Nuclear structure features of Gamow-Teller excitations

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The nuclear shell-model calculations show a clear anticorrelation between the Gamow-Teller strength and the transition rate of the collective quadrupole excitation from the ground state in response to artificial changes of the spin-orbit splitting. The shell-model calculations in the fp space demonstrate that changes of the spin-orbit coupling influence Gamow-Teller and quadrupole modes in the opposite way. This interrelation is discussed in terms of simple symmetry arguments.

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I. INTRODUCTION

Experimental and theoretical studies of weak interactions in general and nuclear Gamow-Teller (GT) transitions specifically are in the focus of modern physics, being important for nuclear structure and reactions, astrophysics, particle physics, and the search for phenomena outside the standard model. In spite of many efforts, some basic problems related to nuclear GT transitions are still controversial. Below we try to address old questions on the crossroads of nuclear structure and mechanisms of the GT dynamics in complex nuclei which still are not convincingly answered.

The earlier shell-model studies [1] discovered a phenomenon of pronounced correlation, or rather anticorrelation, between the GT strength and the low-lying electric quadrupole ($E2$) strength. To the best of our knowledge, this effect is not sufficiently explained. This is the main subject of our discussion. We find that the anticorrelation effect follows naturally as a consequence of isospin invariance, fermionic antisymmetry of the wave functions, and spin-orbit coupling. Due to spin-orbit splitting of single-particle levels, the total orbital momentum L ceases to be an exact quantum number so that the standard GT operator excites a superposition of $L = 0$ and $L = 2$ states. It is not clear if the usual experimental analysis correctly accounts for this fact which, however, should be included in order to guarantee the total model-independent sum rule.

We also confirm that this universal non-energy-weighted sum rule for the GT transitions is fulfilled in the shell-model calculations only through many contributions of very weak transitions which can be hardly visible in an experiment with finite resolution and unavoidable background. The role of complicated configurations in the saturation of the GT sum rule was stressed long ago [2]. Below we show exact results of the shell-model solution in the fp space and add simple arguments based on the symmetry considerations.

Although the important problem of experimentally observed quenching of the GT strength in nuclei [3–7] is not directly touched in this study, in our concluding discussion we mention some aspects of our results (strong fragmentation of the GT strength and admixture of $L = 2$ excitations) which could be related to that problem.

II. TYPICAL SHELL-MODEL RESULTS

We start with the results of typical shell-model calculations for a few nuclei in the fp shell. The normal spin-orbit splitting in the shell-model calculation with FPD6*pn* interaction is 6.5 MeV between $f_{5/2}$ and $f_{7/2}$ levels and 2 MeV between $p_{3/2}$ and $p_{5/2}$ levels. The numerical experiment shown below, similarly to Ref. [1], demonstrates the simultaneous calculation of the total GT excitation probability B^- from the ground state, and the quadrupole excitation rate $B(E2; 0^+ \to 2^+)$ for the lowest quadrupole collective excitation, as a function of the gradually reduced to zero spin-orbit splitting $\Delta\epsilon(f) = \epsilon(f_{5/2}) - \epsilon(f_{7/2})$ (see Table I).

We define the GT operators $\mathbf{V}^{\pm}$ as vectors with respect to spin variables, $\mathbf{s} = (1/2)\vec{\sigma}$, carrying also vector components in the nucleon isospin space $\mathbf{t} = (1/2)\vec{\tau}$, $\tau^{\pm} = \tau_1 \pm i\tau_2$,

$$\mathbf{V}^- = \frac{1}{2}\sum_a \vec{\sigma}_a \tau_a^-, \quad \mathbf{V}^+ = (\mathbf{V}^-)^\dagger = \frac{1}{2}\sum_a \vec{\sigma}_a \tau_a^+, \tag{1}$$

where the sums are taken over nucleons a; some useful algebraic definitions are included in Appendix A.

One can speak of the total GT strength of a given nuclear state $|\nu\rangle$ in the mother nucleus summed over all final daughter states,

$$B^+(\nu) = \tfrac{1}{2}\langle\nu|(\mathbf{V}^- \cdot \mathbf{V}^+)|\nu\rangle, \quad B^-(\nu) = \tfrac{1}{2}\langle\nu|(\mathbf{V}^+ \cdot \mathbf{V}^-)|\nu\rangle. \tag{2}$$

This definition, where the scalar product refers to the spin vectors, leads to the standard universal Ikeda sum rule, independent of the starting state $|\nu\rangle$:

$$B^-(\nu) - B^+(\nu) = \sum_a (\vec{\sigma}_a)^2 (\tau^3)_a = 3(N - Z). \tag{3}$$

Here $|\nu\rangle$ is an arbitrary nuclear state below the meson production threshold. In particular, for nuclei with filled proton shells, such as $^{42-48}$Ca, the B^+ part is quite low, and the sum rule should be fulfilled mainly due to the B^- part.

