Probing nuclear dissipation with first-chance fission probability

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Using the stochastic Langevin model, we investigate the influence of dissipation on first-chance fission properties of nuclei ^{220}Th and ^{240}Cf by calculating the drop of their first-chance fission probability caused by friction over their standard statistical-model value, $P_{f0}^{\rm drop}$, as a function of the presaddle friction strength (β) at different angular momenta and excitation energies. It is shown that the first-chance fission probability may be affected more largely at low angular momentum and that it is sensitive to β and the sensitivity is significantly larger than that of the total fission probability. Furthermore, we find that for heavy ^{240}Cf, while the total fission probability is insensitive to β, the first-chance fission probability depends sensitively on β. Our findings suggest that, to more stringently constrain the presaddle friction strength, the measurement of the first-chance fission probability from those heavy fissioning systems could provide a more sensitive and suitable experimental approach.

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I. INTRODUCTION

Nuclear dissipation plays a crucial role in low-energy nucleus-nucleus collisions. It affects not only the dynamics in entrance channel including deep-inelastic scattering [1–3], fusion [4,5], and quasifission [6–8] but also decay mechanisms of excited nuclei [9–11]. In particular, the influence that dissipation on fission processes has attracted much attention. Numerous measurements for particle prescission emission [12–16] and evaporation residue cross sections [17–19] at high energy have shown a marked deviation from predictions by standard statistical models. This discrepancy has been demonstrated [20–31] to arise from dissipation effects that are not accounted for in the statistical-model calculations.

Different from particle emission which depends on both pre- and postsaddle dissipation effects, evaporation-residue and fission cross sections are governed by presaddle friction (β). In addition to a number of works that have been carried out to constrain β [9,12,15,17,18,26,32–34], various new signals that are identified to be sensitive to β only have been proposed, for instance, the width of fission-fragment charge distributions [35], excitation energy at saddle [36], etc. However, the strength of presaddle friction is still quite uncertain and actively debated [35].

Fission competes with evaporation as a hot nucleus deexcites. As a direct consequence of dissipation effects, fission is retarded, which significantly decreases fission probability. Therefore, the fission probability is presently considered to be among most sensitive indicators of presaddle dynamical effects in fission of highly excited nuclei [37–41].

Due to the multiple emission of particles in the fission process, the total fission probability is composed of first and higher (i.e., second, third, etc.) chance fission probability, commonly referred to as multichance fission. It has been shown experimentally that by matching suitable conditions of excitation energy and angular-momentum population for two produced neighboring isotopes of the same fissioning element, experimental information on the first-chance fission probability can be obtained by measuring fission excitation functions [42] or prescission particle multiplicities [43] of the two neighboring fissioning isotopes.

The present work is devoted to studying whether the new observable, i.e., first-chance fission probability is a sensitive probe of presaddle friction as well as favorable experimental conditions through which presaddle dissipation effects can be better revealed with the first-chance fission probability. Towards that goal, Langevin models will be employed here to calculate the first-chance fission probability. The stochastic approach [20–23,30,33,44,45] has been successfully applied to reproduce a volume of fission data for many compound systems over a broad domain of excitation energy, angular momentum, and fissility.

II. THEORETICAL MODEL

It is well known [31,39,46] that the driving force of a hot system is not simply the negative gradient of the conservative force but should also contain a thermodynamic correction; therefore, the dynamics is described by the Langevin equation that is expressed by free energy. We employ the following one-dimensional Langevin equation to perform the trajectory calculations:

$$\frac{dq}{dt} = \frac{p}{m},$$
$$\frac{dp}{dt} = \frac{p^2}{2m^2}\frac{dm}{dq} - \frac{\partial F}{\partial q} - \beta p + \sqrt{m\beta T}\,\Gamma(t). \qquad (1)$$

Here q is the dimensionless fission coordinate and p is its conjugate momentum. The fission coordinate q is defined as half the distance between the center of mass of the future fission fragments divided by the radius of the compound nucleus. The reduced dissipation coefficient (also called

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the dissipation strength [9,10,12,15,17,19,20,35,37–39,41]) $\beta = \gamma/m$ denotes the ratio of the friction coefficient γ to the inertia parameter m. The inertia parameter is calculated by applying Werner-Wheeler approximation for incompressible irrotational flow [47]. The temperature in Eq. (1) is denoted by T. $\Gamma(t)$ is a time-dependent stochastic variable with a Gaussian distribution. Its average and correlation function are respectively written as $\langle\Gamma(t)\rangle = 0$ and $\langle\Gamma(t)\Gamma(t')\rangle = 2\delta(t - t')$.

