.946	1.092	^{13}N	3.547	1.943	−1.604	−1.566
3^-	^{12}B	3.389	3.37	−0.019	^{12}N	3.132	0.601	−2.531	−2.580
4^-	^{12}B	4.518	3.37	−1.148	^{12}N	4.14	0.601	−3.539	−3.500
$5/2^+$	^{11}Be	1.778	0.502	−1.276	^{11}N			−3.65	−3.67

[a]References [5–9].
[b]In units of MeV, fitting function is $\mathrm{MED}' = 0.9783 + 0.064S_n - 0.0056S_n^2$.

is found to provide comparable agreement. And, it involves nothing about the details of the wave function. The numerical value of the spectroscopic factor is not used—only the fact that it is large. Nothing is input about the remainder of the wave function. Deviations between fitted and experimental MED values are plotted vs A in Fig. 2.

C. Other *d* states

I now apply the QF to several other d states in various light nuclei (Table V). As I discuss each case briefly, the reasons why they were not included in the original fitting will be obvious.

For ^{15}N/^{15}O, the two states that were included in the fitting have $J^\pi = 5/2^+$ and $7/2^+$ and large $\ell = 2$ spectroscopic factors [14]. Coupling d to ^{14}N(g.s.) also produces a $3/2^+$ state, of which three are known in the relevant energy region. The lowest, at $E_x = 7.301$ MeV in ^{15}N, has $S(s) = 0.98(3)$ and a small $S(d)$ [14]. (Here, S stands for spectroscopic factor.)

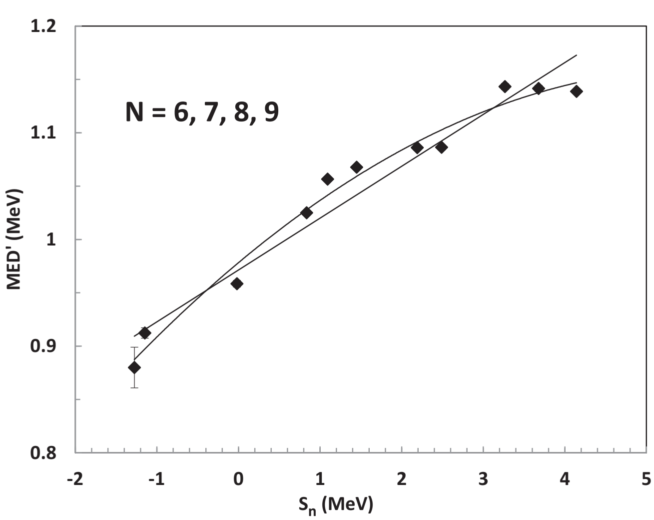

FIG. 1. Plot of $\mathrm{MED}' = \mathrm{MED}(A^{1/3}/Z)$ vs neutron separation energy, with linear and quadratic fits, for $d_{5/2}$ sp states: $\mathrm{MED} = S_n(\text{neutron-excess nucleus}) - S_p(\text{proton-excess mirror})$.

Using the fit parameters from the s analysis [3], the predicted excitation energy of the mirror state in ^{15}O is too low by 98 keV—consistent with the effect of a small d-wave component. The other two, at 8.571 and 10.066 MeV, have $S(d) = 0.13(2)$ and $0.65(2)$, respectively [14], with small s-wave components [15]. Thus, we would expect the higher one to meet the conditions of the current analysis, and the lower one not to. This expectation is borne out in the energies listed in Table V. The fitted ^{15}O energy differs from the experimental one by 143 keV for the 8.571-MeV state, but only by 47 keV for the 10.066-MeV state.

In ^{15}C/^{15}F, the $5/2^+$ resonance energy from the fit agrees with the average [17] of several experimental values to 6 keV, even though the resonance is quite broad.

The ^{14}B/^{14}F case was treated previously [18,19]. The present fit parameters produce a fitted 4^- energy in ^{14}F that differs from the experimental value of 4.35(10) MeV [18] by 0.25 MeV, but the resonance is quite broad [0.55(10) MeV]. A second measurement of this resonance energy might be desirable.

