Updated three-body model of ^{6}He β decay into the $\alpha + d$ continuum

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The β-decay process of the ^{6}He halo nucleus into the $\alpha + d$ continuum is studied in an updated three-body model. The ^{6}He nucleus is described as an $\alpha + n + n$ system in hyperspherical coordinates on a Lagrange mesh. The shape and absolute values of the transition probability per time and energy units of new experiments are reproduced with a modified $\alpha + d$ potential. The obtained total transition probabilities are $(2.48 \pm 0.16) \times 10^{-6}$ s^{-1} for the full energy region and $(2.40 \pm 0.16) \times 10^{-6}$ s^{-1} for the cut-off $E > 150$ keV. The strong cancellation between the internal and halo parts of the β decay matrix element is a challenge for future *ab initio* calculations.

I. INTRODUCTION

The β-delayed deuteron decay of ^{6}He, i.e., the β decay of ^{6}He into ^{4}He and a deuteron,

$$^6\text{He} \rightarrow \alpha + d + e^- + \bar{\nu}_e, \tag{1}$$
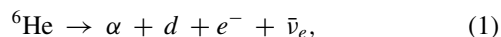
has been measured several times with various results for its very small branching ratio [1–5]. The smallness of the branching was first explained as a cancellation between the internal and halo parts of the Gamow-Teller matrix element by a semimicroscopic model in Ref. [6]. This interpretation was confirmed by later models (see references in Ref. [7]) but all results are very sensitive to tiny details. The branching ratio that we obtained in a three-body model [7,8] agreed with the data of the most recent experiment at that time [3]. This is due to a good description of the ground-state energy and halo of ^{6}He with an $\alpha + n + n$ wave function and to a potential fitting the $\alpha + d$ s-wave phase shift and the normalization of the experimental curve.

Since the publication of our calculation [7,8], two measurements [4,5] were performed, which update the experimental data and challenge our theoretical transition probabilities of the process. The first measurement by the ISOLDE collaboration in 2009 [4] used the technique of implantation into a highly segmented silicon detector. A branching ratio $B = (1.65 \pm 0.10) \times 10^{-6}$ was obtained for deuterons with energies above 350 keV with a 6% error. The corresponding transition probability is $W = (1.42 \pm 0.09) \times 10^{-6}$ s^{-1} for $E_d > 350$ keV. The data slightly underestimate our previous theoretical results, 2.04×10^{-6} s^{-1} for the full energy range or 1.59×10^{-6} s^{-1} for a cutoff $E > 370$ keV, which agreed with the old experimental results [3].

The second measurement [5] was performed in 2015 by the same collaboration at the REX-ISOLDE facility. The ^{6}He ions were implanted into the optical time projection chamber, where the decays with emission of charged particles were recorded. This technique allowed the authors to measure the spectrum down to 150 keV in the $\alpha + d$ center-of-mass frame. The branching ratio for this process amounts to $[2.78 \pm 0.07(\text{stat}) \pm 0.17(\text{sys})] \times 10^{-6}$. The shape of the spectrum is found to be in a good agreement with the three-body model [7,8], while the total transition probability is $[2.39 \pm 0.06(\text{stat}) \pm 0.15(\text{sys})] \times 10^{-6}$ s^{-1} which is about 20% larger than our theoretical prediction of Ref. [8]. The aim of the present paper is to update the theoretical model [7,8] and describe the new experimental data [4,5] with high precision. We also discuss expectations for theoretical progress.

