The $\gamma p \to p\eta\eta$ reaction in an effective Lagrangian model

Bo-Chao Liu* and Shao-Fei Chen
School of Science, Xi'an Jiaotong University, Xi'an, Shannxi 710049, China

(Received 27 October 2016; revised manuscript received 21 August 2017; published 6 November 2017)

In this paper, we investigate the $\gamma p \to p\eta\eta$ reaction within an effective Lagrangian approach and isobar model. We consider the contributions from the intermediate $N^*(1535)$, $N^*(1650)$, $N^*(1710)$, and $N^*(1720)$ isobars which finally decay to the $N\eta$ state. It is found that the excitation of the $N^*(1535)$ dominates this reaction close to threshold and ρ meson exchange plays the most important role for the excitation of nucleon resonances. Therefore, this reaction offers a potentially good place to study the properties of nucleon resonances and their couplings to the $N\rho$ channel. Predictions for angular distributions and invariant mass spectra of final particles are also presented for future comparison with data.

DOI: 10.1103/PhysRevC.96.054001

I. INTRODUCTION

Studying the properties of nucleon resonance and the underlying physics is a central task in hadronic physics. Since nucleon resonances have extremely short lifetimes, in practice the only possible way to study their properties is to make use of the measurements of their decay products. In recent years, such studies have been significantly promoted by the new and accurate data on meson production induced by photons off nucleons. It has been shown that intermediate nucleon resonances play important roles in reactions such as $\gamma p \to p\pi, p\eta, p\omega, K\Sigma, K\Lambda$, etc. [1]. Detailed studies of these reactions have remarkably improved our knowledge of the reaction mechanisms and the properties of the intermediate nucleon resonances [2].

Apart from the studies of the single meson production processes, precise data on some multimeson production processes, such as the $\gamma p \to p\pi^0\pi^0$ and $p\pi^0\eta$ reactions, were also published and used to extract the properties of the intermediate resonances in recent years [3–5]. Even though multimeson production processes are more poorly known and may involve more complicated reaction mechanisms, these reactions are interesting because they offer information about cascade decays of higher-mass resonances. Furthermore, some of these reactions are potentially good places to study the properties of nucleon resonances due to their unique reaction mechanisms. For example, in Ref. [6] the authors showed that the production of the $N^*(1535)$, mainly excited by the η exchange, dominates the reaction $\gamma p \to \phi K\Lambda$ near threshold. Therefore this reaction offers a good place to study the properties of the $N^*(1535)$ and its coupling to the $K\Lambda$ channel. Clearly, such studies are useful to improve our knowledge of nucleon resonances.

In this work, we present studies of the reaction $\gamma p \to p\eta\eta$ within an effective Lagrangian approach and isobar model. The purpose of this work is to make a theoretical attempt to explore this reaction and present the predictions for the observables, which can be checked by future experiments. In the isobar model, the possible mechanisms of this reaction can be depicted by the Feynman diagrams shown in Fig. 1. Usually the s-channel diagrams like Fig. 1(d) give the most important contributions [7]. However, such contributions are assumed to be insignificant in this reaction since there is no evidence that some nucleon resonances have significant coupling to the $N\eta\eta$ channel. The u-channel diagrams like Figs. 1(e) and 1(f) are also expected to be unimportant.[1] Therefore, we will mainly consider the Feynman diagrams in Figs. 1(a)–1(c) in the present work.

Although for now our knowledge of this reaction is still scant, the studies of some relevant reactions, such as $\gamma p \to p\eta$ [11,12] and $pp \to pp\eta$ [13,14], can offer valuable clues for our study. In those works, it has been shown that the well-established nucleon resonances, such as the $N^*(1535)$, $N^*(1650)$, $N^*(1710)$, and $N^*(1720)$, give the most important contributions in their production regions. Therefore, it is reasonable to assume that these four resonances may also play important roles in the present reaction as intermediate states and finally decay to the $N\eta$ channel. Within the effective Lagrangian approach and isobar model, the resonance production process can be depicted by the Feynman diagrams shown in Fig. 1, where nucleon resonances are excited by absorbing or emitting a meson. Since the photon and final η have odd and even C parities respectively, scalar and pseudoscalar meson exchanges are forbidden in this reaction due to C parity conservation. On the other hand, the vector and axial meson exchanges are allowed. Even though axial meson exchanges are allowed by symmetry principles, their couplings to the $\gamma\eta$ channel seem to be weak, since no axial meson of

*liubc@xjtu.edu.cn

[1]In principle, nucleon resonances below the threshold can also contribute. As a reasonableness check of neglecting these diagrams, we calculate the Feynman diagrams in Figs. 1(d) and ig. 1(f) considering the intermediate $N^*(1535)$ contributions. Note that the $N^*N^*\eta$ and $N^*N^*\gamma$ couplings are still not well constrained in previous studies. To have a rough estimate, we adopt the $N^*N^*\eta$ coupling for the $N^*(1535)$ resonance predicted in the chiral unitary approach [8]. The results show that the intermediate $N^*(1535)$ contributions only play an insignificant role in this reaction. Note that the intermediate nucleon contributions are not considered due to the vanishingly small $NN\eta$ coupling [9].