The driving force of the Langevin equation is calculated from the free energy:

$$F(q,T) = V(q) - a(q)T^2. \tag{2}$$

Equation (2) is constructed from the Fermi gas expression [46] with a finite-range liquid-drop potential $V(q)$ [48] that includes q-dependent surface, Coulomb, and rotation energy terms. In our dynamical calculations we use $\{c,h,\alpha\}$ [49] parametrization of the compound nucleus shape. The deformation coordinate q is obtained by the relation $q(c,h) = (3c/8)\{1 + \frac{2}{15}[2h + (c-1)/2]c^3\}$ [20,50], where c and h correspond to the elongation and neck degrees of the freedom of the nucleus, respectively. The present model is one-dimensional (1D), though it is constructed starting from "funny hill" [49], which is 3D in general.

In constructing the free energy, the deformation-dependent level-density parameter $a(q)$ is expressed as

$$a(q) = a_1 A + a_2 A^{2/3} B_s(q), \tag{3}$$

where A is the mass number of the compound nucleus. Coefficients $a_1 = 0.073\ \mathrm{MeV}^{-1}$ and $a_2 = 0.095\ \mathrm{MeV}^{-1}$ are taken from Ignatyuk *et al.*'s prescription [51]. B_s is the dimensionless surface area of the nucleus (for a sphere $B_s = 1$) [52].

In the Langevin model, light-particle evaporation is coupled to the fission mode by a Monte Carlo procedure. The emission width of a particle of kind $\nu\ (= n,p,\alpha)$ is given by [53]

$$\Gamma_\nu = (2s_\nu + 1)\frac{m_\nu}{\pi^2\hbar^2\rho_c(E^*)} \times \int_0^{E^*-B_\nu} d\varepsilon_\nu \rho_R(E^* - B_\nu - \varepsilon_\nu)\varepsilon_\nu \sigma_{\mathrm{inv}}(\varepsilon_\nu), \tag{4}$$

where s_ν is the spin of the emitted particle ν, and m_ν its reduced mass with respect to the residual nucleus. The level densities of the compound and residual nuclei are denoted by $\rho_c(E^*)$ and $\rho_R(E^* - B_\nu - \varepsilon_\nu)$. B_ν are the liquid-drop binding energies. ε is the kinetic energy of the emitted particle and $\sigma_{\mathrm{inv}}(\varepsilon_\nu)$ is the inverse cross sections [53].

As usual [20,22,23,26,27], the discrete emission of light particles is taken into account. The procedure is to calculate the decay widths for light particles at each Langevin time step τ. Then the emission of particle is allowed by asking along the trajectory at each time step τ if a random number ζ $(0 \leqslant \zeta \leqslant 1)$ is less than τ/τ_{dec}, where $\tau_{\mathrm{dec}} = \hbar/\Gamma_{\mathrm{part}}$ with Γ_{part} being the sum of light particle decay widths. If this is the case, then a particle is emitted and we ask for the kind of particle ν by a Monte Carlo selection with the weights $\Gamma_\nu/\Gamma_{\mathrm{part}}$.

After each emission act of a particle, the free energy and the temperature in the Langevin equation are recalculated and the dynamics is continued.

When the fissioning nucleus proceeds from its ground state to its scission point, prescission particles can be evaporated along the Langevin fission trajectory. When the dynamic trajectory reaches the scission point, it is counted as a fission event. Fission probabilities and particle multiplicities are thus calculated by counting the number of corresponding fission and evaporated particle events. The present calculation allows for multiple emissions of light particles and higher-chance fission. So, the first, second, and so on, chance fission probability can be calculated [20] by counting the number of corresponding fission events in which not a single presaddle particle is emitted, only a presaddle particle is emitted.

