Criticality in a hadron resonance gas model with the van der Waals interaction

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(Received 16 September 2017; published 4 January 2018)

The van der Waals interaction is implemented in a hadron resonance gas model. It is shown that this model can describe lattice QCD data of different thermodynamical quantities satisfactorily with the van der Waals parameters $a = 1250 \pm 150$ MeV fm^3 and $r = 0.7 \pm 0.05$ fm. Further, a liquid-gas phase transition is observed in this model with the critical point at temperature, $T = 62.1$ MeV and baryon chemical potential, $\mu_B = 708$ MeV.

DOI: 10.1103/PhysRevC.97.015201

I. INTRODUCTION

Lattice quantum chromodynamics (LQCD) [1–5] provides a first principle approach to study strongly interacting matter at zero chemical potential (μ_B) and finite temperature (T). LQCD calculations indicate a smooth cross over transition [1] from hadronic to a quark-gluon plasma (QGP) phase at zero baryon chemical potential and finite temperature [6]. On the other hand, at high baryon chemical potential and low temperature the nuclear matter is expected to have a first-order phase transition [7] which ends at a critical point, a second-order phase transition point as one moves towards the high temperature and low baryon chemical potential region, in the QCD phase diagram [8,9]. At present, the properties of QCD matter at high temperature and small baryon chemical potential are being investigated using ultrarelativistic heavy ion collisions at the Large Hadron Collider (LHC), CERN and Relativistic Heavy Ion Collider (RHIC), Brookhaven National Laboratory (BNL). The Beam Energy Scan (BES) program of RHIC [10] is currently investigating the location of the critical point [11]. The HADES experiment at GSI, Darmstadt is investigating a medium with very large baryon chemical potential [12]. In future, the compressed baryonic matter (CBM) experiment [13] at the Facility for Antiproton and Ion Research (FAIR) at GSI and the Nuclotron-based Ion Collider fAcility (NICA) [14] at JINR, Dubna will also study nuclear matter at large baryon chemical potential.

The ideal or noninteracting hadron resonance gas (HRG) model is quite successful in reproducing the zero chemical potential LQCD data of bulk properties of the QCD matter at low temperatures $T < 150$ MeV [3–5,15,16]. However, disagreement between LQCD data and ideal HRG model calculations have been observed at higher temperatures. Considering excluded volume correction, which mimics repulsive interaction, in the HRG model, one can improve the picture in the crossover temperature region $T \sim 140$–190 MeV [17,18]. In the excluded volume HRG (EVHRG) model [17–33], effects of van der Waals type hadronic repulsions at short distances are introduced but long distance repulsive interactions are ignored. Recently a van der Waals (VDW) type interaction with both attractive and repulsive parts has been introduced in the HRG model [34–38]. Interestingly the VDWHRG model shows first-order liquid-gas phase transition in nuclear matter at large chemical potentials and small temperatures which was not observed in other HRG models like ideal HRG or EVHRG models. The liquid-gas phase transition in nuclear matter was also predicted in Refs. [39–41] and observed in experiment as well [42]. In Ref. [35] the VDW parameters a and b have been fixed by reproducing the saturation density $n_0 = 0.16\,\text{fm}^3$ and binding energy $E/N = 16\,\text{MeV}$ of the ground state of nuclear matter. The nuclear matter shows the critical point [35] at $T = 19.7$ GeV and $\mu_B = 908$ MeV. Latter it has been shown that using the same interaction parameters a and b as for nuclear matter, for all the baryons in VDWHRG model, LQCD data can be described qualitatively [38] in the cross over region. The motivation of the present work is to carry out the reverse prescription, that is to find out van der Waals parameters a and b that give the best description of LQCD data at zero chemical potential using the VDWHRG model and then extend this work to the finite chemical potential and try to locate the existence of a critical point in the QCD phase diagram.

The paper is organized as follows. In the Sec. II we describe the ideal HRG as well as VDWHRG model. In Sec. III we present our results. Finally in Sec. IV we summarize our findings for this work.

II. MODEL DESCRIPTION

There are varieties of HRG models which exist in the literature. Different versions of this model and some of the recent works using these models may be found in Refs. [17–38,43–74]. Some of the HRG models are noninteracting and some of them consider interaction among the particles. Next we will briefly discuss the noninteracting HRG model and the HRG model with van der Waals type interaction.

