Inclusive $\pi^+ d \to p(\eta p)$ process and the ηN scattering length

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The cross section of the inclusive process $\pi^+ d \to p(\eta p)$ is calculated as a function of the ηp invariant mass when the detected proton is moving in the forward direction. The incident pion has a momentum of $p_{\rm lab} = 898.47$ MeV/c for which the ηp pair are left at rest in the laboratory system which allows one to study the effect of the $\eta p \to \eta p$ final-state interaction in the region of the $N(1535)S_{11}$ resonance. The sensitivity of the inclusive cross section to different parametrizations of the ηN final-state interaction is discussed.

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I. INTRODUCTION

In a previous work [1] I studied the exclusive η-production process $\pi^+ d \to \eta pp$, where the two protons would be detected in coincidence one of them in the forward direction with a momentum almost equal to the incident pion momentum and the other one with very low momentum. This follows from the proposal put forward by Fujioka and Itahashi [2,3] where they point out that for a pion incident momentum $\approx$900 MeV/c the η and one of the protons will be left at rest in the laboratory system so that this process appears to be very well suited to study the ηN final-state interaction in the region of the $N(1535)S_{11}$ resonance, i.e., to study the ηN amplitude very near threshold, in particular, the value of the ηN scattering length.

Although the results of Ref. [1] show large sensitivity of the kinematically complete differential cross section to the η-nucleon low-energy parameters, they require the measurement of the slow proton at very low momenta (100–200 MeV/c) which apparently is difficult to do in the actual experiment [4]. On the other hand, the forward proton has a relatively large momentum ($\sim$900 MeV/c) that can be easily measured in the experiment. This has led to the conclusion that it is better to measure only the forward proton at momenta near 900 MeV/c [4]. Such an approach has been discussed recently for the case of the similar process $\gamma d \to p(\eta n)$ [5,6]. The model used here includes first- and second-order diagrams involving $\pi N \to \pi N$, $\pi N \to \eta N$, $\eta N \to \eta N$, and $NN \to NN$ interactions as shown in Fig. 1.

As I noted in Ref. [1], the ηN scattering length $a_{\eta N}$ is not well known since it must be obtained indirectly from the combined analysis of $\pi N \to \pi N$ and $\gamma N \to \pi N$ together with the available differential and total cross sections data of $\pi N \to \eta N$. Although for the various amplitude analyses [7–11] the imaginary part of the ηN scattering length comes out quite stable at $\sim$0.26 fm, basically as a consequence of the optical theorem, the values for Re $a_{\eta N}$ run from 0.4 to 1.07 fm. This work explores the use of the inclusive $\pi^+ d \to p(\eta p)$ process to try to determine the value of the real part of the ηN scattering length.

The formalism of the inclusive $\pi^+ d \to p(\eta p)$ process is described in Sec. II and the results are presented in Sec. III

II. FORMALISM

Since the formalism to describe the $\pi^+ d \to \eta pp$ amplitude has been presented in full detail in Ref. [1] it will not be repeated here. I will instead just write down the corresponding expression for the inclusive differential cross section and some of the basic elements that enter in the construction of the $\pi^+ d \to \eta pp$ amplitude.

A. The inclusive $\pi^+ d \to p(\eta p)$ cross section

Assuming that the incident pion has a momentum $q_{\rm lab} = 898.47$ MeV/c in the laboratory system and that one of the protons moves along the direction of the incident pion (the z axis) with momentum $k_{\rm lab} \leqslant q_{\rm lab}$, the maximum momentum that the forward proton can reach is $k_{\rm lab} = q_{\rm lab}$, when the second proton and the η meson are left at rest in the laboratory frame.

Following Ref. [6] one should notice that the experimental data are actually given in the form of a ratio, $R_{\rm exp}$ of the measured cross section for $\pi^+ d \to p(\eta p)$ divided by those of $\pi^+ n \to \eta p$ convoluted with the proton momentum distribution in the deuteron. This is for removing systematic uncertainties of the acceptance from the detector coverage. Thus, from the theoretical side, the corresponding quantity to calculate is

$$R = \frac{d\sigma_{\rm full}/dM_{\eta p}}{d\sigma_{\rm imp}/dM_{\eta p}}, \quad (1)$$

where $M_{\eta p}$ is the ηp invariant mass, $\sigma_{\rm full}$ is calculated with the full model of Fig. 1, and $\sigma_{\rm imp}$ is calculated using only the single scattering diagram, Fig. 1(a).