II. MODEL

The ^{6}He nucleus is described as an $\alpha + n + n$ system in hyperspherical coordinates on a Lagrange mesh (see Ref. [9] for details). The ground-state wave function $\Psi_{^6\text{He}}(r,R)$ is then expressed and normalized in Jacobi coordinates: r between the neutrons and R between the α core and the center of mass of these neutrons. When neglecting higher-order effects, the dominant contribution to the transition probability per time and energy units is given by the Gamow-Teller approximation [10]

$$\frac{dW}{dE} = \frac{m_e c^2}{\pi^4 v \hbar^2} G_\beta^2 f(Q - E) B_{\text{GT}}(E), \tag{2}$$

where m_e is the electron mass, v and E are the relative velocity and energy in the center of mass frame of α and deuteron, and G_β is the dimensionless β-decay constant. The Fermi integral $f(Q - E)$ depends on the total kinetic energy $Q - E$ of the electron and antineutrino. The mass difference Q is 2.03 MeV. The Gamow-Teller reduced transition probability reads

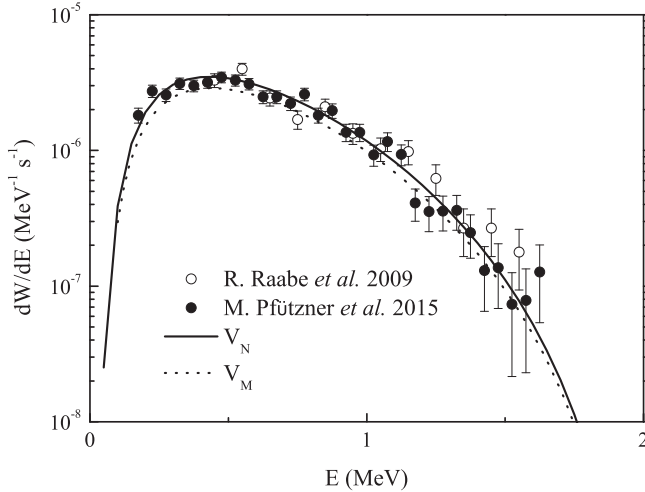$$B_{\text{GT}}(E) = 6\lambda^2 [I_E(\infty)]^2, \tag{3}$$

FIG. 1. Transition probability per time and energy units dW/dE of the ^{6}He β decay into the $\alpha + d$ continuum for the new (V_N, full line) and previous (V_M, dotted line) $\alpha + d$ potentials. The experimental data are from Refs. [4] (open circles) and [5] (full circles).

where λ is the ratio of the axial-vector to vector coupling constants and $I_E(R)$ is the integral [7]

$$I_E(R) = \int_0^R u_{\text{eff}}(R') u_E(R') dR'. \qquad (4)$$

The effective function

$$u_{\text{eff}}(R) = R \int_0^\infty \psi(r, R) u_d(r) r \, dr \qquad (5)$$

is the overlap of the $l_x = l_y = L = S = 0$ component

$$\psi(r, R) = \langle [[Y_0(\Omega_R) \otimes Y_0(\Omega_r)]_0 \otimes \chi_0]^{00} | \Psi_{^6\text{He}} \rangle, \qquad (6)$$

and the s-wave radial function of the deuteron $u_d(r)$ (see Ref. [7] for details). The $l = 0$ scattering wave function $u_E(R)$ is calculated with a simple Gaussian potential which reproduces the binding energy of ^{6}Li and the $\alpha + d$ phase shift δ_0 of the s wave. Its asymptotic behavior is $\cos \delta_0 F_0(kr) + \sin \delta_0 G_0(kr)$, where k is the wave number and F_0 and G_0 are the $l = 0$ regular and irregular Coulomb wave functions.