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In the ideal HRG model, the thermal system consists of noninteracting point like hadrons and resonances. The logarithm of the partition function of a hadron resonance gas in the grand canonical ensemble can be written as

$$\ln Z^{id} = \sum_i \ln Z_i^{id}, \tag{1}$$

where the sum is over all the hadrons and resonances. id refers to ideal, i.e., noninteracting HRG model. For particle species i,

$$\ln Z_i^{id} = \pm \frac{V g_i}{2\pi^2} \int_0^\infty p^2\, dp \ln[1 \pm \exp(-(E_i - \mu_i)/T)], \tag{2}$$

where V is the volume of the thermal system, g is the degeneracy, $E = \sqrt{p^2 + m^2}$ is the single particle energy, m is the mass of the particle, and $\mu_i = B_i\mu_B + S_i\mu_S + Q_i\mu_Q$ is the chemical potential. In the last expression, B_i, S_i, Q_i are, respectively, the baryon number, strangeness, and electric charge of the particle, μ's are the corresponding chemical potentials. The upper and lower sign of $\pm$ corresponds to fermions and bosons, respectively. We have incorporated all the hadrons and resonances listed in the particle data book up to a mass of 3 GeV [75]. The pressure p^{id}, the energy density ε^{id}, and the number density n^{id} of the thermal system are given by the following equations:

$$p^{id} = \sum_i (\pm) \frac{g_i T}{2\pi^2} \int_0^\infty p^2\, dp \ln[1 \pm \exp(-(E_i - \mu_i)/T)], \tag{3}$$

$$\varepsilon^{id} = \sum_i \frac{g_i}{2\pi^2} \int_0^\infty \frac{p^2\, dp}{\exp[(E_i - \mu_i)/T] \pm 1} E_i, \tag{4}$$

$$n^{id} = \sum_i \frac{g_i}{2\pi^2} \int_o^\infty \frac{p^2 dp}{\exp[(E_i - \mu_i)/T \pm 1}. \tag{5}$$

Once we know the partition function or the pressure of the system we can calculate other thermodynamic quantities.

The van der Waals equation in the canonical ensemble is given by [76]

$$\left(p + \left(\frac{N}{V}\right)^2 a\right)(V - Nb) = NT, \tag{6}$$

where p is the pressure of the system, V is the volume, T is the temperature, N is the number of particles, and a,b (both positive) are the van der Waals parameters. The parameters a and b describe the attractive and repulsive interaction, respectively. Equation (6) can be written as

$$p(T,n) = \frac{NT}{V - bN} - a\left(\frac{N}{V}\right)^2 \equiv \frac{nT}{1 - bn} - an^2, \tag{7}$$

where $n \equiv N/V$ is the number density of particles. The first term in the right hand side of Eq. (7) corresponds to the excluded volume correction where the system volume is replaced by the available volume $V_{av} = V - bN$, where $b = \frac{16}{3}\pi r^3$ is the proper volume of particles with r being corresponding hard sphere radius of the particle. The second term in Eq. (7) corresponds to the attractive interaction between particles. The importance of van der Waals equation is that this analytical model can describe first-order liquid-gas phase transition of a real gas which ends at the critical point. Such a feature is also an expectation for the QCD phase diagram.

The van der Waals equation of state in the grand canonical ensemble can be written as [34,35]

$$p(T,\mu) = p^{id}(T,\mu^*) - an^2,$$
$$\mu^* = \mu - bp(T,\mu) - abn^2 + 2an, \tag{8}$$

where $n \equiv n(T,\mu)$ is the particle number density of the van der Waals gas:

$$n \equiv n(T,\mu) \equiv \left(\frac{\partial p}{\partial \mu}\right)_T = \frac{n^{id}(T,\mu^*)}{1 + bn^{id}(T,\mu^*)}. \tag{9}$$

The entropy density (s) for van der Waals gas can be written as

$$s(T,\mu) \equiv \left(\frac{\partial p}{\partial T}\right)_\mu = \frac{s^{id}(T,\mu^*)}{1 + bn^{id}(T,\mu^*)}. \tag{10}$$

Further, the energy density can be calculated as

$$\varepsilon(T,\mu) = Ts + \mu n - p \tag{11}$$

and is given by

$$\varepsilon(T,\mu) = \frac{\varepsilon^{id}(T,\mu^*)}{1 + bn^{id}(T,\mu^*)} - an^2. \tag{12}$$

For a single component nuclear matter ($g = 4, m = 938$ MeV) the values of van der Waals parameters were obtained as $a = 329$ MeV fm^3 and $b = 3.42$ fm^3 ($r = 0.59$ fm) [35] from the properties of the ground state of the nuclear matter.