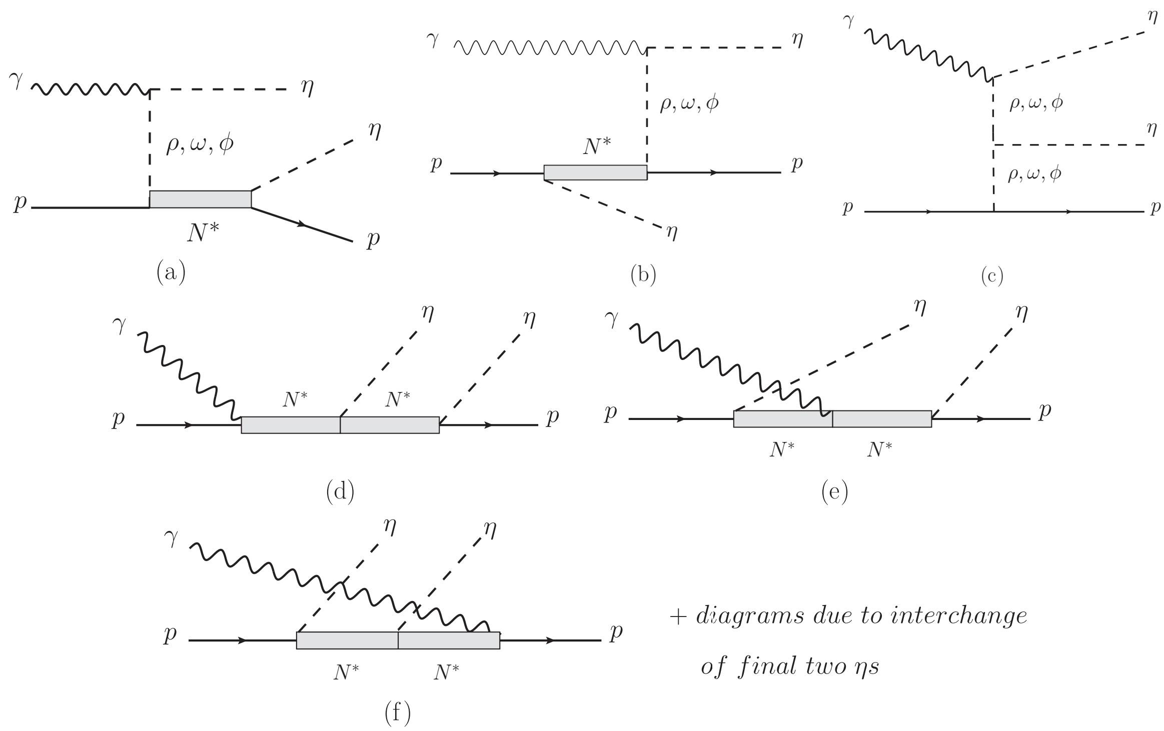

FIG. 1. Feynman diagrams for the reaction $\gamma p \to p\eta\eta$.

mass less than 1.5 GeV has a significant decay branch ratio to $\gamma\eta$ channel [10]. At the same time, their couplings to nucleon resonances are also poorly known and were usually ignored in previous studies [13,14]. The same arguments also hold for the mesons with higher spin, and their contributions are further suppressed because of their relatively large mass. Therefore, we ignore their contributions. The possibility that the final η pair are from the decay of a meson resonance can also be ignored, since there is no clear evidences for the existence of such a state in the energies under study [10]. If the above arguments are correct, it means that the vector meson exchange plays the most important role for the excitation of nucleon resonance in this reaction. Compared with the $NN \to NN\eta$ reaction, where various meson exchanges are allowed, the present reaction offers a relatively clean place to study the role of vector mesons in the excitation of nucleon resonances. In fact, there were some controversies related to the relative roles of π and ρ exchanges for the excitation of the $N^*(1535)$ in the $NN \to NN\eta$ reaction [13,15]. It is thus interesting to expect that future measurements on this reaction will offer a further test of the $N^*N\rho$ couplings and help us to better understand the role of vector meson exchange in relevant reactions.

This paper is organized as follows. In Sec. II, the formalism and ingredients used in the calculation are given. In Sec. III, the results are presented and discussed. A short summary is given in the concluding part. Given in the Appendix are the formulas used to estimate the coupling constants in the interaction Lagrangians.

II. THEORETICAL FORMALISM

In this work, we investigate the reaction $\gamma p \to p\eta\eta$ within an effective Lagrangian approach and isobar model. The basic Feynman diagrams are depicted in Fig. 1, where panels (a)–(c) are assumed to give the dominant contributions and will be considered in the following part. As discussed in the Introduction, the nucleon resonances are expected to be excited mainly due to the exchange of vector mesons. For the intermediate nucleon resonances, we consider the well-established $N^*(1535)$, $N^*(1650)$, $N^*(1710)$, and $N^*(1720)$ because of their relatively large couplings with both η and vector mesons [10] and their important roles in other $N\eta$ production reactions [11–14].

The effective interaction Lagrangians for the vertices not involving nucleon resonances are adopted as [11,12,16]

$$\mathcal{L}_{\rho\eta\gamma} = \frac{e}{m_\rho} g_{\rho\eta\gamma} \epsilon^{\mu\nu\alpha\beta} \partial_\mu \rho_\nu \partial_\alpha A_\beta \eta, \tag{1}$$

$$\mathcal{L}_{\omega\eta\gamma} = \frac{e}{m_\omega} g_{\omega\eta\gamma} \epsilon^{\mu\nu\alpha\beta} \partial_\mu \rho_\nu \partial_\alpha A_\beta \eta, \tag{2}$$

$$\mathcal{L}_{\rho\rho\eta} = \frac{g_{\rho\rho\eta}}{2m_\rho} \epsilon^{\mu\nu\alpha\beta} \partial_\mu \rho_\nu \partial_\alpha \rho_\beta \eta, \tag{3}$$

$$\mathcal{L}_{\omega\omega\eta} = \frac{g_{\omega\omega\eta}}{2m_\omega} \epsilon^{\mu\nu\alpha\beta} \partial_\mu \omega_\nu \partial_\alpha \omega_\beta \eta, \tag{4}$$

where e can be obtained through the fine-structure constant $\alpha = e^2/4\pi = 1/137.036$ and $\varepsilon_{\mu\nu\alpha\beta}$ is the Levi-Civita antisymmetric tensor with $\varepsilon_{0123} = 1$. For the coupling constants $g_{\rho\rho\eta}$ and $g_{\omega\omega\eta}$, we take the values from Ref. [13], which were determined from a systematic analysis of the radiative decay of pseudoscalar and vector mesons. As for the coupling constants $g_{\rho\eta\gamma}$ and $g_{\omega\eta\gamma}$, the values are determined from the experimental decay widths. A summary of the values of various coupling constants and the parameters adopted are given in Table I.