Like previous Langevin calculations [20,22,23,27,45], in the present study the initial conditions for the dynamical Eq. (1) are assumed to correspond to a spherical compound nucleus with an excitation energy E^* and the thermal equilibrium momentum distribution. For starting a Langevin trajectory an orbital angular momentum value is sampled from the fusion spin distribution, which reads

$$\frac{d\sigma(\ell)}{d\ell} = \frac{2\pi}{k^2}\frac{2\ell + 1}{1 + \exp[(\ell - \ell_c)/\delta\ell]}. \tag{5}$$

The parameters ℓ_c and $\delta\ell$ are the critical angular momentum for fusion and diffuseness, respectively.

For decay systems ^{220}Th and ^{240}Cf, whose excitation energy is $E^* = 50$ MeV and 80 MeV and critical angular momentum is $\ell_c = 10\hbar$ and $40\hbar$, their $\delta\ell$ values are predicted to be around $5\hbar$ by applying the scaling formula of $\delta\ell$ [20]. So, to facilitate a theoretical investigation, $\delta\ell = 5\hbar$ is used in our calculations. We have checked that the influence of taking a slightly different value of $\delta\ell$ on the calculated results is minor.

III. RESULTS

In this work, fissioning nuclei ^{220}Th and ^{240}Cf are chosen as representatives for investigating first-chance fission characteristics. Moreover, to better survey the variation of the first-chance fission probability with the strength of presaddle friction (β), dynamical calculations are performed considering different values of β.

Dissipation affects the competition between evaporation channel and fission channel. A delayed fission process enhances particle evaporation, which leads to a deviation of the measured first-chance fission probability and total fission probability from that predicted by statistical models (SMs), and the amplitude of the deviation is extremely sensitive to β. A study for the deviation thus provides a method of determining presaddle friction. For this purpose, we adopt a definition similar to that suggested by Lazarev *et al.* [54], and define the relative drop of the first-chance fission probability P_{f0} calculated by SMs over the value by taking into account the dissipation and fluctuations of collective nuclear motion

$$P_{f0}^{\mathrm{drop}} = \frac{\langle P_{f0}^{\mathrm{SM}}\rangle - \langle P_{f0}^{\mathrm{dyn}}\rangle}{\langle P_{f0}^{\mathrm{SM}}\rangle}. \tag{6}$$

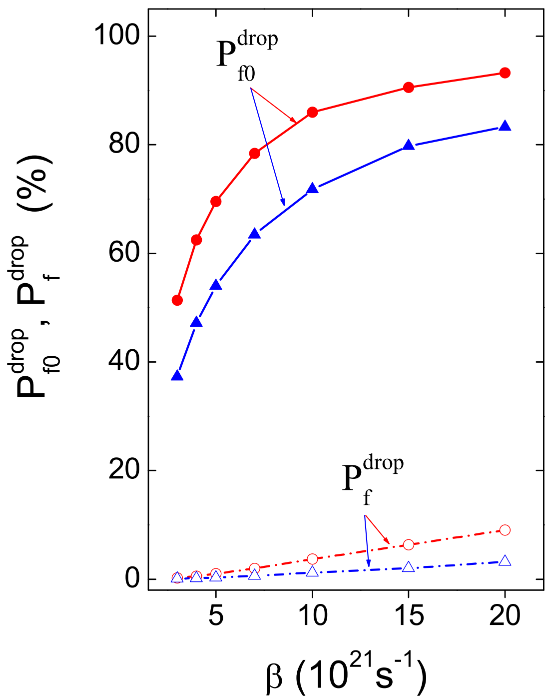

FIG. 1. Dynamical drop of the first-chance fission probability (denoted by P_{f0}^{drop}) and of the total fission probability (denoted by P_f^{drop}) for nucleus ^{220}Th relative to that predicted by SMs as a function of the presaddle dissipation strength β at excitation energy $E^* = 50$ MeV and at two critical angular momenta $\ell_c = 10\hbar$ (circles connected with red lines) and $40\hbar$ (triangles connected with blue lines).