For a hadronic system, we assume that interactions exist between all pairs of baryons and all pair of antibaryons. We ignore the interaction for mesons in order to avoid divergence of their number densities when modified chemical potentials become close to the masses of the particles. The baryon-antibaryon interaction is also ignored because the short range repulsive interaction between baryon and antibaryon may be dominated by the annihilation processes [17]. These are the limitations of the current model and leaves the scope for further improvement in the future. Hence the pressure of VDWHRG model can be written as [38]

$$p(T,\mu) = p_M(T,\mu) + p_B(T,\mu) + p_{\bar{B}}(T,\mu) \tag{13}$$

with

$$p_M(T,\mu) = \sum_{k\in M} p_k^{id}(T,\mu_k), \tag{14}$$

$$p_B(T,\mu) = \sum_{k\in B} p_k^{id}\left(T,\mu_k^{B*}\right) - an_B^2, \tag{15}$$

and

$$p_{\bar{B}}(T,\mu) = \sum_{k\in \bar{B}} p_k^{id}\left(T,\mu_k^{\bar{B}*}\right) - an_{\bar{B}}^2, \tag{16}$$

where $M, B, \bar{B}$ stand for mesons, baryons, and antibaryons, respectively. The modified chemical potential for baryons and antibaryons are given by

$$\mu_k^{B(\bar{B})*} = \mu_k - bp_{B(\bar{B})} - abn_{B(\bar{B})}^2 + 2an_{B(\bar{B})}, \tag{17}$$

where n_B and $n_{\bar{B}}$ are particle number densities of baryons and antibaryons, respectively. Once we know the pressure of the system, we can calculate different thermodynamic quantities. The derivative of $p_{B(\bar{B})}$ with respect to the baryon chemical potential will give us the corresponding number densities:

$$n_{B(\bar{B})} = \frac{\sum_{k\in B(\bar{B})} n_k^{id}\left(T,\mu_k^{B(\bar{B})*}\right)}{1 + b\sum_{k\in B(\bar{B})} n_k^{id}\left(T,\mu_k^{B(\bar{B})*}\right)}. \tag{18}$$

From pressure, we can calculate entropy density, energy density using Eqs. (10) and (11). Further one can calculate specific heat at constant volume as

$$C_V = \left(\frac{\partial \varepsilon}{\partial T}\right)_V \tag{19}$$

and the susceptibilities of conserved charges as

$$\chi_{\rm BSQ}^{xyz} = \frac{\partial^{x+y+z}(p/T^4)}{\partial(\mu_B/T)^x\partial(\mu_S/T)^y\partial(\mu_Q/T)^z}. \tag{20}$$

In the VDWHRG model if we put $a = 0$ and $b = 0$ we will get the results of the ideal HRG model. While with $a = 0$ in the VDWHRG model it corresponds to the excluded volume HRG (EVHRG) model [17–32], where only the repulsive interaction is included. Both the ideal HRG model and EVHRG model do not show any kind of phase transition. Still these models are quite successful in describing LQCD data of the bulk properties of hadronic matter in thermal and chemical equilibrium [3–5,15–18,60,61]. This model is also successful in describing the ratios of hadron yields, at chemical freeze-out, created in central heavy ion collisions from SIS up to LHC energies [43,44,56–58,62–64]. The heavy ion collisions at RHIC and LHC have established the quark-hadron phase transition.

