

Generalized isobaric multiplet mass equation and its application to the Nolen-Schiffer anomaly

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The Wigner isobaric multiplet mass equation (IMME) is the most fundamental prediction in nuclear physics with the concept of isospin. However, it was deduced based on the Wigner-Eckart theorem with the assumption that all charge-violating interactions can be written as tensors of rank two. In the present work, the charge-symmetry breaking (CSB) and charge-independent breaking (CIB) components of the nucleon-nucleon force, which contribute to the effective interaction in nuclear medium, are established in the framework of Brueckner theory with AV18 and AV14 bare interactions. Because such charge-violating components can no longer be expressed as an irreducible tensor due to density dependence, its matrix element cannot be analytically reduced by the Wigner-Eckart theorem. With an alternative approach, we derive a generalized IMME (GIMME) that modifies the coefficients of the original IMME. As the first application of GIMME, we study the long-standing question of the origin of the Nolen-Schiffer anomaly (NSA) found in the Coulomb displacement energy of mirror nuclei. We find that the naturally emerged CSB term in GIMME is largely responsible for explaining the NSA.

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Introduction. The similarity of proton and neutron masses and approximate symmetry of nucleon-nucleon interactions under the exchange of the two kinds of nucleons lead to the concept of isospin [1,2]. At the isospin-symmetry limit, the charge-symmetry requires that the free proton-proton interaction v_{pp} excluding the Coulomb force is equal to the neutron-neutron interaction v_{nn} , while the charge independence requires that the neutron-proton interaction $v_{np} = (v_{nn} + v_{pp})/2$ [3]. However, the nucleon-nucleon scattering data suggest that v_{nn} is slightly more attractive than v_{pp} , and v_{np} is stronger than $(v_{nn} + v_{pp})/2$ [4,5]. In real nuclear systems where many-body effects are important [6], isospin symmetry breaking has long been an active research theme connected to different subfields, for example, in understanding the precise values of the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix elements between the u and d quarks [7,8], the changes in nuclear structure near the $N = Z$ line due to charge-violating nuclear force [9–12], and the influence in nova nucleosynthesis [13].

Isobaric nuclei with the same mass number A , total isospin T , and spin-parity J^π , but different $T_z = (N - Z)/2$, form an isobaric multiplet. The Wigner isobaric multiplet mass equation (IMME) [14]

$$ME(A, T, T_z) = a + bT_z + cT_z^2 \quad (1)$$

provides a relationship for mass excesses of an isobaric multiplet, where a , b , and c are the coefficients depending on T and reduced matrix elements. This quadratic form of IMME turns

out to work remarkably well for almost all isobaric multiplets where data exist [15–17]. Hence it becomes a powerful tool to predict unknown masses, particularly those of very neutron-deficient nuclei important to the astrophysical rp -process [18]. Modern radioactive beam facilities can provide the testing grounds of the validity of the IMME [19,20], from which one may learn about the effective forces for nuclear many-body systems [21–25].

The IMME is regarded to be valid for *any* charge-violating interactions, with the Coulomb interaction being the dominant contributor. The values of b and c in Eq. (1), which are determined experimentally, can potentially yield individual information on violations of the charge symmetry and charge independence [3]. However, the proven validity of the IMME does not in itself provide any direct information on the nature of the charge-violating nuclear interaction. In shell-model calculations, such interaction [26,27] are *added* to an isospin-conserving Hamiltonian, with the charge-symmetry breaking (CSB) or charge-independent breaking (CIB) components in the strong nuclear force fitted to data. In this Rapid Communication, we consider the contributions of CSB and CIB derived from nuclear medium in the effective nucleon-nucleon interaction. Due to density dependence of the charge-violating components, additional terms emerge as compared to the Wigner original IMME, leading to a generalized IMME (GIMME). As the first application of GIMME, the binding-energy difference between two members of a multiplet, defined as the Coulomb displacement energy (CDE), is examined. The long-standing problem of the Nolen-Schiffer anomaly (NSA) [28] in CDE is addressed by using our new formulas, without the need of any empirical terms.

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Effective CSB and CIB interactions in nuclear matter. In the study of nuclear matter with the assumption of isospin conservation in nuclear forces, the energy per nucleon is generally given as a function of density $\rho = \rho_n + \rho_p$ and isospin asymmetry $\beta = (\rho_n - \rho_p)/\rho$, via $E(\rho, \beta) = E(\rho, 0) + S_2(\rho)\beta^2 + O(\beta^4)$ [29–32], where the density-dependent $S_2(\rho)$ is the widely studied 2nd-order symmetry energy coefficient. If one does not neglect the CSB and CIB components, additional terms appear

$$E(\rho, \beta) = E(\rho, 0) + S_0^{(\text{CIB})}(\rho) + S_1^{(\text{CSB})}(\rho)\beta + [S_2(\rho) + S_2^{(\text{CIB})}(\rho)]\beta^2 + O(\beta^3). \quad (2)$$

Specifically, the effective CSB interaction, namely, the CSB component of the effective nucleon-nucleon interaction, gives rise to the 1st-order symmetry energy coefficient, defined as $S_1(\rho) = \partial E(\rho, \beta)/\partial \beta|_{\beta=0}$, while the CIB interaction solely contributes to even-order ones. In other words, $S_1^{(\text{CSB})}$ ($S_2^{(\text{CIB})}$) measures the CSB (CIB) effect in nuclear medium.