For the 3^- state at 4.56 MeV in ^{12}Be, the present fit predicts an energy of $E_p = 4.075$ MeV for the mirror in ^{12}O, where the state is unknown.

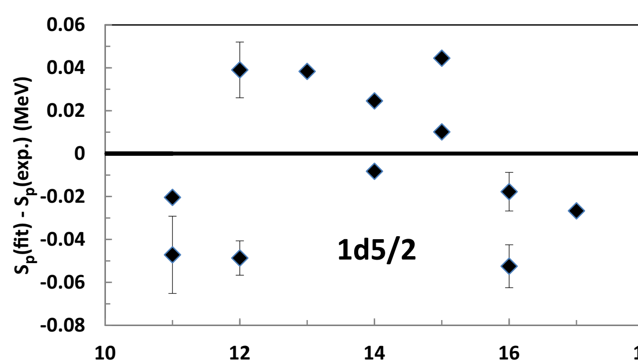

FIG. 2. Differences between calculated and experimental proton separation energies for $d_{5/2}$ sp states plotted vs A.

TABLE V. Energies (MeV) and J^π of additional $d_{5/2}$ states discussed herein.

J^{π} [a]	Nucl.	$E_x(n)$ [a]	S_n(g.s.)	$S_n(E_x)$	Mirror	$E_x(p)$ [a]	S_p(g.s.)	S_p(expt.)	S_p(fit) [d]
$3/2^+$	^{15}N	8.571	10.833	2.191	^{15}O	8.284	7.297	−0.987	−0.844
$3/2^+$	^{15}N	10.07	10.833	3.678	^{15}O	9.484	7.297	−2.187	−2.140
$5/2^+$	^{15}C	0.74	1.218	−0.019	^{15}F			−2.785 [b]	−2.791
4^-	^{14}B	2.04	0.97	−1.148	^{14}F			−4.35 [c]	−4.10
3^-	^{12}Be	4.56	3.171	1.092	^{12}O				−4.075
$9/2^+$	^{11}B	11.265	11.454	0.189	^{11}C	10.679	8.689	−1.990	−2.037
3^-	^{10}Be	7.371	6.812	0.835	^{10}C				−2.742
$5/2^+$	^{9}Be	3.049	1.665	−1.276	^{9}B	2.788	−0.186	−2.974	−3.074

[a]References [5–9].
[b]Reference [17].
[c]Reference [18].
[d]In units of MeV, fitting function is $\mathrm{MED}' = 0.9783 + 0.064S_n - 0.0056S_n^2$.

For ^{11}B/^{11}C, the 3^+ g.s. of ^{10}B means that states with $J^\pi = 5/2^+$ or $7/2^+$ could also have $\ell = 0$ parentage. States with $1/2^+$ to $5/2^+$ will also contain some parentage to 0^+ and 2^+ $T = 1$ cores. Thus, I have chosen the lowest $9/2^+$ state as a candidate for pure d structure. From the table, we note that the fit misses the experimental energy by 47 keV.

For the 3^- state at 7.371 MeV in ^{10}Be, the predicted energy of its mirror in ^{10}C is $E_p = 2.742$ MeV. The state is not known in ^{10}C.

The $5/2^+$ states in ^{9}Be and ^{9}B both have appreciable widths—282(11) keV in ^{9}Be and 550(40) keV in ^{9}B. More importantly, the daughters of their decays (^{8}Be and ^{5}He or ^{5}Li) also have large widths. These large widths of the daughter nuclei cause a distortion of the peak shape in the parent, thereby changing its energy. The predicted ^{9}B energy from the present fit is $E_p = 3.074$ MeV, compared to the experimental value of 2.974(30) MeV. In a potential model, in which the profiles of the daughters were integrated over, the calculated energy was $E_p = 3.04$ MeV [20]. The difference is not substantial.