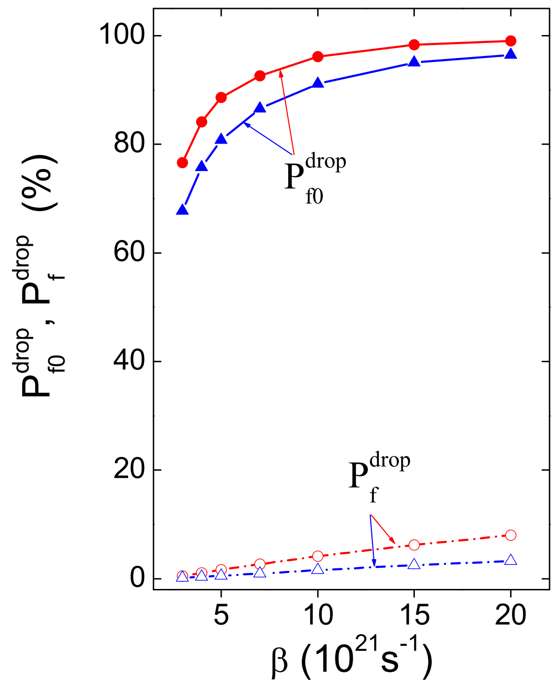

FIG. 2. Same as in Fig. 1 but at excitation energy $E^* = 80$ MeV.

Analogously, we define the relative drop for the total fission probability P_f caused by friction by

$$P_f^{\mathrm{drop}} = \frac{\langle P_f^{\mathrm{SM}} \rangle - \langle P_f^{\mathrm{dyn}} \rangle}{\langle P_f^{\mathrm{SM}} \rangle}. \tag{7}$$

We present in Fig. 1 the drop of the first-chance fission probability relative to SM estimation, P_{f0}^{drop}, as a function of presaddle friction strength (β) for ^{220}Th nuclei. It is seen that P_{f0}^{drop} rises rapidly with an increase of β, showing that the first-chance fission probability is a quite sensitive observable of the presaddle friction strength.

Furthermore, we note that the calculated P_{f0}^{drop} at $\ell_c = 40\hbar$ is below that at $\ell_c = 10\hbar$, demonstrating a stronger dissipation effect on the first-chance fission probability at small ℓ_c. This can be physically understood as follows. Fission barriers drop with raising angular momentum, which increases the first-chance fission probability. So, with a rise in angular momentum, while the dissipation effects cause a change in the magnitude of the first-chance fission probability with respect to the SMs value, the first-chance fission probability estimated by SMs, P_{f0}^{SM}, becomes larger. As a consequence, a high ℓ_c yields a low P_{f0}^{drop} [see Eq. (6)].

The result shows that experimentally, populating fissioning systems with a low spin slightly favors an examination of dissipation effects with first-chance fission probability.

In Fig. 1, the drop of the total fission probability, P_f^{drop}, at different β is also plotted. Compared to the first-chance fission probability, two prominent features of P_f^{drop} are observed. First, it is obvious that P_f^{drop} is much smaller than P_{f0}^{drop}, meaning that dissipation effects are significantly greater for the first-chance fission probability than for the total fission probability. Second, the slope of the curve of P_f^{drop} versus β, which reflects the sensitivity of the total fission probability to the variation of the friction strength, is less steep than the case of the first-chance fission probability, showing a lower sensitivity to β than the latter. This comparison clearly illustrates that the first-chance fission probability is not only a complementary observable to the total fission probability for pinning down dissipation properties in fission, but it could be a more sensitive signature of nuclear friction. Therefore, systematic and detailed experimental studies of the first-chance fission probability could place a tighter constraint on the strength of presaddle friction.

A picture like Fig. 1 is seen at another excitation energy $E^* = 80$ MeV (Fig. 2).

Currently, experimental measurements of the total fission probability have been performed over a wide range of fissioning systems, see, e.g., Refs. [10,12,40]. So, we carry out further calculations for heavier nucleus ^{240}Cf. Figure 3 reveals that the influence of dissipation on P_{f0}^{drop} of ^{240}Cf at different ℓ_c is like it in the case of ^{220}Th.

Besides, we notice that for this heavier nucleus, irrespective of the magnitude of angular momentum, P_f^{drop} is almost unchanged with β, exhibiting an insensitivity to friction. In contrast, P_{f0}^{drop} has a quick change as β varies; that is, it depends sensitively on friction. The contrast evidently demonstrates that for heavier nucleus, the first-chance fission probability