III. RESULTS

In order to extract the van der Waals parameters a and r (or b) in the VDWHRG model that best describe the LQCD data at $\mu_B = 0$, we use the χ^2 minimization technique where χ^2 is defined as

$$\chi^2 = \frac{1}{N}\sum_{i,j}\frac{\left(R_{i,j}^{\rm LQCD}(T_j) - R_{i,j}^{\rm model}(T_j)\right)^2}{\left(\Delta_{i,j}^{\rm LQCD}(T_j)\right)^2}, \tag{21}$$

where $R_{i,j}^{\rm model}(T_j)$ is the ith observable with $R_{i,j}^{\rm LQCD}(T_j)$ and $\Delta_{i,j}^{\rm LQCD}(T_j)$ are its values and errors, respectively, at jth temperature calculated in LQCD, N is the number of LQCD data points. Here, we assume that van der Waals parameters a and r are independent of temperature and chemical potential. Errors on the parameters are obtained by knowing their values at $\chi^2_{\rm min} + 1$. In this work we use latest continuum limit LQCD data [2,16] of p/T^4,ε/T^4,s/T^3,C_V/T^3, and χ_B^2 at $\mu = 0$ within the temperature range 130–180 MeV to calculate χ^2 using Eq. (21). We assume that the HRG model is valid up to $T = 180$ MeV because the transition at $\mu_B = 0$ is a crossover. Hence thermodynamic observables do not exhibit sharp changes. LQCD results of quantities like p/T^4,ε/T^4, and s/T^3 have the smooth crossover at temperature range up to 180 MeV [5]. Depending on the choice of order parameter the QCD crossover temperature (T_c) has a range of 155 MeV to 175 MeV. For example LQCD calculation with chiral condensate gives $T_c = 155$ MeV [77]. However if one chooses strange quark number susceptibility the $T_c \sim 170$ MeV [78]. Typical error including systematics and due to the choice of order parameter on T_c is ∼20 MeV. Lowest temperature is taken as $T = 130$ MeV since the LQCD data of susceptibilities are not available below $T = 130$ MeV in Ref. [16]. The best fit in terms of χ^2 is achieved for parameter values of $a = 1250 \pm 150$ MeV fm^3 and $r = 0.7 \pm 0.05$ fm. Relatively smaller parameter values, $a = 329$ MeV fm^3 and $r = 0.59$ fm, were obtained by [35]. With these parameters only a qualitative description of LQCD data at $\mu = 0$ is possible [38] which we have already stated. In some previous works a hardcore radius has been estimated in the EVHRG model. In Ref. [22] hardcore radii of pion and other hadrons were obtained as 0.62 fm and 0.8 fm, respectively, by fitting the experimental data of hadronic ratios at Atlternating Gradient Synchrotron and Super Proton Synchrotron (SPS) energies. While the value of the hardcore radius was estimated as 0.3 fm in Ref. [56] using the experimental data of hadronic ratios at SPS energies. Also in Refs. [17,18] it was shown that the LQCD data of different thermodynamic quantities can be described in EVHRG model with the radius parameter between 0.2–0.3 fm. Our present estimate of the radius parameter is comparable to that of Ref. [22]. However, it should be noted that in all those works [17,18,22,56] only the repulsive interaction was considered for all mesons and baryons and there was no attractive interaction. To check the sensitivity of the value of the parameters on the temperature range we have refitted the LQCD data up to $T = 165$ MeV (the typical chemical freeze-out temperature from the RHIC top energy) and found the new a value to be 1210 MeV fm^3 which is within the uncertainty of the a value obtained by fitting up to $T = 180$ MeV, i.e., 1250 MeV fm^3 ± 150 MeV fm^3. There is no change in the value of r parameter.

Figure 1 shows a variation of p/T^4,ε/T^4, $(\varepsilon - 3p)/T^4$, $c_s^2 = \partial p/\partial\varepsilon$, s/T^3,C_V/T^3, and χ_B^2 with temperature at $\mu = 0$. Blue lines show the results of the VDWHRG model using the parameters $a = 1250$ MeV fm^3 and $r = 0.7$ fm. The bands are due to the errors on the parameters a and r. Results of the ideal HRG model along with the LQCD data of the Wuppertal-Budapest (WB) Collaboration [2] and the Hot QCD Collaboration [3] are also shown in this figure. Our estimations of all these observables in the VDWHRG model are in good agreement with LQCD calculations in the temperature range studied. Compared to the ideal HRG model, improvement of the results in the VDWHRG model is observed which indicates the interacting nature of baryons especially at high temperature regions. Among all these observables, the behavior of c_s^2 is most interesting in the VDWHRG model. The c_s^2 is a quantity that is sensitive to the phase transition effect. While in ideal HRG model c_s^2 decreases with increasing temperature, in the VDWHRG model it shows a minimum near $T = 150$ MeV which is consistent with the LQCD data. The minimum of the c_s^2 is known as the softest point where the expansion of