TABLE I. Resonances and parameters used in the calculation. Widths and branching ratios are from Ref. [10] and [17].

State	Width (MeV)	Decay channel	Adopted branching ratios	$g^2/4\pi$
ρ	147.8	$\gamma\eta$	3×10^{-4}	1.2×10^{-1}
ω	8.49	$\gamma\eta$	4.6×10^{-4}	9.85×10^{-3}
ϕ	4.27	$\gamma\eta$	1.31×10^{-2}	4.0×10^{-2}
$N^*(1535)$	150	$N\rho$	0.02[a]	1.93
		$N\omega$		3.06
		$N\eta$	0.42	2.75×10^{-1}
$N^*(1650)$	140	$N\rho$	0.08[a]	4.96×10^{-1}
		$N\omega$		3.83×10^{-2}
		$N\eta$	0.18	6.07×10^{-2}
$N^*(1710)$	100	$N\rho$	0.15[a]	1.35
		$N\omega$	0.03	12.2
		$N\eta$	0.3	1.44
$N^*(1720)$	250	$N\rho$	0.775	105.8
		$N\omega$		0.73
		$N\eta$	0.03	7.87×10^{-2}

[a]These $N\rho$ branching ratios are taken from Ref. [17] since they are not offered in the new version of the PDG book. We have checked that the $N^*N\rho$ coupling constants derived from these $N\rho$ branching ratios are closed to the ones obtained in the VMD model within a factor of 2.

The interaction Lagrangians for the vertices that involve nucleon resonances are given as follows [14,15,18]:

$$\mathcal{L}_{N^*(1535)N\rho} = -\frac{g_{N^*(1535)N\rho}}{2m_N}\bar{\Psi}_{N^*}\gamma_5\sigma_{\mu\nu}\partial^\nu\vec{\tau}\cdot\vec{\rho}^\mu\Psi_N + \text{H.c.}, \tag{5}$$

$$\mathcal{L}_{N^*(1535)N\omega} = -\frac{g_{N^*(1535)N\omega}}{2m_N}\bar{\Psi}_{N^*}\gamma_5\sigma_{\mu\nu}\partial^\nu\omega^\mu\Psi_N + \text{H.c.}, \tag{6}$$

$$\mathcal{L}_{N^*(1535)N\eta} = -ig_{N^*(1535)N\eta}\bar{\Psi}_N\Psi_{N^*}\Phi_\eta + \text{H.c.}, \tag{7}$$

$$\mathcal{L}_{N^*(1650)N\rho} = -\frac{g_{N^*(1650)N\rho}}{2m_N}\bar{\Psi}_{N^*}\gamma_5\sigma_{\mu\nu}\partial^\nu\vec{\tau}\cdot\vec{\rho}^\mu\Psi_N + \text{H.c.}, \tag{8}$$

$$\mathcal{L}_{N^*(1650)N\omega} = -\frac{g_{N^*(1650)N\omega}}{2m_N}\bar{\Psi}_{N^*}\gamma_5\sigma_{\mu\nu}\partial^\nu\omega^\mu\Psi_N + \text{H.c.}, \tag{9}$$

$$\mathcal{L}_{N^*(1650)N\eta} = -ig_{N^*(1650)N\eta}\bar{\Psi}_N\Psi_{N^*}\Phi_\eta + \text{H.c.}, \tag{10}$$

$$\mathcal{L}_{N^*(1710)N\rho} = -\frac{g_{N^*(1710)N\rho}}{2m_N}\bar{\Psi}_{N^*}\sigma_{\mu\nu}\partial^\nu\vec{\tau}\cdot\vec{\rho}^\mu\Psi_N + \text{H.c.}, \tag{11}$$

$$\mathcal{L}_{N^*(1710)N\omega} = -\frac{g_{N^*(1710)N\omega}}{2m_N}\bar{\Psi}_{N^*}\sigma_{\mu\nu}\partial^\nu\omega^\mu\Psi_N + \text{H.c.}, \tag{12}$$

$$\mathcal{L}_{N^*(1710)N\eta} = -ig_{N^*(1710)N\eta}\bar{\Psi}_N\gamma_5\Psi_{N^*}\Phi_\eta + \text{H.c.}, \tag{13}$$

$$\mathcal{L}_{N^*(1720)N\rho} = i\frac{g_{N^*(1720)N\rho}}{m_{N^*}+m_N}\bar{\Psi}_\mu\vec{\tau}\cdot(\partial^\nu\vec{\rho}^\mu - \partial^\mu\vec{\rho}^\nu) \times\gamma_\nu\gamma_5\Psi_N + \text{H.c.}, \tag{14}$$

$$\mathcal{L}_{N^*(1720)N\omega} = i\frac{g_{N^*(1720)N\omega}}{m_{N^*}+m_N}\bar{\Psi}_\mu(\partial^\nu\omega^\mu - \partial^\mu\omega^\nu)\gamma_\nu \times\gamma_5\Psi_N + \text{H.c.}, \tag{15}$$

$$\mathcal{L}_{N^*(1720)N\eta} = \frac{g_{N^*(1720)N\eta}}{m_\eta}\bar{\Psi}_\mu\partial^\mu\Phi_\eta\times\Psi_N + \text{H.c.}, \tag{16}$$

where the coupling constants $g_{N^*N\eta}$ and $g_{N^*N\rho}$ can be determined by calculating the corresponding partial decay widths. For $g_{N^*N\omega}$, we also employ the vector meson dominance model to estimate their values for those nucleon resonances which lie below the $N\omega$ threshold or have very little phase space to decay. The corresponding calculations are described in detail in the Appendix. It should be stressed that in this way only the magnitudes of the coupling constants can be determined. To fix the relative signs, we have followed the results of Ref. [12,14].

