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Pure spin-3/2 propagator for use in particle and nuclear physics

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We propose the use of a pure spin-3/2 propagator in the $(3/2,0) \oplus (0,3/2)$ representation in particle and nuclear physics. To formulate the propagator in a covariant form we use the antisymmetric tensor spinor representation and we consider the Δ resonance contribution to the elastic πN scattering as an example. We find that the use of a conventional gauge-invariant interaction Lagrangian leads to a problem: the obtained scattering amplitude does not exhibit the resonance behavior. To overcome this problem we modify the interaction by adding a momentum dependence. As in the case of the Rarita-Schwinger formalism, we find that a perfect resonance description could be obtained in the pure spin-3/2 formulation only if hadronic form factors were considered in the interactions.

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For decades the most commonly used propagator for spin-3/2 particles (e.g., the N and Δ resonances) in particle and nuclear physics has been the Rarita-Schwinger (RS) one [1], although it is also well known that the RS propagator has an intrinsic and long-standing problem [2,3]: it contains the unphysical extra degrees of freedom (DOF) or lower spin background. To be more precise, let us begin with the RS field that is formed by the tensor product of a vector and a Dirac field represented by $(1/2,1/2)$ and $(1/2,0) \oplus (0,1/2)$, respectively. This tensor product yields [4]

$$\left(\tfrac{1}{2},\tfrac{1}{2}\right) \otimes \left[\left(\tfrac{1}{2},0\right) \oplus \left(0,\tfrac{1}{2}\right)\right] = \left(1,\tfrac{1}{2}\right) \oplus \left(\tfrac{1}{2},1\right) \oplus \left(\tfrac{1}{2},0\right) \oplus \left(0,\tfrac{1}{2}\right), \quad (1)$$

which shows that the RS field consists of two fields: the $(1,1/2) \oplus (1/2,1)$ and the Dirac field. The orthogonality relation can be used to eliminate the Dirac field. The $(1,1/2) \oplus (1/2,1)$ is, however, still not free from the Dirac background.

So far, the popular choice for the interaction Lagrangian is given, e.g., in Eq. (16) of Ref. [5], which contains the so-called off-shell parameter. This parameter is required to maintain the invariance of the RS Lagrangian under point transformations [6]. In the phenomenological study of nuclear physics, however, the physical meaning of the off-shell parameter raised a serious problem, i.e., the coupling constants could heavily depend on the off-shell parameter [7] and the Δ contribution for the Compton amplitude is obscured by this parameter [8]. There is also some infamous fundamental problem regarding the interaction with the electromagnetic field, i.e., the Johnson-Sudarshan [9] and Velo-Zwanziger [10] problems. The origin of these problems lies in the unphysical degree of freedom that arises in the interaction for whatever choice we make for the off-shell parameter.

Furthermore, it is shown in Ref. [5] that this interaction does not possess any local symmetry of RS field and, as a consequence, it violates the constraints for reducing the number of independent components of the field to the correct value and involves the unphysical lower-spin DOF. The pathologies of this interaction can be removed by introducing the gauge-invariant (GI) or consistent interaction to decouple the unphysical spin-1/2 background from the Δ-exchanges amplitude [5].

Nevertheless, for the practical use of spin-3/2 propagators and couplings, e.g., in the isobar or coupled-channels model for meson-nucleon scattering or meson-induced reaction, a pure spin-3/2 propagator is strongly desired. In particle physics there are seven nucleon and five Δ resonances with spins 3/2 [11]. These resonances are also intensively used in nuclear physics. Obviously, a solid formulation of the spin-3/2 amplitude is inevitable in these sectors.

From the theoretical point of view, a particle with spin 3/2 can be represented by the pure spin-3/2 representation, $(3/2,0) \oplus (0,3/2)$, which is free from the spin-1/2 field. However, in the $(3/2,0) \oplus (0,3/2)$ representation the formulation involves an eight-dimensional spinor because the spin-3/2 operator is represented by 4×4 matrices. Such an eight-dimensional spinor representation is not the popular choice because it cannot be written in a covariant form. As a consequence it is hard to construct the corresponding interaction Lagrangian.