Contributions of the CSB and CIB components in a bare potential to the effective two-body interaction in nuclear matter can be obtained by solving the Bethe-Goldstone equation in the Brueckner theory with the AV18 interaction. AV18 contains explicit charge dependence and charge asymmetry on top of the AV14 potential [33]. To achieve a reliable accuracy, we determine the $S_1^{(\text{CSB})}$ term with the formula

$$\frac{E(\rho, \beta) - E(\rho, -\beta)}{2\beta} \Big|_{\text{AV18}} = S_1^{(\text{CSB})}(\rho) \quad (3)$$

in order to cancel out the systematical uncertainty effectively. In addition, the n - p mass difference in nucleonic kinetic energy leads to a small part of 1st-order symmetry energy [34], and is incorporated into the CSB effect in the present discussion. Similarly, the even-order symmetry energy coefficients originating from the CIB interaction are extracted by adopting both the AV18 and AV14 potentials via

$$\begin{aligned} & \frac{E(\rho, \beta) + E(\rho, -\beta)}{2} \Big|_{\text{AV18}} - E(\rho, \beta) \Big|_{\text{AV14}} \\ &= S_0^{(\text{CIB})}(\rho) + S_2^{(\text{CIB})}(\rho)\beta^2. \end{aligned} \quad (4)$$

$S_0^{(\text{CIB})}$ is an additional energy induced by the CIB interactions in symmetric nuclear matter, referred to as the zeroth-order symmetry energy coefficient. As a constant for an isobaric multiplet, $S_0^{(\text{CIB})}$ can be absorbed into $E(\rho, 0)$, playing no role in the present discussion. Figure 1 illustrates the density-dependent $S_1^{(\text{CSB})}(\rho)$ and $S_2^{(\text{CIB})}(\rho)$, which are found to be much smaller than the widely investigated 2nd-order one $S_2(\rho)$, and therefore, have been completely neglected in the study of nuclear matter. For the discussion below, we perform polynomial fittings for $S_1^{(\text{CSB})}(\rho)$ and $S_2^{(\text{CIB})}(\rho)$ obtained from the Brueckner theory

$$S_1^{(\text{CSB})}(\rho) = a_0\rho, \quad (5)$$

$$S_2^{(\text{CIB})}(\rho) = a_1\rho + a_2\rho^2 + a_3\rho^3, \quad (6)$$

with the resulting coefficients listed in Table I.

CSB and CIB effects in finite nuclei. With the above results derived for nuclear matter, we now build a Skyrme energy

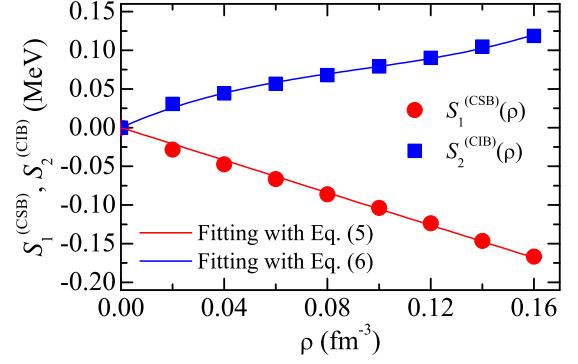


FIG. 1. Density-dependent $S_1^{(\text{CSB})}(\rho)$ (dots) and $S_2^{(\text{CIB})}(\rho)$ (squares) of nuclear matter obtained with the Brueckner-Hartree-Fock approach adopting the AV18 along with AV14 bare interactions. The curves represent the fittings with Eqs. (5) and (6).

density functional for the effective CSB and CIB interactions. Considering Eqs. (5) and (6), we construct the effective two-body CSB and CIB interactions by

$$v_{\text{CSB}} = -2a_0 P_{ij}^\sigma (\tau_{3,i} + \tau_{3,j}) \delta(\vec{r}_i - \vec{r}_j), \quad (7)$$

$$v_{\text{CIB}} = -4(a_1 + a_2\rho + a_3\rho^2) P_{ij}^\sigma \delta(\vec{r}_i - \vec{r}_j), \quad (8)$$

where the P_{ij}^σ is the spin exchange operator and τ_3 is the third component of the Pauli operator. The local density ρ is evaluated at $(\vec{r}_i + \vec{r}_j)/2$, with \vec{r}_i and \vec{r}_j being, respectively, the spacial coordinates of the i th and j th nucleons. Accordingly, the corresponding energy density are given as

$$\mathcal{H}_{\text{CSB}} = a_0(\rho_n^2 - \rho_p^2), \quad (9)$$

$$\mathcal{H}_{\text{CIB}} = (a_1 + a_2\rho + a_3\rho^2)(2\rho_n^2 + 2\rho_p^2 - \rho^2), \quad (10)$$

and hence the energy per nucleon $\mathcal{H}_{\text{CSB}}/\rho$ ($\mathcal{H}_{\text{CIB}}/\rho$) is exactly the symmetry energy term $S_1^{(\text{CSB})}(\rho)\beta$ ($S_2^{(\text{CIB})}(\rho)\beta^2$) in Eq. (2). Note that the isospin exchange operator $P_{12}^q = \delta_{q_1, q_2}$ is assumed since the charge mixing is quite weak. Therefore, the 1st- and 2nd-order symmetry energy coefficients $a_{\text{sym},1}^{(\text{CSB})}(A, T_z)$ and $a_{\text{sym},2}^{(\text{CIB})}(A, T_z)$ for finite nuclei can be calculated as corresponding density functionals

$$a_{\text{sym},1}^{(\text{CSB})}(A, T_z) = \frac{1}{IA} \int_0^\infty 4\pi r^2 \rho(r) S_1^{(\text{CSB})}(\rho) \beta(r) dr, \quad (11)$$

$$a_{\text{sym},2}^{(\text{CIB})}(A, T_z) = \frac{1}{I^2 A} \int_0^\infty 4\pi r^2 \rho(r) S_2^{(\text{CIB})}(\rho) \beta^2(r) dr. \quad (12)$$