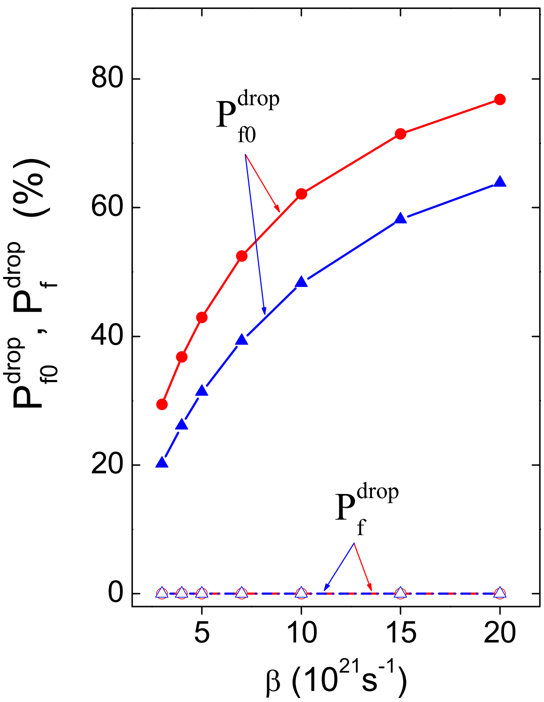

FIG. 3. Calculated P_{f0}^{drop} and P_f^{drop} of heavier nucleus ^{240}Cf as a function of β at $E^* = 50$ MeV and at $\ell_c = 10\hbar$ and $40\hbar$. The meaning of symbols is the same as that in Fig. 1.

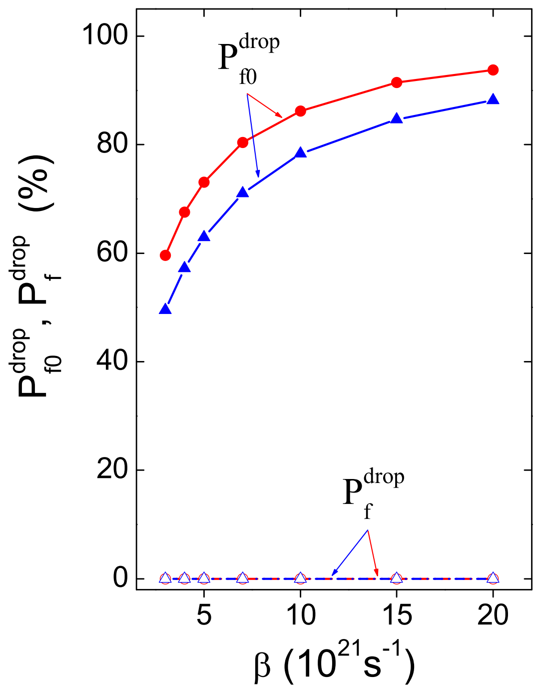

FIG. 4. Same as in Fig. 3 but at excitation energy $E^* = 80$ MeV.

is a more suitable tool for exploring nuclear dissipation than the total fission probability. It means that, in experiments, to put stricter limits on the presaddle friction strength through the measurement of fission observables coming from heavy decaying systems, it is a preferable option to measure the first-chance fission probability.

Displayed in Fig. 4 are calculations for ^{240}Cf at another energy $E^* = 80$ MeV. As can be seen, a change in excitation energy does not alter the main feature revealed in Fig. 3 for the heavier nucleus; that is, the total fission probability has no sensitivity to friction, but the first-chance fission probability shows a sensitive dependence on β.

The different evolving behaviors of the first-chance fission probability and the total fission probability of the heavy ^{240}Cf nucleus with β can be explained in the following way. ^{240}Cf is a very fissile system, thus it is committed to fission, i.e., insensitive to β. On the contrary, the number of neutrons the compound nucleus will emit before fissioning shows some sensitivity to β, and this is why the the first-chance fission probability is seen to be sensitive to β, since it is related to the 0n fission channel.

The different sensitivity of ^{220}Th (shown in Figs. 1 and 2) as compared to ^{240}Cf (shown in Figs. 3 and 4) could be related to a difference in the fissility of the two systems [55,56].

IV. DISCUSSION

Present calculations are performed with the 1D model. We discuss the possible influence of model dimensionality on the results. Both 1D and 3D models yield similar predictions on the changing trend of fission probability with the friction strength [57,58], and a difference in the fission probability (which is due to a difference in fission rates) and its sensitivity to the friction strength calculated with the two models is seen.