Fortunately, Acosta *et al.* [12] show that the components of the eight-dimensional spinor can be embedded into a totally antisymmetric tensor of second rank. The representation is called the antisymmetric tensor spinor (ATS) representation, which is formed by the tensor product of an antisymmetric tensor and the Dirac field. The pure spin-3/2 representation is projected from the ATS representation.

ATS is formed by a tensor product of an antisymmetric tensor field and the Dirac field, which is represented by $(1,0) \oplus (0,1)$ and $(1/2,0) \oplus (0,1/2)$, respectively. This tensor product may be expressed as [12]

$$\begin{aligned}&[(1,0) \oplus (0,1)] \otimes \left[\left(\tfrac{1}{2},0\right) \oplus \left(0,\tfrac{1}{2}\right)\right] \\ &= \left[\left(\tfrac{3}{2},0\right) \oplus \left(0,\tfrac{3}{2}\right)\right] \oplus \left[\left(1,\tfrac{1}{2}\right) \oplus \left(\tfrac{1}{2},1\right)\right] \\ &\oplus \left[\left(\tfrac{1}{2},0\right) \oplus \left(0,\tfrac{1}{2}\right)\right]. \quad (2)\end{aligned}$$

Thus, the ATS representation consists of two fields, i.e., the $(3/2,0) \oplus (0,3/2)$ and the RS field. The RS field can be removed from the ATS by operating the projection operator. To this end, one can define the Casimir operator $F = \frac{1}{4} J_{\mu\nu} J^{\mu\nu}$, where $J^{\mu\nu}$ is the angular momentum operator for the

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representation. The eigenstate equation of the Casimir operator for the field (a,b) reads

$$F|(a,b)\rangle = C(a,b)|(a,b)\rangle, \tag{3}$$

where $C(a,b) = a(a+1) + b(b+1)$. The $(1,1/2) \oplus (1/2,1)$ and $(1/2,0) \oplus (0,1/2)$ fields are removed from the ATS by the projection operator

$$\mathcal{P} = \frac{\left[F - C\left(1,\frac{1}{2}\right)\right]\left[(F - C\left(\frac{1}{2},0\right)\right]}{\left[C\left(\frac{3}{2},0\right) - C\left(1,\frac{1}{2}\right)\right]\left[C\left(\frac{3}{2},0\right) - C\left(\frac{1}{2},0\right)\right]}. \tag{4}$$

Reference [12] shows that the projection operator is equal to

$$\mathcal{P}_{\alpha\beta\gamma\delta} = \tfrac{1}{8}(\sigma_{\alpha\beta}\sigma_{\gamma\delta} + \sigma_{\gamma\delta}\sigma_{\alpha\beta}) - \tfrac{1}{12}\sigma_{\alpha\beta}\sigma_{\gamma\delta}\,, \tag{5}$$

where

$$\sigma_{\alpha\beta} = \tfrac{i}{2}[\gamma_\alpha,\gamma_\beta]\,. \tag{6}$$

This projection operator ensures that the ATS has only the $(3/2,0) \oplus (0,3/2)$ representation. The pure spin-3/2 ATS is obtained by operating a pure spin-3/2 projection operator to the GI RS spinor, i.e., [12]

$$w^{\mu\nu}(\mathbf{p},\lambda) = 2\mathcal{P}^{\mu\nu}{}_{\alpha\beta}U^{\alpha\beta}(\mathbf{p},\lambda), \tag{7}$$

where $\lambda = -3/2,-1/2,+1/2,+3/2$ are the z components of the spin-3/2 operator eigenvalues and $U^{\alpha\beta}(\mathbf{p},\lambda)$ is the GI RS spinor, given by

$$U^{\alpha\beta}(\mathbf{p},\lambda) = \frac{1}{2m}[p^\alpha\mathcal{U}^\beta(\mathbf{p},\lambda) - p^\beta\mathcal{U}^\alpha(\mathbf{p},\lambda)], \tag{8}$$

with $\mathcal{U}^\alpha(\mathbf{p},\lambda)$ the RS vector-spinor.