Another set of d ground states includes the nuclei $^{21-25}$Al, considered as mirrors of nuclei from ^{25}Mg to ^{21}O. Results for them are presented in Table VI. It can be noted that agreement for $^{23-25}$Al is extremely good. Experimental proton separation energies are unknown for the lighter Al nuclei. For completeness, I also include ^{19,20}Al. The last column of this table is discussed later, in Sec III.

TABLE VI. Energies (MeV) and J^π of additional $d_{5/2}$ ground states discussed herein.

J^{π} [a]	Nucl.	S_n(g.s.)	Mirror	S_p(expt.) [b]	S_p(fit)	ImKG [a]
$5/2^+$	^{25}Mg	7.331	^{25}Al	2.272	2.276	
4^+	^{24}Na	6.959	^{24}Al	1.863	1.859	
$5/2^+$	^{23}Ne	5.201	^{23}Al	0.141	0.142	
(4^+)	^{22}F	5.230(13)	^{22}Al	0(400) [c]	0.094	0.149(13)
$(5/2^+)$	^{21}O	3.806(12)	^{21}Al	−2.14(40) [c]	−1.315	−1.265(13)
–	^{20}N	7.49(6)	^{20}Al	–	−2.933	−2.89(6)
–	^{19}C	4.76(10)	^{19}Al	–	−4.491	−4.489(104)

[a]Discussed in Sec. III.
[b]Reference [4], unless otherwise noted.
[c]Estimate from systematics.

D. Comparison of s and d fits

A comparison of fit results for s and d are compared in Fig. 3, where the MED′ values are plotted vs S_n. This plot provides a visual representation of the Thomas-Ehrman effect.

E. ^{17}C/^{17}Na

The mirror pair ^{17}C/^{17}Na merits special consideration, because the nature of the mixed s and d parentage is clear. Various calculations [21–24] predict that the g.s. of ^{17}Na will be the mirror of the $1/2^+$ state at 0.21 MeV in ^{17}C. The most recent potential-model calculation [24] predicted the proton energy for $^{17}\mathrm{Na} \to {}^{16}\mathrm{Ne} + p$ to be $E_p = 3.02$ MeV (Table VII). This calculation used spectroscopic factors from a shell-model calculation and experimental energies of six $(sd)^2$ states in ^{16}C and ^{16}Ne. (The energy of the excited 0^+ state was taken from a calculation, because the energy of that state in ^{16}Ne is not known.)

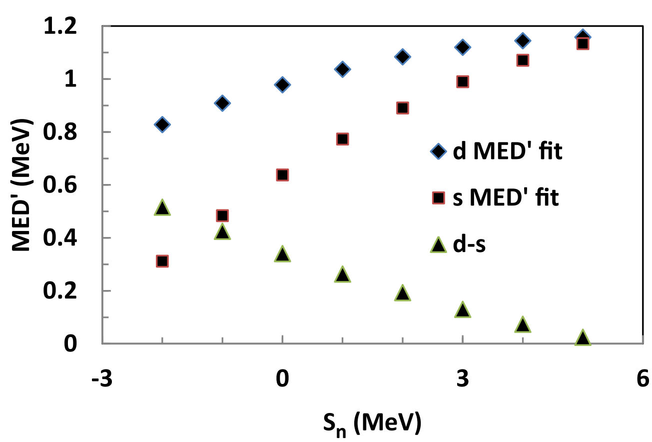

FIG. 3. Plot of fitted s and d MED′ vs S_n, and their difference.

TABLE VII. Predictions for the energy of the $1/2^+$ g.s. of ^{17}Na.

Source	E_p (MeV)	Ref.
QF *s*	2.24	Present
QF *d*	3.40	Present
$(1/3)\,s + (2/3)\,d$	3.01	Present
Potential model	3.02	[24]
Multichannel algebraic scattering	1.03	[23]
Microscopic cluster model	2.40	[21]

Present quadratic fits predict $E_p = 2.24$ MeV for s and 3.40 MeV for d. In the simple shell model, the summed s spectroscopic factor is 1.0—roughly equally split between the two 0^+ core states. For d, the summed S is 2.0—split among three states (two 2^+ and one 3^+). Weighting the QF predictions by these S's produces a result of $E_p = 3.01$ MeV. The agreement with the potential-model calculation is remarkable.