To calculate the Feynman diagrams shown in Fig. 1, a relevant off-shell form factor is also used for the exchanged particles to take into account the internal structure of hadrons and off-shell effects. In our computations, we adopt a monopole form factor for the $N^*N\rho(\omega)$ vertices as

$$F_t(q^2) = \frac{\Lambda_t^2 - m^2}{\Lambda_t^2 - q^2}, \tag{17}$$

where Λ_t, m, and q are the cutoff parameter, mass, and four-momentum of the exchanged meson, respectively. We adopt $\Lambda_t = 1.31$ GeV and $\Lambda_t = 1.5$ GeV for ρ and ω exchanges as the ones used in the CD-Bonn model [9]. For the electromagnetic $\gamma\eta\rho(\omega)$ vertex, we follow Refs. [11,13] and adopt the form factor as

$$F_{V=\rho,\omega}(q^2) = \left(\frac{\Lambda_V^{*2}}{\Lambda_V^{*2} - q^2}\right)^2, \tag{18}$$

where $\Lambda_V^* \sim 1.2$ GeV is determined by fitting experimental data. To take into account the off-shell effects of the intermediate nucleon resonances, we adopt the following form factor:

$$F_B(p^2) = \frac{\Lambda_B^4}{\Lambda_B^4 + \left(p^2 - m_B^2\right)^2} \tag{19}$$

with $\Lambda_B = 1.2$ GeV [13].

The propagator of the nucleon resonance with $S = \frac{1}{2}$ is written as [19–23]

$$G_{N^*_{1/2}}(p) = \frac{i(\not{p} + M_{N^*})}{p^2 - M_{N^*}^2 + iM_{N^*}\Gamma_{N^*}}, \tag{20}$$

and for the $S = \frac{3}{2}$ state it is

$$G_{N^*_{3/2}}(p) = -\frac{i(\not{p} + M_{N^*})}{p^2 - M_{N^*}^2 + iM_{N^*}\Gamma_{N^*}}\left[g_{\mu\nu} - \frac{1}{3}\gamma_\mu\gamma_\nu - \frac{1}{3M_{N^*}}(\gamma_\mu p_\nu - \gamma_\nu p_\mu) - \frac{2}{3M_{N^*}^2}p_\mu p_\nu\right]. \tag{21}$$

The propagator of the vector meson can be written in the form

$$G_V^{\mu\nu}(q) = -i\left(\frac{g^{\mu\nu} - q^\mu q^\nu/q^2}{q^2 - m^2}\right), \tag{22}$$

where q_V is the four-momentum of the intermediate vector meson.

With all relevant effective Lagrangians, coupling constants and propagators given above, the amplitudes for various diagrams can be written straightforwardly by following the Feynman rules. Here we present explicitly the individual amplitudes:

$$\begin{aligned}
\mathcal{M}_a^{\frac{1}{2}^-,V} &= \frac{e g_{V\eta\gamma} g_{N^*NV} g_{N^*N\eta}}{4 m_V m_N} \bar{u}_f G_{N^*_{1/2}}(P)\gamma_5(-\gamma^\rho \not{q} + \not{q}\gamma^\rho) u_i \epsilon_{\sigma\rho\alpha\beta} q^\sigma k^\alpha \varepsilon^\beta F_t(q^2) F_V(q^2) F_B(P^2),\\
\mathcal{M}_a^{\frac{1}{2}^+,V} &= \frac{e g_{V\eta\gamma} g_{N^*NV} g_{N^*N\eta}}{4 m_V m_N} \bar{u}_f \gamma_5 G_{N^*_{1/2}}(P)(-\gamma^\rho \not{q} + \not{q}\gamma^\rho) u_i \epsilon_{\sigma\rho\alpha\beta} q^\sigma k^\alpha \varepsilon^\beta F_t(q^2) F_V(q^2) F_B(P^2),\\
\mathcal{M}_a^{\frac{3}{2}^+,V} &= \frac{e g_{V\eta\gamma} g_{N^*NV} g_{N^*N\eta}}{m_V m_\eta (m_N^* + m_N)} l_{2,\lambda} \bar{u}_f \big(- G^{\lambda\rho}_{N^*_{3/2}}(P)\not{q} + G^{\lambda\mu}_{N^*_{3/2}}(P) q_\mu \gamma^\rho \big)\gamma_5 u_i \epsilon_{\sigma\rho\alpha\beta} q^\sigma k^\alpha \varepsilon^\beta F_t(q^2) F_V(q^2) F_B(P^2),\\
\mathcal{M}_b^{\frac{1}{2}^-,V} &= \frac{e g_{V\eta\gamma} g_{N^*NV} g_{N^*N\eta}}{4 m_V m_N} \bar{u}_f \gamma_5(-\gamma^\rho \not{q} + \not{q}\gamma^\rho) G_{N^*_{1/2}}(P') u_i \epsilon_{\sigma\rho\alpha\beta} q^\sigma k^\alpha \varepsilon^\beta F_t(q^2) F_V(q^2) F_B(P'^2),\\
\mathcal{M}_b^{\frac{1}{2}^+,V} &= \frac{e g_{V\eta\gamma} g_{N^*NV} g_{N^*N\eta}}{4 m_V m_N} \bar{u}_f (-\gamma^\rho \not{q} + \not{q}\gamma^\rho) G_{N^*_{1/2}}(P')\gamma_5 u_i \epsilon_{\sigma\rho\alpha\beta} q^\sigma k^\alpha \varepsilon^\beta F_t(q^2) F_V(q^2) F_B(P'^2),\\
\mathcal{M}_b^{\frac{3}{2}^+,V} &= \frac{e g_{V\eta\gamma} g_{N^*NV} g_{N^*N\eta}}{m_V m_\eta (m_{N^*} + m_N)} l_{2,\lambda} \bar{u}_f \gamma_5 \big(\not{q} G^{\rho\lambda}_{N^*_{3/2}}(P') - q_\mu \gamma^\rho G^{\mu\lambda}_{N^*_{3/2}}(P')\big) u_i \epsilon_{\sigma\rho\alpha\beta} q^\sigma k^\alpha \varepsilon^\beta F_t(q^2) F_V(q^2) F_B(P'^2),\\
\mathcal{M}_c^{V} &= \frac{e g_{V\eta\gamma} g_{VV\eta} g_{NNV}}{2 m_V^2} G_V^{\mu\rho}(q) \epsilon_{\sigma\rho\alpha\beta} q^\sigma l_1^\alpha \varepsilon^\beta G_V^{\gamma\nu}(q') \epsilon_{\omega\nu\delta\mu} q'^\omega l_2^\delta \bar{u}_f \left(\gamma_\gamma + i\frac{\kappa_V}{2 m_N}\sigma_{\gamma\kappa} q'^\kappa\right) u_i F_t(q'^2) F_V(q^2).
\end{aligned} \tag{23}$$