In these equations, $I = (N - Z)/A = 2T_z/A$ denotes isospin asymmetry of a nucleus, and $\beta(r) = [\rho_n(r) - \rho_p(r)]/\rho(r)$ is the local isospin asymmetry, with $\rho_p(r)$ and $\rho_n(r)$ being the proton and neutron density distribution, respectively.

We comment on how excited states in a given multiplet are calculated, although these states do not appear in the discussion

TABLE I. Fitted coefficients of Eqs. (5) and (6).

a_0 (MeV fm ⁻³)	a_1 (MeV fm ⁻³)	a_2 (MeV fm ⁻⁶)	a_3 (MeV fm ⁻⁹)
-1.05132	1.49199	-10.96773	39.58976

TABLE II. 1st-order symmetry energy coefficient $a_{\text{sym},1}^{(\text{CSB})}(A, T_z)$ (keV) (columns 2–4) and 2nd-order one $a_{\text{sym},2}^{(\text{CIB})}(A, T_z)$ (keV) (columns 5–7) for finite nuclei obtained by Eqs. (11) and (12).

Nuclei	SLy4	SLy5	KDE	SLy4	SLy5	KDE
^{20}O	-40.7	-40.0	-43.1	20.3	19.9	21.1
^{53}Ni	-107.5	-106.2	-109.7	86.8	85.7	86.4
^{208}Pb	-111.9	-112.0	-116.1	79.9	80.1	82.5

of the present work. For an isobaric analog state (IAS) with $N - 1$ neutrons and $Z + 1$ protons ($N > Z$) whose T is greater than $|T_z|$, its wave function can be obtained by [35,36] $|IAS\rangle = |T, T_z = T - 1\rangle = \frac{1}{\sqrt{2T}} T_- |0\rangle$, where T_- is the isospin lowering operator and $|0\rangle$ is the ground state of the parent nucleus belonging to a multiplet with $T = T_z$ (N neutrons and Z protons). Due to the above isospin-symmetry conserving operation, it naturally leads to $(\rho_n + \rho_p)_{IAS} = (\rho_n + \rho_p)_{\text{parent}}$. However, $T_- |0\rangle / \sqrt{2T}$ describes the IAS with the zeroth-order approximation only which conserves isospin. Because of the core polarization induced by the charge-violating interactions, corresponding corrections should be introduced [36]. Consequently, $(\rho_n - \rho_p)_{IAS} - (\rho_n - \rho_p)_{\text{parent}} = -\frac{1}{T} \rho_{n,\text{parent}}^{\text{exc}}$ is obtained, where $\rho_{n,\text{parent}}^{\text{exc}}$ is the density of the excess neutrons in the parent nucleus. Thus with the obtained nucleonic density distributions, the symmetry energies of the IAS can be also computed by the above density functionals.

Since $a_{\text{sym},1}^{(\text{CSB})}(A, T_z)$ and $a_{\text{sym},2}^{(\text{CIB})}(A, T_z)$ are related solely to the nuclear force, one should perform many-body calculations excluding the Coulomb force, which leads to $a_{\text{sym},1}^{(\text{CSB})}(A, T_z) = a_{\text{sym},1}^{(\text{CSB})}(A, -T_z)$ and $a_{\text{sym},2}^{(\text{CIB})}(A, T_z) = a_{\text{sym},2}^{(\text{CIB})}(A, -T_z)$ for mirror nuclei within an isobaric multiplet. Furthermore, considering the fact that the CSB and CIB effects are small, we treat them as perturbations. Consequently, both $a_{\text{sym},1}^{(\text{CSB})}(A, T_z)$ and $a_{\text{sym},2}^{(\text{CIB})}(A, T_z)$ are completely isolated from the rest of the energy, and thus can be reliably extracted.

We now briefly discuss the calculated $a_{\text{sym},1}^{(\text{CSB})}(A, T_z)$ and $a_{\text{sym},2}^{(\text{CIB})}(A, T_z)$ for finite nuclei. The Skyrme-Hartree-Fock-BCS approach with three interactions studied in our previous work [37], i.e., the SLy4, SLy5, and KDE interactions [38], are employed to calculate the quantities of Eqs. (11) and (12), in which the empirical gaps from Ref. [39] are applied. Table II lists the calculated results, taking ^{20}O (a member of $A = 20$ quintet), ^{53}Ni (a member of $A = 53$ quartet), and a heavy nucleus ^{208}Pb as examples. Both $a_{\text{sym},1}^{(\text{CSB})}(A, T_z)$ and $a_{\text{sym},2}^{(\text{CIB})}(A, T_z)$ are found to be weakly model dependent because different interactions generate nearly identical nucleonic density profiles. For the members of isobaric multiplets, such as ^{53}Ni , the values of the 1st-order symmetry energy term $E_{\text{sym},1}^{(\text{CSB})}(A, T_z) = a_{\text{sym},1}^{(\text{CSB})}(A, T_z)IA$ are very small due to their low isospin asymmetries I and the undersized $S_1^{(\text{CSB})}(\rho)$. On the other hand, $E_{\text{sym},1}^{(\text{CSB})}(A, T_z)$ for ^{208}Pb can be as large as -5 MeV. Apparently, the 2nd-order ones, $E_{\text{sym},2}^{(\text{CIB})}(A, T_z) = a_{\text{sym},2}^{(\text{CIB})}(A, T_z)I^2A$, are smaller.

