

High-mass twin stars with a multipolytrope equation of stateD. E. Alvarez-Castillo^{1,2,3,*} and D. B. Blaschke^{1,4,5,†}¹*Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, Joliot-Curie str. 6, 141980 Dubna, Russia*²*GSI Helmholtzzentrum für Schwerionenforschung GmbH, Planckstraße 1, 64291 Darmstadt, Germany*³*Instituto de Física, Universidad Autónoma de San Luis Potosí Av. Manuel Nava 6, San Luis Potosí, S.L.P. 78290, México*⁴*Institute for Theoretical Physics, University of Wrocław, Max Born Pl. 9, 50-204 Wrocław, Poland*⁵*National Research Nuclear University (MEPhI), Kashirskoe Shosse 31, 115409 Moscow, Russia*

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We show that in the three-polytrope model of Hebeler *et al.* [*Astrophys. J.* **773**, 11 (2013)] for the neutron star equation of state (EoS) at supersaturation densities a third family of compact stars can be obtained which confirms the possibility of high-mass twin stars that have coincident masses $M_1 = M_2 \approx 2M_\odot$ and significantly different radii $|R_1 - R_2| > 2\text{--}3$ km. We show that the causality constraint puts severe limitations on the maximum mass of the third family sequence which can be relaxed when this scheme is extended to four polytropes thus mimicking a realistic high-density matter EoS.

DOI: [10.1103/PhysRevC.96.045809](https://doi.org/10.1103/PhysRevC.96.045809)**I. INTRODUCTION**

Compact stars (CS) represent one possible endpoint of stellar evolution with conditions of very high density in their interiors for which the equation of state (EoS) is currently unknown. In particular, the composition of CS interiors is puzzling: are they composed of exotic forms of matter like hyperonic matter or phases of quark matter which even may be color superconducting [1]? In this connection arises the question for the order of the transition to these exotic phases. If it was to be a first-order transition, this would imply the existence of at least one critical endpoint in the QCD phase diagram, since we know from lattice QCD simulations that at vanishing baryon density and finite temperatures the QCD transition is a crossover [2].

In 2013, we suggested [3,4] that there was a possibility to decide this question by CS observations following the classification scheme [5] of mass-radius (M - R) diagrams for hybrid star EoS in dependence on the characteristic features of the transition, the jump in energy density $\Delta\varepsilon$ and the critical pressure P_{crit} at the onset of the transition. For instance, if the M - R diagram of CS would feature the so-called *third family* branch that is separated from the second family branch of neutron stars (NS) as purely hadronic CS by a sequence of unstable configurations, this would indicate the EoS of CS matter has a phase transition with a sufficiently large $\Delta\varepsilon$. Since the conversion of the core matter to a phase with higher density at the same pressure entails a compactification of the star accompanied with a release of gravitational binding energy and therefore a lowering of the gravitational mass, the second and third family branches do overlap in a certain mass window. This situation is called the mass twin phenomenon: two stars of the same mass would be located in the second and third family branches, respectively, having different radii and internal composition. If this occurs at a high mass of $\sim 2M_\odot$

one speaks of the high-mass twin (HMT) scenario. The most prominent example being due to the hadron-to-quark-matter phase transition resulting in hybrid CS composed of a hadronic mantle and a quark matter core, see [6,7] and references therein. The HMT scenario is, in principle, accessible to observational verification, e.g., by satellite missions like NICER [11] or ground-based programs as SKA [12]. All it takes is to measure radii of NS with sufficiently similar high masses such as PSR J0348 + 0432 with $M = 2.01 \pm 0.04M_\odot$ [8] and PSR J1614-2230 with $M = 1.928 \pm 0.017M_\odot$ [9,10], and to find out that their radii are significantly different.

In fact, the discussion of a third family branch in the M - R diagram due to a phase transition in dense matter goes back to Gerlach [13] and has subsequently been considered, e.g., by Kämpfer [14] before Glendenning and Kettner [15] coined the term neutron star twins in a work that used a Relativistic Mean Field model for the hadronic phase together with a bag model for the quark matter phase. Schertler *et al.* [16,17] considered strange quark matter. However, due to the soft hadronic equations of state used there, the resulting twin stars cannot reach the $M = 2.01M_\odot$, a measurement at that time unknown. On the other hand, [5,18] feature a fixed hadronic EoS together with a constant speed of light quark matter parametrization and present a phase diagram for all masses and different mass-radius topologies, namely, connected and disconnected branches. Another alternative description is reported in [19] where a SU(3) chiral-meson model [20] together with a bag constant quark matter model is implemented and leads to very large CS radii incompatible with the Hebeler constraints. Last but not least, a few other realizations of HMTs have been reviewed in [7] where the connection with heavy ion collision experiments has been underlined.