Up to the normalization constant $(2m)^{-1}$ the GI RS spinor $U^{\alpha\beta}(\mathbf{p},\lambda)$ given in Eq. (8) is identical to the GI RS field tensor $\Delta^{\mu\nu} = \partial^\mu\Delta^\nu - \partial^\nu\Delta^\mu$ given in Ref. [5]. Therefore, the ATS representation differs from the GI RS field tensor in the projection operator. Note that this projection operator is different from the common projection operator in a RS field. This projection operator projects out the $(3/2,0) \oplus (0,3/2)$ field from the ATS representation instead of the $(1,1/2) \oplus (1/2,1)$ field. The propagator of pure spin-3/2 representation reads

$$S_{\alpha\beta\gamma\delta}(p) = \frac{\Delta_{\alpha\beta\gamma\delta}(p)}{p^2 - m^2 + i\epsilon}, \tag{9}$$

where

$$\Delta_{\alpha\beta\gamma\delta}(p) = \left(\frac{p^2}{m^2}\right)\mathcal{P}_{\alpha\beta\gamma\delta} - \left(\frac{p^2 - m^2}{m^2}\right)1_{\alpha\beta\gamma\delta}, \tag{10}$$

and $1_{\alpha\beta\gamma\delta}$ is the identity in ATS space with

$$1_{\alpha\beta\gamma\delta} = \tfrac{1}{2}(g_{\alpha\gamma}g_{\beta\delta} - g_{\alpha\delta}g_{\beta\gamma})1_{4\times4}. \tag{11}$$

Based on the orthogonality relation for the projection operator $\gamma^\mu\mathcal{P}_{\mu\nu\rho\sigma} = 0$, one can prove that the pure spin-3/2 spinor satisfies $\gamma_\mu w^{\mu\nu}(\mathbf{p},\lambda) = 0$. This relation reduces the number of DOF of the ATS representation, i.e., $6 \times 4 = 24$, by $4 \times 4 = 16$. As expected, the pure spin-3/2 field in the ATS representation has $24 - 16 = 8$ DOF.

The free Lagrangian for a pure spin-3/2 field in the ATS representation is given by [12]

$$\mathcal{L} = (\partial^\mu\bar{\Psi}^{\alpha\beta})\Gamma_{\mu\nu\alpha\beta\gamma\delta}(\partial^\nu\Psi^{\gamma\delta}) - m^2\bar{\Psi}^{\mu\nu}\Psi_{\mu\nu}\,, \tag{12}$$

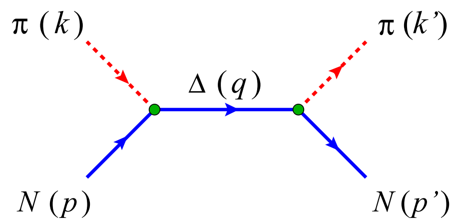

FIG. 1. Feynman diagram for the elastic πN scattering with a Δ resonance in the intermediate state.

where $\Gamma_{\mu\nu\alpha\beta\gamma\delta} = 4g^{\sigma\rho}\mathcal{P}_{\alpha\beta\rho\mu}\mathcal{P}_{\sigma\nu\gamma\delta}$, $\Psi^{\mu\nu}$ is the $(3/2,0) \oplus (0,3/2)$ field and m is the mass of particle. The kinetic term of the Lagrangian is invariant under the following gauge transformation:

$$\Psi^{\mu\nu} \to \Psi^{\mu\nu} + \xi^{\mu\nu}\,, \tag{13}$$

where $\xi^{\mu\nu}$ is an antisymmetric tensor containing the γ matrices

$$\xi^{\mu\nu} = \gamma^\mu\partial^\nu\xi - \gamma^\nu\partial^\mu\xi\,. \tag{14}$$

