Spin and pseudospin symmetries in the single-Λ spectrum

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We explore spin and pseudospin symmetries in the single-Λ spectrum within the framework of relativistic mean-field theory. We find that the spin symmetry of the single-Λ spectrum maintains, which is consistent with experimental results of small spin-orbit splitting, whereas the approximate pseudospin symmetry is quite similar to that of nucleons. More interestingly, the $\omega\Lambda\Lambda$ tensor coupling has opposite effects on these symmetries which makes the spin symmetry better but the pseudospin symmetry worse. This can be understood by the negative value of the $\omega\Lambda\Lambda$ tensor potential, which reduces the spin-orbit potential while increasing the pseudospin-orbit potential.

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I. INTRODUCTION

Symmetries in the single-particle (s.p.) spectrum of atomic nuclei are of great importance to nuclear structure and have been extensively discussed in the literature for nucleons and antinucleons (see Refs. [1,2] and references therein). In ordinary nuclei, the s.p. spectra are characterized by an obvious violation of spin symmetry (SS) and an approximate pseudospin symmetry (PSS). The breaking of SS, i.e., the remarkable spin-orbit (SO) splitting of the spin doublets $(n,l,j=l\pm 1/2)$ caused by the SO potential, lays the foundation for explaining the traditional magic numbers in nuclear physics [3,4]. The conservation of PSS, i.e., the quasidegeneracy between two s.p. states with quantum numbers $(n,l,j=l+1/2)$ and $(n-1,l+2,j=l+3/2)$ redefined by the pseudospin doublets $(\tilde{n}=n,\tilde{l}=l+1,\ j=\tilde{l}\pm 1/2)$ [5,6], has been used to explain a number of phenomena in nuclear structures, including deformation [7], superdeformation [8], magnetic moment [9], and identical rotational bands [10].

Since the recognition of PSS in the nuclear spectrum, comprehensive efforts have been made to understand its origin. Apart from relabeling of quantum numbers, the explicit transformations from the normal scheme to the pseudospin scheme have been proposed in Refs. [11–13]. In 1997, Ginocchio made substantial progress and clearly pointed out that PSS is a relativistic symmetry in the Dirac Hamiltonian and becomes exact when the scalar and vector potentials have the same size but opposite sign, i.e., $\Sigma(r)\equiv S(r)+V(r)=0$ [14]. He also revealed that the pseudo-orbital angular momentum $\tilde{l}$ is nothing but the orbital angular momentum of the lower component of the Dirac wave function [14] and the occurrence of approximate PSS in nuclei is connected with certain similarities in the relativistic single-nucleon wave functions of the corresponding pseudospin doublets [15]. With a more general condition, Meng *et al.* pointed out that $d\Sigma(r)/dr=0$ can be approximately satisfied in exotic nuclei with highly diffuse potentials [16,17]. They also related the onset of the pseudospin symmetry to a competition between the pseudocentrifugal barrier (PCB) and the pseudospin-orbit (PSO) potential. Afterward, the SS and PSS in nuclear spectra were studied extensively such as PSS in the deformed nuclei [18–23], SS in antinucleon spectra [24–28], PSS in the s.p. resonate states [29–36], perturbative interpretation of SS and PSS [28,37–40], and PSS in supersymmetric quantum mechanics [41,42].

In hypernuclei, the studies of SS and PSS in the single-Λ spectra are still inadequate. To our present knowledge, only some work on the SS in the anti-Λ spectra has been performed [43–45]. In fact, since the first discovery of Λ hypernucleus by Danysz and Pniewski in 1953 [46], the study of hypernuclei has been attracting great interest among nuclear physicists [47–50]. One of the important goals of hypernuclear physics is to extract information on the baryon-baryon interactions including the strangeness of freedom, which is crucial not only for hypernuclear structures [51–54] but also for neutron stars [55–58]. Experimentally, it has been found that the SO splitting for Λ hyperons is much smaller than that for nucleons [59–62], which is because the s (strange) quark contributes little to the nuclear force. For example, the experimentally observed SO splitting of the p_Λ state in hypernucleus ${}^{13}_{\Lambda}$C is 0.152 ± 0.090 MeV, which is much smaller than that in ordinary nuclei by a factor of 20–30 [62]. Theoretically, to understand this small SO splitting, the $\omega\Lambda\Lambda$ tensor coupling has been suggested, and it has had a considerable success [38,63–68]. Therefore, it is fascinating to study the SS and PSS in the single-Λ spectra and clarify the effect of $\omega\Lambda\Lambda$ tensor coupling on these symmetries.

During the past decades, the relativistic mean-field (RMF) model has achieved great success not only in ordinary nuclei [69–76] but also in hypernuclei [43–45,52,67,77–88]. The relativistic approach is suitable for the discussion of SO splitting, as the SO interaction is naturally emerged within the relativistic framework. Having adopted the RMF model, many works on SS and PSS [2] have achieved great success.

In this work, the SS and PSS in the single-Λ spectra are studied within the framework of the RMF model. The corresponding mechanisms are given and the $\omega\Lambda\Lambda$ tensor

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coupling effects on these symmetries are discussed. The paper is organized as follows. In Sec. II, we present the RMF model for Λ hypernuclei and the formalism of SS and PSS in the single-Λ spectrum. After the numerical details in Sec. III, we present the results and discussions in Sec. IV. Finally, a summary is drawn in Sec. V.