Furthermore, as pointed out in [21], the HMT phenomenon is of great relevance for the study of the NS EoS not only because it can provide evidence for a first-order phase transition and thus for the very existence of a critical endpoint in the QCD diagram, but also because it provides a resolution to several issues: the hyperon puzzle [22], the reconfinement problem, and the masquerade case (see [21] and references

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therein). In addition, the HMT may be discussed in the context of explaining the origin of fast radio bursts [23] as possible intermediate metastable states due to a sudden change in the internal structure of a fastly rotating supramassive neutron star [24,25] created, e.g., in a NS merger event before its final collapse to a black hole [26].

The purpose of this work is to point out that the HMT case is not the result of the construction of a rather exotic case of an EoS but may be obtained even within the rather conservative scheme of Hebeler *et al.* [27]. It consists of a multi-polytrope description of the NS EoS [28] in line with constraints derived from a chiral effective field theory describing nuclear few- and many-particle systems at densities up to nuclear saturation.

II. PIECEWISE POLYTROPE EOS WITH A FIRST-ORDER PHASE TRANSITION

We would like to investigate the question whether in the scheme of Hebeler *et al.* [27] with a piecewise polytrope EoS at supersaturation densities it would be possible to describe the HMT phenomenon. To this end one should define one of the polytropes as a constant pressure region with $P = P_{\text{crit}}$ with a jump in energy density $\Delta\varepsilon$ due to a first-order phase transition that would fulfill the Seidov constraint [29]

$$\frac{\Delta\varepsilon}{\varepsilon_{\text{crit}}} \geq \frac{1}{2} + \frac{3}{2} \frac{P_{\text{crit}}}{\varepsilon_{\text{crit}}} \quad (1)$$

for the occurrence of an instability in the M - R relation of compact stars. Such a sequence of unstable configurations is precondition for a disconnected (third family) branch of stable hybrid stars that would furthermore require a sufficiently stiff high density EoS to allow for a maximum mass fulfilling the constraint from the measurement of the mass $M = 2.01 \pm 0.04 M_{\odot}$ for the pulsar PSR J0348 + 0432 [8]. According to [27], the supersaturation density region is split into three regions

$$\begin{aligned} i = 1 : n_1 \leq n \leq n_{12}, \quad i = 2 : n_{12} \leq n \leq n_{23}, \\ i = 3 : n \geq n_{23}, \end{aligned} \quad (2)$$

where $n_1 = 1.1n_0$ is just above the nuclear saturation density $n_0 = 0.15 \text{ fm}^{-3}$ and the polytrope EoS pieces fulfill

$$P_i(n) = \kappa_i n^{\Gamma_i} \quad (3)$$

in the corresponding regions. For our setting of the problem, in all our EoS models the first region is taken from the Hebeler *et al.* paper [27] and corresponds to a polytrope fit to the stiffest EoS ($n > 1.1n_0$) of their Table V together with an intermediate homogeneous phase in β equilibrium ($0.5n_0 < n < 1.1n_0$), presented in their section III, and the BPS EoS for the outer NS crust ($n < 0.5n_0$) of their Table VII. Therefore, the resulting fit gives polytrope parameter values for this density region of $\Gamma_1 = 4.92$ and $\kappa_1 = 17906.60 \text{ MeV fm}^{3(\Gamma_1-1)}$. Furthermore, the region $i = 2$ shall correspond to the phase coexistence region of the first-order phase transition with constant pressure, so that $\Gamma_2 = 0$ and $P_2 = \kappa_2 = P_{\text{crit}}$. The boundaries of this region shall be obtained from a Maxwell construction [30] which requires the pressure as a function of the chemical potential μ . To facilitate this construction for a pair of

polytropes at zero temperature, we utilize the formulas given in the Appendix of Ref. [31],

$$P(n) = n^2 \frac{d(\varepsilon(n)/n)}{dn}, \quad (4)$$

$$\varepsilon(n)/n = \int dn \frac{P(n)}{n^2} = \int dn \kappa n^{\Gamma-2} = \frac{\kappa n^{\Gamma-1}}{\Gamma-1} + C, \quad (5)$$