We thus conclude that the 1st-order symmetry energy term should not always be neglected in the calculations for

neutron-rich nuclei. We note, for example, that nuclear masses can be presently predicted by employing macroscopic-microscopic mass models [40,41] with an accuracy of several hundred keV. Furthermore, the CSB interaction has been shown to play an important role in nuclear structure [10,11]. Our obtained effective interactions including the symmetry-breaking components could be employed to explore the relevant problems such as the charge-exchange reactions, Gamow-Teller transitions, and β decays. Up to now, shell-model calculations for these quantities can only be performed by introducing phenomenological symmetry-breaking terms with the strengths fitted to data [42].

A generalized IMME including effective CSB and CIB interactions. In his derivation of Eq. (1), Wigner assumed $|\alpha TT_z\rangle$ to be the eigenstate of the charge-independent Hamiltonian H_0 , with α for all additional quantum numbers to specify this state. All charge-violating two-body interactions, including the Coulomb interaction H_C among protons and $H_{\text{CSB}+\text{CIB}}$ of CSB and CIB interactions, are treated by the 1st-order perturbation. The total negative binding energy is given by

$$-\text{BE}(\alpha TT_z) = \langle \alpha TT_z | H_0 + H_C + H_{\text{CSB}+\text{CIB}} | \alpha TT_z \rangle, \quad (13)$$

where H_C and $H_{\text{CSB}+\text{CIB}}$ are assumed to be written as tensors of rank two. With help of the Wigner-Eckart theorem for irreducible tensor, the perturbing terms can be neatly expressed as reduced matrix elements and the coefficients involving only T and T_z .

However, in nuclear medium, $H_{\text{CSB}+\text{CIB}}$ becomes a density-dependent effective interaction. As a result, it can no longer be expressed as an irreducible tensor, and the corresponding perturbation energy $\langle \alpha TT_z | H_{\text{CSB}+\text{CIB}} | \alpha TT_z \rangle$ does not have analytic forms as in the case of the Coulomb interaction. When the effective CSB and CIB interactions are present, the perturbation energy in the present work is expressed as the symmetry energy terms

$$\langle \alpha TT_z | H_{\text{CSB}+\text{CIB}} | \alpha TT_z \rangle = a_{\text{sym},1}^{(\text{CSB})}(A, T_z)IA + a_{\text{sym},2}^{(\text{CIB})}(A, T_z)I^2A, \quad (14)$$

with the zeroth-order symmetry energy coefficient absorbed into a . One thus ends up with a generalized IMME (GIMME) in the form of

$$\text{ME}(A, T, T_z) = a + (b_c + \Delta_{\text{nH}} + 2a_{\text{sym},1}^{(\text{CSB})}(A, T_z))T_z + \left(c_c + \frac{4}{A}a_{\text{sym},2}^{(\text{CIB})}(A, T_z) \right) T_z^2, \quad (15)$$

with $\Delta_{\text{nH}} = 0.782$ MeV being the neutron-hydrogen mass difference. As a mass equation beyond the original IMME, the contribution from the effective charge-violating nuclear interactions is now completely separated from that of the Coulomb force, while the T_z -independent b_c and c_c in Eq. (15) are induced solely by the Coulomb interaction. The coefficients of T_z and T_z^2 are no longer constants for a given multiplet. The T_z dependence of the new $a_{\text{sym},1}^{(\text{CSB})}(A, T_z)$ and $a_{\text{sym},2}^{(\text{CIB})}(A, T_z)$ terms, originating from the CSB and CIB components of nuclear medium, are an explicit indication of the breakdown of the original IMME. We remark that this T_z dependence is quite weak, supporting the general validity of the original IMME

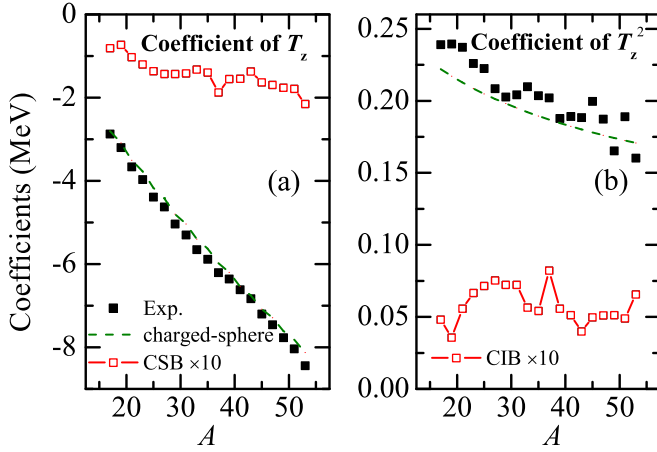


FIG. 2. Coefficients of (a) T_z and (b) T_z^2 in Eq. (1) extracted from the experimental data [45] for the $T = 3/2$ quartets. The calculated coefficients of the Coulomb contribution (plus Δ_{np}) from a simple nonuniformly charged sphere with Eq. (17) (dashed curves) are shown for comparison. The contributions of the CSB and CIB effects in Eq. (15), taking $T_z = T$ nuclei as examples, are calculated by using Eqs. (11) and (12) with the SLy4 interaction.