F. The $1f_{7/2}$ fit

I now skip to the next major shell, viz., $1f_{7/2}$ states coupled to cores with no $1f_{7/2}$ occupancy. Energies [16,25–31] for eight mirror pairs have been included in the fit (see Table VIII and Fig. 4). Note the greatly expanded vertical scale in the figure. Here a linear fit suffices. The average value of the absolute energy differences is 19 keV.

G. Other $1f_{7/2}$ states

Results for other $1f_{7/2}$ states are listed in Table IX. In ^{36}Cl, the excited-state energies are only tentative. For most of the others, the experimental g.s. proton separation energies are unknown and are only estimated from systematics [4].

III. COMPARISON WITH OTHER PREDICTIONS

Many of the states discussed here are excited states, but still of sp character. Most models for predicting nuclear masses refer only to ground states. Thus, comparisons with other work can be made only for nuclei whose g.s. is a sp state. An improved Kelson-Garvey (ImKG) model has recently appeared [32]. In Table X, I compare the predictions of my simple linear

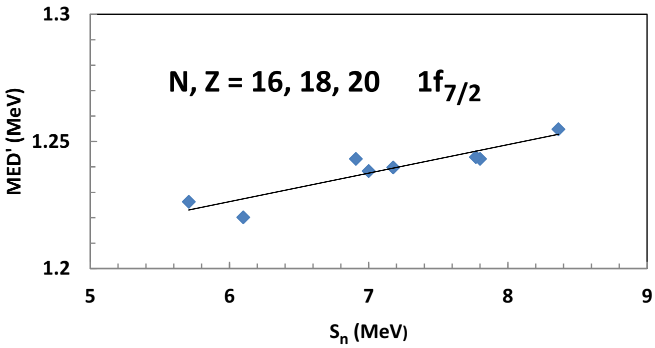

FIG. 4. Plot of MED′ vs neutron separation energy, with linear fit, for $f_{7/2}$ sp states.

fit with those of the ImKG model for most of the ground states in Table IX. The agreement between the two sets of predictions is astounding, because the two approaches are quite different. My present analysis depends only on the neutron separation energy of the mirror, whereas the ImKG results involve the masses of several nearby nuclei. The reason for this close agreement is unknown to me.

Comparison of my analysis with the ImKG predictions for $2s_{1/2}$ ground states in $^{23-26}$P and $d_{5/2}$ ground states in $^{19-22}$Al are given in Tables III and VI. From these comparisons, it appears that the ImKG analysis may not fully reproduce the Thomas-Ehrman effect. The two sets of predictions are nearly identical for d states, but they differ considerably for s states.

I note that for ^{17}Na (discussed in Sec. IIE above), the ImKG prediction is $E_p = 3.561(26)$ MeV, but that is presumably for the mirror of the $3/2^+$ g.s. of ^{17}C.

A paper by Vogt *et al.* [33] concerns formulas for neutron separation energies. It is therefore not relevant here. Starting with an improved Weizsacker mass formula [34], Bao *et al.* [35] obtained three-parameter expressions for MED's. Their parameters are different for different $N-Z$, and are different for various major shells. Whenever possible, I have compared their predictions with the results of my analysis. I emphasize that such a comparison is possible only for a nucleus whose g.s. is predominantly sp in character. This is because I have considered only sp states here (but both ground and excited states), and Bao *et al.* considered only ground states. Such a compari-

TABLE VIII. Energies (MeV) and J^π of predominantly $f_{7/2}$ states included in the fit.