II. THEORETICAL FRAMEWORK

The starting point of the meson-exchange RMF model for Λ hypernuclei is the covariant Lagrangian density

$$\mathscr{L} = \mathscr{L}_N + \mathscr{L}_\Lambda, \tag{1}$$

where $\mathscr{L}_N$ is the standard RMF Lagrangian density for nucleons [69–76], in which the couplings with the scalar σ, vector ω_μ, and vector-isovector $\vec{\rho}_\mu$ mesons, and the photon A_μ are included. The Lagrangian density $\mathscr{L}_\Lambda$ represents the contribution from hyperons [79]. Since Λ hyperons are charge neutral with isospin $\vec{\tau} = 0$, only the couplings with the σ and ω meson are included. The Lagrangian density $\mathscr{L}_\Lambda$ for the single-Λ hypernuclei reads

$$\mathscr{L}_\Lambda = \bar{\psi}_\Lambda\bigg[i\gamma^\mu\partial_\mu - m_\Lambda - g_{\sigma\Lambda}\sigma - g_{\omega\Lambda}\gamma^\mu\omega_\mu - \frac{f_{\omega\Lambda\Lambda}}{2m_\Lambda}\sigma^{\mu\nu}\partial_\nu\omega_\mu\bigg]\psi_\Lambda, \tag{2}$$

where m_Λ is the mass of the Λ hyperon, $g_{\sigma\Lambda}$ and $g_{\omega\Lambda}$ are the coupling constants with the σ and ω meson, respectively. The last term in $\mathscr{L}_\Lambda$ is the $\omega\Lambda\Lambda$ tensor coupling related to the small SO splitting [65,66], and $f_{\omega\Lambda\Lambda}$ is the coupling constant.

For a system with time-reversal symmetry, the space-like components of the vector fields ω_μ and $\vec{\rho}_\mu$ vanish, leaving only the time components ω_0 and $\vec{\rho}_0$. Meanwhile, charge conservation guarantees that only the third component $\rho_{0,3}$ in the isospin space of $\vec{\rho}_0$ exists. With the mean-field and no-sea approximations, the s.p. Dirac equations for baryons and the Klein-Gordon equations for mesons and photons can be obtained by the variational procedure. And they are solved by iterations in the coordinate space.

To study the SS and PSS in the single-Λ spectrum with the RMF theory, we will examine the Dirac equations governing the motion of Λ hyperons. In the spherical case, the Dirac spinor for Λ hyperons can be expanded as

$$\psi_{\kappa m}(\boldsymbol{r}) = \frac{1}{r}\begin{pmatrix} iG_\kappa(r)Y^l_{jm}(\theta,\phi) \\ -F_\kappa(r)Y^{\tilde{l}}_{jm}(\theta,\phi)\end{pmatrix}, \tag{3}$$

where $G_\kappa(r)/r$ and $F_\kappa(r)/r$ are the upper and lower components of the radial wave functions, $Y^l_{jm}(\theta,\phi)$ is the spinor spherical harmonic, $\tilde{l} = l - \mathrm{sign}(\kappa)$ is pseudo-orbital angular momentum, and the quantum number κ is defined as $\kappa = (-1)^{j+l+1/2}(j+1/2)$.

With the radial wave functions, the Dirac equation for the Λ hyperon can be written as

$$\begin{pmatrix} V+S & -\frac{d}{dr}+\frac{\kappa}{r}+T \\ \frac{d}{dr}+\frac{\kappa}{r}+T & V-S-2m_\Lambda \end{pmatrix}\begin{pmatrix} G_\kappa \\ F_\kappa\end{pmatrix} = \varepsilon_\kappa \begin{pmatrix} G_\kappa \\ F_\kappa\end{pmatrix}, \tag{4}$$

with the s.p. energy ε_κ and the mean-field scalar potential S, vector potential V, and tensor potential T,

$$S = g_{\sigma\Lambda}\sigma, \tag{5a}$$

$$V = g_{\omega\Lambda}\omega_0, \tag{5b}$$

$$T = -\frac{f_{\omega\Lambda\Lambda}}{2m_\Lambda}\partial_r\omega_0. \tag{5c}$$

Starting from the Dirac equation (4), one can go a step further and get a second-order Schrödinger-like equation for either the upper or lower component. To study the SS, we will analyze the Schrödinger-like equation for the upper component $G(r)$, and to study the PSS, we will analyze that for the lower component $F(r)$.

For the upper component $G(r)$, by substituting

$$F(r) = \frac{1}{\varepsilon + 2m_\Lambda - V + S}\left(\frac{d}{dr} + \frac{\kappa}{r} + T\right)G(r), \tag{6}$$

one can obtain

$$\bigg\{-\frac{1}{M_+}\frac{d^2}{dr^2} + \frac{1}{M_+^2}\frac{dM_+}{dr}\frac{d}{dr} + V + S + \frac{1}{M_+}\frac{\kappa(\kappa+1)}{r^2} + \frac{1}{M_+}\left(T^2 - \frac{dT}{dr} + \frac{1}{M_+}\frac{dM_+}{dr}T\right) + \frac{1}{M_+}\left(\frac{1}{M_+}\frac{dM_+}{dr} + 2T\right)\frac{\kappa}{r}\bigg\}G = \varepsilon G, \tag{7}$$