Let us consider the $\pi N \to \pi N$ scattering with a Δ resonance in the intermediate state as an example. The corresponding Feynman diagram is depicted in Fig. 1, where the momenta of the involved particles are also shown for our convention. The popular choice for $\pi N\Delta$ Lagrangian interaction is [5]

$$\mathcal{L}_{\pi N\Delta} = \left(\frac{g_{\pi N\Delta}}{m_\pi}\right)\bar{\Delta}^\mu\Theta_{\mu\nu}(z)N\partial^\nu\pi + \text{H.c.}, \tag{15}$$

where $\Theta_{\mu\nu}(z)$ is given by

$$\Theta_{\mu\nu}(z) = g_{\mu\nu} - \left(z + \tfrac{1}{2}\right)\gamma_\mu\gamma_\nu, \tag{16}$$

the constant z is arbitrary and conventionally called the off-shell parameter. Furthermore, Δ^μ, N, and π in Eq. (16) denote the Δ-baryon vector-spinor, nucleon spinor, and pion field respectively. As stated before this Lagrangian does not possess any local symmetries of the RS field, and as a consequence it induces the unphysical lower-spin DOF, which is called the spin-1/2 background [5]. To decouple this unphysical background from the Δ-exchange amplitude Pascalutsa and Timmermans introduced a GI interaction which is given by [5]

$$\mathcal{L}_{\pi N\Delta} = \left(\frac{g_{\pi N\Delta}}{m_\pi m_\Delta}\right)\bar{N}\gamma_5\gamma_\mu\tilde{\Delta}^{\mu\nu}\partial_\nu\pi + \text{H.c.}, \tag{17}$$

where $\tilde{\Delta}^{\mu\nu}$ is the dual tensor of the GI RS field tensor $\Delta^{\mu\nu}$ defined as

$$\Delta_{\mu\nu} = \partial_\mu\Delta_\nu - \partial_\nu\Delta_\mu. \tag{18}$$

This GI interaction yields the Δ-exchange amplitude

$$\Gamma^\mu(k')S_{\mu\nu}(q)\Gamma^\nu(k) = \frac{(g_{\pi N\Delta}/m_\pi)^2}{\not{q} - m_\Delta}\frac{q^2}{m_\Delta^2}P^{(3/2)}_{\mu\nu}(q)k'^\mu k^\nu, \tag{19}$$

with $P^{(3/2)}_{\mu\nu}$ the spin-3/2 projection operator in the RS field given by

$$P^{(3/2)}_{\mu\nu}(q) = g_{\mu\nu} - \frac{1}{3}\gamma_\mu\gamma_\nu - \frac{1}{3q^2}(\not{q}\gamma_\mu q_\nu + q_\mu\gamma_\nu\not{q}). \tag{20}$$

Analogous to the GI interaction, one can construct the $\pi N\Delta$ interaction by changing the GI RS field tensor to the $(3/2,0)\oplus(0,3/2)$ representation

$$\mathcal{L}_{\pi N\Delta} = g_{\pi N\Delta}\bar{N}\gamma_5\gamma_\mu\tilde{\Psi}^{\mu\nu}\partial_\nu\pi + \text{H.c.}, \tag{21}$$

where $\Psi^{\mu\nu}$ is the $(3/2,0)\oplus(0,3/2)$ field tensor and $\tilde{\Psi}^{\mu\nu}$ is its dual tensor. By using the vertex factor

$$\Gamma_{\mu\nu}(k) = g_{\pi N\Delta}\gamma_5\gamma_\mu k_\nu, \tag{22}$$

the corresponding Δ-exchange amplitude becomes $\Gamma_{\mu\nu}(k')\tilde{S}^{\mu\nu\rho\sigma}(q)\Gamma_{\rho\sigma}(k)$, where $\tilde{S}^{\mu\nu\rho\sigma}$ is defined by

$$\Gamma_{\mu\nu}(k')\tilde{S}^{\mu\nu\rho\sigma}(q)\Gamma_{\rho\sigma}(k) = \tfrac{1}{4}g^2_{\pi N\Delta}\epsilon^{\mu\nu\alpha\beta}\epsilon^{\rho\sigma\kappa\lambda}\gamma_5\gamma_\mu \times S_{\alpha\beta\kappa\lambda}(q)\gamma_\rho\gamma_5 k'_\nu k_\sigma\,. \tag{23}$$