Table I and Fig. 1 show the anticorrelation mentioned in the Introduction. In the isospin-symmetric nucleus ^{44}Ti, the total GT strength linearly falls to zero while $B(E2)$ grows when the spin-orbit splitting $\Delta\epsilon(f)$ is gradually reduced to zero.

TABLE I. The evolution of the total GT excitation probability from the ground state and the quadrupole excitation rate $B(E2;0^+\to 2^+)$ from the ground state to the lowest quadrupole collective excitation in ^{44}Ti and ^{46}Ti, in the process of gradually changing the spin-orbit splitting $\Delta\epsilon(\ell)=\epsilon(j=\ell-1/2)-\epsilon(j=\ell+1/2)$ from its realistic value to zero. To relate to the upper line for ^{46}Ti, we included in the calculation 1000 final states.

	$\epsilon(2p_{3/2})$	$\epsilon(2p_{1/2})$	$\Delta\epsilon(p)$	$\epsilon(1f_{7/2})$	$\epsilon(1f_{5/2})$	$\Delta\epsilon(f)$	^{44}Ti			^{46}Ti			
							B^-	$B(E2)$	$E(2^+)$	B^-	B^+	$B(E2)$	$E(2^+)$
0	−6.495	−4.478	2.017	−8.388	−1.897	6.491	1.26	699	1.30	6.92	0.93	781	0.98
1	−6.495	−4.478	2.017	−8.088	−2.197	5.891	1.05	734	1.27	6.73	0.76	835	0.96
2	−6.495	−4.478	2.017	−7.788	−2.497	5.291	0.87	764	1.22	6.61	0.64	879	0.92
3	−6.495	−4.478	2.017	−7.488	−2.797	4.691	0.71	793	1.17	6.50	0.52	924	0.88
4	−6.495	−4.478	2.017	−7.188	−3.097	4.091	0.56	820	1.14	6.40	0.42	967	0.85
5	−6.495	−4.478	2.017	−6.888	−3.397	3.491	0.43	843	1.11	6.31	0.32	1008	0.82
6	−6.495	−4.478	2.017	−6.588	−3.697	2.891	0.32	864	1.09	6.23	0.25	1045	0.79
7	−6.495	−4.478	2.017	−6.288	−3.997	2.291	0.23	880	1.07	6.17	0.19	1078	0.77
8	−6.495	−4.478	2.017	−5.988	−4.297	1.691	0.17	893	1.06	6.12	0.14	1106	0.76
9	−6.495	−4.478	2.017	−5.688	−4.597	1.091	0.12	902	1.05	6.09	0.11	1127	0.75
10	−6.495	−4.478	2.017	−5.388	−4.897	0.491	0.09	907	1.05	6.07	0.09	1141	0.75
11	−6.495	−4.478	2.017	−5.088	−5.197	−0.109	0.09	909	1.04	6.06	0.07	1149	0.75
12	−6.495	−4.478	2.017	−5.134	−5.134	0.000	0.09	909	1.04	6.06	0.08	1149	0.75
13	−5.486	−5.486	0.000	−5.134	−5.134	0.000	0.04	873	1.13	6.02	0.04	1101	0.81
14	−5.134	−5.134	0.000	−5.134	−5.134	0.000	0.04	837	1.22	6.02	0.04	1059	0.87

The weakening of the spin-orbit coupling is harmful to the GT strength [in the limit of no such coupling, both GT strengths (3) for $N=Z$ vanish; see below]. The energy of the quadrupole phonon 2^+ state goes down (Fig. 2), which is also reflected by the resulting increase of the quadrupole strength (Fig. 3). Qualitatively, we see a similar evolution for ^{46}Ti, where the sum rule (3) gives 6.

Figures 4 and 5 show that the changes of the summed GT strength and low-lying $B(E2)$ transition probability as a function of the spin-orbit splitting $\Delta\epsilon(f)$ in ^{44}Ti are almost exactly parabolic and opposite to each other. They do not depend on the sign of the spin-orbit coupling. Figure 6 illustrates the distribution of the GT$^-$ strength from the ground state of the $^{46}_{22}$Ti$_{24}$ nucleus as a function of the excitation energy in the daughter states of ^{46}V. The same process of accumulating the total GT strength along the excitation energy of ^{46}Ti is shown by Fig. 7; it works faster at small spin-orbit splitting, while in the realistic situation the accumulation of the total strength is going slower.