[14] that has been tested against many experimental data. Yet, under certain circumstances, the quadratic form of the IMME may break down, and the underlying mechanism will be discussed in further detail in a forthcoming paper.

We now discuss how much the corrections are actually introduced by the CSB and CIB effects, and examine their systematic behavior. With the assumption that the nucleus is treated as a nonuniformly charged sphere [43], the Coulomb energy E_c can be written as

$$E_c = \frac{3e^2}{5r_0A^{1/3}(1+\Delta)} \{Z(Z-1) - 0.25[1 - (-1)^Z]\}, \quad (16)$$

with $r_0 = 1.2$ fm, where the correction due to the last unpaired proton [44] is supplemented. The parameter $\Delta = 5\pi^2 d^2 / (6r_0^2 A^{2/3})$ with $d \approx 0.55$ fm [43] is introduced to describe the effect of the surface diffuseness on the Coulomb energy, which is a correction to the uniformly charge sphere model [35], and the Coulomb interaction on the surface asymmetry is ignorable for the $N \approx Z$ nuclei. Hence the contributions of the Coulomb energy to the coefficients of T_z and T_z^2 are simply derived as

$$b_c = \frac{3e^2}{5r_0A^{1/3}(1+\Delta)} \left[(1-A) + \frac{(-1)^{A/2-T} - (-1)^{A/2+T}}{8T} \right],$$

$$c_c = \frac{3e^2}{5r_0A^{1/3}(1+\Delta)} \left[1 + \frac{(-1)^{A/2-T} + (-1)^{A/2+T}}{4(2T-1)} \right]. \quad (17)$$

Figure 2 illustrates the coefficients of the T_z and of T_z^2 terms extracted from the masses of the $T = 3/2$ isobaric quartets [45], and compares them with those given by a nonuniformly charged sphere [43]. The contributions of the CSB and CIB effects in Eq. (15), taking $T_z = T$ nuclei as examples, are also presented in Fig. 2 for comparison. The contribution of the CSB effect to the coefficient of T_z term increases roughly from -80 to -220 keV when A goes from 17 to 53, which

is found to be consistent with the estimations for the $T = 1$ multiplets given in Table 5.4 of Ref. [46]. In general, the CSB effect results in a reduction of the coefficient of T_z term by 2.0%–3.1%, and the CIB effect enhances the coefficient of T_z^2 term by 1.6%–4.4%. Note that while the energy splitting among the isobaric multiplet is predominately attributed to the Coulomb interaction, clearly the corrections to the IMME have the CSB and CIB origin.

Nolen-Schiffer anomaly (NSA). The NSA [28] is a long-standing historical problem. The Coulomb displacement energy (CDE)—the difference in binding energy between two members of a multiplet—is directly related to the IMME coefficients in Eq. (1): for adjacent members of a multiplet one has $CDE(A, T, T_z) = -b - c(2T_z + 1) + \Delta_{nH}$ [3], where T_z is taken for the isobar with the larger proton number. It is an anomaly because when all the corrections were taken into account, there remained a consistent underestimate of the CDE by about a few to ten percent [28,35,47]. Now with our GIMME, the CDE expression is modified as

$$CDE(A, T, T_z) = -b_c - c_c(2T_z + 1) + \Delta_{NSA}, \quad (18)$$

where the new last term arising from the CSB and CIB components of the nuclear medium is given by

$$\begin{aligned} \Delta_{NSA} &= -2a_{\text{sym},1}^{(\text{CSB})}(A, T_{z>}) - \frac{4(2T_z + 1)}{A} a_{\text{sym},2}^{(\text{CIB})}(A, T_{z>}) \\ &\quad + 2T_z [a_{\text{sym},1}^{(\text{CSB})}(A, T_z) - a_{\text{sym},1}^{(\text{CSB})}(A, T_{z>})] \\ &\quad + \frac{4T_z^2}{A} [a_{\text{sym},2}^{(\text{CIB})}(A, T_z) - a_{\text{sym},2}^{(\text{CIB})}(A, T_{z>})], \\ &\simeq -2a_{\text{sym},1}^{(\text{CSB})}(A, T_{z>}) - \frac{4(2T_z + 1)}{A} a_{\text{sym},2}^{(\text{CIB})}(A, T_{z>}), \end{aligned} \quad (19)$$

with $T_{z>} = T_z + 1$. With Δ_{NSA} , it becomes clear that the CDE has contributions from CSB and CIB, in addition to the Coulomb force. The CSB effect contributes predominately in Eq. (19), whereas the CIB effect is much smaller, particularly for heavier masses due to the $1/A$ dependence. According to Fig. 2, Δ_{NSA} accounts for 2%–3% of the CDE for isobaric quartets, which, according to our calculation, can add to CDE with 100–200 keV for $T_z = \pm 1/2$ and 300–600 keV for $T_z = \pm 3/2$ mirror pairs. These amounts are qualitatively consistent with what is needed to account for the NSA, as discussed in Ref. [3].