By evaluating Eq. (23) we find that the nonvanishing Δ-exchange amplitude originates from the contraction with $1_{\alpha\beta\kappa\lambda}$, since the contraction with $\mathcal{P}_{\alpha\beta\kappa\lambda}$ vanishes due to the orthogonality relation $\gamma^\alpha\mathcal{P}_{\alpha\beta\kappa\lambda}=0$ and $\tilde{\sigma}^{\mu\nu}=-\gamma_5\sigma^{\mu\nu}$. After some calculations we obtain the Δ-exchange amplitude in the form of

$$\Gamma_{\mu\nu}(k')\tilde{S}^{\mu\nu\rho\sigma}(q)\Gamma_{\rho\sigma}(k) = \frac{g^2_{\pi N\Delta}\left(q^2-m^2_\Delta\right)}{m^2_\Delta\left(q^2-m^2_\Delta+i\epsilon\right)} \times\left(g^{\nu\sigma}+\tfrac{1}{2}\gamma^\nu\gamma^\sigma\right)k'_\nu k_\sigma\,. \tag{24}$$

Based on Eq. (24) we can conclude that this Δ-exchange amplitude cannot describe the contribution of a resonance, since at the resonance pole ($q^2=m^2_\Delta$) the amplitude is equal to zero instead of being maximum. As a consequence, the Lagrangian given in Eq. (21) cannot be used for calculating the resonance contribution.

The problem originates from the GI interaction Lagrangian given in Eq. (21), since the contraction between γ matrices and the pure spin-3/2 field tensor vanishes. To overcome this problem one can modify the interaction Lagrangian by replacing the γ matrix with a partial derivative, i.e.,

$$\mathcal{L}_{\pi N\Delta} = \left(\frac{g_{\pi N\Delta}}{m_\Delta}\right)\bar{N}\gamma_5\partial^\mu\Psi_{\mu\nu}\partial^\nu\pi + \text{H.c.} \tag{25}$$

By using the vertex factor

$$\Gamma^{\mu\nu}(k) = \left(\frac{g_{\pi N\Delta}}{m_\Delta}\right)\gamma_5 q^\mu k^\nu, \tag{26}$$

the Δ-exchange amplitude corresponding to this interaction reads

$$\Gamma^{\mu\nu}(k')S_{\mu\nu\rho\sigma}(q)\Gamma^{\rho\sigma}(k) = \left(\frac{g_{\pi N\Delta}}{m_\Delta}\right)^2\gamma_5 q^\mu \times S_{\mu\nu\rho\sigma}(q)\gamma_5 q^\rho k'^\nu k^\sigma. \tag{27}$$

After some calculations we obtain

$$\Gamma^{\mu\nu}(k')S_{\mu\nu\rho\sigma}(q)\Gamma^{\rho\sigma}(k) = \frac{g^2_{\pi N\Delta}k'^\nu k^\sigma}{q^2-m^2_\Delta+i\epsilon}\left[\frac{q^4}{4m^4_\Delta}P^{(3/2)}_{\nu\sigma}(q) - \left(\frac{q^2-m^2_\Delta}{2m^4_\Delta}\right)\left(q^2 g_{\nu\sigma}-q_\nu q_\sigma\right)\right], \tag{28}$$

which differs from the result of the GI interaction given by Eq. (19) by the second term. However, this result is very interesting, because at the resonance pole, i.e., $q^2=m^2_\Delta$, the second term vanishes and the Δ-exchange amplitude is proportional to the RS spin-3/2 projection operator.