This picture is practically universal, always the specific daughter states with large GT strength do not give the full sum rule. Moving along the excitation energy of the daughter nucleus and collecting the GT strength from the mother state we can see the gradual filling of the total strength required by the GT sum rule. Apart from a few significant peaks in a cumulative sum, the convergence to the required value slowly proceeds through a large number of quite small increments. This can be seen in detail in Fig. 8, where both the cumulative strengths B^- and B^+ for ^{46}Ti are shown. The B^+ strength here is relatively small and quickly saturates, while the B^- strength

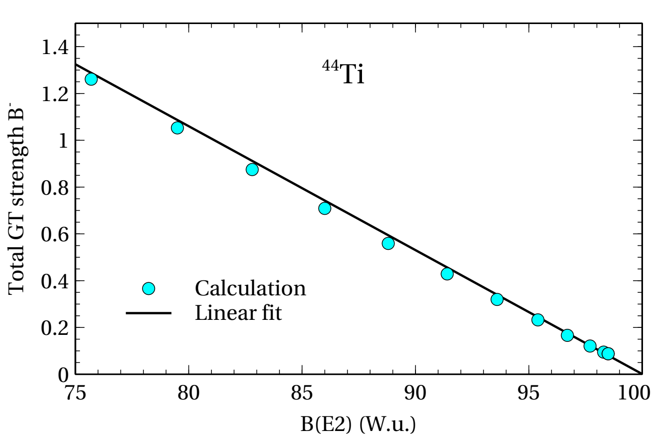

FIG. 1. The total GT strength from the ground state of ^{44}Ti is linearly anticorrelated with the transition rate $B(E2)$ (shown in Weisskopf units, W.u.) from the ground state to the collective 2^+ phonon state.

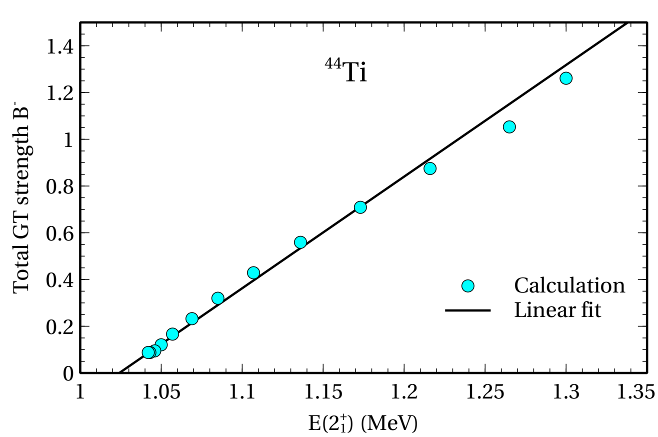

FIG. 2. Energy of the lowest quadrupole excitation in ^{44}Ti is almost linearly anticorrelated with the total GT strength from the ground state when both are changed by the gradual elimination of the spin-orbit splitting.

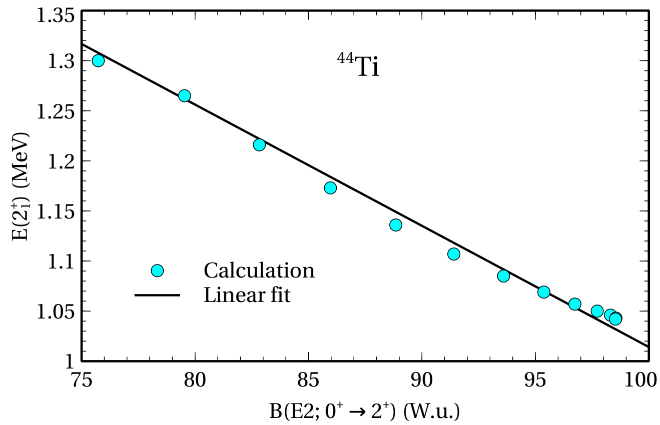

FIG. 3. Energy of the first collective quadrupole state in ^{44}Ti is reduced while the corresponding quadrupole transition probability grows.

grows slowly until the difference sum rule (3) is satisfied. Including in the calculation more final states (up to 1000 states, at an excitation energy of about 20 MeV), we checked that the accuracy of satisfying the sum rule grows. To relate to the upper line for ^{46}Ti of Table I, we obtain $B^- - B^+ = 6.922 - 0.930 = 5.992$, which gives a precision of 0.13% of the exact value. This confirms that the sum rule is fulfilled by addition of very many little contributions.