I will calculate the differential cross section in the center of mass frame. If one calls the two nucleons as particles 1 and 2 and the η meson as particle 3, the kinematically complete differential cross section in the c.m. frame when particle 1

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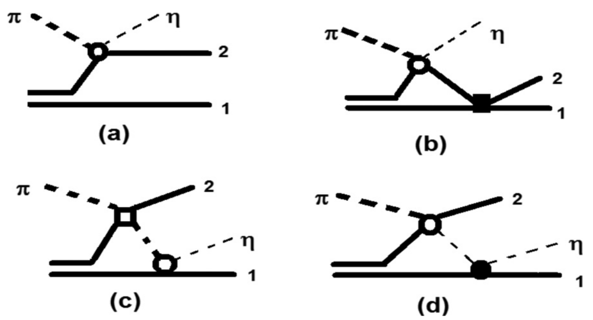

FIG. 1. Model of the $\pi d \to \eta NN$ process: (a) single-scattering diagram, (b) N-exchange diagram, (c) π-exchange diagram, and (d) η-exchange diagram.

moves in the forward direction is given by

$$\left.\frac{d\sigma}{dk_1 dk_3 d\cos\theta_1}\right|_{\cos\theta_1=1} = \frac{M^2 k_1 k_3}{48(2\pi)^3 q_{\text{c.m.}} W E_1 \omega_3} \frac{2}{3} \sum_{\text{spins}} |A|^2, \tag{2}$$

The factor $\frac{2}{3}$ in Eq. (2) comes from the isospin and the sum is over the spin projections of the two nucleons and deuteron. M is the proton mass, W is the invariant energy of the system, k_1 (E_1) and k_3 (ω_3) are the momenta (energies) of particles 1 and 3 while $q_{\text{c.m.}}$ is the pion momentum, all of them measured in the three-body c.m. frame.

The inclusive differential cross section is obtained as

$$\frac{d\sigma}{dk_1} = \int_{k_3^-}^{k_3^+} dk_3 \left.\frac{d\sigma}{dk_1 dk_3 d\cos\theta_1}\right|_{\cos\theta_1=1}, \tag{3}$$

where k_3^- and k_3^+ are the solutions of

$$W = \sqrt{M^2 + k_1^2} + \sqrt{M^2 + (k_1 \pm k_3)^2} + \sqrt{m_\eta^2 + k_3^2}. \tag{4}$$

Finally, since the ηp invariant mass squared is given by

$$M_{\eta p}^2 = W^2 + M^2 - 2W\sqrt{M^2 + k_1^2}, \tag{5}$$

one gets that

$$\frac{d\sigma}{dM_{\eta p}} = \frac{d\sigma}{dk_1} \frac{M_{\eta p}\sqrt{M^2 + k_1^2}}{k_1 W}. \tag{6}$$

B. The $\pi^+ d \to \eta pp$ amplitude

The model of the $\pi^+ d \to \eta pp$ amplitude shown diagrammatically in Fig. 1 includes single- and double-scattering terms to which one must add the corresponding diagrams where the final nucleons 1 and 2 are interchanged. Thus, the amplitude is given by

$$A = A_0 + A_N + A_\pi + A_\eta + (1 \leftrightarrow 2). \tag{7}$$

The single-scattering term A_0 is represented by Fig. 1(a) where nucleon 2 in the deuteron undergoes the elementary process $\pi N \to \eta N$ while nucleon 1 remains as spectator. The term A_N is represented by Fig. 1(b) where after the elementary η production process a nucleon is exchanged. Similarly, the terms A_π and A_η represent the processes depicted by Figs. 1(c) and 1(d) where a pion or an η are exchanged. As I pointed out in Ref. [1], Fig. 1(a) is given by

$$A_0 = \bar{u}_2(\vec{k}_2) t_{\pi N \to \eta N} \frac{\not{K} - \not{q}_\pi - \not{k}_1 + M}{(K - q_\pi - k_1)^2 - M^2 + i\epsilon} V_{dNN} v_1(\vec{k}_1), \tag{8}$$

where K is the total four-momentum and k_1 and k_2 are the two final proton momenta. V_{dNN} is the deuteron-nucleon-nucleon vertex with one nucleon off the mass shell, $v_1 = i\gamma_2 u_1^*$ is a charge conjugate spinor for nucleon 1, and $t_{\pi N \to \eta N}$ is the elementary pion-induced η-production amplitude.