J^{π} [a]	Nucl.	$E_x(n)$ [a]	S_n(g.s.) [b]	$S_n(E_x)$	Mirror	$E_x(p)$ [a]	S_p(g.s.) [b]	S_p(expt.)	S_p(fit)
$7/2^-$	^{41}Ca	0	8.363	8.363	^{41}Sc	0	1.085	1.085	1.097
4^-	^{40}K	0	7.800	7.8	^{40}Sc	0	0.53	0.53	0.511
3^-	^{40}K	0.030	7.800	7.77	^{40}Sc	0.034	0.53	0.496	0.483
2^-	^{40}K	0.800	7.800	7	^{40}Sc	0.772	0.53	−0.242	−0.237
5^-	^{40}K	0.892	7.800	6.908	^{40}Sc	0.892	0.53	−0.362	−0.323
$7/2^-$	^{39}Ar	0	6.599	6.599	^{39}Sc	0	−0.597	−0.597	−0.640
$7/2^-$	^{37}Ar	1.611	8.787	7.176	^{37}K	1.379	1.858	0.479	0.481
$7/2^-$	^{33}S	2.934	8.642	5.708	^{33}Cl	2.686	2.277	−0.409	−0.393

[a]References [16,25–31].
[b]Reference [4].

TABLE IX. Energies (MeV) and J^{π} of additional $f_{7/2}$ states discussed herein.

J^{π} [a]	Nucl.	E_x	S_n(g.s.)	$S_n(E_x)$	Mirror	E_x	S_p(g.s.)[a]	S_p(expt.)	S_p(fit)
2^-	^{40}Cl	0	5.83(3)	5.83	^{40}V	0	−2.40(45)[b]		−2.046
2^-	^{38}Cl	0	6.108	6.108	^{38}Sc	0	−1.3(2)[b]	−1.3	−1.194
2^-	^{36}Cl	1.952	8.58	6.628	^{36}K	(1.706)	1.659	−0.047	−0.095
3^-	^{36}Cl	2.469	8.58	6.111	^{36}K	(2.197)	1.659	−0.538	−0.580
$7/2^-$	^{37}S	0	4.304	4.304	^{37}Sc	0	−2.65(30)[b]	−2.65	−2.942
(4^-)	^{36}P	0	3.465	3.465	^{36}Sc	0	−3.27(36)[b]	−3.27	−3.790
$(7/2^-)$	^{35}Si	0	2.47	2.47	^{35}Sc				−4.786
(4^-)	^{34}Al	0	2.67	2.67	^{34}Sc				−4.670

[a]Reference [4], unless otherwise noted.
[b]Estimate from systematics.

son for 11 nuclei is given in Table XI. For ease of comparison, I have converted my proton separation energies (Ref. [3] and Sec. II here) into mass excesses. For Bao *et al.*'s predictions, the average uncertainty in the theoretical masses for these 11 nuclei is 240 keV, and the average of the absolute deviations between experimental and theoretical masses is 311 keV. However, the average of the absolute deviations in my analysis for these 11 nuclei is 74 keV.

Earlier, I performed a different analysis for MED's of core + pp nuclei [36,37]. I found that a three-parameter fit (3PF) worked well, and the MED's depended on both neutron separation energy and $2s_{1/2}$ occupancy. A comparison of those results with those of Bao *et al.* is given in Table XII. Here, the average uncertainty in Bao *et al.*'s predictions is 243 keV, and the average absolute deviation is 209 keV. For my analysis, the latter is 31 keV.

IV. SUMMARY

I find that mirror energy differences of predominantly $1d_{5/2}$ states in light nuclei can be fitted by a simple expression, as was the case previously [3] for $2s_{1/2}$ states. Quadratic and linear fits give comparable agreement, with a slight preference for quadratic. I then used this formula to compute expected energies of $1d_{5/2}$ states in other nuclei. Agreement is generally good. Application of the same procedure to $1f_{7/2}$ sp states indicates also good agreement, with only a linear fit needed. Predictions for several other $1f_{7/2}$ ground states are made, and it is noted that they are in amazing agreement with predictions of a recent improved Kelson-Garvey model.

TABLE X. Comparison of energies (MeV) from present fit and from ImKG model [32].