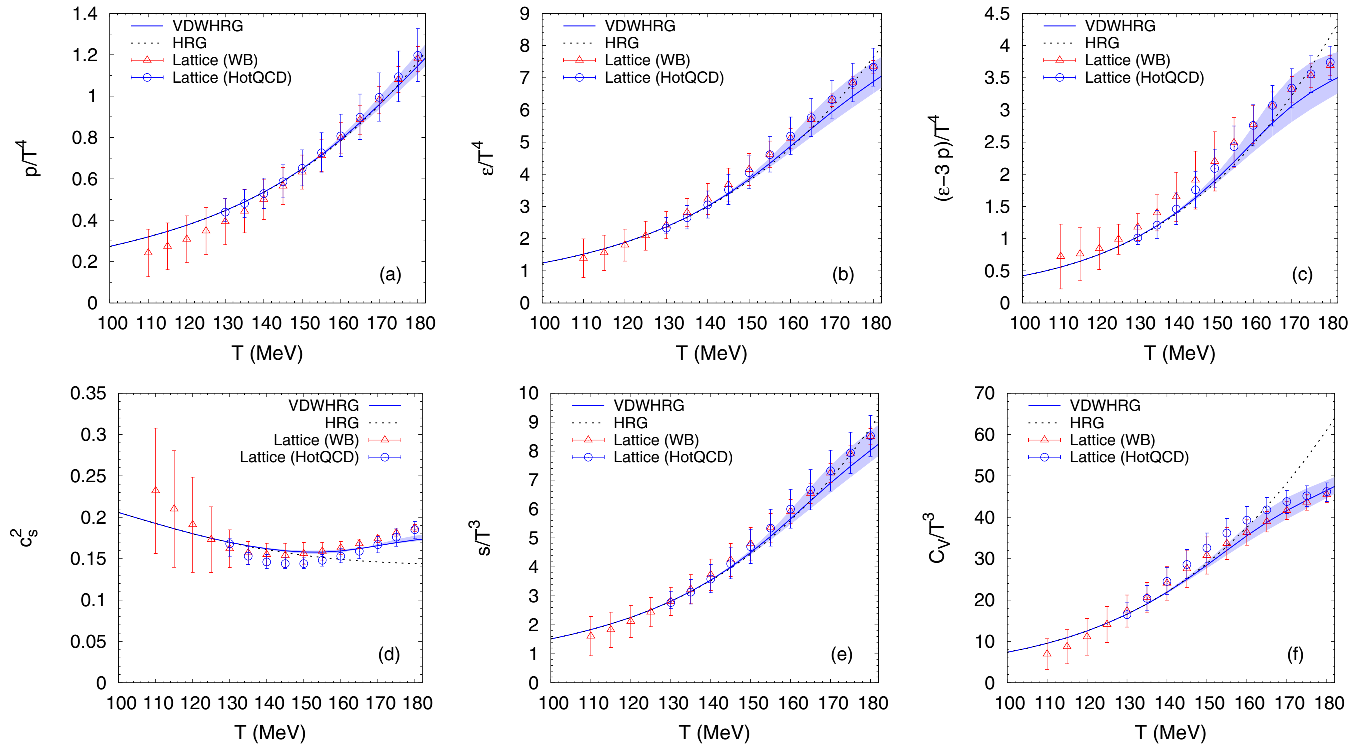

FIG. 1. The variation of different thermodynamical quantities with the temperature at $\mu = 0$. Blue lines show the results of VDWHRG model using the parameters $a = 1250$ MeV fm^3 and $r = 0.7$ fm. Blue bands are due to the errors on the van der Waals parameters in the VDWHRG model. The continuum extrapolated LQCD data are taken from Refs. [2] (WB) and [3] (HotQCD).

the system slows down. As a result the system spends a longer time in this temperature range which may be a crucial indicator of the quark-hadron transition of the system observed in heavy ion collisions [3,79].