One- and multi-dimensional models give different first-chance fission probabilities. This is because a higher fission rate predicted in the multi-dimensional model [59] than in the one-dimensional model shortens fission lifetimes, which decreases multichance fission and slightly increases first-chance fission. So the results based on one dimension may have a change by considering a multi-dimensional model.

The total 3D fission probability shows a milder dependence on the friction strength than the total 1D fission probability. Based on the observation in 1D, displayed in Figs. 1–4, that the first-chance fission probability is more sensitive to the friction strength than the total fission probability, we estimate that after extension to 3D, the observation would still be that the first-chance fission probability is more sensitive than the total fission probability in 3D.

Moreover, for heavy fissioning nuclei such as ^{240}Cf, both 1D and 3D models give the same fission probability (i.e., $\sim 100\%$) for the nucleus independently of the friction strength. This implies that an insensitivity of the total fission probability of the heavy nucleus to β is predicted in 1D and 3D models. By contrast, the emitted neutron number before the decaying system fissions, which is related to first- and higher-chance fission probabilities, illustrates a sensitivity to β, as mentioned previously. Thus, for a very heavy fissioning system, its first-chance fission probability has a greater sensitivity to β than the total fission probability independently of the model dimensionality.

There are two known experimental approaches to obtaining information of the first-chance fission probability. The first approach is by measuring fission excitation functions of two neighboring fissioning isotopes. In this respect, one can clearly see from Fig. 1 in Ref. [60] that the amplitudes of the error bars of the extracted first-chance fission probability data are similar to that of the total fission probability data (where the experimental excitation energy covers a wide range), and they become smaller with decreasing excitation energy.

There is controversial [61,60] about the results for the characteristics of fission hindrance reported in Ref. [62].

In Refs. [60,62], the obtained transient time τ_{trans} is considered as a limitation to transient effects, but this time does not completely determine the properties of presaddle dissipation effects. The reason is that the concept of the transient effects is not strictly equivalent to that of the presaddle dissipation effects, though they have some connections. Specifically speaking, the presaddle dissipation effects lead to transient effects (i.e., due to dissipation, a time is required for the fission width to attain its quasistationary value) and, in addition, it causes other physical effects, such as a decrease of the asymptotic value of the fission decay width. Thus, making use of transient time only can not completely characterize the presaddle dissipation effects.

Experimental measurements on the total fission probability have been performed for decaying systems from light nuclei up to very heavy nuclei ($A \sim 250$). It is known that these total fission probability data have been used to obtain information on presaddle friction strengths by confronting them with theoretical simulations, as shown in a number of works (see, e.g., Refs. [20,33]).

Because of this reason, it is expected that the presaddle friction could be probed by comparing the first-chance fission probability data and the Langevin model calculations.

In addition to measuring fission excitation functions, it has recently been showed [43] that the first-chance fission probability data can be obtained by measuring prescission particle multiplicities of two neighboring fissioning isotopes. The prescission particle multiplicities can be extracted in experiment by a conventional multi-source model fit method [12] that has been applied to many systems from light to very heavy fissioning nuclei, and they have been frequently used to precisely determine the friction strength in the fission process. This means that the first-chance fission probability can be measured in experiment.

While attempts [60] have been made to get experimental information of first-chance fission probability through the measurement of fission excitation functions, there is challenging in praxis. Currently, there are few such experiments based on the “differential” method. It requires to produce and measure the fission probabilities of the compound nucleus at some excitation energies, and the fission probability of all its possible daughters compound nuclei which undergo fission after emission of 1, 2, etc., neutrons. So, all daughters have to be produced for the good isotope at the corresponding excitation energy, and so on. Also, the use of the differential method (basically, subtracting the signal of the daughter from that of the compound nucleus), requires good enough statistics in order to reduce uncertainties. In addition, the fission probability is sensitive to the level density, so a more precise extraction of β requires good-quality data. This means that more experimental and theoretical efforts are still needed to better address the issue.

TABLE I. Proposed experimental reactions. From left to right, the symbols respectively represent incident energy (E_{lab}), reaction, compound nucleus (CN), excitation energy (E^*), and the average angular momentum ($\bar{L}_{\mathrm{ave}}$) contributing to fission, which is calculated with the fusion spin distribution formula [i.e., Eq. (5)] and the scaling formulas for critical angular momentum and diffuseness for fusion [20]. The unit of E_{lab} and E^* is MeV, and the unit of $\bar{L}_{\mathrm{ave}}$ is $\hbar$.