Nucl.	S_n(g.s.)[a]	Mirror	S_p(expt.)[a]	S_p(fit)	S_p(ImKG)
^{43}Ca	7.933	^{43}V	0.10(4)	0.097	0.094(43)
^{42}K	7.534	^{42}V	−0.79(30)[b]	−0.335	−0.351(64)
^{41}Ar	6.099	^{41}V	−1.76(34)[b]	−1.731	−1.679(68)
^{40}Cl	5.83(3)	^{40}V	−2.40(45)[b]	−2.046	−1.942(72)
^{38}Cl	6.108	^{38}Sc	−1.3(2)[b]	−1.194	−1.155(4)
^{37}S	4.304	^{37}Sc	−2.65(30)[b]	−2.942	−2.961(4)
^{36}P	3.465	^{36}Sc	−3.27(36)[b]	−3.790(13)	−3.800(14)
^{35}Si	2.47	^{35}Sc		−4.786(36)	−4.789(42)
^{34}Al	2.67	^{34}Sc		−4.670(100)	−4.583(92)

[a]Reference [4], unless otherwise noted.
[b]Estimate from systematics.

TABLE XI. Mass excesses (MeV) and uncertainties (keV) for various sp ground states discussed herein.

Nucl.	M (expt.)[a]	Unc. (expt.)[a]	M (fit)[b]	M (th)[c]	Unc. (th)[c]
^{10}N	38.8[d]	400	37.99	38.495	237
^{11}N	24.304	46	24.396	25.141	289
^{14}F	31.964	41	31.988	32.281	238
^{15}F	16.807	62	16.675	17.064	289
^{16}F	10.680	8	10.912	11.051	205
^{26}P			11.031	10.909	237
^{27}P	−0.722	26	−0.731	−0.779	289
^{28}P	−7.148		−7.23	−7.222	205
^{29}P	−16.952		−17.163	−16.903	126
^{31}Cl	−7.066	50	−7.044	−7.172	289
^{32}Cl	−13.335		−13.324	−13.403	205
^{33}Cl	−21.003		−21.005	−20.969	126
^{35}K	−11.173		−11.216	−11.183	289
^{36}K	−17.417		−17.358	−17.398	205
^{37}K	−24.800		−24.825	−24.73	126
^{41}Sc	−28.642		−28.654	−28.482	126
^{40}Sc	−20.523	3	−20.505	−20.534	205
^{39}Sc	−14.173	24	−14.129	−14.114	289
^{38}Sc	−4.55[e]	200	−4.653	−4.661	237
^{43}V	−17.916	43	−17.913	17.526	289
^{42}V	7.62[e]	300	8.074	7.479	237

[a]Reference [4], unless otherwise noted. Uncertainties larger than 1 keV are listed.
[b]Reference [3] and herein.
[c]Reference [32].
[d]New experiment reports $M = 38.1(2)$ MeV.
[e]Estimate from systematics.

TABLE XII. Mass excesses (MeV) and uncertainties (keV) for various core + pp ground states discussed herein.

Nucl.	M (expt.)[a]	Unc. (expt.)[a]	M (3PF)[b]	M (th)[c]	Unc. (th)[c]
^{12}O	31.915	24	32.042	32.371	237
^{13}O	23.115	10	23.118	22.848	289
^{14}O	8.007		8.003	7.885	205
^{16}Ne	23.986	20	23.997	23.979	237
^{17}Ne	16.500		16.495	16.261	290
^{18}Ne	5.318		5.317	5.354	205
^{20}Mg	17.559	27	17.626[d]	17.223	237

[a]Reference [4], unless otherwise noted. Uncertainties larger than 1 keV are listed.
[b]Reference [36] and herein.
[c]Reference [32].
[d]Reference [37].