where a, b, and c indicate the corresponding Feynman diagrams shown in Fig. 1, J^P ($= \frac{1}{2}^\pm$ or $\frac{3}{2}^+$) represents the quantum numbers of intermediate nucleon resonance, and V refers to the ρ (ω) meson. ε^μ, u_i, and $\bar{u}_f$ are the polarization vector of the photon and the spinors of initial and final protons, respectively. The momenta for individual particles are taken as $\gamma(k) + p(p_i) \to p(p_f) + \eta(l_1) + \eta(l_2)$, where the letters in parentheses indicate the momenta of corresponding particles. We also define $q = k - l_1$, $q' = p_i - p_f$, $P = q + p_i$, and $P' = p_i - l_2$. Note that the amplitudes due to the interchange of the final two η's are not shown above to save space, but their contributions should be considered in the calculations.

III. RESULTS AND DISCUSSIONS

With the formulas and ingredients given in the last section, we have studied the reaction $\gamma p \to p\eta\eta$ for beam momentum (p_γ) below 3 GeV. The calculated total cross sections as a function of beam momentum are shown in Fig. 2(a). In this figure, we show the role of various nucleon resonances and background contributions in determining the total cross sections. The dotted, dashed, dot-dashed, and dot-dot-dashed curves represent the contributions of the $N^*(1535)$, $N^*(1650)$, $N^*(1710)$, and $N^*(1720)$, respectively. The coherent sum of all contributions is shown by the solid line.

It can be found that the $N^*(1535)$ gives the dominant contribution at low energies. With increasing energies, the roles of the $N^*(1710)$ and $N^*(1720)$ become more and more important. On the other hand, the contributions from the $N^*(1650)$ and t-channel mesonic terms only play minor roles in this reaction. The dominant role of the $N^*(1535)$ at lower energies is mainly due to its rather large coupling to the $N\eta$ channel and the s-wave coupling nature, while at higher energies the important roles of the $N^*(1710)$ and $N^*(1720)$ are mainly due to their relatively large couplings to $N\rho$ and $N\omega$ channels. The relative importance of the contributions from various meson exchanges is shown in Fig. 2(b), where the dashed and dotted lines represent the contributions of ρ and ω exchanges respectively. It is obvious that the contribution from ρ exchange plays a more important role in this reaction. The relative role of ρ exchange compared with ω exchange can be partially understood by looking at the different strengths of $\rho\eta\gamma$ and $\omega\eta\gamma$ couplings. As can be seen in Table I, the ratio of $g^2_{\rho\eta\gamma}/g^2_{\omega\eta\gamma} \sim 10$ can roughly account for the strength of ρ and ω exchanges at the lower energies. This shows that the present reaction may be helpful in the study of nucleon resonances which have substantial coupling to the $N\rho$ channel. Since the $N^*(1535)$ is expected to dominant in the near-threshold region, this reaction can be used to test the coupling of the $N^*(1535)$ with the $N\rho$ channel [11,15] when experimental data are available.

For future comparisons with experimental data, we also calculate the angular distributions and invariant mass spectra of final particles. In our model, it is found that the $N^*(1535)$ dominates at lower energies, while the $N^*(1710)$ becomes as important as the $N^*(1535)$ at beam momenta larger than 2.4 GeV. Thus we present the differential cross sections calculated at $p_\gamma = 1.89$ GeV (Fig. 3) and $p_\gamma = 2.5$ GeV (Fig. 4) as two representative examples. At a relatively low energy ($p_\gamma = 1.89$ GeV), the $N^*(1535)$ shows as a slight enhancement in the distribution of $M_{p\eta}$ relative to the phase space distribution, and the angular distribution of η in the center of mass frame shows a linear increase in $\cos\theta_\eta$. At $p_\gamma = 2.5$ GeV, a two-peak structure appears in the $M_{N\eta}$ distribution. As can be inferred from Fig. 2(a), this structure is mainly caused by the contributions from the $N^*(1535)$ and $N^*(1710)$. Compared to the case where the $N^*(1535)$ gives the dominant contribution, the interference effects among various N^*'s play an important role here. We have checked that the shapes of the angular distribution and the invariant mass distribution are sensitive to the relative phases among the various amplitudes. Thus the experimental results at this