The CDE for a $T = 1/2$ pair of mirror nuclei, defined as $CDE(A, T = 1/2) = BE(A, T_z = 1/2) - BE(A, T_z = -1/2)$, has been widely used to study the NSA. In our method, the CDE is given by $CDE(A) = -b_c - 2a_{\text{sym},1}^{(\text{CSB})}$, and the CSB effect is simply obtained with $\Delta_{NSA} = -2a_{\text{sym},1}^{(\text{CSB})}$. Our calculated Δ_{NSA} for nuclei near the closed shells with the magic numbers 8 and 20, compared with those based on the SIII Skyrme interaction [48] and a calibrated independent-particle model [49] with inclusion of many corrections for some extensively studied mirror pairs, are listed in Table III. Note that the results from the dominant Coulomb term and the small corrections, such as the finite size of nucleons and short-range correlation, exhibit considerable differences between

TABLE III. Calculated $\Delta_{\text{NSA}} = -2a_{\text{sym},1}^{(\text{CSB})}$ (in MeV) due to the CSB effect for the $T = 1/2$ mirror pairs in the $A = 16$ and 40 regions, compared with other calculations for the study of the NSA.

Nuclide	SLy4	SLy5	KDE	Ref. [48]	Ref. [49]
$^{15}\text{O}-^{15}\text{N}$	0.16	0.16	0.16	0.29	0.16 ± 0.04
$^{17}\text{F}-^{17}\text{O}$	0.11	0.11	0.11	0.11	0.31 ± 0.04
$^{39}\text{Ca}-^{39}\text{K}$	0.15	0.16	0.16	0.44	0.22 ± 0.08
$^{41}\text{Sc}-^{41}\text{Ca}$	0.14	0.14	0.15	0.12	0.59 ± 0.08

Refs. [48] and [49], suggesting a model-dependence character in the results. Moreover, the core-polarization correction, even in its sign, presents a strong model dependence [48,50]. Interestingly, our results are found consistent with those in Ref. [48] ([49]) for particle (hole) nuclei. We emphasize, however, that our Δ_{NSA} is completely separated from the Coulomb energy, and in addition, our results are obtained without tuning any particular parameters.

From our derivation, the CDE of a pair of mirror nuclei with $T = 1$ becomes

$$\text{CDE}(A, T = 1) = -2b_c - 4a_{\text{sym},1}^{(\text{CSB})}(A, T_z = 1). \quad (20)$$

In the above expression, the contribution to the CSB effect is directly obtained as $\Delta_{\text{NSA}} = -4a_{\text{sym},1}^{(\text{CSB})}(A, T_z = 1)$. In order to compare our results with the CDE data, we compute the CDE with the nonuniformly charged sphere model, with and without the second term in Eq. (20). The results together with experimental data are presented in Fig. 3. The difference between the two calculations is obvious. It can be seen that overall, the calculated CDE with inclusion of the CSB effect tends to describe the experimental data, where the CSB effect contributes an amount of 2%–3%.

The origin of the NSA has been studied by many authors (see, for example, Ref. [35]) and is generally expected to result mainly from the CSB effect. The (isospin) symmetry-breaking terms are usually not included in normal shell-model Hamiltonians, and therefore the answer to the anomaly lies

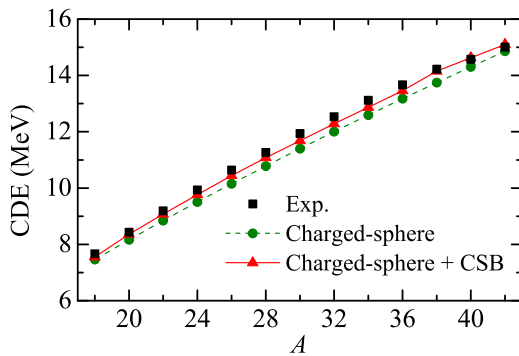


FIG. 3. Comparison of calculated CDE of the $T = 1$ mirror pairs with experimental data [56]. The calculated results with and without the second term in Eq. (20) are plotted as triangles and dots, respectively. The first term in Eq. (20) is calculated using Eq. (17); the second, using Eq. (11) with the SLy4 interaction.

likely in a deeper level [51]. In contrast to the applied models based on the effects of the nucleon mass splitting or meson mixing [52–55], in our framework, the CSB and CIB effects starting from nuclear medium are established by employing the microscopic Brueckner theory without any adjustable parameter. Incidentally, the triplet displacement energy (TDE) [57] is related to our new coefficients through $\text{TDE}(A) = 2c_c + 8a_{\text{sym},2}^{(\text{CIB})}(A, T_z = 1)/A$. However, the c_c coefficient cannot be well achieved with the charged sphere model, as that in Fig. 2. Different from the CDE of a mirror pair discussed above, the TDE originates from the Coulomb force together with the CIB effect, where the latter contributes about 3%. As the NSA for CDE, the Coulomb interaction alone cannot account for TDE, a conclusion consistent with shell-model studies [10,11].

Summary. We have generalized the Wigner IMME by considering the contributions of CSB and CIB derived in nuclear medium to the effective nucleon-nucleon interaction, and used it to study the NSA. The main conclusions are as follows. (i) The density-dependent CSB and CIB interactions in nuclear matter, characterized respectively by the symmetry energy coefficients $S_1^{(\text{CSB})}$ and $S_2^{(\text{CIB})}$, were built within the Brueckner theory with the bare interactions as inputs. Therefore, our work bridges the charge-violating nuclear force in free nucleons and that in nuclear medium. (ii) With these results as calibrations, we established the effective CSB and CIB interactions in the Skyrme functions, and carried out the calculations of their effects in finite nuclei. For neutron-rich nuclei, we found that the 1st-order symmetry energy term $E_{\text{sym},1}(A)$ induced by the CSB effect, which is generally dropped in nuclear mass calculations, should not be neglected. (iii) The perturbative Hamiltonian with the density-dependent effective CSB and CIB interactions is no longer an irreducible tensor, hence its matrix element cannot be analytically reduced via the Wigner-Eckart theorem, as Wigner did [14]. We derived the GIMME which presents new corrections to the original Wigner IMME, where the contribution of the effective CSB and CIB interactions is clearly separated from that of Coulomb force. (iv) As the first application of GIMME, the NSA was naturally elucidated to a large extent to originate from the CSB effect, with the needed correction of several hundred keV being reproduced.