$$\mu(n) = \frac{P(n) + \varepsilon(n)}{n} = \frac{\kappa \Gamma}{\Gamma-1} n^{\Gamma-1} + m_0, \quad (6)$$

where the integration constant C is fixed by the condition that $\varepsilon(n \rightarrow 0) = m_0 n$. Now we have for the polytrope EoS

$$n(\mu) = \left[(\mu - m_0) \frac{\Gamma-1}{\kappa \Gamma} \right]^{1/(\Gamma-1)}, \quad (7)$$

so that the pressure as a function of the chemical potential for the polytrope EoS (3) is

$$P(\mu) = \kappa \left[(\mu - m_0) \frac{\Gamma-1}{\kappa \Gamma} \right]^{\Gamma/(\Gamma-1)}. \quad (8)$$

With EoS in the form (8) one can perform the Maxwell construction of a first-order phase transition.

III. HIGH MASS TWINS FROM MULTI-POLYTROPE EQUATIONS OF STATE

Now we can apply these general relations to the case of a transition from nuclear matter in region 1 (with $m_{0,1}$ being the nucleon mass) to high density matter in region 3. That may correspond, e.g., to hyperon matter or quark matter. From the Maxwell construction

$$P_1(\mu_{\text{crit}}) = P_3(\mu_{\text{crit}}) = P_{\text{crit}}, \quad (9)$$

$$\mu_{\text{crit}} = \mu_1(n_{12}) = \mu_3(n_{23}), \quad (10)$$

follow the two conditions

$$\kappa_3 = \kappa_1 n_{12}^{\Gamma_1} / n_{23}^{\Gamma_3}, \quad (11)$$

$$\begin{aligned} P_{\text{crit}} &= (m_{0,1} - m_{0,3}) \left[\frac{\Gamma_1}{n_{12}(\Gamma_1-1)} - \frac{\Gamma_3}{n_{23}(\Gamma_3-1)} \right]^{-1} \\ &= \kappa_1 n_{12}^{\Gamma_1}. \end{aligned} \quad (12)$$

The above equations (11) and (12), allow for determination of κ_3 and $m_{0,3}$ once the values of n_{12} , n_{23} , $m_{0,1}$, and Γ_3 are fixed. In order to fulfill the compact star mass constraint, we may demand that upon solving the corresponding compact star sequence, the mass at the onset of the transition fulfills $M(n_{12}) \simeq 2 M_{\odot}$ which fixes the value n_{12} , allowing to determine P_{crit} and μ_{crit} from the Maxwell construction. Moreover, since the EoS just above nuclear saturation is fixed according to the stiff limit of Hebeler *et al.* [27], the constants Γ_1 and κ_1 are also fixed. So we are then left with the three equations (11), (12), and (1) for the four unknowns κ_3 , Γ_3 and n_{23} , and $m_{0,3}$. We fix n_{23} such that the equality sign holds in the Seidov criterion (1). Therefore, in order to close the system, we dial Γ_3 as a free parameter. Its maximal value is determined so that the speed of sound shall not exceed the speed of light up to

TABLE I. Parameter values for sets 1, 2, 3, and 4. The EoS in this set share the following properties: $P_{\text{crit}} = 63.177 \text{ MeV fm}^{-3}$, $\varepsilon_{\text{crit}} = 318.26 \text{ MeV fm}^{-3}$, $\Delta\varepsilon = 253.89 \text{ MeV fm}^{-3}$. The second polytrope with $P_2 = P_{\text{crit}}$ and $\Gamma_2 = 0$ lies between the densities $n_{12} = 0.32 \text{ fm}^{-3}$ and $n_{23} = 0.53 \text{ fm}^{-3}$. For set 4 the high-density region has been divided in two polytrope branches: set 4a for densities $n_{23} \leq n \leq n_{34}$ and set 4b for densities $n \geq n_{34} = 0.75 \text{ fm}^{-3}$.

	Γ_3	κ_3 [MeV fm $^{3(\Gamma_3-1)}$]	$m_{0,3}$ [MeV]	M_{max}^{NS} [M_{\odot}]	M_{max}^{HS} [M_{\odot}]	M_{min}^{HS} [M_{\odot}]
set 1	2.50	302.56	991.75	2.01	—	—
set 2	2.80	365.12	1004.88	2.01	1.910	1.909
set 3	3.12	447.16	1014.87	2.01	1.991	1.934
set 4a	4.00	774.375	1031.815	—	—	—
set 4b	2.80	548.309	958.553	2.01	2.106	1.961

the density values reached in the very center of the maximum mass star configurations.