In Fig. 2 temperature dependences of second-order fluctuations of different conserved charges at zero chemical potential have been shown. One can see that the qualitative behaviors of all these fluctuations in the VDWHRG model are similar to the LQCD data at high temperature which are different from the ideal HRG model where all the quantities increase rapidly with increasing temperature. Not only that, the χ_B^2 and χ_S^2 obtained from the VDWHRG model match quantitatively with the LQCD data. However, for χ_Q^2, which is dominated by the noninteracting mesons, hence the VDWHRG model overestimates the LQCD data.

Figure 3 shows correlations among conserved charges. Magnitudes of the χ_{BS}^{11} and χ_{QS}^{11} increase with increasing temperature and at a very high temperature they are expected to reach at $1/3$, the value at the Stefan-Boltzmann limit. χ_{BS}^{11} and χ_{QS}^{11} calculated in the VDWHRG model are close to the LQCD data in the temperature range studied. On the other hand, the LQCD data of χ_{BQ}^{11} shows a hump around $T = 170$ MeV which indicate the crossover transition in this region. Almost a similar qualitative behavior is observed for χ_{BQ}^{11} in the VDWHRG

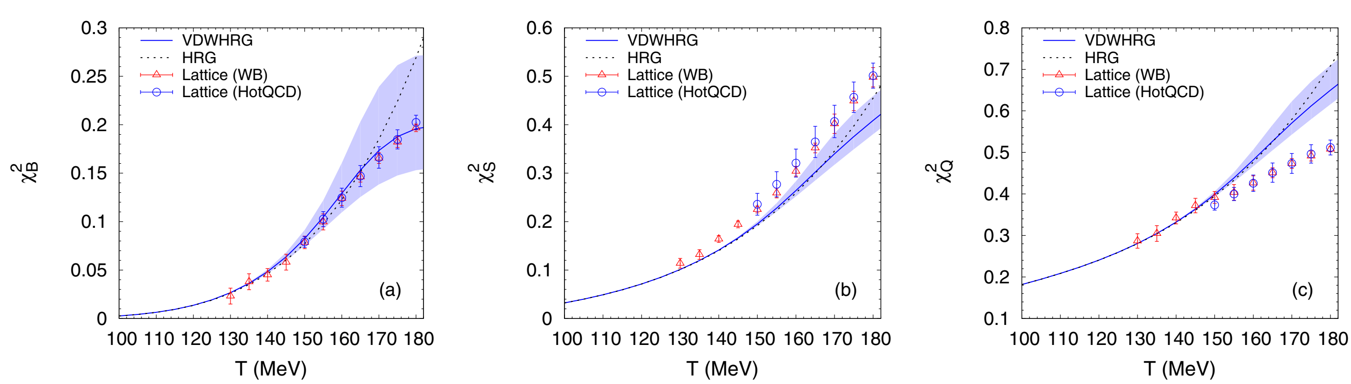

FIG. 2. The variation of second-order fluctuations of conserved charges with the temperature at zero chemical potential. Blue lines show the results of VDWHRG model using the parameters $a = 1250$ MeV fm^3 and $r = 0.7$ fm. Blue bands are due to the errors on the van der Waals parameters in the VDWHRG model. The LQCD data are taken from Refs. [5,16].