with the energy-dependent effective mass $M_+ = \varepsilon + 2m_\Lambda - V + S$. Here and hereafter, the subscript κ is omitted for simplification. For the above equation, in analogy with the Schrödinger equation, $V+S$ is the central potential; the term proportional to $\kappa(\kappa+1) = l(l+1)$ corresponds to the centrifugal barrier (CB); and the last two terms correspond to the SO potential leading to the substantial SO splitting in the single-particle spectrum. Namely,

$$V_{\mathrm{CB}}(r) = \frac{1}{M_+}\frac{\kappa(\kappa+1)}{r^2}, \tag{8a}$$

$$V_{\mathrm{SO}}(r) = \frac{1}{M_+}\left(\frac{1}{M_+}\frac{dM_+}{dr} + 2T\right)\frac{\kappa}{r}. \tag{8b}$$

It is well known that there is no SO splitting if potential V_{SO} vanishes. When neglecting the tensor term T, one can obtain a simple formula of V_{SO},

$$V_{\mathrm{SO}}(r) = \frac{1}{M_+^2}\frac{dM_+}{dr}\frac{\kappa}{r}, \tag{9}$$

and $\frac{dM_+}{dr} = -\frac{d(V-S)}{dr} = 0$ corresponds to the SS limit. Note that this condition is equivalent to the mean-field potentials $V - S = 0$ in the whole r space if they go to zero at infinity.

For the lower component $F(r)$, by substituting

$$G(r) = \frac{1}{\varepsilon - V - S}\left(-\frac{d}{dr} + \frac{\kappa}{r} + T\right)F(r), \tag{10}$$

one can obtain

$$\left\{-\frac{1}{M_-}\frac{d^2}{dr^2}+\frac{1}{M_-^2}\frac{dM_-}{dr}\frac{d}{dr}+V-S-2m_\Lambda\right.$$
$$+\frac{1}{M_-}\frac{\kappa(\kappa-1)}{r^2}+\frac{1}{M_-}\left(T^2+\frac{dT}{dr}-\frac{1}{M_-}\frac{dM_-}{dr}T\right)$$
$$\left.-\frac{1}{M_-}\left(\frac{1}{M_-}\frac{dM_-}{dr}-2T\right)\frac{\kappa}{r}\right\}F=\varepsilon F, \quad (11)$$

with the energy-dependent effective mass $M_- = \varepsilon - V - S$. The term in the above equation proportional to $\kappa(\kappa-1) = \tilde{l}(\tilde{l}+1)$ is regarded as the PCB and the last two terms correspond to the PSO potential, which leads to substantial PSO splitting. Namely,

$$V_{\mathrm{PCB}}(r) = \frac{1}{M_-}\frac{\kappa(\kappa-1)}{r^2}, \quad (12a)$$

$$V_{\mathrm{PSO}}(r) = -\frac{1}{M_-}\left(\frac{1}{M_-}\frac{dM_-}{dr}-2T\right)\frac{\kappa}{r}. \quad (12b)$$

When neglecting the tensor term, the potential V_{PSO} is reduced as

$$V_{\mathrm{PSO}}(r) = -\frac{1}{M_-^2}\frac{dM_-}{dr}\frac{\kappa}{r}, \quad (13)$$

and $\frac{dM_-}{dr} = \frac{d(V+S)}{dr} = 0$ corresponds to the PSS limit. Also note that this condition is equivalent to the mean-field potentials $V + S = 0$ in the whole r space if they go to zero at infinity.

III. NUMERICAL DETAILS

In this work, hypernucleus $^{209}_{\Lambda}$Pb is taken as an example to study the SS and PSS in the single-Λ spectra. The Dirac equation (4) in the RMF model is solved in the coordinate space with a box size of $R = 20$ fm and a step size of 0.05 fm. For the NN interaction, parameter set PK1 [89] is adopted. For the ΛN interaction, the scalar coupling constant $g_{\sigma\Lambda}/g_{\sigma N} = 0.618$ is constrained by reproducing the experimental binding energies of Λ hyperon in the $1s_{1/2}$ state of hypernucleus $^{40}_{\Lambda}$Ca [90]; the vector coupling constant $g_{\omega\Lambda}/g_{\omega N} = 0.666$ is determined according to the naive quark model [91]. Here, $g_{\sigma N}$ and $g_{\omega N}$ are respectively the coupling constants between nucleons and the σ and ω mesons in the Lagrangian $\mathscr{L}_N$. These NN and ΛN interactions describe well the single-Λ spectra of hypernuclei ranging from $^{12}_{\Lambda}$C to $^{208}_{\Lambda}$Pb [88]. Especially, for the hypernucleus $^{208}_{\Lambda}$Pb, the single-Λ binding energy in the $1s_{1/2}$ state by the RMF model is 26.75 MeV, which is very close to the experimental data 26.3 ± 0.8 MeV [47]. To compare with the SS and PSS in the single-nucleon spectra, we first neglect the tensor term T and take $f_{\omega\Lambda\Lambda} = 0$ to study symmetries in the single-Λ spectra. Then we focus on the $\omega\Lambda\Lambda$ tensor coupling effects on the SS and PSS by changing the coupling constant $f_{\omega\Lambda\Lambda}$ from $0.0g_{\omega\Lambda}$ to $-1.0g_{\omega\Lambda}$.