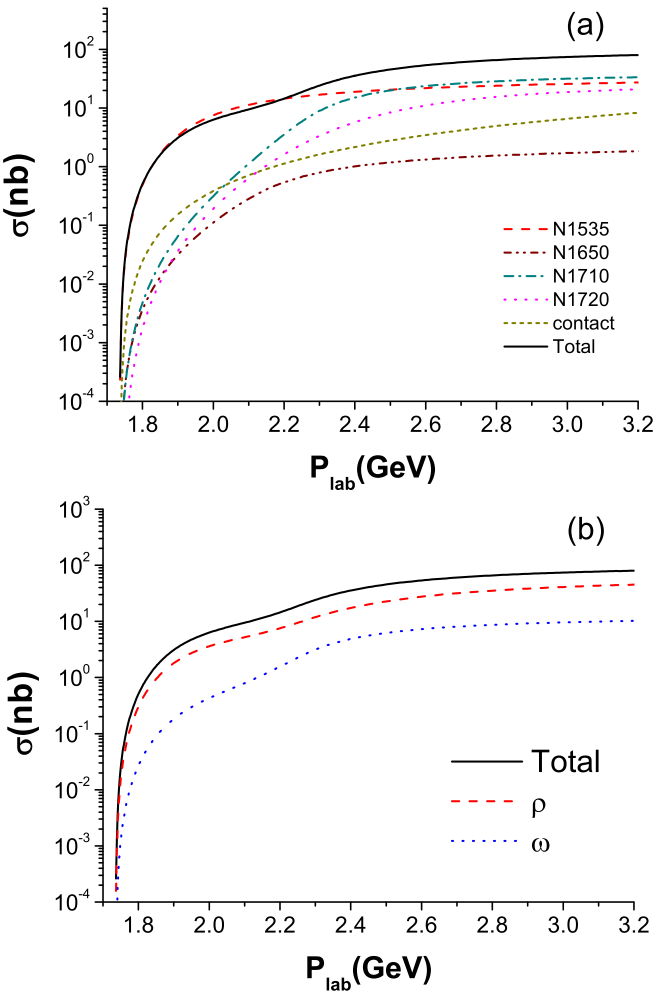

FIG. 2. Total cross section for the $\gamma p \to p\eta\eta$ reaction as a function of beam momentum within our model: (a) contribution from various nucleon resonances and mesonic term; (b) contribution from ρ and ω exchanges.

energy can put strong constraints on the roles of various N^*'s. These model predictions can be checked by future experimental data.

Note that in our calculations we do not consider the final state interactions among final particles, since our work is not focused on the very near threshold region. At higher energies, the effects due to final state interactions are expected to be small. For example, the N-η final state interactions are expected to be significant only at excess energies below 20 MeV [13,24]. Therefore, the inclusion of such effects will not cause significant changes to the results shown in Fig. 3. However, the predictions for the total cross sections in the very near threshold region should be interpreted with caution.

In summary, we have investigated the reaction $\gamma p \to p\eta\eta$ within an effective Lagrangian approach in this work. By assuming that this reaction proceeds mainly through the contact term and the excitations of the $N^*(1535)$, $N^*(1650)$, $N^*(1710)$, and $N^*(1720)$ in the intermediate states, we calculate various contributions to this reaction. It is found that the excitation of the $N^*(1535)$ dominates this reaction in

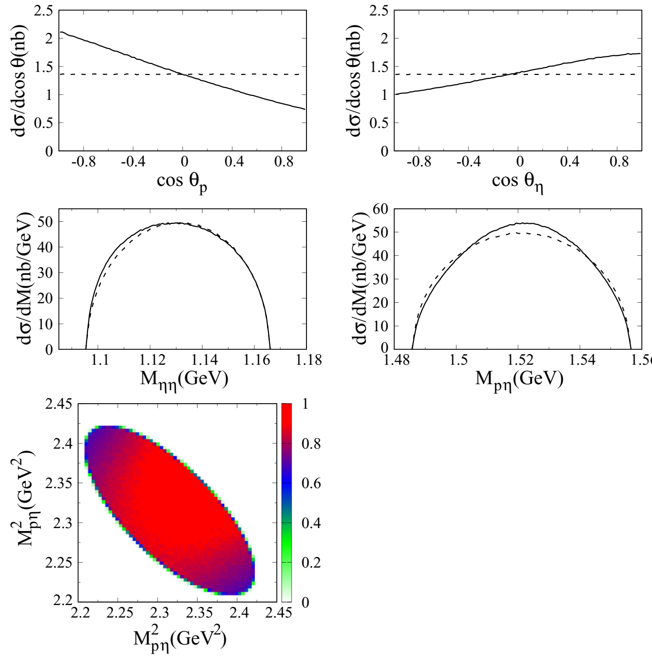

FIG. 3. Predictions of angular distribution, invariant mass spectrum, and Dalitz plot at $p_\gamma = 1.89$ GeV. The solid line represents the full results and the dashed line represents the phase space distribution. θ denotes the angle of the outgoing particles relative to the incident beam direction in the center-of-mass frame.

the near threshold region. With increasing energies, the roles of the $N^*(1710)$ and $N^*(1720)$ become important gradually. Since we expect that the nucleon resonances are mainly excited by the exchange of vector mesons in this reaction, we also

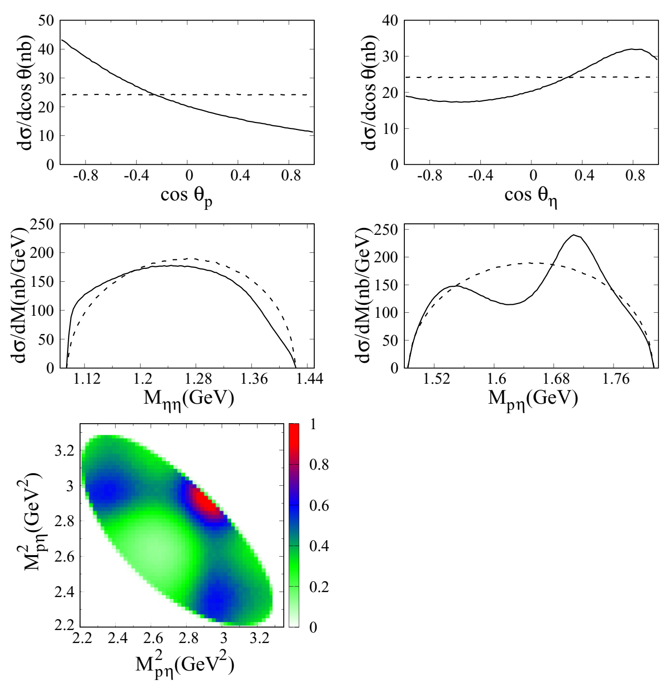

FIG. 4. Same as Fig. 3, but at $p_\gamma = 2.5$ GeV.

study the relative roles of the ρ and ω exchanges. We find that the ρ exchange is much more important because of the relatively large value of $g_{\rho\eta\gamma}$. Therefore, this reaction offers a potentially good place to study nucleon resonances which have large couplings to the $N\rho$ and $N\eta$ channels. The predicted total cross section, angular distribution, and invariant mass spectrum of final particles can be tested by future experiments.