With the above scheme, we are thus able to compute the EoS of hybrid compact stars. Next, we obtain the corresponding star sequences by solving the Tolman-Volkoff-Oppenheimer (TOV) equations describing a static, nonrotating, spherically symmetric star [32,33]

$$\frac{dP(r)}{dr} = -\frac{G(\varepsilon(r) + P(r))(M(r) + 4\pi r^3 P(r))}{r(r - 2GM(r))}, \quad (13)$$

$$\frac{dM(r)}{dr} = 4\pi r^2 \varepsilon(r) \quad (14)$$

with $P(r=R) = 0$ and $P_c = P(r=0)$ as boundary conditions for a star with mass M and radius R . The complete

NS sequence is determined by increasing the chosen central pressure P_c up to a maximum mass.

The enclosed baryonic mass is obtained by integrating

$$\frac{dN_B(r)}{dr} = 4\pi r^2 \left(1 - \frac{2GM(r)}{r}\right)^{-1/2} n(r). \quad (15)$$

It plays an important role in the description of the dynamics of CS evolution scenarios that conserve the baryonic mass, like spinning down into the hybrid twin configuration [25]. In our numerical calculations, we vary the values of the polytrope index Γ_3 in order to discuss the effect of the stiffness of the high-density EoS using sets 1–4 of model parameters given in Table I.

The hybrid star EoS corresponding to sets 1–4 share the same hadronic branch, onset, and density jump at the phase transition, see panel (a) of Fig. 1. In panel (b) of Fig. 1 we demonstrate that the causality constraint is fulfilled for all these sets. Increasing the quark matter stiffness through the parameter Γ_3 results in an increase of the maximum mass on the hybrid star branch. The EoS for set 3 reaches the causality limit just at the maximum mass of the hybrid star sequence, which is denoted by the plus symbols on panels (b) and (c) of that figure. It shows that a third family of stable hybrid stars can be obtained within the multi-polytrope scheme of Hebeler *et al.* [27] using just three polytropes. This is the main result of this paper. The hybrid star branch is more compact than the purely hadronic one by about 2–3 km. In Fig. 2 we illustrate the case of HMT stars by showing for set 3 the energy density profiles of two stars with the same mass of $1.99M_{\odot}$, one from the hadronic branch and one from the hybrid star branch. The latter has an extended quark matter core and is more compact