For future consideration we need to point out here that the GI electromagnetic interaction reads [5]

$$\mathcal{L}_{\gamma N\Delta} = e\bar{N}(g_1\tilde{\Delta}_{\mu\nu}+g_2\gamma_5\Delta_{\mu\nu}+g_3\gamma_\mu\gamma^\rho\tilde{\Delta}_{\rho\nu} + g_4\gamma_5\gamma_\mu\gamma^\rho\Delta_{\rho\nu})F^{\mu\nu}+\text{H.c.}, \tag{29}$$

whereas the non-GI interactions are [5]

$$\mathcal{L}^{(1)}_{\gamma N\Delta} = \frac{ieG_1}{2m}\bar{\Delta}^\rho\Theta_{\rho\mu}(z_1)\gamma_\nu\gamma_5 NF^{\mu\nu}+\text{H.c.},$$
$$\mathcal{L}^{(2)}_{\gamma N\Delta} = -\frac{eG_2}{(2m)^2}\bar{\Delta}^\rho\Theta_{\rho\mu}(z_2)\gamma_5\partial_\nu NF^{\mu\nu}+\text{H.c.}, \tag{30}$$

where z_1 and z_2 are the off-shell parameters. Interestingly, the pure spin-3/2 interaction is given by

$$\mathcal{L}_{\gamma N\Delta} = e\bar{N}(f_1\Psi_{\mu\nu}+f_2\gamma_\mu\partial^\rho\Psi_{\rho\nu})F^{\mu\nu}+\text{H.c.}, \tag{31}$$

which differs from the GI one by the number of coupling constants, i.e., the pure spin-3/2 representation has only two couplings because $\tilde{\Psi}_{\mu\nu}=-\gamma_5\Psi_{\mu\nu}$. Thus, the interaction of the photon with the pure spin-3/2 resonance has the same number of couplings as in the case of the non-GI model.

To visualize the behavior of the pure spin-3/2 propagator we will compare the contributions of the spin-3/2 $\Delta(1232)$ resonance amplitudes obtained from the pure spin-3/2 propagator and from the Rarita-Schwinger one to the total cross section of elastic πN scattering. To this end we include the resonance width Γ in the resonance propagator by replacing $i\epsilon\to i\Gamma m_\Delta$ and write the scattering amplitude in the form of

$$\mathcal{M} = \bar{u}(p',s')(A+B\not{Q})u(p,s), \tag{32}$$

with $Q=(k+k')/2$. For the RS propagator with GI interaction we obtain

$$A = G\{m_N\left(3k'\cdot k-2p\cdot k-m^2_\pi-2q\cdot k'q\cdot k/q^2\right) + m_\Delta\left(3k'\cdot k-2p\cdot k-m^2_\pi-2m^2_\pi q\cdot Q/q^2\right)\}, \tag{33}$$

$$B = G\{3k'\cdot k-m^2_\pi+2m^2_N-2q\cdot k'q\cdot k/q^2 + q\cdot(k'-k)+2m_\Delta m_N(1-q\cdot Q/q^2)\}, \tag{34}$$

with

$$G = q^2\,g^2_{\pi N\Delta}/\left[3m^2_\pi m^2_\Delta\left(q^2-m^2_\Delta+i\Gamma m_\Delta\right)\right]. \tag{35}$$

In the case of the pure spin-3/2 propagator we have

$$A = G\left[\left(q^4/12m^4_\Delta\right)\left(3k'\cdot k-m^2_\pi-2p\cdot k - 2m^2_\pi\,q\cdot Q/q^2\right)-\left\{\left(q^2-m^2_\Delta\right)/2m^4_\Delta\right\} \times(q^2k'\cdot k-q\cdot kq\cdot k')\right], \tag{36}$$