IV. RESULTS AND DISCUSSION

In Fig. 1, the calculated single-Λ spectra and the mean-field potential in $^{209}_{\Lambda}$Pb are presented. Figures 1(a) and 1(b) correspond to spin doublets and pseudospin doublets, respectively. In Fig. 1(a), six sets of spin doublets are displayed, i.e., $1p$ ($1p_{1/2}$ and $1p_{3/2}$), $1d$ ($1d_{3/2}$ and $1d_{5/2}$), $1f$ ($1f_{5/2}$ and $1f_{7/2}$), $1g$ ($1g_{7/2}$ and $1g_{9/2}$), $2p$ ($2p_{1/2}$ and $2p_{3/2}$), and $2d$ ($2d_{3/2}$ and $2d_{5/2}$). Almost all these spin doublets are found to be quasidegenerate. And we can see that the splitting of spin doublets with the same main quantum number is enlarged with the angular momentum l increasing, which is because the centrifugal barrier $V_{\mathrm{CB}} = \frac{1}{M_+}\frac{l(l+1)}{r^2}$ keeps the particle away from the center so that a big overlap between the wave function $G(r)$ and the spin-orbit potential always happens for larger l [92]. This has been observed in single-Λ states in $^{89}_{\Lambda}$Y by the (π^+,K^+) reactions at KEK [62]. In Fig. 1(b), four sets of pseudospin doublets are exhibited, i.e., $1\tilde{p}$ ($1d_{3/2}$ and $2s_{1/2}$), $1\tilde{d}$ ($1f_{5/2}$ and $2p_{3/2}$), $1\tilde{f}$ ($1g_{7/2}$ and $2d_{5/2}$), and $2\tilde{p}$ ($2d_{3/2}$ and $3s_{1/2}$). It is clear that the PSS of pseudospin doublet $2\tilde{p}$ near the threshold is well preserved. From the above discussions, SO splittings and PSO splittings in the single-Λ spectra of $^{209}_{\Lambda}$Pb are found to be quite different, i.e., considerable good SS for almost all the spin doublets versus approximate PSS for the pseudospin doublets.

To show the SO and PSO splittings more clearly, the reduced SO splittings $\Delta E_{\mathrm{SO}} = (\varepsilon_{j_<} - \varepsilon_{j_>})/(2l+1)$ and reduced PSO splittings $\Delta E_{\mathrm{PSO}} = (\varepsilon_{j_<} - \varepsilon_{j_>})/(2\tilde{l}+1)$ versus

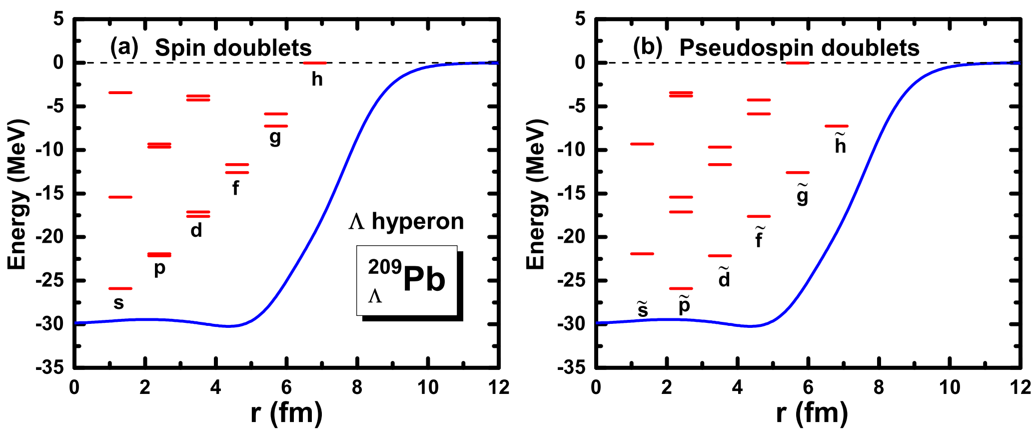

FIG. 1. Single-particle spectra for the Λ hyperon in $^{209}_{\Lambda}$Pb for spin and pseudospin doublets. Potential $V + S$ is shown as the blue solid line. The $\omega\Lambda\Lambda$ tensor coupling is omitted.