ACKNOWLEDGMENTS

We acknowledge support from the National Natural Science Foundation of China under Grant No. 11375137, the Natural Science Foundation of Shaanxi Province under Grant No. 2015JQ1003, and the Fundamental Research Funds for the Central Universities (Grant No. xjj2017109).

APPENDIX: COUPLING CONSTANTS AND DECAY AMPLITUDES

In this appendix, we present the formulas needed for the determination of various coupling constants. The relevant coupling constants $g_{N^*N\eta}$ are determined from the experimentally observed partial decay widths of the nucleon resonances. With the Lagrangians given in Sec. II, the partial decay widths can be easily obtained as

$$\Gamma_{N^*(1535)\to N\eta} = \frac{g^2_{N^*(1535)N\eta}(m_N+E_N)p_\eta^{c.m.}}{4\pi M_{N^*(1535)}}, \tag{A1}$$

$$\Gamma_{N^*(1650)\to N\eta} = \frac{g^2_{N^*(1650)N\eta}(m_N+E_N)p_\eta^{c.m.}}{4\pi M_{N^*(1650)}}, \tag{A2}$$

$$\Gamma_{N^*(1710)\to N\eta} = \frac{g^2_{N^*(1710)N\eta}(E_N-m_N)p_\eta^{c.m.}}{4\pi M_{N^*(1710)}}, \tag{A3}$$

$$\Gamma_{N^*(1720)\to N\eta} = \frac{g^2_{N^*(1720)N\eta}(m_N+E_N)\left(p_\eta^{c.m.}\right)^3}{12\pi M_{N^*(1720)}m_\eta^2}. \tag{A4}$$

The coupling constants $g_{N^*N\rho}$ can be extracted from the experimental partial decay width $\Gamma_{N^*\to N\rho\to N\pi\pi}$, which can be calculated through the following formula:

$$\begin{aligned} d\Gamma_{N^*(p)\to N\rho\to N(p_1)\pi(p_2)\pi(p_3)} &= \overline{|\mathcal{M}_{N^*\to N\rho\to N\pi\pi}|^2}\frac{d^3p_1}{(2\pi)^3}\frac{m_1}{E_1}\frac{d^3p_2}{(2\pi)^3}\frac{1}{2E_2}\frac{d^3p_3}{(2\pi)^3}\frac{1}{2E_3} \\ &\quad\times(2\pi)^4\delta^4(p-p_1-p_2-p_3). \end{aligned} \tag{A5}$$

The decay amplitude $\mathcal{M}_{N^*\to N\rho\to N\pi\pi}$ of various N^*s can be constructed in the standard way using the Lagrangian $\mathcal{L}_{\rho\pi\pi} = g_{\rho\pi\pi}(\vec{\pi}\times\partial^\mu\vec{\pi})\cdot\vec{\rho}_\mu$ for the $\rho\pi\pi$ vertex and the Lagrangian densities offered above. As an example, we present here the explicit decay amplitude of $N^*(1710)$:

$$\begin{aligned} \mathcal{M}_{N^*(1710)\to N\rho\to N\pi\pi} &= g_{\rho\pi\pi}g_{N^*(1710)N\rho}F_\rho\left(p_\rho^2\right) \\ &\quad\times\bar{u}_N(p_1,s_1)\sigma_{\mu\nu}k_\rho^\nu G_\rho^{\mu\alpha} \\ &\quad\times(p_2-p_3)_\alpha u_{N^*(1710)}, \end{aligned} \tag{A6}$$

where $F_\rho(q^2) = (\frac{\Lambda^2}{\Lambda^2+|p_\rho^2-m_\rho^2|})^2$ with $\Lambda = 1.0$ GeV [15] is introduced to take into account the off-shell effects and $G_\rho^{\mu\alpha}$ is the propagator of the ρ meson [see Eq. (22)]. The value of $g_{\rho\pi\pi}$ is well determined as $g_{\rho\pi\pi} = 3.02$. Using these formulas, the coupling constant can be extracted by comparing with the experimental value. The adopted values of the partial decay width in the calculations and the obtained coupling constants are shown in Table I.

For the coupling constants $g_{N^*N\omega}$, since the resonances considered in this work either lie below the threshold or have very limited phase space to decay, knowledge of their values is still rather poor. The Particle Data Group (PDG) estimation for the decay branch ratio $R_{N\omega}$ is only available for $N^*(1710)$, through which we can determine the coupling constant $g_{N^*(1710)N\omega}$ by the decay process $N^*(1710)\to N\omega\to N\pi^0\gamma$. The corresponding decay amplitude can be written as

$$\begin{aligned} \mathcal{M}_{N^*(1710)} &= -\frac{e}{m_\omega}\frac{g_{\omega\pi\gamma}g_{N^*(1710)N\omega}}{2m_N}\epsilon^{\mu\nu\alpha\beta}P_\mu \\ &\quad\times G_{\nu\sigma}(P)F_\omega(P^2)k_\alpha\varepsilon_\beta^*\bar{\Psi}_N\sigma^{\sigma\rho}P_\rho\Psi_{N^*}. \end{aligned} \tag{A7}$$

The PDG value for the partial decay width is obtained as $\Gamma_{N^*(1710)\to N\omega\to N\pi^0\gamma} = \Gamma^0_{N^*(1710)}\times\text{Br}(N^*(1710)\to N\omega)\times\text{Br}(\omega\to\gamma\pi) = 0.662$ MeV. Then one can get an estimation of $g_{N^*(1710)N\omega}$ using the same formula as Eq. (A5), and the obtained value is also shown in Table I. To get a rough estimation of the coupling constants $g_{N^*N\omega}$ for other N^*, one possible way is to utilize the vector meson dominance (VMD) model. In the VMD model, the $\omega\gamma$ interaction Lagrangian can be described as

$$\mathcal{L}_{\omega\gamma} = \frac{em_\omega^2}{2\gamma_\omega}\omega_\mu A^\mu, \tag{A8}$$

where the coupling strength $\gamma_\omega = 8.53$ is fixed by the electromagnetic $\omega\to e^+e^-$ [25]. Through a straightforward calculation, the $N^*N\gamma$ partial widths calculated within the VMD model (via ω meson) can be given as