The double-scattering terms depicted by Figs. 1(b), 1(c) and 1(d), are evaluated by putting the spectator nucleon on the mass shell in the loop of the corresponding diagrams. Thus, the term where a nucleon is exchanged [Fig. 1(b)] is given by

$$\begin{aligned} A_N = {} & \frac{1}{(2\pi)^3} \int \frac{M}{E_k} d\vec{k}\, \bar{u}_2(\vec{k}_2) \bar{v}_1(\vec{k}) t_{NN \to NN} v_1(\vec{k}_1) \\ & \times \frac{\not{K} - \not{q}_\eta - \not{k} + M}{(K - q_\eta - k)^2 - M^2 + i\epsilon} \\ & \times t_{\pi N \to \eta N} \frac{\not{K} - \not{q}_\pi - \not{k} + M}{(K - q_\pi - k)^2 - M^2 + i\epsilon} V_{dNN} v_1(\vec{k}), \end{aligned} \tag{9}$$

while the terms where a meson b ($b = \pi, \eta$) is exchanged [Figs. 1(c) and 1(d)] are given by

$$\begin{aligned} A_b = {} & \frac{1}{(2\pi)^3} \int \frac{M}{E_k} d\vec{k}\, \bar{u}_1(\vec{k}_1) t_{bN \to \eta N} u_1(\vec{k}) \\ & \times \frac{1}{(K - k_2 - k)^2 - m_b^2 + i\epsilon} \\ & \times \bar{u}_2(\vec{k}_2) t_{\pi N \to bN} \frac{\not{K} - \not{q}_\pi - \not{k} + M}{(K - q_\pi - k)^2 - M^2 + i\epsilon} \\ & \times V_{dNN} v_1(\vec{k}); \quad b = \pi, \eta. \end{aligned} \tag{10}$$

As I pointed out in Ref. [1], the deuteron-nucleon-nucleon vertex with one nucleon off the mass shell is of the form [12,13]

$$\begin{aligned} V_{dNN} = {} & F(k'^2)\not{\epsilon}_d + \frac{1}{M} G(k'^2) \epsilon_d \cdot k \\ & + \frac{\not{k}' - M}{M} \left[H(k'^2)\not{\epsilon}_d + \frac{1}{M} I(k'^2) \epsilon_d \cdot k \right], \end{aligned} \tag{11}$$

where ϵ_d is the polarization vector of the deuteron and k and k' are the four-momenta of the on-shell and off-shell nucleons, respectively. The connection between the form factors F, G, H, and I and the components of the deuteron wave function (the familiar s- and d-wave states u and w plus the two relativistic v_t and v_s p-wave states) is given in Refs. [1,12,13]. As I explained in Ref. [1], the four components of the deuteron wave function u, w, v_t, and v_s, have been constructed by Buck and Gross [12] by considering different models of the NN interaction. These models were fitted to reproduce the static properties of the deuteron. For the πNN vertex these models consider a linear combination of pseudovector and pseudoscalar coupling as

$$\Gamma_{\pi NN} = \lambda\gamma_5 + (1 - \lambda)\frac{1}{2M}\gamma_5 \not{q}_\pi, \tag{12}$$

TABLE I. Low-energy parameters of the three models of the ηN amplitude Eq. (13). Model A is that of Ref. [11], model B is that of Ref. [7], and model C is that of Ref. [9].

Model	a (fm)	r_0 (fm)	s (fm^3)
A	$0.407 + i0.255$	$-3.442 + i0.320$	$0.202 - i0.124$
B	$0.717 + i0.264$	$-1.594 - i0.028$	$-0.014 - i0.015$
C	$1.07 + i0.26$	$-1.25 - i0.25$	$-0.20 - i0.05$

where $\lambda = 0$ corresponds to pure pseudovector coupling and $\lambda = 1$ corresponds to pure pseudoscalar coupling.