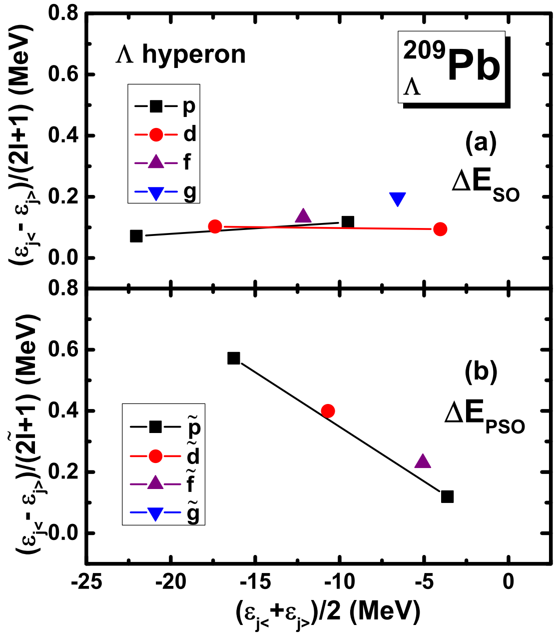

FIG. 2. (a) Reduced SO splitting $\Delta E_{\rm SO} = (\varepsilon_{j_<} - \varepsilon_{j_>})/(2l+1)$ and (b) reduced PSO splitting $\Delta E_{\rm PSO} = (\varepsilon_{j_<} - \varepsilon_{j_>})/(2\tilde{l}+1)$ versus their average s.p. energies $E_{\rm av} = (\varepsilon_{j_<} + \varepsilon_{j_>})/2$ in s.p. spectrum for Λ hyperon of $^{209}_{\Lambda}$Pb. For the spin doublets, $j_< = l - 1/2$ and $j_> = l + 1/2$; and for the pseudospin doublets, $j_< = \tilde{l} - 1/2$ and $j_> = \tilde{l} + 1/2$. Spin (pseudospin) doublets with the same l ($\tilde{l}$) are linked by lines. The $\omega\Lambda\Lambda$ tensor coupling is omitted.

the average s.p. energies $E_{\rm av} = (\varepsilon_{j_<} + \varepsilon_{j_>})/2$ are respectively plotted in Figs. 2(a) and 2(b). For the spin doublets, all the energy splittings are less than 0.2 MeV, which are much smaller than those in single-nucleon spectra. This can be understood from Eq. (9), where the SO splitting $V_{\rm SO}$ for hyperons is reduced when compared to that for nucleons due to a larger effective mass M_+ in the denominator or the larger Λ-hyperon mass m_Λ compared to the nucleon mass and due to the smaller couplings to σ and ω mesons. Besides, SS for Λ hyperons are less energy dependent, which is also caused by the large Λ-hyperon mass m_Λ in the effective mass $M_+ = \varepsilon + 2m_\Lambda - V + S$, where $2m_\Lambda$ is much larger than either the potential $S - V$ or the s.p. energy ε. For the pseudospin doublets, however, obvious energy dependence can be seen. For the $1\tilde{p}$ doublets, the reduced PSO splitting is 0.57 MeV, but 0.12 MeV for the $2\tilde{p}$ doublets. The PSS in the single-Λ spectra is quite similar to that in the single-nucleon spectra.

To analyze the mechanism of the SS and PSS in the single-Λ spectrum shown in Figs. 1 and 2, we compare the centrifugal barrier $V_{\rm CB}(r)$ with spin-orbit potential $V_{\rm SO}(r)$ and pseudocentrifugal barrier $V_{\rm PCB}(r)$ with pseudospin-orbit potential $V_{\rm PSO}(r)$. It was pointed out in Ref. [16] that if

$$|V_{\rm PSO}(r)| \ll |V_{\rm PCB}(r)|, \tag{14}$$

the pseudospin symmetry will be good. Analogously, the relation for the spin symmetry can be deduced that if

$$|V_{\rm SO}(r)| \ll |V_{\rm CB}(r)|, \tag{15}$$

the spin symmetry will be good. However, unfortunately, it is difficult to compare potentials $V_{\rm PSO}(r)$ and $V_{\rm PCB}(r)$ in Eq. (12) directly, as they have a singularity at $M_-(r) = 0$. Since the two potentials have a common factor $1/M_-(r)$ and we are only interested in the relative magnitude, a factor of $M_-(r)$ can be multiplied for comparison. In this way, effective PCB and effective PSO potential are introduced,

$$V^{\rm eff}_{\rm PCB}(r) = \frac{\kappa(\kappa-1)}{r^2}, \tag{16a}$$

$$V^{\rm eff}_{\rm PSO}(r) = -\frac{1}{M_-(r)}\frac{dM_-(r)}{dr}\frac{\kappa}{r}. \tag{16b}$$

However, the singularity in potential $V_{\rm PSO}(r)$ is not yet solved due to another $1/M_-(r)$ in $V^{\rm eff}_{\rm PSO}(r)$. To solve this problem, a corrected effective PSO potential is proposed as in Ref. [93], which is a continuous function in the whole r space,

$$V^{\rm c\text{-}eff}_{\rm PSO}(r) = \frac{\kappa}{[r - r_0 + C(r-r_0)^3]r_0}\frac{F^2(r_0)}{F^2(r)} - \frac{1}{M_-(r)}\frac{dM_-(r)}{dr}\frac{\kappa}{r}, \tag{17}$$

with the position r_0 where $M_-(r) = 0$ and a constant C. In the limit of $C \to \infty$, the first term approaches zero and the potential $V^{\rm c\text{-}eff}_{\rm PSO}(r)$ reproduces exactly the uncorrected one.

The problem of singularity does not exist for the potentials $V_{\rm SO}(r)$ and $V_{\rm CB}(r)$ in Eq. (8) because $M_+(r) \neq 0$ for all values of r. In this case, the effective CB and the effective SO potentials,

$$V^{\rm eff}_{\rm CB}(r) = M_+(r)\frac{\kappa(\kappa+1)}{r^2}, \tag{18a}$$

$$V^{\rm eff}_{\rm SO}(r) = \frac{dM_+(r)}{dr}\frac{\kappa}{r}, \tag{18b}$$

are introduced by multiplying a factor of $M_+^2(r)$ in Eq. (8). Note that the tensor potential T is omitted here.