III. EFFECT OF SPIN-ORBIT SPLITTING

Here we comment on the spin-orbit coupling part of the mean field as an appropriate intermediary agent influencing both low-lying collective quadrupole vibrations and the Gamow-Teller mode based on the spin excitation. Because of this coupling, the total orbital momentum L of the excitation is not conserved, and one of the specific effects of spin-orbit coupling is the mixing of $L = 0$ and $L = 2$ excitations.

In agreement with the findings of Ref. [1], in the limit of switched-off spin-orbit coupling, the GT strength vanishes in $N = Z$ nuclei, $B^- = B^+ = 0$. This can be understood in terms of isospin invariance and the LS coupling scheme instead of the jj coupling. Indeed, neutrons and protons occupy here the

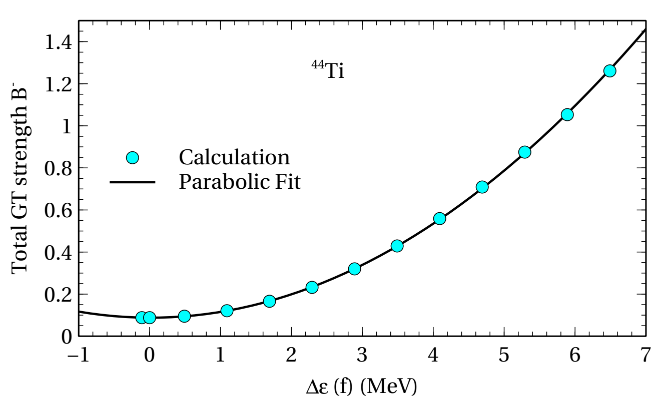

FIG. 4. The parabolic dependence of the GT strength on the spin-orbit splitting in ^{44}Ti.

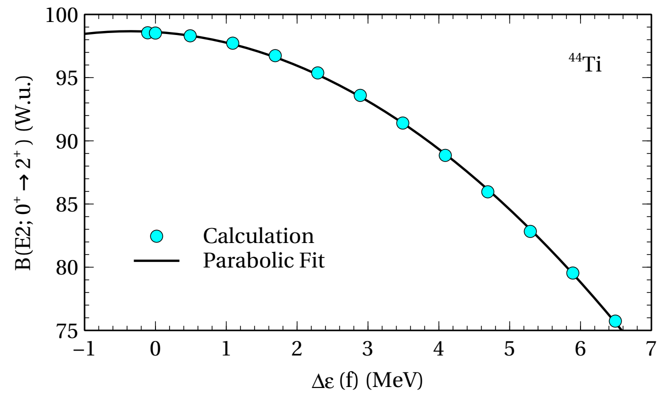

FIG. 5. The parabolic dependence of the quadrupole strength $B(E2; 0^+ \to 2^+)$ on the spin-orbit splitting in ^{44}Ti.

same orbitals, so that the $n \leftrightarrow p$ transformations require the spin flip. This changes the spin symmetry of the corresponding nucleon pair which could be compensated by the change of orbital symmetry. However, if there is no coupling between orbital and spin momenta the process turns out to be forbidden.

To illustrate this by the simplest example, consider the shell-model state of a valence np pair that should satisfy $(-)^{T+L+S} = -1$. For example, take quantum numbers $L = 0$, $S = 0$, $T = 1$, and $T_3 = 0$ of the mother state $|i\rangle$, $N = Z$,

$$|i\rangle = \frac{1}{\sqrt{2}}\,(p^\dagger_{1/2}n^\dagger_{-1/2} - p^\dagger_{-1/2}n^\dagger_{1/2})|0\rangle, \qquad (4)$$

where only spin projections of proton and neutron creation operators are indicated. The zero spin component GT^-_0 of the GT^- operator acts as

$$(\mathrm{GT})^-_0|i\rangle = \frac{1}{\sqrt{2}}\,[p^\dagger_{1/2}(s_z p^\dagger_{-1/2}) - p^\dagger_{-1/2}(s_z p^\dagger_{1/2})]|0\rangle$$
$$= \frac{1}{2\sqrt{2}}[-p^\dagger_{1/2}p^\dagger_{-1/2} - p^\dagger_{-1/2}p^\dagger_{1/2}]|0\rangle. \qquad (5)$$