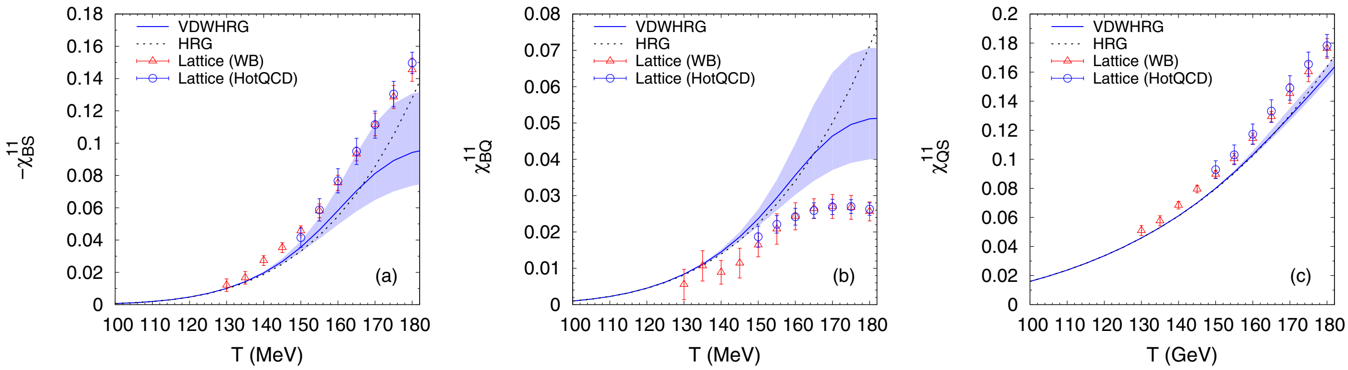

FIG. 3. The variation of correlations between conserved charges with the temperature at zero chemical potential. Blue lines show the results of the VDWHRG model using the parameters $a = 1250$ MeV fm^3 and $r = 0.7$ fm. Blue bands are due to the errors on the van der Waals parameters in the VDWHRG model. The LQCD data are taken from Refs. [5,16].

model as well although the model values overestimate the LQCD data.

The HRG model does not have QGP phase but with attractive and repulsive interaction for baryons the VDWHRG model explains LQCD data which have QGP phase. So if interactions are the sole driving force behind the physics of phase transitions one expects a similar phase transition effect in the VDWHRG model. Figure 4 shows a variation of pressure with number density at a fixed temperature in the VDWHRG model. The parameters a and r are fixed from the best fit values of the VDWHRG model to LQCD data at $\mu_B = 0$. For simplicity, we assume the nature of the interaction is similar to both nonzero and zero μ_B regions of the phase diagram. We observe the value of critical temperature to be $T = 62.1$ MeV. Below this temperature the number density changes discontinuously which resembles a hadron-liquid first-order phase transition. The picture will be more clear in Fig. 5, where we show variations of $(\partial p/\partial n)_T$ with respect to μ_B and n, respectively. One can see that at $T = 62.1$ MeV and $\mu_B = 708$ MeV, $(\partial p/\partial n)$ becomes zero and above $T = 62.1$ MeV, $(\partial p/\partial n)$ is always greater than zero. Since we have used the van der Waals interaction it is expected that the phase transition which we observed is a liquid-gas phase transition and the critical point (CP) ($T = 62.1^{+25.4}_{-19.1}$ MeV, $\mu_B = 708^{+90}_{-146}$ MeV) so obtained is that of a liquid-gas transition. Errors on the critical point is due to the uncertainties on the parameters a and r. A similar result of critical point with $T = 89$ MeV and $\mu_B = 724$ MeV is also obtained by using the holographic gauge/gravity correspondence to map baryon number fluctuations in QCD to the charge fluctuations of holographic black holes [80].

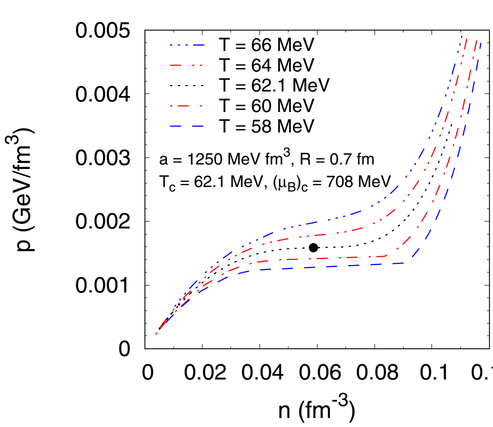

FIG. 4. The variation of pressure with the number density of the hadronic medium at different temperature. The black dot indicates the critical point.