I use for the $\pi N \to \eta N$, $\pi N \to \pi N$, and $\eta N \to \eta N$ amplitudes, the variable-mass isobar model [14,15] in which the spin-$\frac{1}{2}$ and spin-$\frac{3}{2}$ isobars have a mass equal to the invariant mass of the system $\sqrt{s}$ and the meson-nucleon-isobar couplings are chosen such as to generate scattering in the orbital angular momenta $\ell_\pm = j \pm \frac{1}{2}$. Since in the region that is being considered the $\eta N \to \eta N$ amplitude is dominated by the $N(1535)S_{11}$ resonance, it suffices to consider only that channel with the corresponding expression of the amplitude given by

$$t_{\eta N}^{-1} = 1/a + \tfrac{1}{2}r_0 q_0^2 + s q_0^4 - i q_0, \qquad (13)$$

where the low-energy parameters a, r_0, and s obtained from three different analyses are given in Table I. I take into account the fact that the particles can go off the mass shell by including form factors in the meson-nucleon-isobar vertices [16] through the substitution

$$t_{aN\to bN} \to e^{(q_b^2 - m_b^2)/\Lambda^2} e^{({k'_N}^2 - M^2)/\Lambda^2} t_{aN\to bN} \times e^{(q_a^2 - m_a^2)/\Lambda^2} e^{(k_N^2 - M^2)/\Lambda^2}, \qquad (14)$$

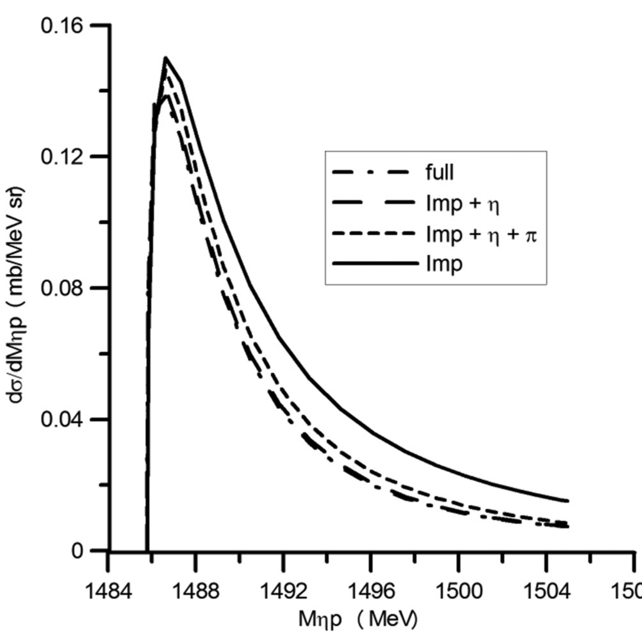

FIG. 2. Differential cross section $d\sigma/dM_{\eta p}$ for model B of the ηN amplitude considering different contributions to the process.

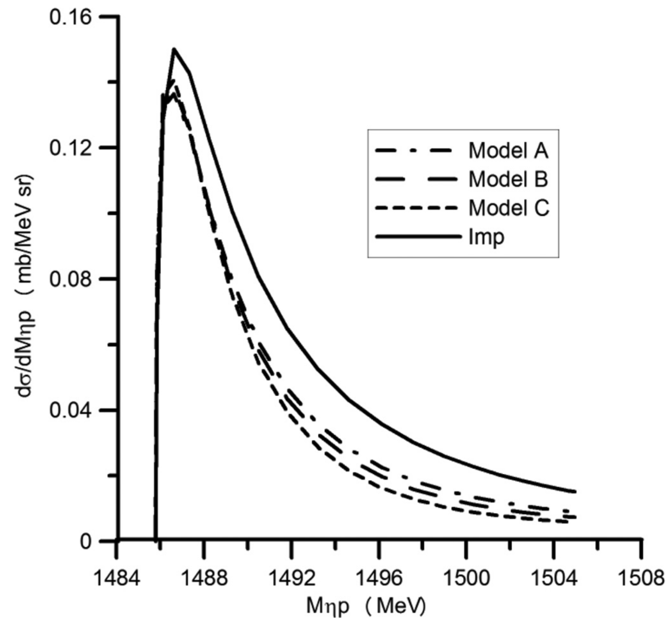

FIG. 3. Differential cross section for models A, B, and C as well as considering only the impulse approximation.

where, according to Ref. [17], the cutoff parameter Λ should lie between 1000 MeV$/c$ and 1200 MeV$/c$.