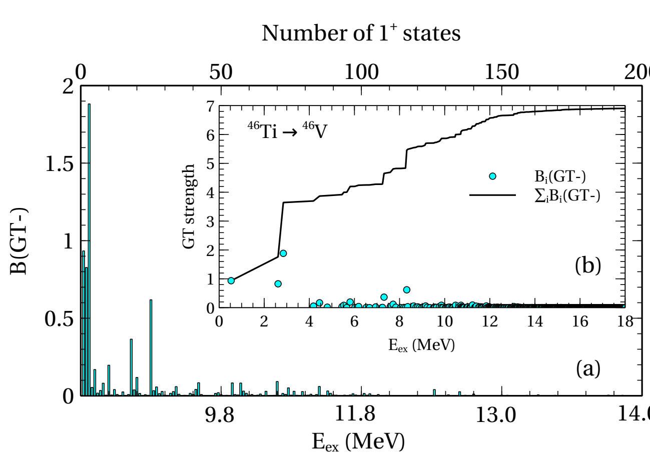

FIG. 6. (a) Distribution of the GT^- strengths from the ground state of ^{46}Ti along the excitation energy in the daughter state ^{46}V. (b) Cumulative sum of the GT^- strengths growing as a function of the excitation energy in the daughter nucleus ^{46}V.

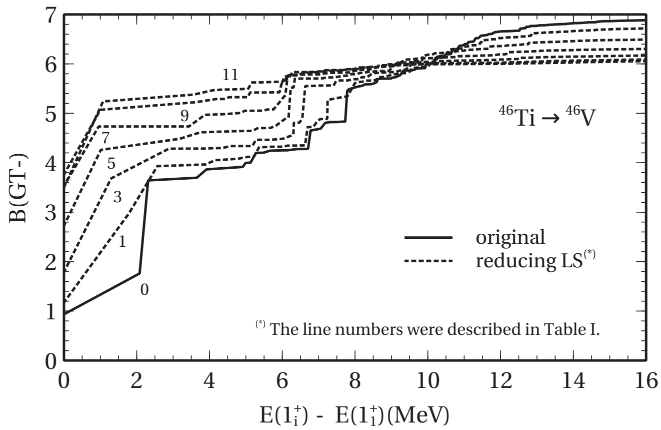

FIG. 7. Cumulative sum of the GT$^-$ strengths growing as a function of the excitation energy in the daughter nucleus ^{46}V. The consecutive lines (labeled as the lines of Table I) illustrate the accumulation process for several values of the spin-orbit splitting.

Using the anticommutator of proton operators, we get zero. The "down" component, GT^-_-, and "up" component, GT^-_+, of the GT operator do not act either:

$$(\mathrm{GT})^-_-|i\rangle = -\frac{1}{\sqrt{2}}[p^\dagger_{-1/2}(s_- p^\dagger_{1/2})] = 0. \tag{6}$$

Therefore, in this case the GT strength vanishes, and it turns out the same for any even L. Now, for odd L and $S = 1$, we take $M_L = 0$, $S_z = 0$, and the result is the same. This mechanism works in a general case of $N = Z$ in the absence of spin-orbit coupling.

Table I and Figs. 1 and 2 show the monotonous growth of the GT strength for the $N = Z$ nucleus $^{44}_{22}\mathrm{Ti}_{22}$ in the shell-model calculation for two valence np pairs as a function of the increasing energy splitting between $f_{7/2}$ and $f_{5/2}$ orbitals. This splitting serves as a measure of the spin-orbit coupling strength. At the same time, the $B(E2)$ transition rate from the ground state naturally grows with the change of this splitting

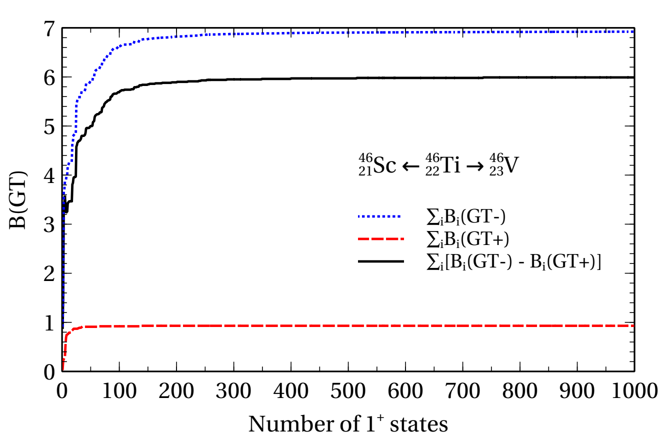

FIG. 8. Cumulative sums of the GT$^-$ and GT$^+$ strengths growing as a function of the number of 1^+ states in the daughter nuclei ^{46}V and ^{46}Sc.

in the opposite direction due to the increasing softening of all simple transitions coupled into the collective mode.