In Figs. 3 and 4, the comparison of the CB with SO potential for the $1p$ and $2p$ spin doublets and the PCB with PSO potential for the $1\tilde{p}$ and $2\tilde{p}$ pseudospin doublets of the Λ hyperon in hypernucleus $^{209}_{\Lambda}$Pb are presented, respectively. In Fig. 4, the potentials $V^{\rm eff}_{\rm PCB}$ and $V^{\rm c\text{-}eff}_{\rm PSO}$ are multiplied by the square of the wave function $F(r)$ of the lower component, by which the first term of potential $V^{\rm c\text{-}eff}_{\rm PSO}(r)$ in Eq. (17) becomes an odd function of $r - r_0$ and can be integrated out. The constant C is taken to be 20 fm^{-2} to calculate the corrected effective PSO potential $V^{\rm c\text{-}eff}_{\rm PSO}(r)$. In Fig. 3, it is clear that the effective CB is much bigger than the effective SO potential, which leads to the well-conserved SS in the single-Λ spectrum. Moreover, the effective CB for $1p$ doublets and $2p$ doublets are found almost identical, which gives an explanation for the approximate energy independence of the SS. In Fig. 4, the effective PCB is much bigger than the corrected effective PSO potential in the coordinate space with $r < 5$ fm, which leads to the approximate PSS. However, around the nuclear surface with $r > 5$ fm, the corrected effective PSO is bigger than the effective PCB. Different from the effective CB shown in Fig. 3, the effective PCB for $1\tilde{p}$ doublets are smaller than that for $2\tilde{p}$

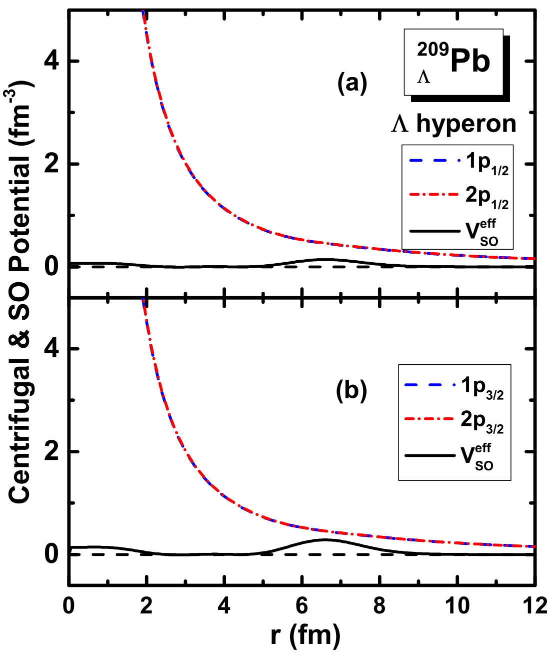

FIG. 3. Comparison of the Λ hyperon effective CB $V_{\rm CB}^{\rm eff}$ (dashed and dot-dashed lines) with the effective SO potential $V_{\rm SO}^{\rm eff}$ (solid line) for (a) $p_{1/2}$ and (b) $p_{3/2}$ states in $^{209}_{\Lambda}$Pb. The dashed lines are for $1p_{1/2}$ and $1p_{3/2}$ states, and the dot-dashed lines are for $2p_{1/2}$ and $2p_{3/2}$ states. The $\omega\Lambda\Lambda$ tensor coupling is omitted.

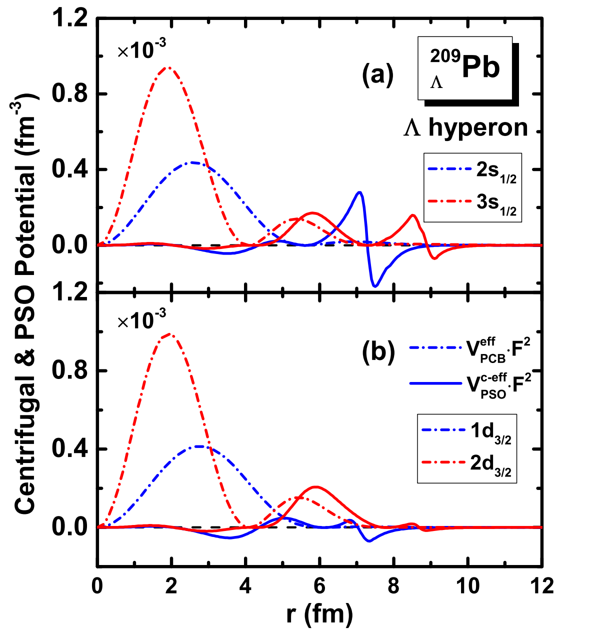

FIG. 4. Comparison of the Λ hyperon effective PCB $V_{\rm PCB}^{\rm eff}$ (short dot-dashed lines) and the corrected effective PSO potential $V_{\rm PSO}^{\rm c\text{-}eff}$ (solid lines) multiplied by the square of the wave function F of the lower component for the (a) $s_{1/2}$ and (b) $d_{3/2}$ states in $^{209}_{\Lambda}$Pb. All the potentials are multiplied by a factor of 10^3. The $\omega\Lambda\Lambda$ tensor coupling is omitted.

doublets, which has the consequence that the PSS is much better for the weakly bound pseudospin doublets.