III. NUMERICAL RESULTS

The new data [5] can be described by a refitted $\alpha + d$ potential. We slightly modify the Gaussian potential $V_M(r) = -79.4 \exp(-0.21 r^2)$ MeV from Ref. [8] into $V_N(r) = -V_0 \exp(-0.2135 r^2)$ with $V_0 = 80.55$ MeV which describes equally well the binding energy 1.474 MeV of the ^{6}Li ground state and the s-wave phase shift of the $\alpha + d$ scattering up to 4 MeV, an energy exceeding the threshold energy 2.03 MeV of the β decay. At the scale of Fig. 1 of Ref. [7], the phase shifts of both potentials V_M and V_N are not distinguishable from the phase shift obtained with the Gaussian potential V_a in that figure. Both potentials possess a bound state below the ^{6}Li ground state which simulates a Pauli forbidden state in the s wave.
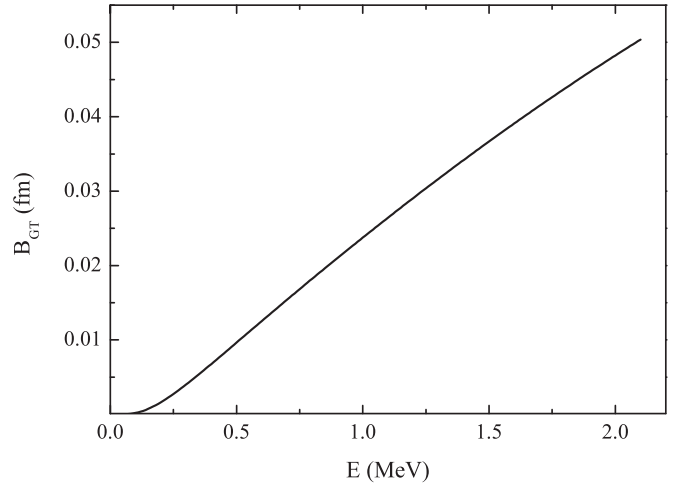

FIG. 2. Gamow-Teller reduced transition probability.

In Fig. 1, the transition probabilities per time and energy units dW/dE of the ^{6}He β decay into the $\alpha + d$ continuum for potentials V_M and V_N are displayed in comparison with the experimental data from Refs. [4,5]. A three-body hyperspherical wave function with hypermomentum components up to $K = 24$ is used for the ^{6}He ground state. As can be seen from the figure, the modified potential V_N describes the new experimental data [4,5] pretty well. The data of Ref. [5] are fitted with a χ^2 per datum of 0.98 while both sets are fitted with a χ^2 per datum of 1.13. The corresponding numbers for potential V_M are 2.08 and 2.18, respectively. When the coefficient V_0 of potential V_N is allowed to vary, a fit of the upper and lower limits of the error bars leads to $V_0 = 80.55 \pm 0.04$ MeV.

The total transition probability calculated with the V_N potential and its variation domain is estimated as $W = (2.48 \pm 0.16) \times 10^{-6} \text{ s}^{-1}$, while for the cut-off $E > 150$ keV we obtain $W[E > 150 \text{ keV}] = (2.40 \pm 0.16) \times 10^{-6} \text{ s}^{-1}$, which is very consistent with Ref. [5].

The shape of the theoretical curve agrees with the new data at low deuteron energies. This agreement over an extended energy range again confirms the cancellation mechanism of the internal and halo parts since it can reproduce both the order of magnitude and energy dependence of the data. The Gamow-Teller reduced transition probability is depicted in Fig. 2 as a function of the energy. It is very small under the Coulomb barrier, which explains the fast decrease of the transition probability at low energies in Fig. 1. Above 0.1 MeV, it increases almost linearly. The decrease of the transition probability above 0.5 MeV is entirely due to the phase-space factor.

The shape of the reduced transition probability B_{GT} can be understood with Fig. 3. Though integral $I_E(R)$ is only observable asymptotically when R tends to infinity, its shape contains important physical information about the cancellation mechanism. In Fig. 3, this integral is represented at three energies: $E = 0.5, 1,$ and 1.5 MeV. Its absolute value reaches a maximum near 5 fm before a decrease due to a change of sign of the $l = 0$ scattering wave. This decrease continues to
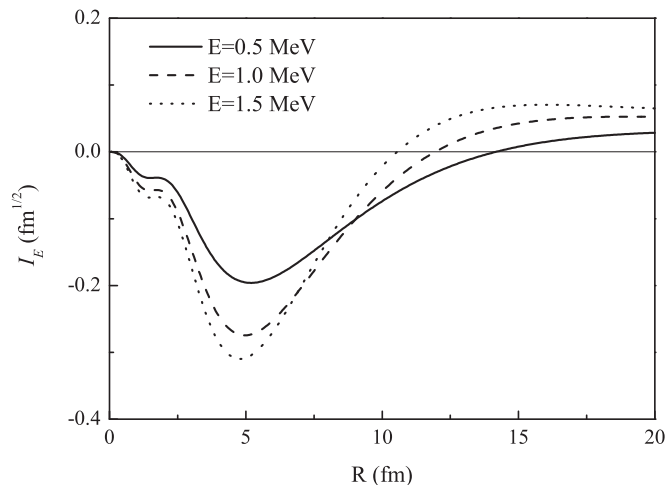
FIG. 3. Integrals $I_E(R)$ at $E = 0.5$ (full line), 1 (dashed line), and 1.5 MeV (dotted line) calculated with the V_N potential.