Finally, we note that our obtained CIB interaction in an effective Skyrme energy density functional describes only the ground-state properties for finite nuclei. The J dependence of CIB (see, for example Refs. [42,58]), which cannot be discussed here, is, however, an important aspect of CIB, and should be investigated within the present theory.

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- [1] W. Heisenberg, *Z. Phys.* **77**, 1 (1932).
- [2] E. Wigner, *Phys. Rev.* **51**, 106 (1937).
- [3] M. A. Bentley and S. M. Lenzi, *Prog. Part. Nucl. Phys.* **59**, 497 (2007).
- [4] E. M. Henley, in *Isospin in Nuclear Physics*, edited by D. H. Wilkinson (North-Holland, Amsterdam, 1969).
- [5] R. Machleidt, *Phys. Rev. C* **63**, 024001 (2001).
- [6] D. D. Warner, M. A. Bentley, and P. Van Isacker, *Nat. Phys.* **2**, 311 (2006).
- [7] J. C. Hardy and I. S. Towner, *Phys. Rev. C* **71**, 055501 (2005); **79**, 055502 (2009).
- [8] I. S. Towner and J. C. Hardy, *Phys. Rev. C* **77**, 025501 (2008).
- [9] A. P. Zuker, S. M. Lenzi, G. Martínez-Pinedo, and A. Poves, *Phys. Rev. Lett.* **89**, 142502 (2002).
- [10] K. Kaneko, Y. Sun, T. Mizusaki, and S. Tazaki, *Phys. Rev. Lett.* **110**, 172505 (2013).
- [11] K. Kaneko, Y. Sun, T. Mizusaki, and S. Tazaki, *Phys. Rev. C* **89**, 031302(R) (2014).
- [12] J. Henderson, D. G. Jenkins, K. Kaneko, P. Ruotsalainen, P. Sarriguren, K. Auranen, M. A. Bentley, P. J. Davies, A. Gorgen, T. Grahm, P. T. Greenlees, A. Hay, T. W. Henry, A. Herzan, U. Jakobsson, R. Julin, S. Juutinen, J. Konki, M. Leino, C. McPeake, S. Milne, A. J. Nichols, J. Pakarinen, P. Papadakis, J. Partanen, P. Peura, P. Rakhila, E. Sahin, M. Sandzelius, J. Saren, C. Scholey, M. Siciliano, L. Sinclair, J. Sorri, S. Stolze, J. Uusitalo, R. Wadsworth, and M. Zielinska, *Phys. Rev. C* **90**, 051303(R) (2014).
- [13] M. B. Bennett, C. Wrede, B. A. Brown, S. N. Liddick, D. Pérez-Loureiro, D. W. Bardayan, A. A. Chen, K. A. Chipps, C. Fry, B. E. Glassman, C. Langer, N. R. Larson, E. I. McNeice, Z. Meisel, W. Ong, P. D. O'Malley, S. D. Pain, C. J. Prokop, H. Schatz, S. B. Schwartz, S. Suchyta, P. Thompson, M. Walters, and X. Xu, *Phys. Rev. Lett.* **116**, 102502 (2016).
- [14] E. P. Wigner, in *Proceedings of the R.A. Welch Foundation Conference on Chemical Research, Houston*, edited by W. O. Millikan (Welch Foundation, Houston, 1957), Vol. 1; S. Weinberg and S. B. Treiman, *Phys. Rev.* **116**, 465 (1959).
- [15] W. Benenson and E. Kashy, *Rev. Mod. Phys.* **51**, 527 (1979).
- [16] J. Britz, A. Pape, and M. S. Antony, *At. Data Nucl. Data Tables* **69**, 125 (1998).
- [17] Y. H. Lam *et al.*, *At. Data Nucl. Data Tables* **99**, 680 (2013).
- [18] A. Parikh, J. José, C. Iliadis, F. Moreno, and T. Rauscher, *Phys. Rev. C* **79**, 045802 (2009).
- [19] Y. H. Zhang, H. S. Xu, Y. A. Litvinov, X. L. Tu, X. L. Yan, S. Typel, K. Blaum, M. Wang, X. H. Zhou, Y. Sun, B. A. Brown, Y. J. Yuan, J. W. Xia, J. C. Yang, G. Audi, X. C. Chen, G. B. Jia, Z. G. Hu, X. W. Ma, R. S. Mao, B. Mei, P. Shuai, Z. Y. Sun, S. T. Wang, G. Q. Xiao, X. Xu, T. Yamaguchi, Y. Yamaguchi, Y. D. Zang, H. W. Zhao, T. C. Zhao, W. Zhang, and W. L. Zhan, *Phys. Rev. Lett.* **109**, 102501 (2012).
- [20] A. T. Gallant, M. Brodeur, C. Andreoiu, A. Bader, A. Chaudhuri, U. Chowdhury, A. Grossheim, R. Klawitter, A. A. Kwiatkowski, K. G. Leach, A. Lennarz, T. D. Macdonald, B. E. Schultz, J. Lassen, H. Heggen, S. Raeder, A. Teigelhöfer, B. A. Brown, A. Magilligan, J. D. Holt, J. Menéndez, J. Simonis, A. Schwenk, and J. Dilling, *Phys. Rev. Lett.* **113**, 082501 (2014).