[1] C. R. Hoffman, B. P. Kay, and J. P. Schiffer, Phys. Rev. C **89**, 061305(R) (2014).
[2] C. R. Hoffman, B. P. Kay, and J. P. Schiffer, Phys. Rev. C **94**, 024330 (2016).
[3] H. T. Fortune, Phys. Rev. C **88**, 024309 (2013).
[4] M. Wang, G. Audi, A. H. Wapstra, F. G. Kondev, M. MacCormick, X. Xu, and B. Pfeiffer, Chin. Phys. C **36**, 1603 (2012).
[5] D. R. Tilley, H. R. Weller, and C. M. Cheves, Nucl. Phys. A **564**, 1 (1993).
[6] F. Ajzenberg-Selove, Nucl. Phys. A **523**, 1 (1991).
[7] F. Ajzenberg-Selove, Nucl. Phys. A **506**, 1 (1990).
[8] D. R. Tilley, H. R. Weller, C. M. Cheves, and R. M. Chasteler, Nucl. Phys. **595**, 1 (1995).
[9] D. R. Tilley, J. H. Kelley, J. L. Godwin, D. J. Millener, J. E. Purcell, C. G. Sheu, and H. R. Weller, Nucl. Phys. A **745**, 155 (2004).
[10] J. Hooker *et al.*, Phys. Lett. B **769**, 62 (2017).
[11] R. Sherr and H. T. Fortune, Phys. Rev. C **87**, 054333 (2013).
[12] C. Angulo *et al.*, Phys. Rev. C **67**, 014308 (2003).
[13] M. Assie *et al.*, Phys. Lett. B **712**, 198 (2012).
[14] C. E. Mertin, D. D. Caussyn, A. M. Crisp, N. Keeley, K. W. Kemper, O. Momotyuk, B. T. Roeder, and A. Volya, Phys. Rev. C **91**, 044317 (2015).
[15] H. T. Fortune, Phys. Rev. C **94**, 024339 (2016).
[16] P. M. Endt and C. Van Der Leun, Nucl. Phys. A **310**, 1 (1978).
[17] H. T. Fortune, Phys. Rev. C **74**, 054310 (2006).
[18] V. Z. Goldberg *et al.*, Phys. Lett. B **692**, 307 (2010).
[19] R. Sherr and H. T. Fortune, Phys. Lett. B **699**, 281 (2011).
[20] R. Sherr and H. T. Fortune, Phys. Rev. C **70**, 054312 (2004).
[21] N. K. Timofeyuk and P. Descouvemont, Phys. Rev. C **81**, 051301(R) (2010).
[22] H. T. Fortune and R. Sherr, Phys. Rev. C **82**, 027310 (2010).
[23] K. Amos *et al.*, Nucl. Phys. A **879**, 132 (2012).
[24] H. T. Fortune, Phys. Rev. C **90**, 067302 (2014).
[25] P. M. Endt, Nucl. Phys. A **633**, 1 (1998).
[26] C. D. Nesaraja and E. A. McCutchan, Nucl. Data Sheets **133**, 1 (2016).
[27] B. Singh and J. A. Cameron, Nucl. Data Sheets **107**, 225 (2006).
[28] J. Chen, Nucl. Data Sheets **140**, 1 (2017).
[29] J. A. Cameron and B. Singh, Nucl. Data Sheets **109**, 1 (2008).
[30] N. Nica, J. Cameron, and B. Singh, Nucl. Data Sheets **113**, 1 (2012).
[31] J. Chen, J. Cameron, and B. Singh, Nucl. Data Sheets **112**, 2715 (2011).
[32] J. L. Tian, N. Wang, C. Li, and J. J. Li, Phys. Rev. C **87**, 014313 (2013).
[33] K. Vogt, T. Hartmann, and A. Zilges, Phys. Lett. B **517**, 255 (2001).
[34] N. Wang, M. Liu, and X. Z. Wu, Phys. Rev. C **81**, 044322 (2010); N. Wang, Z. Y. Liang, M. Liu, and X. Z. Wu, *ibid.* **82**, 044304 (2010); M. Liu, N. Wang, Y. G. Deng, and X. Z. Wu, *ibid.* **84**, 014333 (2011).
[35] M. Bao, Y. Lu, Y. M. Zhao, and A. Arima, Phys. Rev. C **94**, 044323 (2016).
[36] H. T. Fortune, Phys. Lett. B **718**, 1342 (2013).
[37] H. T. Fortune, Phys. Rev. C **94**, 044305 (2016).