The typical spin-orbit term in the mean-field approximation can be written as a sum of single-particle contributions,

$$H^{(ls)} = \sum_a h(r_a)(\vec{\ell}\cdot\mathbf{s})_a, \tag{7}$$

where the radial form factor of spin-orbit coupling contains the radial derivative of the mean nuclear potential and can be evaluated on average as

$$\bar{h} \approx -\frac{20}{A^{2/3}}\ \mathrm{MeV}. \tag{8}$$

$|\bar{h}|$ is slightly bigger in the shell-model description of the fp-shell nuclei used in our calculations.

The isoscalar quadrupole moment of the nucleus is taken as a sum over particles,

$$Q_{kl} = \sum_a (q_{kl})_a = \sum_a (3x_k x_l - r^2\delta_{kl})_a. \tag{9}$$

The shift of the collective quadrupole excitation due to the spin-orbit splitting can be estimated with the help of general arguments, for example, using a simple model of factorizable (in this case quadrupole-quadrupole) forces, $H_Q = -\kappa(Q\cdot Q)$. As discussed in textbooks (see, for example, Ref. [8], Sec. 18.1), in the case of an attractive residual interaction, $\kappa > 0$, the energy ω of a collective excitation is lower than the centroid of energies $\bar{\epsilon}$ of independent (mean-field) excitations with the same quantum numbers, $\omega \approx \bar{\epsilon} - \kappa\mathcal{N}\overline{q^2}$, where $\mathcal{N}$ is a characteristic collectivity factor (a number of simple excitations coherently coupled to a collective mode) and $\overline{q^2}$ their typical strength. A simplified model in Appendix B illustrates the main features of the behavior of the collective frequency and transition rate seen in Table I.

The GT strength from the ground state is, to a good approximation, a quadratic function of the spin-orbit splitting. This is exactly what we should expect for transitions induced by a time-odd operator (magnetic dipole or GT). As follows from the symmetry arguments (Ref. [8], Sec. 13.11), in such cases the matrix element for the transition between orbitals λ and λ' is proportional to the combination

$$P^{(-)}_{\lambda\lambda'} = u_\lambda v_{\lambda'} - u_{\lambda'} v_{\lambda'}, \tag{10}$$

where the factors u and v describe the occupancies (n_λ between zero and 1) of corresponding orbitals:

$$v^2_\lambda = n_\lambda, \quad u^2_\lambda = 1 - n_\lambda. \tag{11}$$

For degenerate levels, the occupancies in equilibrium filling are equal, and the transition probability vanishes. With spin-orbit splitting growing, the difference of occupancies grows quadratically with this splitting, in agreement with what is given by the numerical calculation of Fig. 4.

The spin-dependent contribution to the equation of motion for the quadrupole moment is found as

$$[H^{(ls)}, Q_{kl}] = -3i\sum_a h(r_a)([\mathbf{s}\times\mathbf{r}]_k x_l + [\mathbf{s}\times\mathbf{r}]_l x_k)_a. \tag{12}$$

Looking for the physical overlap of GT and quadrupole modes, we evaluate the double commutator typical for the sum rules,

$$[V_l^-,[H^{(ls)},Q_{kl}]] = 3M_k^-, \tag{13}$$

where the sum over repeated Cartesian subscripts is assumed. The vector operators $M_k^\pm$ are spin-quadrupole moments for the two opposite directions of the GT excitation,

$$M_k^\pm = \sum_a \tau_a^\pm h_a(3(\mathbf{s}\cdot\mathbf{r})x_k - r^2 s_k)_a. \tag{14}$$

The physical effect of this dynamics is the appearance of the quadrupole component in the GT excitation so that the part of the GT strength is now transferred to the $L = 2$ channel. In a crude estimate, vectors (14) are proportional to the original GT amplitudes.