$$\begin{aligned} \Gamma_{N^*(1535)N\gamma} &= \frac{\alpha g^2_{N^*(1535)N\omega}p^3}{4\gamma_\omega^2m^2}\frac{1}{\left(1+\Gamma_\omega^2/m_\omega^2\right)} \\ &\quad\times\frac{1}{\left(1+m_\omega^2/\Lambda^2\right)^4}, \end{aligned} \tag{A9}$$

$$\begin{aligned} \Gamma_{N^*(1650)N\gamma} &= \frac{\alpha g^2_{N^*(1650)N\omega}p^3}{4\gamma_\omega^2m^2}\frac{1}{\left(1+\Gamma_\omega^2/m_\omega^2\right)} \\ &\quad\times\frac{1}{\left(1+m_\omega^2/\Lambda^2\right)^4}, \end{aligned} \tag{A10}$$

$$\begin{aligned} \Gamma_{N^*(1720)N\gamma} &= \frac{\alpha g^2_{N^*(1720)N\omega}(4M_{N^*}+2m-p)p^3}{6\gamma_\omega^2M_{N^*}(m+M_{N^*})^2} \\ &\quad\times\frac{1}{\left(1+\Gamma_\omega^2/m_\omega^2\right)}\frac{1}{\left(1+m_\omega^2/\Lambda^2\right)^4}, \end{aligned} \tag{A11}$$

where α is the fine-structure constant and p is the momentum of final particles in the N^* rest frame. Note that the form factor $F_\omega(q^2) = (\frac{\Lambda^2}{\Lambda^2+|p_\omega^2-m_\omega^2|})^2$ with $\Lambda = 1$ GeV is considered in the calculations as in Ref. [15]. The experimental value of radiative decay width of N^* can be expressed by helicity amplitudes

$$\Gamma^s_{N^*N\gamma} = \frac{q^2}{\pi}\frac{2m_N}{(2J+1)m_{N^*}}\left(\left|\mathcal{A}^s_{\frac{1}{2}}\right|^2+\left|\mathcal{A}^s_{\frac{3}{2}}\right|^2\right), \tag{A12}$$

where the $\mathcal{A}^s_{\frac{1}{2}}$ and $\mathcal{A}^s_{\frac{3}{2}}$ represent the scalar parts of the decay amplitudes and can be obtained through $\mathcal{A}_s = \frac{1}{2}(\mathcal{A}_p + \mathcal{A}_n)$ [26]. With the central value of the helicity decay amplitudes of the N^*'s in the PDG book [10] and the assumption that the ω meson dominates the scalar part of the N^* radiative decay, it is then easy to get $g_{N^*N\omega}$ values using the above formulas. Values of $g_{N^*N\omega}$ estimated in this way are also listed in Table I.

[1] V. D. Burkert, Few-Body. Syst. **57**, 873 (2016).
[2] V. Crede and W. Roberts, Rep. Prog. Phys. **76**, 076301 (2013).
[3] V. Sokhoyan *et al.* (CBELSA/TAPS Collaboration), Eur. Phys. J. A **51**, 95 (2015).
[4] E. Gutz *et al.* (CBELSA/TAPS Collaboration), Eur. Phys. J. A **50**, 74 (2014).
[5] A. Käeser *et al.* (A2 Collaboration), Eur. Phys. J. A **52**, 272 (2016).
[6] Q.-F. Lü, R. Wang, J.-J. Xie, X.-R. Chen, and D.-M. Li, Phys. Rev. C **91**, 035204 (2015).
[7] A. Fix, V. L. Kashevarov, A. Lee, and M. Ostrick, Phys. Rev. C **82**, 035207 (2010).
[8] J. C. Nacher, A. Parreño, E. Oset, A. Ramos, A. Hosaka, and M. Oka, Nucl. Phys. A **678**, 187 (2000).
[9] R. Machleidt, Phys. Rev. C **63**, 024001 (2001).
[10] C. Patrignani *et al.* (Particle Data Group), Chin. Phys. C **40**, 100001 (2016).
[11] K. Nakayama, Y. Oh, and H. Haberzettl, J. Kor. Phys. Soc. **59**, 224 (2011).
[12] R. Shyam and O. Scholten, Phys. Rev. C **78**, 065201 (2008).
[13] K. Nakayama, J. Speth, and T.-S. H. Lee, Phys. Rev. C **65**, 045210 (2002).
[14] R. Shyam, Phys. Rev. C **75**, 055201 (2007).
[15] J. J. Xie, C. Wilkin, and B. S. Zou, Phys. Rev. C **77**, 058202 (2008).
[16] C. Aydin and A. H. Yilmaz, Acta Phys. Pol. **B34**, 4145 (2003).
[17] K. A. Olive *et al.* (Particle Data Group) Chin. Phys. C **38**, 090001 (2014).
[18] M. Benmerrouche, N. C. Mukhopadhyay, and J. F. Zhang, Phys. Rev. D **51**, 3237 (1995).
[19] T. Bauer *et al.*, in *The 8th International Workshop on the Physics of Excited Nucleons: NSTAR 2011, May 2011, Newport News, VA*, edited by V. Burkert, M. Jones, M. Pennington, and D. Richards, AIP Conf. Proc. No. 1432 (AIP, New York, 2011), p. 269.
[20] A. Denner and S. Dittmaier, Nucl. Phys. B. Proc. Suppl. **160**, 22 (2006).
[21] El. Amiri *et al.*, Nucl. Phys. A **543**, 673 (1992).
[22] H. Garcilazo and E. Moya de Guerra, Nucl. Phys. A **562**, 521 (1993).
[23] C. Fernandez-Ramirez *et al.*, Ann. Phys. (NY) **321**, 1408 (2006).
[24] S. Schneider *et al.*, Eur. Phys. J. A **18**, 421 (2003).
[25] A. I. Titov and T.-S. H. Lee, Phys. Rev. C **66**, 015204 (2002).
[26] M. Post and U. Mosel, Nucl. Phys. A **688**, 808 (2001).