The Dirac wave functions for the (pseudo)spin doublets will provide another way to check the (pseudo)spin approximation in nuclei [15]. Taking as examples, several single-particle wave functions are plotted in Fig. 5 from the RMF calculations without the tensor couplings. The curves in the upper panels denote upper and lower components of the wave functions for the $1p$, $2p$, and $1f$ spin doublets, and those in lower panels are for the $1\tilde{p}$, $2\tilde{p}$, and $1\tilde{f}$ pseudospin doublets, respectively. As seen in the figures, for the spin doublets, the upper components $G(r)$ are quite similar and almost identical. And for the pseudospin doublets, the lower components $F(r)$ are almost identical. Such similarities of the wave functions reflect the corresponding conserved SS and PSS.

In the following, we focus on the $\omega\Lambda\Lambda$ tensor coupling effects on the SS and PSS. In Fig. 6, we present the single-Λ spectra of $^{209}_{\Lambda}$Pb calculated with different tensor coupling constants $f_{\omega\Lambda\Lambda}$ ranging from $0.0g_{\omega\Lambda}$ to $-1.0g_{\omega\Lambda}$. We can see that $\omega\Lambda\Lambda$ tensor coupling can shift the single-Λ levels apparently. With the increase of $f_{\omega\Lambda\Lambda}/g_{\omega\Lambda}$, remarkable reduction of the SO splittings are observed for all the spin doublets. This is consistent with the studies in Ref. [79].

To clarify the changes of the energy splittings by the $\omega\Lambda\Lambda$ tensor couplings, we plot the reduced SO splitting $\Delta E_{\rm SO}$ and reduced PSO splitting $\Delta E_{\rm PSO}$ as functions of $f_{\omega\Lambda\Lambda}/g_{\omega\Lambda}$ in Figs. 7(a) and 7(b), respectively. It can be clearly seen that the SO splitting is gradually decreasing while the PSO splitting is increasing with the increase of $f_{\omega\Lambda\Lambda}/g_{\omega\Lambda}$. Quantitatively, the SO splitting is reduced by a factor of $5 \sim 7$ when $f_{\omega\Lambda\Lambda}/g_{\omega\Lambda}$ varies from 0 to -1. This can be understood by the negative tensor potential T which reduces the SO potential $V_{\rm SO}$ in Eq. (8b). With the increase of $f_{\omega\Lambda\Lambda}/g_{\omega\Lambda}$, the tensor potential T increases a lot while the term $\frac{1}{M_+}\frac{dM_+}{dr}$ in Eq. (8b) keeps almost constant. The tensor coupling of the ω to the Λ hyperon predicted by the quark model is negative, and its contribution is comparable with the original σ and ω parts. This results in a cancellation in the SO potential. However, the tensor potential T increases the PSO potential $V_{\rm PSO}$ in Eq. (12b) and results in a worse PSS.

V. SUMMARY

We study SS and PSS in the single-Λ spectrum within the framework of RMF theory. And we discuss the $\omega\Lambda\Lambda$ tensor coupling effects on these symmetries and make some analyses in details.

First, by neglecting the tensor coupling, SS and PSS in the single-Λ spectrum of hypernucleus $^{209}_{\Lambda}$Pb are studied. Well-conserved SS are found, which is consistent with experimental results of the small SO splitting, but different from the breaking of SS in the single-nucleon spectrum. Besides, for the spin doublets with the same main quantum number n, the SO splitting always increases with larger angular momentum l. However, PSS in the single-Λ spectrum is approximately conserved, which is quite similar to that in a single-nucleon spectrum. Compared with the less energy dependent SO splitting, the PSO splitting behaves with an obvious energy dependence and the pseudospin doublets close to the threshold are quasidegenerate.