- [21] F. Wienholtz *et al.*, *Nature* **498**, 346 (2013).
- [22] J. D. Holt, J. Menéndez, and A. Schwenk, *Phys. Rev. Lett.* **110**, 022502 (2013).
- [23] E. M. Ramirez *et al.*, *Science* **337**, 1207 (2012).
- [24] D. Steppenbeck *et al.*, *Nature* **502**, 207 (2013).
- [25] K. Blaum, *Phys. Rep.* **425**, 1 (2006).
- [26] W. E. Ormand and B. A. Brown, *Nucl. Phys. A* **491**, 1 (1989).
- [27] Y. H. Lam, N. A. Smirnova, and E. Caurier, *Phys. Rev. C* **87**, 054304 (2013).
- [28] J. A. Nolen and J. P. Schiffer, *Annu. Rev. Nucl. Part. Sci.* **19**, 471 (1969).
- [29] A. W. Steiner, M. Prakash, J. M. Lattimer, and P. J. Ellis, *Phys. Rep.* **411**, 325 (2005).
- [30] V. Baran, M. Colonna, V. Greco, and M. Di Toro, *Phys. Rep.* **410**, 335 (2005).
- [31] B. A. Li, L. W. Chen, and C. M. Ko, *Phys. Rep.* **464**, 113 (2008).
- [32] J. M. Lattimer and M. Prakash, *Phys. Rep.* **621**, 127 (2016).
- [33] R. B. Wiringa, V. G. J. Stoks, and R. Schiavilla, *Phys. Rev. C* **51**, 38 (1995).
- [34] J. M. Dong, W. Zuo, and J. Z. Gu, *Chin. Phys. Lett.* **33**, 102101 (2016).
- [35] N. Auerbach, *Phys. Rep.* **98**, 273 (1983), and references therein.
- [36] N. Auerbach and N. Van Giai, *Phys. Rev. C* **24**, 782 (1981).
- [37] J. Dong, W. Zuo, and J. Gu, *Phys. Rev. C* **91**, 034315 (2015).
- [38] E. Chabanat, P. Bonche, P. Haensel, J. Meyer, and R. Schaeffer, *Nucl. Phys. A* **635**, 231 (1998); B. K. Agrawal, S. Shlomo, and V. K. Au, *Phys. Rev. C* **72**, 014310 (2005).
- [39] P. Möller, J. R. Nix, W. D. Myers, and W. J. Swiatecki, *At. Data Nucl. Data Tables* **59**, 185 (1995).
- [40] P. Möller, W. D. Myers, H. Sagawa, and S. Yoshida, *Phys. Rev. Lett.* **108**, 052501 (2012).
- [41] N. Wang, M. Liu, X. Wu, and J. Meng, *Phys. Lett. B* **734**, 215 (2014).
- [42] K. Kaneko, Y. Sun, T. Mizusaki, S. Tazaki, and S. K. Ghorui, *Phys. Lett. B* **773**, 521 (2017).
- [43] P. Danielewicz, *Nucl. Phys. A* **727**, 233 (2003).
- [44] S. Sengupta, *Nucl. Phys.* **21**, 542 (1960).
- [45] M. MacCormick and G. Audi, *Nucl. Phys. A* **925**, 61 (2014).
- [46] N. Auerbach, J. Hufner, A. K. Kerman, and C. M. Shakin, *Rev. Mod. Phys.* **44**, 48 (1972).
- [47] N. Auerbach, V. Bernard, and N. Van Giai, *Phys. Rev. C* **21**, 744 (1980); *Nucl. Phys. A* **337**, 143 (1980).
- [48] A. Poves, A. L. Cedillo, and J. M. G. Gómez, *Nucl. Phys. A* **293**, 397 (1977).
- [49] S. Shlomo, *Rep. Prog. Phys.* **41**, 957 (1978).
- [50] A. Barroso, *Nucl. Phys. A* **281**, 267 (1977).
- [51] J. Schiffer (private communication).
- [52] T. Hatsuda, H. Hogaasen, and M. Prakash, *Phys. Rev. C* **42**, 2212 (1990).
- [53] T. D. Cohen, R. J. Furnstahl, and M. K. Banerjee, *Phys. Rev. C* **43**, 357 (1991).
- [54] L. N. Epele, H. Fanchlotti, C. A. G. Canal, and G. A. G. Sprlnberg, *Phys. Lett. B* **277**, 33 (1992).
- [55] M. H. Shahnas, *Phys. Rev. C* **50**, 2346 (1994).
- [56] M. Wang *et al.*, *Chin. Phys. C* **36**, 1603 (2012).
- [57] P. E. Garrett, W. E. Ormand, D. Appelbe, R. W. Bauer, J. A. Becker, L. A. Bernstein, J. A. Cameron, M. P. Carpenter, R. V. F. Janssens, C. J. Lister, D. Seweryniak, E. Tavukcu, and D. D. Warner, *Phys. Rev. Lett.* **87**, 132502 (2001).
- [58] M. A. Bentley, S. M. Lenzi, S. A. Simpson, and C. Aa. Digt, *Phys. Rev. C* **92**, 024310 (2015).