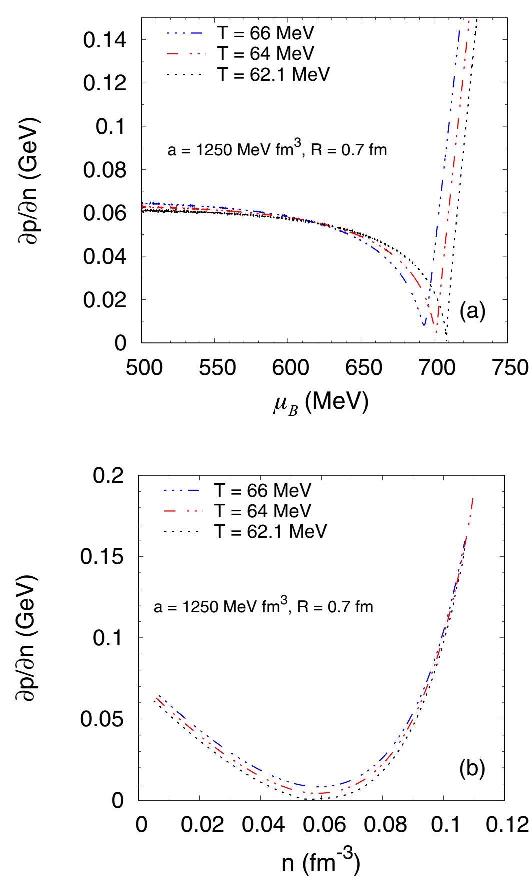

FIG. 5. Variations of $(\partial p/\partial n)_T$ with respect to μ_B and n, respectively.

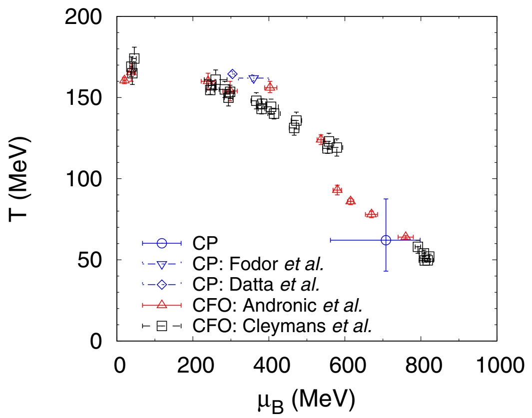

FIG. 6. The critical point of the liquid-gas transition of the present work in the QCD phase diagram. Critical points calculated in lattice are taken from Refs. [81] (Fodor *et al.*) and [82] (Datta *et al.*). Chemical freeze-out (CFO) parameters shown in this figure are taken from Refs. [63] (Andronic *et al.*) and [83] (Cleymans *et al.*).

In Fig. 6 we have plotted a collection of (T,μ_B) points to make a comparison of (i) liquid-gas CP from our present analysis (ii) CP from LQCD (iii) chemical freeze-out parameters from heavy ion collision experiments. Blue circular point in Fig. 6 shows the critical point ($T = 62.1^{+25.4}_{-19.1}$ MeV, $\mu_B = 708^{+90}_{-146}$ MeV) of the liquid-gas transition as estimated within the current model calculations in the QCD phase diagram. Critical points calculated in lattice [81,82] and the chemical freeze-out parameters obtained by different groups [63,83] at various energies are also shown in this plot.

IV. SUMMARY

To summarize, we have used LQCD data of $p/T^4, \varepsilon/T^4, s/T^3, C_V/T^3$, and χ_B^2 at $\mu = 0$ to extract the van der Waals parameters in the VDWHRG model. We assume that baryons are interacting whereas mesons are noninteracting. We get $a = 1250 \pm 150$ MeV fm^3 and $r = 0.7 \pm 0.05$ fm in our present work which best describes the LQCD data at $\mu = 0$ within the temperature range 130–180 MeV. The values of the VDWHRG model parameters are obtained using a χ-square minimization procedure. With these parameters which explains the QCD matter simulated by lattice, we observe a phase transition in the VDWHRG model at large potential with a critical point in the (T,μ_B) phase diagram at $T = 62.1$ MeV and $\mu_B = 708$ MeV. Our result of critical point is comparable with that of Ref. [80] where the critical point is obtained by using the holographic gauge/gravity correspondence to map baryon number fluctuations in QCD to the charge fluctuations of holographic black holes. Several improvements in the future can be carried out to our present idea and work. One of them includes incorporating the mesonic interaction of the system. Another is to incorporate other missing resonances in the hadronic spectrum [84].

ACKNOWLEDGMENTS

We are thankful to Sourendu Gupta, Victor Roy, and Nu Xu for carefully reading the manuscript and for their valuable comments. We thank Paolo Alba for suggesting useful references related to the work. B.M. acknowledges financial support from J C Bose National Fellowship of DST, Government of India. S.S. acknowledges financial support from DAE-SRC, Government of India.

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