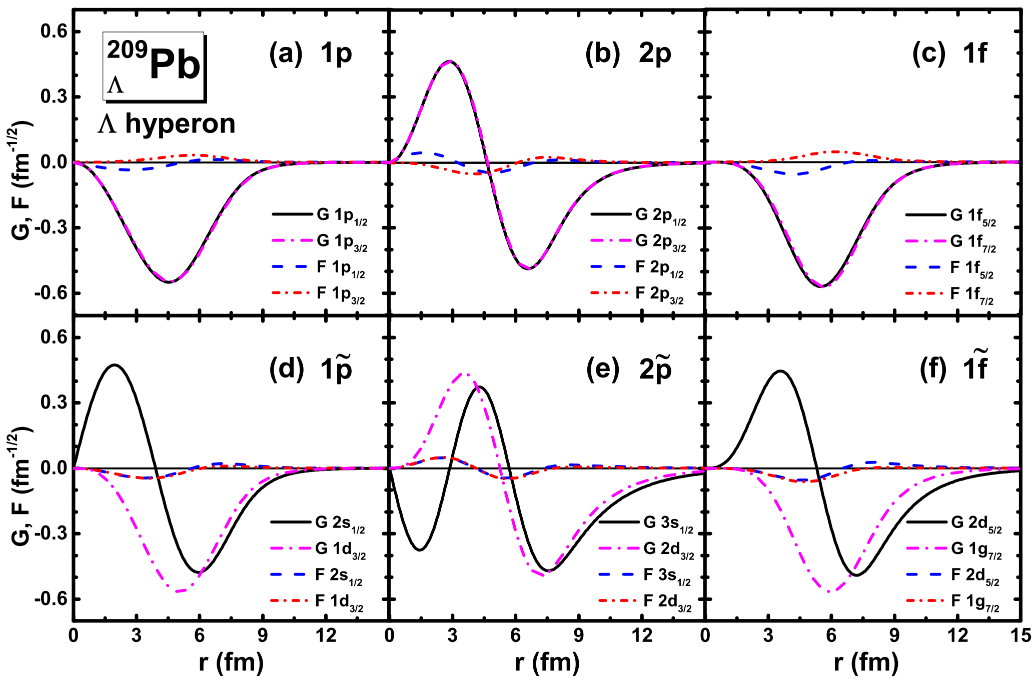

FIG. 5. Single-particle wave functions for the Λ hyperon of the $1p$, $2p$, and $1f$ spin doublets (upper panels) and $1\tilde{p}$, $2\tilde{p}$, and $1\tilde{f}$ pseudospin doublets (lower panels) in ${}^{209}_{\Lambda}$Pb calculated by the RMF theory. The $\omega\Lambda\Lambda$ tensor coupling is omitted.

Second, to understand the mechanism of such behaviors of the (P)SS in the single-Λ spectrum, comparison of the (P)CB with the (P)SO potentials of single-Λ states are performed. We find that the CB is much larger than the SO potential, which leads to the well-conserved SS. While PCB is much larger than the PSO with $r < 5$ fm but smaller with $r > 5$ fm, which leads to approximate PSS. As another way to check, the Dirac wave functions of several spin and pseudospin doublets in ${}^{209}_{\Lambda}$Pb are also investigated. For the spin doublets, the upper components $G(r)$ of the Dirac spinor are almost identical, and for the pseudospin doublets, the lower components $F(r)$ are almost identical.

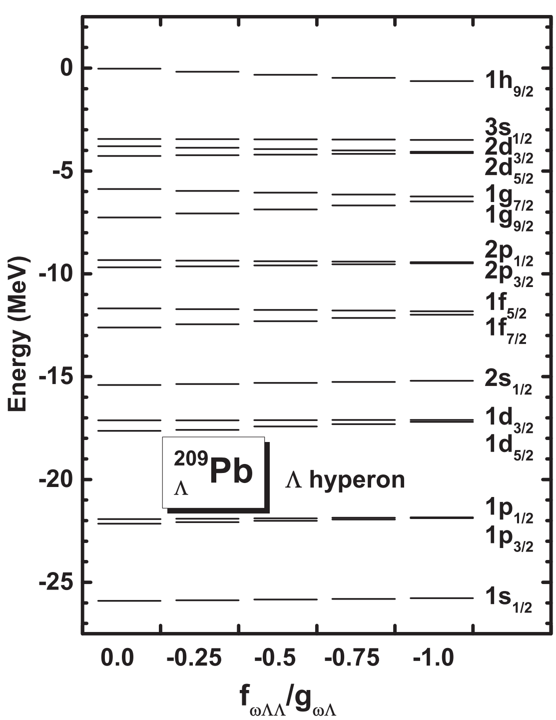

FIG. 6. Single-particle spectrum for Λ hyperon in ${}^{209}_{\Lambda}$Pb as a function of the $\omega\Lambda\Lambda$ tensor coupling strength $f_{\omega\Lambda\Lambda}/g_{\omega\Lambda}$.

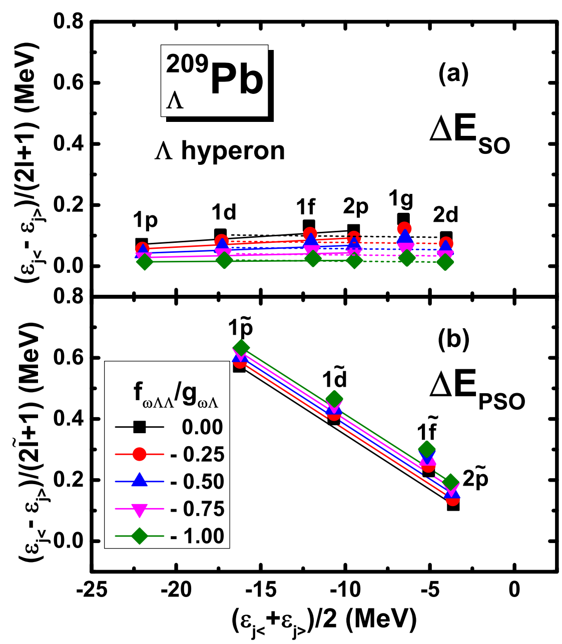

FIG. 7. Same as Fig. 2, but with different $\omega\Lambda\Lambda$ tensor coupling strengths $f_{\omega\Lambda\Lambda}/g_{\omega\Lambda}$.

Finally, the $\omega\Lambda\Lambda$ tensor coupling effects on the SS and PSS are considered. With the increase of the tensor coupling constant $f_{\omega\Lambda\Lambda}/g_{\omega\Lambda}$, the SO splitting is gradually decreasing while the PSO splitting is increasing, i.e., the SS becomes better and the PSS becomes worse due to the $\omega\Lambda\Lambda$ tensor coupling. This is caused by the negative value of the $\omega\Lambda\Lambda$ tensor potential, which can reduce the SO term and increase the PSO term.

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