For an estimate by order of magnitude we assume that the soft quadrupole mode with its direction of slowly changing deformation generates on average the same directional character of the fast GT excitation, so that $Q_{kl} \propto 3n_k n_l - \delta_{kl}$ and $V_k^\pm \propto v^\pm n_k$ in terms of the unit vector $\mathbf{n}$. Then

$$[V_k^+V_l^-,[H^{(ls)},Q_{kl}]] \Rightarrow 6\bar{h}\bar{q}_{kl}n_k n_l\langle v^+v^-\rangle = 12\bar{h}\bar{q}\langle v^+v^-\rangle, \tag{15}$$

where the bar means the average over relevant single-particle transitions, and the matrix elements q_{kl} were defined in Eq. (9). However, the sum rule following from the original equation of motion with our auxiliary Hamiltonian gives for the expectation value of the left hand side of Eq. (15) the estimate $4\Delta\omega Q\langle v^+v^-\rangle$. Here $\Delta\omega$ is the displacement of the collective quadrupole excitation energy because of the spin-orbit splitting, and Q is the phonon amplitude, $Q = \mathcal{N}\bar{q}$, where $\mathcal{N}$ is the factor of collectivity of the phonon excitation. The comparison of two estimates gives

$$\Delta\omega \approx \frac{3\bar{h}}{Q/q} \approx -\frac{60}{A^{2/3}\mathcal{N}}\ \text{MeV}. \tag{16}$$

This quantity is of the order 200–300 keV, which is in agreement with Fig. 2.

In this oversimplified approach, being mediated by the spin-orbit interaction, the centroid of the GT excitation and the low-lying collective quadrupole excitation follow each other, in a qualitative agreement with exact results of shell-model computation. Figure 6 shows that the GT sum rule is getting fulfilled earlier in the process of gradual switching off the spin-orbit coupling when, as mentioned earlier [see Eq. (16) and Appendix B], the quadrupole frequency diminishes. The realistic spin-orbit interaction moves the GT final states up, slowing the approach to the sum rule limit and making this process more fine grained.

The whole interplay here can be considered a result of the effective interaction between quadrupole and GT degrees of freedom, which, in the lowest order, can be written as $H_{\text{eff}} \propto Q_{kl}V_k^+V_l^-$. This is somewhat similar to the correlation between collective octupole and quadrupole modes also described by the cubic anharmonic terms. That correlation was predicted theoretically [9] and found experimentally [10] to work practically exactly for the chain of xenon isotopes. Later this effect was qualitatively observed in the data for other isotope chains [11]. The same idea was useful in the theoretical search [12] for the enhancement of the nuclear Schiff moment, important in the problem of the electric dipole moment, due to the combined action, and therefore correlation, of soft quadrupole and octupole modes [13].

IV. CONCLUSION

We discussed some features of the nuclear GT processes which are not clearly formulated in the literature. The phenomenon of anticorrelation between the GT strength and collectivity of the lowest quadrupole excitation was studied numerically by exact shell-model calculations for the fp orbital space and with the help of simple clarifying models. The physics of this phenomenon is based on Fermi statistics, isospin invariance, and spin-orbit interaction.

In self-conjugate nuclei ($N = Z$) without spin-orbital splitting, the GT strengths in both directions would vanish under exact isospin symmetry. This interrelation is illustrated by the shell-model calculations for consecutive intermediate values of spin-orbit splitting. As follows from the general physics of low-lying collective excitations, in the same process of eliminating spin-orbit splitting, the quadrupole frequency goes down and the corresponding transition rate grows.

Our study might be useful for the problem of experimental quenching of the GT strength, which was not touched directly in the text above. For a long time it was claimed that the GT strength exciting the ground or a low-lying nuclear state is significantly quenched compared to the standard estimates [3–6]. The experimental studies typically find only about 60–70 % of the total strength. When such a reduction factor is introduced, the advanced shell-model calculations, including the Monte Carlo studies, agree with what is observed, for example, in the ^{56}Ni(p,n) charge-exchange reaction [14]. This subject was broadly discussed in the literature and, as stated in the old review article [15], "Both detailed nuclear structure calculations and extensive analysis of the scattering data suggest that the nuclear configuration mixing effect is the more important quenching mechanism, although subnuclear degrees of freedom cannot be ruled out." One argument in favor of nuclear mechanisms behind the quenching is that the GT strength considerably grows for the processes started in excited states $|\nu\rangle$. The shell-model analysis of the GT strength for the ^{24}Mg nucleus [16] shows a steady increase of this strength as a function of excitation energy of the initial state. Apart from the statistical effect of the level density, a considerable part of this increase comes from the suppression of spatial symmetry and corresponding progress towards the Wigner $\mathcal{SU}(4)$ symmetry.

Two aspects of the current study could be useful in the quenching problem. With restoration of the spin-orbit interaction, the GT strength centroid moves to higher energies with increasing fragmentation. This process is anticorrelated with the enhancement of the collective quadrupole mode. The limiting value of the universal GT sum rule is reached through growing fragmentation into many weak transitions. The understanding of this process can again (see, for example, Ref. [7]) raise the question of better evaluation of experimental results on GT quenching with the detailed consideration of the significantly fragmented strength.