large distances because of the large extension of the halo. The integral vanishes at a location where the internal and external parts of the integrand exactly cancel each other. Beyond this zero of $I_E(R)$, the halo part dominates and the integral changes sign. The cancellation mechanism is very sensitive to the location of the s-wave node. It is stronger at small energies where this node is at a larger distance. Hence, $B_{\mathrm{GT}}(E)$ progressively increases when this node moves to the left with increasing energy. The sensitivity of $B_{\mathrm{GT}}(E)$ to the exact location of this node make *quantitative* predictions of the transition probability very difficult. The transition matrix element for the β-delayed deuteron decay of ^{6}He promises to be a very stringent test for such predictions with *ab initio* calculations. In the present model, experimental data on the transition probability are needed to fix the phenomenological $\alpha + d$ potential which is not constrained enough by the phase shifts. If a new experiment leads to a more accurate normalization of these data, this potential may have to be refitted.

The description of the delayed β decay is accessible to *ab initio* calculations since both ^{6}He [11] and the $\alpha + d$ scattering [12] have been studied in this way. Since these models have no free parameter, their results will be very sensitive to the delicate cancellation mechanism and small inaccuracies may lead to large disagreements with experiment. Moreover, it is not clear whether these models are able yet to accurately describe the halo of ^{6}He up to about 20 fm as required by the cancellation mechanism. In these models, the

energy-dependent integral would be defined by

$$I_E(R) = \frac{k}{\sqrt{2\pi}} \int_0^R \left\langle \Psi_{\alpha+d}^{10+} \middle| \delta(\rho - R') \sum_{j=1}^6 t_{j-} s_{jz} \middle| \Psi_{^6\mathrm{He}}^{00+} \right\rangle dR',$$

(7)

where s_j and t_j are the dimensionless spin and isospin operators of nucleon j and ρ is the relative coordinate between the ^{4}He and deuteron centers of mass. The six-nucleon wave functions $\Psi_{^6\mathrm{He}}^{00+}$ and $\Psi_{\alpha+d}^{10+}$ represent the ^{6}He ground state and the $J^\pi = 1^+$ partial wave of an $\alpha + d$ scattering wave normalized asymptotically to $\exp(ik \cdot \rho)$, respectively. It is important to verify whether such a microscopic calculation confirms at least the internal part of $I_E(R)$ depicted in Fig. 3. Information about the location of the s-wave node would also be essential.

IV. CONCLUSION

In conclusion, the three-body model based on the hyperspherical Lagrange-mesh method [7,8] is updated for the description of new experimental data [4,5]. It is shown that the new data can be described pretty well with the help of a modification of the $\alpha + d$ potential in the s wave, while keeping the descriptions of the binding energy of the ^{6}Li ground state and s-wave $\alpha + d$ phase shift and, importantly, the presence of a Pauli forbidden state. The modification of the potential results in a shift of the nodal position of the $l = 0$ relative scattering wave function, which affects the values of the effective integral for the β-decay matrix elements. Nevertheless, the main conclusion of the three-body model of Ref. [7] remains valid, i.e., that the lowering of the β-decay transition probability occurs due to a cancellation effect of the internal and external parts of the Gamow-Teller matrix element. An important open question is how to fix the potential of the $\alpha + d$ relative motion without fitting its parameters to β-decay data. The answer could come from microscopic approaches. We suggest that the effective integral $I_E(R)$ should provide an important link between partly phenomenological three-body models and *ab initio* descriptions.

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