As I explained in Ref. [1], the $NN \to NN$ amplitude with one of the initial nucleons off the mass shell that enters into Eq. (9) were constructed in a similar way as the meson-nucleon amplitudes. The spin-0 and spin-1 isobars have a mass equal to the invariant mass of the system $\sqrt{s}$ and the

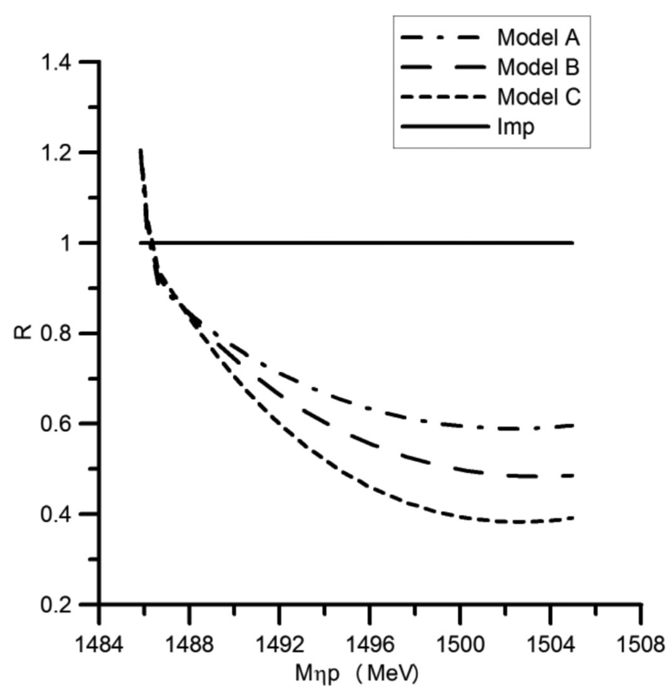

FIG. 4. The ratio R given by Eq. (1) for models A, B, and C as well as considering only the impulse approximation.

nucleon-nucleon-isobar couplings are chosen such that they generate scattering in the orbital angular momentum states $\ell = 0$ and $\ell = 1$, i.e., in the $^1S_0$, $^3P_0$, and $^3P_1$ channels. For the $NN \to NN$ amplitude with one of the initial nucleons off the mass shell [16] the solutions obtained from the Paris potential [18] are used by applying the minimal-relativity transformation [19].

III. RESULTS

The predictions of the model will be obtained by taking $\Lambda = 1200\,\mathrm{MeV}/c$ for the cutoff parameter of Eq. (14) [17] and the model of the deuteron wave function with $\lambda = 0$ in Eq. (12), which corresponds to a pure pseudovector πNN vertex [12]. As pointed out in [1], using instead $\Lambda = 1000\,\mathrm{MeV}/c$ [17] the behavior of the results is pretty much the same. Similarly, with respect to the deuteron wave function, using $\lambda = 1$ in Eq. (12) which corresponds to a pure pseudoscalar πNN vertex [12] it was found that the results are basically independent of the model of the deuteron wave function used.

I have calculated the inclusive differential cross section using for the ηN amplitude the three models A, B, and C shown in Table I which correspond to values of Re $a = 0.407$ fm, Re $a = 0.717$ fm, and Re $a = 1.07$ fm, respectively. Figure 2 shows the contribution of different terms of the amplitude to the cross section for the case of model B. This figure shows a behavior similar to that of Fig. 4 of Ref. [6] for the $\gamma d \to p(\eta n)$ process.

Figure 3 shows the results of the full model for the three cases of the ηN amplitude A, B, and C, as well as the result of the impulse term. Finally, Fig. 4 shows the corresponding results for the ratio R defined by Eq. (1). From Fig. 4 one can see that there is good sensitivity in the parameter R to allow one to pin down the value of the real part of the ηN scattering length.

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