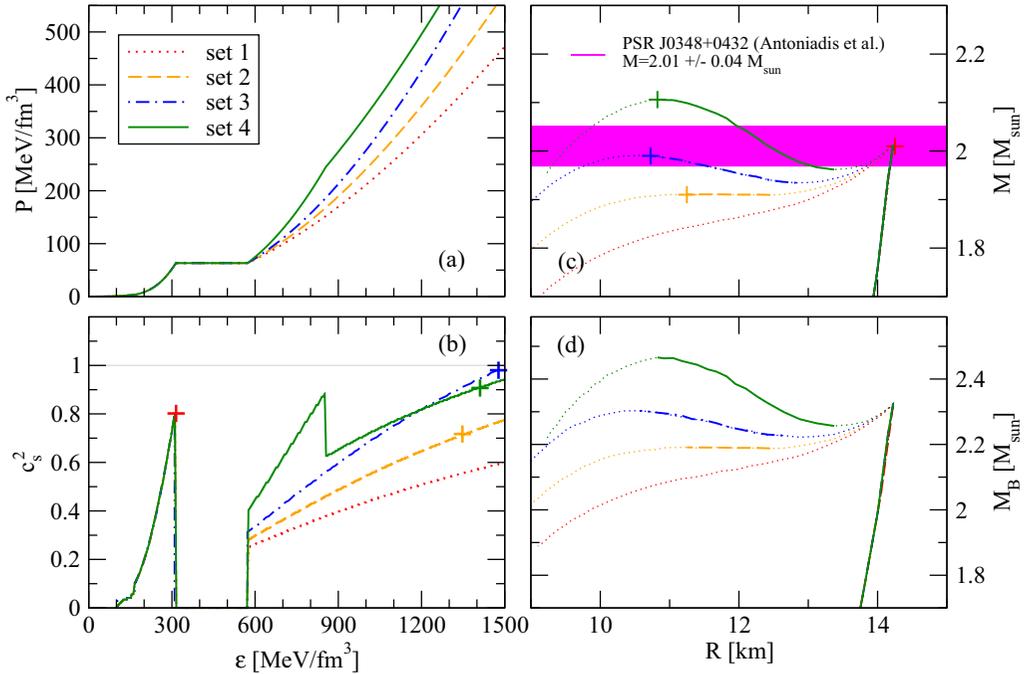


FIG. 1. EoS [(a)— P vs. ε ; (b)— c_s^2 vs. ε] and sequences of compact stars [(c)— M vs. R ; (d) — M_B vs. R] for sets 1–4 of Table I. The EoS have the same onset density and density jump of the phase transition, but different stiffness of the high density (quark matter) phase. The plus symbols denote values for the maximum mass configurations.

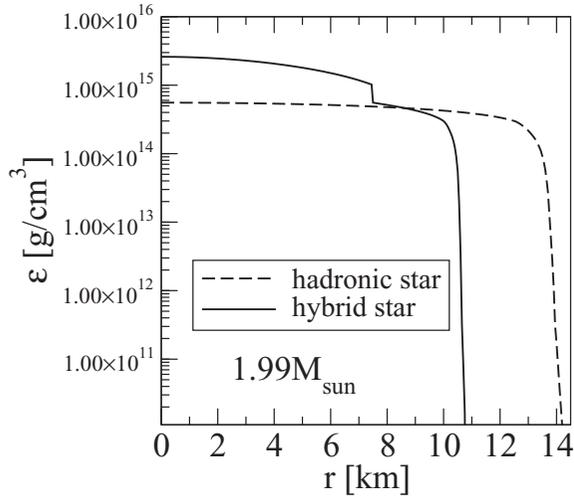


FIG. 2. High mass twins internal energy density profile. The dashed curve corresponds to the pure hadronic star whose radius exceeds the hybrid star by about 3 km.

than its hadronic twin by about 3 km. This is a potentially observable effect.

Another important result is obtained when going beyond the three-polytrope scheme of Hebeler *et al.* [27] by introducing a fourth polytrope at high densities in order to prevent a causality breach. With set 4 of our study we achieve an increase in the hybrid star maximum mass which now even reaches $\sim 2.1M_{\odot}$.

Actually, the physical motivation for adopting a change in the polytrope index Γ_3 of the high-density matter comes from the fact that there can be a sequence of phase transitions in CS matter at high densities [37], which may even lead to the occurrence of a fourth family of CS [38]. Recently we could show that also a fifth family solution is possible for the case of three sequential transitions in CS matter [39], as discussed in [37].

Lowering Γ_3 we obtain a value for which barely a stable hybrid star sequence can be obtained (set 2), and lowering Γ_3 further (set 1) no stable hybrid stars are possible. The four parameter sets for which the EoS and compact star sequences are illustrated in Fig. 1 are given in Table I. Note that for set 4 the high-density region has been divided in two polytrope branches: set 4a for densities $n_{23} \leq n \leq n_{34}$ and set 4b for densities $n \geq n_{34} = 0.75 \text{ fm}^{-3}$.

It is interesting to discuss the present parametrizations of the multi-polytrope EoS in the classification scheme of Ref. [5] where a phase diagram for hybrid CS sequences with a phase transition has been introduced that is spanned by the plane of $\Delta\epsilon$ and P_{crit} , both measured in units of ϵ_{crit} , see Fig. 3. By construction all our parametrizations have the same P_{crit} , ϵ_{crit} , and $\Delta\epsilon$. Therefore, they are represented by one and the same point in the phase diagram of Fig. 3, which also by construction lies on the Seidov border line (1). Changing the stiffness of the high-density phase by increasing the value of Γ_3 moves the border that divides regions for disconnected (D) and absent (A) stable hybrid star branches so that for set 1 our model lies in the domain A, while for the stiffer sets 2–4 it is in region D. The fact that the point representing our parametrizations lies