$$B = \left(q^4 m_N G/6m^4_\Delta\right)(1-q\cdot Q/q^2), \tag{37}$$

with

$$G = g^2_{\pi N\Delta}/\left[q^2-m^2_\Delta+i\Gamma m_\Delta\right]. \tag{38}$$

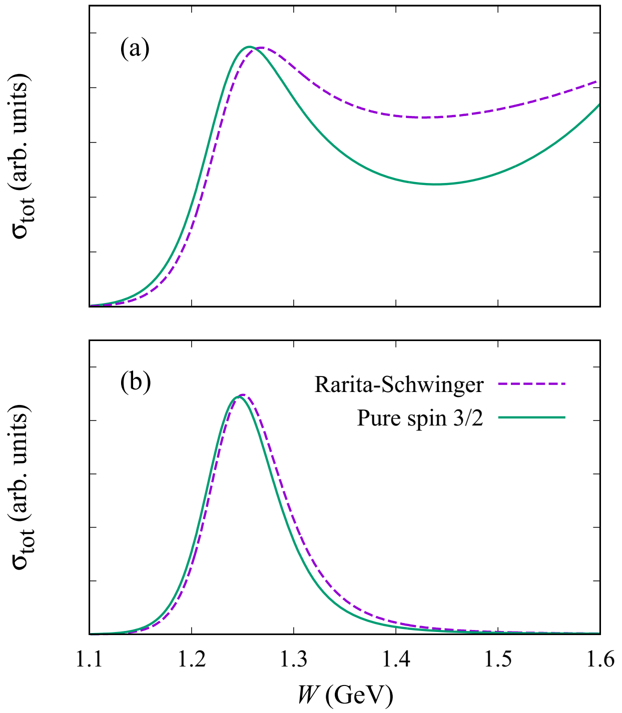

FIG. 2. Contribution of the $\Delta(1232)$ resonance to the $\pi N \to \pi N$ scattering total cross section in arbitrary unit (arb. units) according to the Rarita-Schwinger formalism with GI interaction and the pure spin-3/2 models as a function of total c.m. energy W. Panels (a) and (b) show the contributions if the hadronic form factors are excluded and included, respectively. Note that for the sake of comparison the two models do not use the same value of coupling constant.

The cross section can be obtained from the scattering amplitude $\mathcal{M}$ given by Eq. (32) by means of the standard method [13]. Since the analytic forms of the two amplitudes are completely different we have to use different coupling constants in order to produce comparable results. We believe that this should not raise a problem since the coupling constants are usually fitted to reproduce the experimental data.

The original contributions of both models are depicted in Fig. 2(a), where we can see the resonance behavior centered around $W \approx 1.25$ GeV, followed by the background phenomenon originating from momentum dependence of the numerator of Eq. (28) indicated by a smoothly increasing cross section for $W \gtrsim 1.40$ GeV. Note that this background shifts the resonance peak from the original position at 1.232 GeV to a higher energy region. It is also obvious from Fig. 2(a) that the background obtained from the pure spin-3/2 model is significantly smaller than that of the RS model at $W \approx 1.40$ GeV. This phenomenon originates from the second term in the square bracket of Eq. (28). Above this energy point the first term of Eq. (28) starts to become dominant, since $q^4 = W^4$, and the total contribution starts to diverge.

The large background contribution is natural in the covariant Feynman diagrammatic approach. Alternatively, this could be also interpreted as the contribution of a Z-diagram [14], i.e., the existence of particle and antiparticle in the intermediate state, which is not considered in Fig. 1. The situation is quite different in the multipoles approach, where a relatively perfect resonance structure is parametrized by using the Breit-Wigner function [15]. However, in a covariant isobar model [16] the large background contributions produced by a number of resonances in the model could disturb the nature of the resonance itself and increase the difficulty to fit the experimental data. To suppress this undesired background one usually considers a hadronic form factor (HFF) in each of hadronic vertices of Fig. 1. A related discussion to this end can be found, e.g., in Refs. [17,18]. Nevertheless, here we have to emphasize that the use of HFF is theoretically required to account for the fact that the nucleon is not a point-like object. In the present analysis we use a dipole HFF in the form of [18]

$$F = \Lambda^4/[\Lambda^4 + (q^2 - m_\Delta^2)^2], \tag{39}$$

with the hadronic cutoff $\Lambda = 0.5$ GeV. The choice seems to be trivial, but at present it is solely intended as an example. A detailed study for this purpose can be addressed in the future.

The result obtained after including this HFF is shown in Fig. 2(b). Clearly, we obtain a perfect resonance structure for both models and the fact that the RS structure is slightly shifted to the right can be understood from its original contribution shown Fig. 2(a). Therefore, instead of its different formulation the pure spin-3/2 propagator also exhibits a common resonance structure as in the conventional RS propagator. This result also emphasizes our argument that in order to obtain the natural properties of resonance the use of HFF is mandatory in the covariant Feynman diagrammatic approach.