Another question that might reappear is the role of spin-orbit forces in mixing various values of the total orbital momentum L in GT processes and charge-exchange reactions, including the isovector spin-monopole giant resonance [17]. In the presence of spin-orbit coupling, the total orbital momentum L of the nucleus is not conserved. With the spin-orbit coupling as an intermediary, the GT pseudovector operator in the nuclear medium excites not only $L = 0$ but also $L = 2$ states (these channels are interfering). The experimental treatment of charge-exchange reactions with the help of multipole decomposition typically extracts from the angular distribution only the $L = 0$ strength, which does not reflect the total strength excited by the GT operator inside the target nucleus. This question deserves better attention from both experimental and theoretical viewpoints.

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APPENDIX A: OPERATOR ALGEBRA

The nine operators related to the $\mathcal{SU}(4)$ group are

$$V_i^\alpha = \frac{1}{2}\sum_a (\sigma_i)_a(\tau^\alpha)_a. \tag{A1}$$

They commute (in Cartesian coordinates of vectors) according to

$$\left[V_i^\alpha, V_j^\beta\right] = i(\epsilon_{ijk}\delta^{\alpha\beta}S_k + \delta_{ij}\epsilon^{\alpha\beta\gamma}T^\gamma), \tag{A2}$$

where $\mathbf{S} = \sum_a \mathbf{s}_a$ and $\mathbf{T} = \sum_a \mathbf{t}_a$ are the total spin (Latin subscripts) and total isospin (Greek superscripts) operators, respectively. In particular (vector summation over i in the third equality),

$$[V_i^+, V_j^-] = 2i\epsilon_{ijk}S_k + 2\delta_{ij}T^3, \quad [V_i^\pm, V_j^\pm] = 0,$$
$$[V_i^+, V_i^-] = 3(N - Z), \tag{A3}$$

in agreement with Eq. (A3). There are also simple ladder relations:

$$[V_i^\pm, T^3] = \mp V_i^\pm. \tag{A4}$$

We note that this commutator of two vector operators, Eqs. (A2)–(A4), contains only pseudovector and scalar components with respect to spin coupling while the quadrupole component is absent. The squared vector part is proportional to the GT intensity.

APPENDIX B: SIMPLE MODEL

Here we use an oversimplified but generic model to illustrate the conciliated behavior of the collective quadrupole frequency and corresponding transition probability under the change of spin-orbit splitting. Assume that we have two groups of degenerate single-particle levels (images of our $f_{7/2}$ and $f_{5/2}$ orbitals) with approximately the same single-particle matrix elements q of the collective operator (a quadrupole moment in our problem). The interaction matrix elements H'_{ij} are factorized as $\kappa q_i q_j$, where $\kappa < 0$. The unperturbed Hamiltonian includes degenerate energies inside those groups, $\epsilon_1 = 0$ and $\epsilon_2 > 0$, and pairing forces which create the energy gap Δ so that the characteristic excitation energies in an even system are 2Δ and $2\sqrt{\Delta^2 + \epsilon^2}$.

The secular equation for the collective energy ω contains the two groups of contributions:

$$1 = \frac{S}{\omega - 2\Delta} + \frac{S'}{\omega - 2\sqrt{\Delta^2 + \epsilon^2}}, \tag{B1}$$

where $S = \kappa\sum_k q_k^2$ and $S' = \kappa\sum_{k'} q_{k'}^2$ contain contributions of the first and the second groups of single-particle transitions, respectively. S and S' are quantities of the same order of magnitude and for simplicity we set $S' = S$. For typical numerical values of the upper line of Table I, $\epsilon = 6.5$ MeV, $\omega = 1.3$ MeV, and, in this region of the nuclear chart, $\Delta \approx 1.7$ MeV, we extract $S \approx -1.8$ MeV. Changing the level distance ϵ to zero we increase Δ and decrease the collective frequency ω. Normalizing correctly the collective state [8] we find the collective transition probability at any point of this process:

$$B = \frac{4S}{\kappa}\,\frac{(\Delta + \sqrt{\Delta^2 + \epsilon^2} - \omega)^2}{(2\Delta - \omega)^2 + (2\sqrt{\Delta^2 + \epsilon^2} - \omega)^2}. \tag{B2}$$

The maximum of this probability is reached for degenerate levels, $\epsilon \to 0$, when

$$B_{\max} = \frac{2S}{\kappa}. \tag{B3}$$

The ratio $B/B_{\max}$ for the upper line of Table I is predicted by Eq. (B2) to be 0.67, which agrees with the numerical results in this table.

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