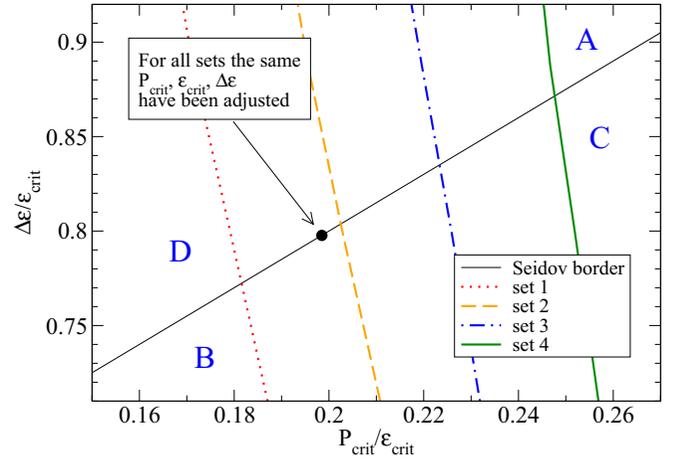


FIG. 3. “Phase diagram” of hybrid star sequences following Ref. [5] for parameter sets 1–4 of this work. For details, see text.

on the Seidov border is merely of academic interest here. It indicates that at the onset of the phase transition, if $\Delta\epsilon$ were just a little smaller, there would be a small connected branch of hybrid stars with a tiny quark matter core before the onset of the unstable branch, which makes out the difference that one has now B and C instead of D and A. If one would disregard this tiny “academic” detail, then the regions just below the Seidov line should also be called D and A, respectively.

All the EoS models in this work are causal and fall inside the EoS region that was given in Hebeler *et al.* [27] for the case supporting a $1.97M_{\odot}$ NS, see Fig. 4.

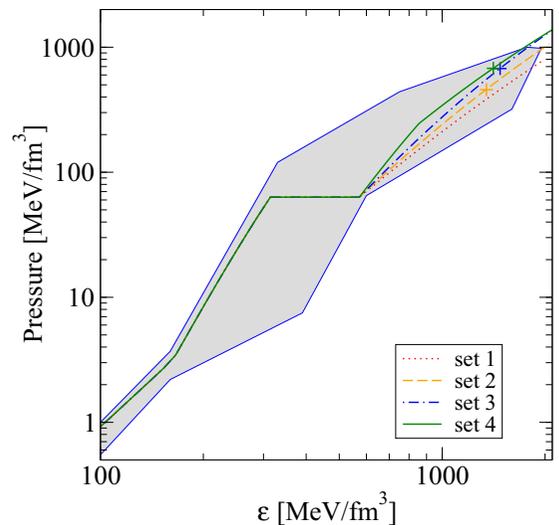


FIG. 4. All multi-polytrope EoS for sets 1–4 from Table I fall within the region given in Hebeler *et al.* [27] for the case supporting a $1.97M_{\odot}$ NS (grey shaded region). These EoS share the hadronic branch EoS, the onset density and jump in energy density at the transition but vary in the stiffness of the high-density phase. The plus symbols denote the pressure and energy density values at the center of the compact star configuration with the maximum mass.

IV. CONCLUSIONS

In this work we have shown that within the multi-polytrope approach by Hebeler *et al.* [27] we can obtain high mass twins in the M - R diagram for compact stars. We have also shown that going beyond the three-polytrope scheme one can achieve an increase in the maximum mass on the third family branch of compact hybrid stars. This feature is of particular interest for scenarios of NS-NS merger events where in the spin-down evolution of the supermassive compact star a phase transition can occur.

The EoS parametrizations presented here obey causality and fall in the constraint region derived in [27] for the condition that a mass of $1.97M_{\odot}$ has to be reached.

In concluding, we mention that the multi-polytrope approach can of course not replace a realistic EoS. While we have shown that extending the three-polytrope scheme to four polytropes gives already a significant improvement of the maximum mass for the hybrid star configurations on the third family branch solutions, the most promising strategy shall be to employ microscopic approaches to high-density quark matter like, e.g., the NJL model with higher order repulsive interactions [6] or the relativistic string-flip model [34] do not have this problem of multi-polytropes. The multi-polytrope scheme can give, however, interesting heuristic guidance along this path.

Finally, we would like to emphasize the importance of the HMT phenomenon detection and its relation to the QCD critical point [4,35]. If a first-order phase transition exists in the QCD diagram it should feature a critical end up that borders the

crossover region. Thus, this transition shall extend into isospin asymmetric matter covering the low temperature region where compact star matter is located. As we have presented in this work, if the strength of this transition satisfies the Seidov conditions and quark matter is sufficiently stiff, the high mass twins phenomenon shall occur. Potential detection relies on accurate radius measurements which, for the models presented here, should be capable of resolving a 2–3 km difference with confidence. Bayesian techniques that utilize astronomical measurements can provide model parameter estimation useful to probe the compact star equation of state [36].

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