Finally, one could also raise questions about the consistency of the interaction Lagrangian given in Eq. (25). According to Ref. [19], an interaction is said to be consistent if the interaction Lagrangian has the same symmetry as in the free Lagrangian, i.e., the invariance under the same transformation as in the free Lagrangian. The Lagrangian given in Eq. (25) is not invariant under the gauge transformation given by Eq. (13). However, it is not difficult to construct a consistent Lagrangian for this purpose. The general form of the interaction Lagrangian reads

$$\mathcal{L} = g\bar{J}_{\mu\nu}\Psi^{\mu\nu} + \text{H.c.} \tag{40}$$

The invariance of this interaction under the gauge transformation given in Eq. (13) requires that

$$\bar{J}_{\mu\nu}\xi^{\mu\nu} = 0, \tag{41}$$

whereas $J_{\mu\nu}$ must not be a symmetric tensor because in general one does not expect that $J_{\mu\nu}\Psi^{\mu\nu}$ vanishes. One of the possible choices for the consistent Lagrangian is

$$\mathcal{L}_{\pi N\Delta} = \left(\frac{g_{\pi N\Delta}}{m_\Delta}\right)\bar{N}\gamma_5\mathcal{P}_{\mu\nu\rho\sigma}\partial^\rho\Psi^{\mu\nu}\partial^\sigma\pi + \text{H.c.} \tag{42}$$

By using the vertex factor

$$\Gamma_{\mu\nu}(k) = \left(\frac{g_{\pi N\Delta}}{m_\Delta}\right)\gamma_5\mathcal{P}_{\mu\nu\rho\sigma}q^\rho k^\sigma\,, \tag{43}$$

the corresponding Δ-exchange amplitude for this interaction reads

$$\Gamma_{\mu\nu}(k')S^{\mu\nu\rho\sigma}(q)\Gamma_{\rho\sigma}(k) = \left(\frac{g_{\pi N\Delta}}{m_\Delta}\right)^2 q^\alpha k'^\beta \mathcal{P}_{\mu\nu\alpha\beta} S^{\mu\nu\rho\sigma}(q)\mathcal{P}_{\rho\sigma\gamma\delta} q^\gamma k^\delta . \quad (44)$$

With the idempotent relation of the projection operator $\mathcal{P}^{\mu\nu\alpha\beta}\mathcal{P}_{\alpha\beta\rho\sigma} = \mathcal{P}^{\mu\nu}{}_{\rho\sigma}$, one can prove the relation $\mathcal{P}^{\mu\nu\alpha\beta}\Delta_{\alpha\beta\rho\sigma} = \mathcal{P}^{\mu\nu}{}_{\rho\sigma}$. As a result, the transition amplitude becomes

$$\Gamma_{\mu\nu}(k')S^{\mu\nu\rho\sigma}(q)\Gamma_{\rho\sigma}(k) = \frac{g^2_{\pi N\Delta}}{m^2_\Delta(q^2 - m^2_\Delta + i\epsilon)} \times [q^\alpha k'^\beta \mathcal{P}_{\alpha\beta\gamma\delta} q^\gamma k^\delta] = \frac{g^2_{\pi N\Delta}}{q^2 - m^2_\Delta + i\epsilon}\left[\frac{q^2}{4m^2_\Delta} P^{(3/2)}_{\beta\delta} k'^\beta k^\delta\right]. \quad (45)$$

Surprisingly, this transition amplitude contains only the RS spin-3/2 projection operator term. This transition amplitude differs from the result of Pascalutsa and Timmermans [5] by the $(\not{q} + m_\Delta)$ factor.

In conclusion we have proposed the use of a pure spin-3/2 propagator to describe the properties of spin-3/2 particles in the study of particle and nuclear physics. We used the ATS representation to describe the corresponding projection operator. We have shown that in the ATS formalism we have to redefine the interaction Lagrangian, otherwise the obtained scattering amplitude cannot display the resonance behavior. By calculating its contribution to the elastic πN scattering total cross section we have shown that this pure spin-3/2 propagator also exhibits the natural properties of a resonance, as in the conventional RS one, if the hadronic form factors were considered in its hadronic interactions.

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