E_{lab}	Reaction	CN	E^*	$\bar{L}_{\mathrm{ave}}$
67.3	^{4}He + ^{248}Cm	^{252}Cf	60.0	14.9
42.9	^{3}He + ^{248}Cm	^{251}Cf	50.6	11.9
67.6	^{4}He + ^{241}Am	^{245}Bk	60.0	14.9
43.1	^{3}He + ^{241}Am	^{244}Bk	49.8	12.0
67.3	^{4}He + ^{239}Pu	^{243}Cm	60.0	14.9
42.8	^{3}He + ^{239}Pu	^{242}Cm	51.0	11.9
66.4	^{4}He + ^{235}U	^{239}Pu	60.0	14.8
41.9	^{3}He + ^{235}U	^{238}Pu	51.0	11.7

On the experimental side, as noted above, an alternative approach [43], that is, measuring prescission particle multiplicities of two neighboring fissioning isotopes, has been recently proposed and successfully applied to extract the experimental information of first-chance fission characteristics.

For this method, one still needs to perform two experiments; that is two times beam time, the need of finding appropriate combinations.

In Ref. [43], it is proposed to investigate the characteristics of first-chance fission with the experimentally determined survival probability for first-chance fission (which is a complementary quantity to first-chance fission probability) by measuring prescission neutron multiplicities (which have a dependence on pre- and postsaddle friction) in two matched reactions. Their method is different from the previous approaches used to survey presaddle dissipation properties, where the measured observable (e.g., fission probability) depends on presaddle friction only.

To facilitate to use first-chance fission probabilities to explore the presaddle friction, several concrete reactions are proposed here. They could be used as a reference in future experiments and their relevant information is compiled in Table I.

Here we propose to explore the experimental approach proposed by Loveland and collaborators [43]. The principal reason is that it has been recently applied to study first-chance fission characteristics of heavy nuclei. In addition, in this approach, the prescission particle multiplicities of two neighboring fissioning isotopes produced in two corresponding reactions can be measured accurately.

The incident energies of the projectile ^{4}He in different reactions are obtained by choosing an excitation energy of 60 MeV for all formed heavy compound nuclei (CNs) ^{252}Cf, ^{245}Bk, ^{243}Cm, and ^{239}Pu.

After these CNs evaporate one neutron, the resulting residual nuclei are just those CNs populated in ^{3}He-induced reactions, because both ^{3}He and ^{4}He hit on the same targets. The excitation energies of various residual nuclei can be calculated with the method in Ref. [43], that is, subtracting the neutron separation energy S_n and its kinetic energy $2T$ (where T is temperature) from the excitation energy of the corresponding mother nuclei, i.e., 60 MeV. Assuming the excitation energies of these residual nuclei to be equal to those CNs formed in ^{3}He-induced reactions, one can get the corresponding incident energies of ^{3}He in different reactions.

For these reactions collected in Table I, on one hand, we have considered that targets and projectiles are available in experiment, since they were widely used in many previous reaction experiments, see, e.g., Refs. [63–66]. On the other hand, we choose light projectiles, i.e., using ^{4}He to bombard targets ^{248}Cm, ^{241}Am, ^{239}Pu, and ^{235}U to yield the suited heavy compound systems with low spins, mainly because our model calculations show that low angular momentum may favor an investigation of the presaddle friction with first-chance fission probabilities.

V. CONCLUSIONS

Based on the dynamical Langevin equations coupled to a statistical decay model, we have evaluated the evolution of the drop of the first-chance fission probability of ^{220}Th and ^{240}Cf systems with respect to SM values due to friction effects, $P_{f0}^{\rm drop}$, with the presaddle friction strength β. It has been shown that the first-chance fission probability is not only a sensitive probe of β, but the sensitivity is substantially higher than the total fission probability. Low angular momentum may yield a larger dissipation effect on the first-chance fission probability. Moreover, we have found that for heavier nucleus ^{240}Cf, the first-chance fission probability is apparently advantageous over the total fission probability for exploiting nuclear dissipation. These findings are helpful for the choice of the observables to be measured in future experiments. In other words, they suggest that, on the experimental side, to accurately probe presaddle dissipation strength, it is best to measure the first-chance fission probability of those heavy fissioning systems.

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